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0. Inflation as a 4D EFT

0.1 Classical inflationary EKG

\[ \int d^4x \sqrt{-g} \left( \frac{R}{2} - \frac{1}{2} (\partial \phi)^2 - V(\phi) \right) \]

with FRW EKG:

- Positive, expanding Universe
- Converging distances
- No constant Universe expansion

and e.o.m.:

\[ H^2 = 3H^2 \dot{H} + \frac{1}{2} \dot{\phi}^2 + V(\phi) \]

\[ \dot{\phi} + 3H \phi = -V'(\phi) \]

\[ H \equiv \frac{\dot{a}}{a} \] Hubble parameter

0. Inflation = accelerated expansion $\ddot{a} > 0 \Rightarrow$

\[ (aH)^{-1} = (\dot{a})^{-1} \] shrinking Hubble radius

Converging distance, particles can communicate in a given time.
\[ E = \frac{-\dot{H}}{H^2} = \frac{\dot{\phi}^2}{M_{Pl}^2 H^2} < \frac{3}{2} \frac{\dot{\phi}^2}{V(\phi)} < 1 \]

\[ H \approx \text{const} \quad \dot{a}^2 = -\ddot{a}^2 + 2H \dddot{a} \]

1. Geometry is quasi-de Sitter for \[ H \approx \text{const} \]
2. Energy scale of inflation
3. Amount of inflation:

\[ N \equiv \int_{a_*}^{a} \frac{da}{aH(a)} \]

\[ \sim \int_{a_*}^{a} \text{de} \text{ne} \]

\[ \text{can use } t, \phi \text{ or } N \text{ as a clock to measure progress of inflation.} \]

a) Slow roll inflation when

\[ E = \frac{H_{Pl}^2}{2} (\frac{V'}{V})^2 \ll 1 \] and

\[ 17 \sim \frac{H_{Pl}^2 (V')^2}{V} \ll 1 \]

b) Monomial or axion monodromy

\[ V = \frac{1}{2} m^2 \phi^2 \]

\[ \Phi \sim 15M_{Pl} \]

\[ \Phi \sim 7M_{Pl} \]

\[ V = V_0 (1 - e^{-1/3}) \]

\[ \text{Slow roll-like} \]

\[ \Phi \sim 5M_{Pl} \]

\[ \Phi \ll H_{Pl} \]

\[ V = V_0 + \lambda_1 \phi^4 \phi_{Pl}^2 + \lambda_3 \phi^3 \frac{\phi^3}{3 \phi_{Pl}^2} \]

\[ \text{inflation pt} \sim \text{D-term inflation} \]

Observitionally favoured.
Quantum Perturbations \to CMB Observables

\[ S_{\gamma j} = a^2 2 R \delta \gamma j + a^2 h_{\gamma j} \quad \text{(comoving gauge \( \delta T_{0i} = 0 \))} \]

Scalar perturbations \( R(t, \mathbf{x}) \)

- Converted to Fourier transform into momentum space

\[ R(t, \mathbf{x}) = \int d^3k \left[ \hat{R}_k(t) a_k e^{i \mathbf{k} \cdot \mathbf{x}} + c.c. \right] \]

- Quantize \( \hat{R}_k(t) \) a quantum field in classical inflation.

Quantum field in classical inflation.

- High superhorizon

\[ \delta T = (aH)^{-1} \ln(aH) \]

CMB observation today probe quantum origins of universe.
Compute "2-pt $P_k$" or "primordial power spectrum"

$$\langle 0| \hat{R}_k \hat{R}_{k'} | 10 \rangle = P_k S(k+k')$$

Result: \[ \Delta^2(k) = \frac{k^3}{2\pi^2} P_k \approx \frac{H^2}{\epsilon} \left( \frac{k}{aH} \right) \]

Directly relate correlation functions at horizon exit to observables at late times.

For de Sitter $\Delta^2(k) = \text{const.}$, slowly varying $H$ \[ \Rightarrow \text{almost scale-invariant power spectrum:} \]

- For $k < aH$, \[ \Delta^2(k) \approx A_s \left( \frac{k}{k_p} \right)^{n_s-1} + \frac{1}{2} \delta_s \ln (k/k_p) + \ldots \]

- Chosen pivot scale $k_p$.

- Time dependence of $H$.

- $n_s = 1 - 6\epsilon + 2\eta$

- $\alpha_s = -24\epsilon^2 + 16\epsilon\eta - 2\delta$

- For single field, slow roll inflation, fluctuations are Gaussian - fully characterised by 2-point function with non-trivial interactions or derivative terms, higher order 3-point function contains new info.

