Inflation in string theory

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Reference:  Baumann, McAllister “Inflation and String Theory”  1404.2601
Outline

• Overview/Introduction
  – Why inflation in string theory?
  – Large field inflation in string theory: Challenges and advantages
  – Natural Inflation and Axion Monodromy

• Moduli Stabilization in string theory (on the white board)
  – Flux compactifications in string theory
  – GKP, KKLT and LVS, dS vacua in string theory
  – the landscape of dS vacua [if time permits]
Inflation

The slow-roll models of inflation

\[ \frac{1}{\sqrt{g}} L = -\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \]

\[ V_{\text{today}} \]

\[ V_{\text{inf}} \]

\[ \rho = \frac{1}{2} \dot{\phi}^2 + V(\phi) \]

\[ p = \frac{1}{2} \dot{\phi}^2 - V(\phi) \]

For \( \frac{1}{2} \dot{\phi}^2 \ll V(\phi) \) we have

\[ \rho = -p \iff w = -1 \]
The slow-roll models of inflation

\[
\frac{1}{\sqrt{g}} L = -\frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - V(\phi)
\]

Inflation requires that the following two slow-roll parameters are sufficiently small:

\[
\epsilon_V = \frac{M_P^2}{2} \left( \frac{V'}{V} \right)^2
\]

\[
\eta_V = M_P^2 \frac{V''}{V}
\]

Experiments can measure these 50-60 e-folds before the end of inflation at \( \phi_{CMB} \).
Planck 2015

$n_s \approx 1 - 6 \epsilon_V + 2 \eta_V$

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Planck 2015

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Large field inflationary models

• The $r$ value is directly related to the value of the first derivative divided by the value of the potential

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• We need $\varepsilon_V \ll 1$ for inflation. This requires that $r$ is also smallish.
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- One has $r \approx .1 \iff M_P V' \approx .1 V$
  $r \approx .001 \iff M_P V' \approx .01 V$
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- One has
  \[ r \approx .1 \iff M_P V' \approx .1 V \]
  \[ r \approx .001 \iff M_P V' \approx .01 V \]

$\Rightarrow$ Reasonable to expect measurable $r$ value
E-modes and B-modes

Only gravitational waves can generate B-mode polarization in the CMB photons.
Primordial B-modes can tell us the energy scale of inflation:

\[
\Delta_t^2 \approx \frac{2}{3\pi^2} \frac{V}{M_{Pl}^4}
\]

\[
r = \frac{\Delta_t^2}{\Delta_s^2} \approx 16\epsilon_V \quad \Rightarrow \quad (V_{inf})^{\frac{1}{4}} \approx 2 \times 10^{16} \text{GeV} \quad \left(\frac{r}{1}\right)^{\frac{1}{4}}
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**B-modes**

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12 orders of magnitude larger than LHC!
Two challenges for model builders

When trying to build models that can explain potential future observations of a non-zero $r$ value we face two challenges:

1. Inflation happens around the GUT scale which is close to the Planck scale

2. The inflaton moves over many Planck distances in field space $\Rightarrow$ `large field models’

$$\frac{\Delta \phi}{M_{Pl}} \geq \sqrt{\frac{r}{.01}}$$
The $\eta$-problem

How can we trust a low energy expansion?

\[ V(\phi) = V(\phi_0) \left( 1 + \sum_{n \geq 1}^{} c_n \left( \frac{\phi - \phi_0}{M_{Pl}} \right)^n \right) \]

\[ \Rightarrow \quad \eta_V = M_{Pl}^2 \frac{V''(\phi)}{V(\phi)} \approx \sum_{n \geq 2}^{} n(n-1)c_n \left( \frac{\phi - \phi_0}{M_{Pl}} \right)^{n-2} \]
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We need to control Planck suppressed operators!

For $\phi - \phi_0 \ll M_{Pl}$ we need to know that $c_2 \ll 1$
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We need to control Planck suppressed operators!

For $\phi - \phi_0 > M_{Pl}$ we need to know that $c_n \ll 1, \forall n$!

$\Rightarrow$ String theory very useful!
The $\eta$-problem

How can we trust a low energy expansion?

$$V(\phi) = V(\phi_0) \left( 1 + \sum_{n \geq 1} c_n \left( \frac{\phi - \phi_0}{M_{Pl}} \right)^n \right)$$

- It is difficult to forbid higher order terms using a symmetry (see below for a shift symmetry)
- We need a UV complete theory of quantum gravity to understand these corrections

$\Rightarrow$ String theory
Axions as inflatons

Having a UV complete theory of quantum gravity seems very useful, **but this is not enough**: Usually we expect to have new features in the potential whenever we move by one Planck distance.
Axions as inflatons

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Axions as inflatons

• String compactification usually have 100-1000 scalar fields (see white board lecture)

• We want to move one field over 10 Planck distances without disturbing the other fields (too much)

• The best approach seems to use an axion field with a (broken) shift symmetry as inflaton
Axions as inflatons

- String compactification usually have 100-1000 scalar fields (see white board lecture)
- We want to move one field over 10 Planck distances without disturbing the other fields (too much)
- The best approach seems to use an axion field with a (broken) shift symmetry as inflaton
- In string compactification we find many axion fields
- In critical (i.e. 10d) string theory half of the scalar fields are axions
Axions as inflatons

Usual lore:

There are no continuous global symmetries in a theory of quantum gravity
Axions as inflatons

Usual lore:

There are no continuous global symmetries in a theory of quantum gravity

- This seems to be true in string theory
- The continuous shift symmetry of axions is broken by non-perturbative effects to a discrete symmetry

\[ a \rightarrow a + f n, \quad n \in \mathbb{Z} \]

The discrete shift symmetry still forbids many corrections!
Axions as inflaton

Usual lore:

There are no continuous global symmetries in a theory of quantum gravity

A continuous shift symmetry:

\[ V(a) \propto \text{const.} \]
Axions as inflatons

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A discrete shift symmetry:

\[ V(a) \propto \cos(a/f) \]
$r \approx 16\epsilon_V$

$n_s \approx 1 - 6\epsilon_V + 2\eta_V$

\[
\epsilon_V = \frac{M_P^2}{2} \left( \frac{V'}{V} \right)^2, \quad \eta_V = M_P^2 \frac{V''}{V}
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Natural inflation

- To match the experimental data we need $f > M_{Pl}$
- Many semi-explicit models of this kind exist in string theory
Natural inflation

• To match the experimental data we need $f > M_{Pl}$
• Many semi-explicit models of this kind exist in string theory
• However, recently the very existence of such models (in any kind of quantum gravity) has been questioned
• The concerns are related to the so called weak gravity conjecture
  Arkany-Hamed, Motl, Nicolis, Vafa  hep-th/0601001
• For some this weak gravity conjecture means $f < M_{Pl}$
Natural inflation

“We better figure this out before the experiments are done.” – Juan Maldacena
Axion monodromy inflation

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Axion monodromy inflation

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The axion potential after breaking the symmetry:

\[ V(a) \propto a^p + \cos(a/f) \approx a^p \]
Axion monodromy inflation

- Quantum tunneling between branches puts an upper bound on the field range during inflation
- String theory does not seem to allow arbitrary large field range for inflaton (cf. weak gravity conjecture)
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