- Non-Gaussianities are measured by "3-point function" bispectrum

$$\langle 0| \hat{R}_{k_1} \hat{R}_{k_2} \hat{R}_{k_3} | 1 \rangle = (2\pi)^3 B_{k_1, k_2, k_3} S(k_1+k_2+k_3)$$

- momenta dependence $\Rightarrow$ amount of $\Delta^2$ associated with $\Delta^2$'s of different shapes.

Different models give different templates which can be searched for in data.

Useful measure in $f_{NL} = \frac{5}{18} \frac{B_3(k, k, k)}{P^2(k)}$ - a number.
EFT Case Study:

Starobinsky Inflation

\[
V = \frac{H_{pe}^4}{4\lambda} \left( 1 - e^{-\sqrt{3} \frac{\Phi}{H_{pe}}} \right)^2
\]

\[
N_* = \int_{a_{end}}^{a_0} da a n_a = \int_{\Phi_{end}}^{\Phi_0} \frac{d\Phi}{\sqrt{2e}}
\]

\[
\Rightarrow \eta = H_{pe}^2 \frac{V''}{V} = -\frac{4}{3} e^{-\sqrt{3} \frac{\Phi}{H_{pe}}}
\]

\[
e = -\frac{1}{2} \left( \frac{V'}{V} \right)^2 = 3/4 \eta^2
\]

\[
N_* = -\frac{1}{\eta}
\]

\[
\Rightarrow n_s - 1 = -6e + 2\eta \times 2\eta = -2/\eta
\]

\[
N_0 = 50 \Rightarrow n_s = 0.96
\]

\[
N_0 = 60 \Rightarrow n_s = 0.97
\]

\[
r = 16e \approx 12\eta^2 = 12/\eta^2
\]

\[
N_0 = 50 \Rightarrow r = 0.005
\]

\[
N_0 = 60 \Rightarrow r = 0.003
\]

\[\Delta \Phi \sim 5 H_{pe}\]

\[H_{rad} \sim 10^{16} \text{ GeV}\]

\[n_s \text{ negligible}
\]

\[NGS \text{ negligible}\]
Tensor perturbations can be computed in an analogous way.

\[ \Delta^2_{R}(k) = \frac{2}{\pi^2} \frac{H^2}{H_{\text{pe}}^2} \left| \frac{\dot{\chi}}{\chi} \right| \text{if } k = aH \]

→ Tensor-to-scalar ratio

\[ r = \frac{\Delta^2_{T}}{\Delta^2_{R}} = 16 \epsilon \]

and \[ \frac{H}{H_{\text{pe}}} = \pi \Delta^2_{R}(k) \sqrt{\frac{r}{2}} \]

measured ~ 4.7 x 10^{-5}

\[ \Rightarrow \text{H}_{\text{inf}} \equiv \text{V}_{\text{inf}} \simeq \left( 3 H^2 H_{\text{pe}} \right)^{1/4} = 1.8 \times 10^{-16} \text{ GeV} \left( \frac{r}{0.1} \right)^{1/4} \]

measuring \( r \) Would fix \( \text{H}_{\text{inf}} \)!

Moreover:

\[ N_2 = \int \frac{d\alpha}{\alpha} = \int \frac{\dot{a}}{a} dt = \int \frac{H}{\dot{\phi}} d\phi = \int \frac{H_{\text{pe}}}{\dot{\phi}} \frac{d\phi}{H_{\text{pe}}} = \sqrt{8 \pi} n^{1/2} \Delta \phi \]

\[ \Rightarrow \frac{\Delta \phi}{H_{\text{pe}}} \gg 6 \times \left( \frac{r}{0.1} \right)^{1/2} \]

Current bound \( r \leq 0.07 \) ⇒ observable tensors require ultra-Planckian-field ranges and \( \text{H}_{\text{inf}} \ll \text{H}_{\text{out}} \)!

Summary:

- \( \phi V_{\text{inf}}(\phi) \) gives \( \phi > 0 \) and solves attractor flatness and monopole problem.
- Quantum fluctuations seed CBH temp fluctuations & LSS.
- In excellent agreement w/ extnt.

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B = bounds are getting tighter—should be shown soon if model is correct.
Inflation stems from a specific potential:

\[ V = \frac{m^2}{2} (1 - e^{-\frac{2\phi}{m^2}}) \]

in FRW background:

\[ \frac{dc^2}{d^2} = -d^2 + a(t)^2 d^2 \]

\[ H^2 = \frac{8\pi G}{3} \frac{\rho}{a^2} \]

\[ \dot{H} = \frac{9}{4} \frac{H^2}{a^2} \]

\[ \dot{\phi} + 3H\phi = -V'(\phi) \]

Inflation \( a > 0 \) when \( (aH)^{-1} = (a) - 1 \)

\[ e = \frac{H_p c}{2} \left( \frac{V'}{V} \right)^2 = 3/4 \eta^2 < 1 \quad \text{and} \]

\[ \eta = \frac{H_p c V''}{-\frac{4}{3} e^{-\frac{3}{2} \phi}} \phi / H_p \ll 1 \]

Classical limit - explains large-scale anisotropy

Quantum fluctuations in inflaton \( \Rightarrow \) temp. fluctuations in CMB

\[ R(\ell) = \text{c} \text{mb} R(\ell) \approx e^{2\ell^2} \quad \text{or} \quad c = \text{quadratic} \]

\[ \text{observed in CMB} \]

Given \( \phi, V(\phi) \) \( \Rightarrow \) compute power-spectrum (and higher n-correlation for)

fluctuations at time they exited horizon and compare to CMB observations.

Inflation \( \Rightarrow \) almost scale invariant power spectrum:

\[ \Delta^2 \simeq (k/k_0)^{n_s - 1} + \frac{1}{2} \alpha_s k \frac{k}{k_0}^{n_s - 1} \]

\[ n_s = 1 - 6E + 2\zeta \]

\[ |E| < 0.005 H_p \text{ at horizon crossing} \]

\[ r = \frac{\Delta^2}{\Delta^2} = 16E \]

\[ \text{Hmfv} = 1.8 \times 10^{16} \text{ GeV} \left( \frac{m}{10^{-2}} \right)^{1/4} \]

\[ \frac{\Delta^2}{H_p^2} > 0.05 \left( \frac{m}{10^{-2}} \right)^{1/2} \]
Time that scales observed in the CMB exited horizon depend on whole history of Universe, in particular, including reheating 50-60 e-folds before end of inflation.

\[ N_0 = \int \frac{d\phi}{e^{\phi} \sqrt{V_0 / V}} \]

eg. for Starobinsky inflation

\[ N_0 = -\frac{1}{2} \]

\[ n_s - 1 \approx -2/N_0 \]

\[ r \approx 12/N_0 \]

\[ N_0 = 50 \quad n_s = 0.96, \quad r = 0.005 \]

\[ N_0 = 600 \quad n_s = 0.97, \quad r = 0.003 \]

\( \Delta \Phi \sim 5 \text{ Hpe} \)

High-scale inflation

\[ H_{\text{inf}} \sim 10^{16} \text{ GeV} \]

Summary of Constraints
I. Inflation from String Theory - Overview

I.1 Why?

Cosmology described v. well by 4D LEEFT

But how well potential is v. sensitive to corrections from high energy dofs:

$$\text{Leff}[\phi] = L_0[\phi] + \sum_i C_i \frac{\Delta \phi_i^2}{\Lambda_{\text{soft}}}$$

Radiative corrections generate all terms allowed by symmetries!

$$\eta \text{- problem: } \eta = \frac{M_{\text{pl}}}{V}\sqrt{\frac{V''}{V}} \sim \frac{m_{\phi}^2}{H^2} > 1$$

$$\Rightarrow m_{\phi}^2 \sim 1_{\text{EW}} \gg H^2$$

Is inflation robust against quantum gravity corrections?

Note: Inflaton has a shift symmetry $\phi \rightarrow \phi + \text{const}$ broken mildly by almost flat potential $\Rightarrow$ potential is radiatively stable against doop corrections "technically natural" naively,

But how do high energy dofs contribute to $V(\phi)$? Are these protected by any symmetry?

String theory - a well-developed, precise framework to answer this question.

Opportunities - connect quantum gravity to exp!

* What is inflaton $\phi$ and how is $V(\phi)$ protected?

New mechanisms of inflation $\rightarrow$ new patterns in data to search for

* Falsifiable models from string theory & QFT.
Rely on mass hierarchies

\[ H_{\text{inf}} < H_{\text{MK}} < H_S < H_{\text{pl}} \]

Hand especially for large field, high scale models! w/ observable physics!

\[ n = 3.1 \times 10^8 \left( \frac{H_{\text{inf}}}{H_{\text{pl}}} \right)^4 \]

\[ = 3.1 \times 10^8 \left( \frac{H_{\text{inf}}}{H_{\text{MK}}} \right)^4 \left( \frac{H_{\text{MK}}}{H_S} \right)^4 \left( \frac{H_S}{H_{\text{pl}}} \right)^4 \]

\[ \text{Use } \frac{1}{2k_{10}} \int d^{10}x \sqrt{-g_{10}} R_{10} \rightarrow \frac{H_{\text{pl}}^2}{2} \int d^4x \sqrt{-g_4} R_4 \]

\[ \Rightarrow H_{\text{pl}}^2 = \frac{V_0}{K_{10}^2} \Rightarrow \frac{1}{2} (2\pi)^4 (\chi')^4 \]

Assume \[ V_0 = (2\pi L)^6 \]

\[ H_{\text{MK}} = \frac{1}{L} \]

Also \[ \chi' = L_S^2 = \frac{1}{H_S^2} \]

\[ \Rightarrow \quad H_{\text{inf}} \lesssim 0.3 \quad H_{\text{MK}} \lesssim 0.3 \quad H_S \]

\[ n \lesssim 5.5 \times 10^{-7} \]

\[ H_{\text{inf}} \lesssim 0.45 \quad H_{\text{MK}} \lesssim 0.45 \quad H_S \]

\[ n \lesssim 0.002 \]

\[ \Rightarrow \text{Large field inflation & observable physics at limits of validity of EFT}. \]
Module stabilisation - moduli and inflaton dynamics cannot be decoupled.

- Inflaton potential must be around
  - Metastable de vacuum other moduli must be stabilised!
  - NOGOs \( \implies \) achieved by balancing delicately many sources
    - \( p \)-form fluxes
    - \( D \)-branes \( / \) \( D \)-planes
    - \( \text{pert} / \text{non-pert. quantum corrections} \)
  - There must be no steep runaway directions
    - \( \implies \) multiple flat directions
  - Moduli must not be destabilised during inflation

\[
V_{\text{eff}} = \frac{e^{\pm \phi}}{\text{vol}} + \frac{V(\phi)}{\text{vol}^2} \quad n \neq 2, 3
\]

\[
V_{\text{vol}} = H \phi^3 + m_{32}^2 \phi^2 \quad \text{KLT from } m_{32}^2 \approx \text{TeV}^2 \quad \text{all sources}
\]

\[
V_{\text{volume}} = H \phi^3 + m_{32}^2 \phi^2 \quad \text{from } m_{32}^2 \approx \text{TeV}^2 \quad H \phi^3 \quad \text{KLT of gravity}
\]

- \( \eta \)-problem hard to steep inflaton light \( M_\phi \ll H \)

When \( H_{\text{inf}} < H_{\text{pl}} \), string description of K"{a}W"{a}fa

\[
\mathcal{L}_{\text{inflaton}} = - K e^{\phi} \bar{\omega} e^{\phi} e^{- \Phi} - \frac{\Phi^{H_{\text{pl}}}}{H_{\text{pl}}^2} \left[ K e^{\phi} \Delta W \Delta W - 3 \Delta W^2 \right]
\]

\( \Phi \) assume inflaton lives in complex de Sitter manifold chiral supermultiplet.

Expand around \( \Phi = 0 \):

\[
K = K(0) + K e^{\phi} (0) e^{- \Phi} + \ldots
\]

\[
\Rightarrow \mathcal{L}_{\text{inflaton}} = - 3 \Phi \phi \bar{\omega} e^{- \Phi} - V(0) (1 + \frac{\Phi}{H_{\text{pl}}} + \ldots)
\]

where \( \phi e^{- \Phi} = K e^{\phi} (0) e^{- \Phi} \) anomalously normalised \( \phi \)

\[
\Rightarrow m_\phi^2 \approx \frac{V(0)}{H_{\text{pl}}^2} + \ldots = 3 H^2 + \ldots \Rightarrow \eta = 1
\]

So a generic \( W(\phi) \) has \( m_\phi^2 = O(H^2) \)

\* Non-generic \( W(\phi) \) eg. \( K = \phi \bar{\omega}, W = \Phi \) \( \Rightarrow V = e^{\phi} \Phi (1 + \Phi^2)^2 - 3 \Phi e^{- \Phi} = 1 + \frac{\Phi}{H_{\text{pl}}^2} + \ldots
\]

\* Shift symmetry \( \phi \rightarrow \Phi + \Phi + \text{const} \Rightarrow K = (\Phi - \bar{\omega})^2 \) Winding of \( \phi \)

\( \text{proteces } \Phi \text{ only } \quad \text{mod } \Phi \text{ only }\)
Multi-field dynamics: if $\Phi$ is light, why are other moduli $X$ not light? Very field $m \lesssim H$ are classically and quantum mechanically active during inflation. 

- Bkgd inflationary trajectory
- parts and observables - "closed curvature" parts & NGPs

Backreaction of heavy moduli: even when $m > H$, time-dependent inflationary energy can induce evolution of moduli which can then change form of inflaton potential.

\[ V(\Phi, X) = -\frac{1}{2} \Phi^2 X^2 - \frac{1}{2} H^2 (X - m)^2 \]

\[ V_{\text{infl}} = \frac{1}{2} \Phi^2 \frac{m^2}{\Phi^2 + m^2} \]

Integrate out $X$, assuming it adiabatically follows its min: $\dot{X}/V = 0$ - solve for $X$ and plug into $V(\Phi, X)$

\[ V(\Phi, X(\Phi)) = -H^2 m^2 \frac{1}{2} \Phi^2 \frac{1}{\frac{1}{2} \Phi^2 + m^2} \]

Reheating: inflation must end and energy transferred to SM fields to initiate BB; avoid overproduction of relics from other "unobserved" moduli, KK modes, excited "strings", axions, hidden matter, hidden radiation.

"Cosmological moduli problem": moduli inflation evolves, $\min V_{\text{mod}}$ for module shift.

\[ V_{\text{mod}} = -\frac{1}{2} m^2 X^2 - \frac{1}{2} H^2 (X - \frac{1}{2} \Phi)^2 \quad m < H \]

\[ V_{\text{mod}} \uparrow \]

During inflation, Hubble friction dominates evolution up to $X = \frac{1}{2} \Phi \sim 0 (H_{\text{pe}})$

\[ X = \frac{1}{2} \Phi \sim 0 (H_{\text{pe}}) \]

Before inflation, $H_{\text{infl}} > m > H \rightarrow$ moduli roll down to min. and oscillates.
**Epoch of moduli domination - Moduli must decay before BBN**

\[ m_X > 10 \text{ TeV} \]

- Observables depend on \( N_\ast \), \( N_\ast \) depends on history of Universe, including reheating.

\[ \text{(aH)}' \]

\[ \ln k' \]

\[ \text{(chose pivot scale)} \]

\[ a_{\text{end}} = \frac{a_{\text{end}}}{a_{\text{end}}} \]

\[ \text{from CHB} \]

\[ \text{has long decay reheating lasts} \]

\[ \text{last to decay} \]

\[ \text{accurate to CHB exiting} \]

\[ N_\ast \approx 55 \pm 5 \]

**CMB** = after inflationary reheating there is additional modulus dominated epoch and second reheating.

\[ N_\ast = (55 - \frac{1}{4} N_{\text{mod}}) + 5 \]

\[ \text{altars is at 170 level} \]

- Use string to understand robustness of inflation against QG effects
- Method - Identify string compactification of 4D LEFT of interesting cosmology - what is inflator, what is its potential, what protects its potential?
- Must understand dynamics of moduli - during and after inflation.
Recap

0. Inflation: \((aH)^{-1} = (a')^{-1} \Rightarrow \beta \lesssim \frac{\beta H_0}{2 \sqrt{V'}} < 1\)

\[ q \equiv \frac{H_0^2 V''}{V} << 1 \]

Classical evolution = Large scale topology of Universe.
Quantum fluctuations = Small scale anisotropies in CMB.
Power spectrum of quantum fluctuations: \(n_s = \frac{d n_s}{d \ln k}\)

Test against CMB observations.

1. Embed in string theory: \(\Phi\) is a modulus
   - Can compute \(V(\Phi)\)
   - Can understand suppression of quantum corrections to \(V(\Phi)\)

As \(|\Phi| < 1\) after quantum gravity corrections.

2. \(\Phi\) couples to other moduli, \(X_i\) - understand moduli
   - Are other moduli stable during inflation and after inflation.
   - If \(m_X < H\) - multi-field
   - If \(m_X > H\) - cannot just truncate when
     must compute \(V(\Phi, X, \phi(\Phi))\)

Reheating: Inflation must end and energy transferred to
S5 drops to emulate standard Big Bang cosmology.

Avoid overproduction of relics from other sectors which
would oversize Universe.

E.g. Cosmological moduli problem

As inflaton evolves, min for modulus shift.

\[ V_{\text{mod}} = -\frac{m^2}{2} \chi^2 - \frac{1}{2} H^2 (\chi - \chi_0)^2 \]

\(m^2 < H^2\) (flattening effect)

- Backreaction of \(\phi\) on \(X\)
- Axion production a displaced from min.

\(\chi = \chi_0\) after inflation.

\(H, m > H\)

- Modules rolls down to min
- \(x = 0\) and accelerates
- Moduli particle production
- End of module domination.

\(\Rightarrow\) decay \(X\) at end of inflation that forces observed in
the CMB today (CMB data).
II. Inflation from String Theory - Case Studies

II.1 D3 Inflation

Deals & Tye '98
KKLT '03
Baumann, Dymarsky, Klebanov, Modesti '07

Setup: TIIB KKLT flux compactification wk "wrapped absent" regular

A: Position modulus of slowly moving probe D3-brane

V(\phi): Interaction of D3-brane and modulus stabilizing effects

Prelude: in string th. we have Dp-branes localized in spacetime \( X^A(\sigma_1, \ldots, \sigma_p) \)

\[ S_{D3} \sim -T_3 \int d^4 \sigma \sqrt{-\text{det } G_{ab}} + \frac{1}{T_3} \int C_4 \]

\[
\begin{align*}
T_3 &= \frac{1}{(2\pi)^3} x^2 = \mu_3 \\
\mu_3 &= \frac{1}{2\pi^2} x^2 = \mu_3
\end{align*}
\]

Taking D3 + \overline{D3} in flat space: Mink4 \times T^6

\[ \text{Coulomb force due to exchange of graviton, dilaton, and C}_4 \]

\[ V(\phi) = 2T_3 \left(1 - \frac{1}{2\pi^2} \frac{T_3}{H_5^8} \phi^2 + \right) \]

\[ \phi = \sqrt{T_3} \eta \]

\[ \text{when } \eta \sim \frac{1}{M_5} \text{ tachyon productionoble} \]

\[ 4 \text{ D3} \times \overline{D3} \text{ and } 4 \]
\[ S_{100} = \int d^4x \int d^4y \sqrt{-g_{10}} \left[ \frac{M_S^{\text{eff}}}{2} R^{(10)} + \ldots \right] \]

\[ = \int d^4x \sqrt{-g_{14}} \left[ \frac{M_P^{\text{eff}}}{2} R^{(14)} + \ldots \right] \]

\[ \Rightarrow M_P^{\text{eff}} = \frac{M_S^{\text{eff}}}{g_{14}/g_{10}} \]

\[ \Rightarrow | \mathcal{M} | \approx \frac{M_P^{\text{eff}} V''}{V} \approx 0.3 \left( \frac{\mathcal{M}}{r} \right)^6 \]

\[ \Rightarrow \text{need a } \mathcal{M} \text{ impossible!} \]

Compactification effects spoil us the roll! Although the thing is very rich, it is also highly constrained.

- **Bkgd geometry - warped throat**

Consider a stack of $N$ D3-branes in 10D Mink.

1. Spacetime curved as

\[ ds^2 = e^{2A(r)} g_{\mu\nu} dx^\mu dx^\nu + e^{-2A} (dr^2 + r^2 d\Omega_5^2) \]

\[ -4A(r) = 1 + L^+/r^4 \quad \text{with} \quad L^+ = 4\pi g_5 N \]

\[ \Phi = \text{const.} \quad x(r) = (C_4) + x_1 x_2 x_3 = e^{4A(r)} \]

On $r < L$,

\[ ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + \frac{L^+}{r^2} dr^2 + L^2 d\Omega_5^2 \]

Other solutions possible:

- replace $S^5$ of the Einstein manifold ($R_{\mu\nu} g_{\mu\nu} \neq 0$)

  - e.g. $T^{1,1}$ (topologically $S^2 \times S^3$) $\rightarrow$ warped manifold.

  - Smooth out singularity at $r=0$ $\rightarrow$ warped deformed manifold (KS)
So far 6D internal space is non-compact. (No Vol)

need finite volume for infinite $H_{\phi}^2 = H_5^8 V_6$

$D_3$'s extended along extra dirn
$D_3$'s wrap DS bends/fluxes universe warping
$D_3$ brane in warped throat
feels no force - position
modulus has flat potential

- $D_3$-brane minimizes energy near $R$ at IR cutoff $\phi = r_\text{IR}$
- Coulomb force between brane and anti-brane:

$V = \frac{\alpha \phi_0^4}{N} \left( 1 - \frac{\phi_0^4}{N} \right)$

$\phi_0^2 \equiv T_3 r_\text{IR}^2$

- Warping --> potential
- If flat even for small $\phi$

But moduli stabilising effects give extra contribution to $V(\phi)$

Volume modulus coupling

$K(p, \phi, \epsilon, \epsilon) = -3 \log (p + \frac{\phi}{\epsilon} - k(\epsilon, \epsilon))$

$W(p) = W_0 + A \epsilon^{-a_p}$

$KKLT$

Assume $\phi$ stabilised at $p = \phi_0$ if $\epsilon, \epsilon > 0$ and $6D$ universe.

$\Rightarrow$ additional contribution to $V(\phi)$

$V(\phi) = \frac{V_0(\phi_0)}{(1 - \frac{\phi_0}{\epsilon^3})^2} \approx V_0(\phi_0) \left( 1 + \frac{2}{3} \phi \phi \right)$

$\Rightarrow m_\phi^2 = \frac{V_0}{\phi_0} = 2H^2 \Rightarrow \phi^{3/2}$
Fine-tune other contributions to $V(\phi)$ that can cancel $K$-contribution and allow flat potential:

\[ V(\phi) = V_0 + \frac{\lambda_1}{M_{P1}} (\phi - \phi_0)^2 + \frac{\lambda_2}{3! M_{P1}^2} (\phi - \phi_0)^3 + \ldots \]

Fine-tuning $|W| < 1$ near $\phi$ implies "inflection pt" inflection

\[ V(\phi) = V_0 + \lambda_1 (\phi - \phi_0)^2 + \frac{\lambda_2}{3! M_{P1}^2} (\phi - \phi_0)^3 + \ldots \]

Class of potential $\lambda$ coefficients unknown

- try to match to data (suitably choosing coefficients)

- understand suppression of quantum corrections
  - $W$ chosen for $V$
  - $\lambda$ large volume, weak coupling expansion 95 suppressed contributions
  - $\phi$

Observables

- $r_5 - 1 = -4/N_{\text{conf}} \approx 0.93$ (on $N_{\text{tot}} > N_{\text{conf}}$)
- $k_5 \sim 10^{-5}$

Summing compactification effects limit field range:

\[ (\Delta \phi)^2 \lesssim \frac{1}{G_{T G}} \]

\[ \Rightarrow \Delta \phi^2 / H_{P1}^2 < 2 / \sqrt{N} \]

\[ H_{P1} = \frac{V_0}{\xi_0} \]

Summary

- compactification effects limit field range as dipole
- Warping fluctuations don't cancel
  - $r < 4/N \times 0.01 < 0.01$
  - $n \sim 10^{-7}$; Hints $\sim 10^{12}$ GeV

\[ \Psi = \psi \rightarrow - 5 \]
2. DBI Inflation

**Setup:** IIIB KKLT flux compactification in warped throat region

- Operator:
- Position modulus of relativistically moving D3-brane
- \( V(\phi) \): Interaction of D3-brane and moduli-stabilising effects

- Start again from DBI action in warped \( \mathbb{R}_{6} \):

\[
\mathcal{L} = -T_3 e^{-\Phi} \sqrt{1 + \left( \frac{2\phi}{T_3 e^{\Phi}} \right)^2} - V(\phi) \quad \phi = \sqrt{T_3 F_6}
\]

- \( V(\phi) = V_0 - \frac{1}{2} \beta H^2 \phi^2 \) from moduli stabilising effects

Define "density factor":

\[
\gamma = \left( 1 - \frac{\phi^2}{T_3 e^{\Phi}} \right)^{-1/2}
\]

\( \gamma \) - maximal speed for probe D3

\[
\phi^2 = T_3 e^\Phi A(\phi)
\]

- Couple to gravity \( \Rightarrow \) \( \sqrt{\gamma} \) - kinetic term

\[
\gamma H^2 = (\gamma - 2) T_3 e^\Phi A(\phi) + V(\phi)
\]

- Potential is steep but

\[
\mathcal{E}_H = -\frac{H^2}{H^2} = \frac{2 H^2}{\gamma^2} \left( \frac{H'}{H} \right)^2
\]

\[
\eta_H = \frac{E_H}{H E_H} = \frac{2 H^2}{\gamma^2} \left[ 2 \frac{H'}{H} \left( \frac{H'}{H} \right)^2 - 2 \frac{H''}{H} + \frac{H'}{H} \frac{H''}{H} \right]
\]

- The Hubble parameter factor, \( \gamma^2 \) suppression!

- Despite large derivatives, quantum corrections negligible due to symmetries in \( \mathbb{R}_{6} \) spacetime.

- Inflation requires \( P_E \gg K_E \Rightarrow \gamma \gg 1 \) difficult!

Difficult to realise explicitly in string theory but interesting as EFT mechanism

- Distinctive signatures:
  - Field dependent sound speed \( c_s^2 = \gamma^2(\phi) \)
  - \( \Omega \Sigma \): flat \( \Rightarrow -\frac{35}{108} \gamma^2 \Rightarrow \) Planck \( \Rightarrow \delta < 24 \)
  - \( \rho \sim 10^{-7}, \ M_{\text{inf}} \sim 10^{12} \text{ GeV} \)

- Overall goals: whatever gives rise to potential will kick out on sigma inflation.
II.3 Fibre Inflation

Setup: II B Large Volume Scenario flux compactification

- $\phi$: Kähler moduli - closed string
- $V(\phi)$: Perturbative & non-perturbative corrections

- Setting: $K3$ fibration: $K3 \rightarrow CV_3 \rightarrow CP^1$
  - choose $C_1$
  - $y$ of 3 Kähler moduli and vol given by
  $$ J = x (\sqrt{t_1, t_2} - \lambda_3 t_3^{3/2}) $$
  - basis chosen
  - $t_1$ is volume of $K3$ fibre
  - $t_2$ is blow up cycle (small)

- Kähler potential of leading $\phi'$-correction
  $$ K = -2 \ln J - \frac{\phi}{\sqrt{J}} $$
  - no scale: $<V_P/\phi> = 0$
  - $<V_P/\phi> = 0$ unmixed

  Superpotential from fluxes and NP effects:
  $$ W = W_0 + \lambda t \exp(-\alpha t) $$

  $\Rightarrow$ scalar potential fixes $t_3$ and $\phi$ leaving flat direction in $(t_1, t_2)$ plane

- Flat direction lifted by string loop corrections to Kähler potential
  $$ SK_{(g_3)} \sim \sqrt{V} $$
  - $g_{(g_3)}$ is a leading $g_3$ contribution in $V$ canceling extended no scale structure
  $$ \Rightarrow SV_{(g_3)} = \frac{W_0^2}{\phi_0^2} \left( a \frac{\phi_0^2}{t_1^2} - b \frac{1}{\sqrt{t_2}} + c \frac{g_3^2 t_1}{t_2} \right) $$

  fixed $t_1 \sim g_3^{4/3} \phi_0^{2/3}$

- Suppose $t_3$ and $\phi$ remain fixed at min. While $t_1$ is initially displaced:

  $$ \Rightarrow V(\phi) = V_0 \left( 1 - 4/3 \exp(-\phi) \right)^{2/3} $$

  where $\phi = \sqrt{3}/2 \ln t_1$
Balancing leading order\textsuperscript{a} pert effects against each other

- Higher order loop $\chi^2$ corrections\textsuperscript{b} may be suppressed by additional (possibly fractional) powers of $\mathcal{O}$ and $\omega^{-1}$ and $\mathcal{O}$ is large.

\textbf{But} to match \textit{at amplitude of scalar power spectrum} $\mathcal{O} \approx 10^2 - 10^3$

- Symmetry - in 10D volume limit there are enhanced symmetries - 10D general covariance

  $\Rightarrow$ inflation has weakly broken non-compact shift symmetry $\Phi \rightarrow \Phi + \text{const}$.

- Cosmology:

  $V(\Phi) \approx V_0(1 + \Phi e^{-\Phi/\eta})$

  \[
  \epsilon = \frac{H^2_{\text{re}}}{2} (\frac{V'}{V})^2 \approx \frac{1}{2} \eta^2 f^2 \quad \eta = \frac{H^2_{\text{re}}}{V} \approx -\frac{\epsilon}{f^2}
  \]

  $\Rightarrow \epsilon \ll \eta$

  $n_s = 1 + 2\eta$

  $r = 2f^2/M_{\text{re}}^2 (n_s - 1)^2 \approx 0.005 \quad f = \sqrt{3}$ and $n_s = 0.97$

  $\Delta s = -\frac{1}{2} (n_s - 1)^2 \approx 5 \times 10^{-4}$

  $\Delta \Phi \approx 8 H_{\text{re}}$ and $M_{\text{re}} \approx 10^{16} \text{ GeV}$

  $\Rightarrow$ Difficult to achieve $M_{\text{re}} \ll M_{s} \ll M_{s}$

  $\frac{M_{s}}{M_{s}} \approx 0.4$

  At limits of EFT\textsuperscript{c}, validity.
IV The Future

- CB and LSS raise important questions
- Inflationary scenario provides compelling answers
- Very sensitive to high energy effects
  - need to embed in consistent qd. of quantum gravity!
- So far, string theory has provided general mechanisms
  for inflation and its robustness, which can be
  tested against observations
- Expect already favours some models over others
- Expect further improvements in experiment and theory!

* Increasing precision over a wide range of scales:
  - LSS curves
    - $n_s \rightarrow$ We, reheating & history of Universe
    - $r_s \rightarrow$ slow roll vs. features in potential
    - $N_{\text{e}} \rightarrow$ single field vs. multifield
    - slow roll vs. custo. interactions
  - CHB polarization (ground based / balloon / space)
    - $r \rightarrow$ large field, high scale vs. small field, low scale
    - $n_t = \frac{2H}{H_0} < 0$ and $n_t = -1/8$ for single field
  - GWs - inflationary quite unlikely, but other poss.
    (e.g. cosmic strings could be seen)

* Theoretical questions:
  - UV completion of inflation that is well understood
    and explain flatness of $\Omega_R$ potential (e.g. axisymmetric
    mechanism)
  - Effect of moduli on inflationary phd. and observability
  - End of inflation and reheating
  - Primordial costs.