The Weak Gravity Conjecture, Black Holes, and Cosmology

Gary Shiu
Based on work with:


+ work in progress
String Landscape and the Swampland

- A vast landscape in string phenomenology & cosmology:
  - Field theoretical scenarios found their UV realizations.
  - New scenarios have been uncovered along the way.
- The other side of the question is equally interesting:
  - Are there low energy effective theories that are not UV completable in quantum gravity (aka in the Swampland)?

Vafa '05; Ooguri, Vafa, '06
Where to draw the Fences?
Where to draw the Fences?
Inflation & Gravity Waves
A distinguishing parameter is the tensor-to-scalar ratio $r$.

Many experiments including BICEP/KECK, PLANCK, ACT, PolarBeaR, SPT, SPIDER, QUEIT, Clover, EBEX, QUaD… can potentially detect primordial B-mode at the sensitivity $r \sim 10^{-2}$.

LiteBIRD & PIXIE may have the sensitivity to detect $r \sim 10^{-3}$. 
B-mode and UV Sensitivity

Large $r$ not only observationally interesting, but theoretically challenging:

✦ Energy scale of inflation is around the GUT scale

$$E_{\text{inf}} \simeq 0.75 \times \left( \frac{r}{0.1} \right)^{1/4} \times 10^{-2} M_{\text{Pl}}$$

✦ The inflaton field excursion was super-Planckian

$$\frac{\Delta \phi}{M_{\text{Pl}}} \gtrsim 2 \times \left( \frac{r}{0.01} \right)^{1/2}$$

✦ This strong UV sensitivity motivates inflation in string theory.
After a large field excursion, the inflaton potential deviates significantly from that at the origin.

\[
\mathcal{L}_{\text{eff}}[\phi] = \frac{1}{2}(\partial \phi)^2 - \frac{1}{2} m^2 \phi^2 \left( 1 + \sum_{i=1}^{\infty} c_i \frac{\phi^{2i}}{\Lambda^{2i}} + \cdots \right)
\]

Consider e.g., Chaotic inflation \( \text{Linde '86} \)

\[
c_i \sim \mathcal{O}(1)
\]

\( \Lambda \)
Axions & Large field inflation

- Natural inflaton candidates as they enjoy a shift symmetry that is only broken by non-perturbative effects:

\[ V(\phi) = \sum_k c_k e^{-km} \left[ 1 - \cos \left( \frac{k\phi}{f} \right) \right] \]

- Controlled, slow-roll potential:

\[ e^{-m} \ll 1, \quad f > M_p \]

\[ \text{decay constant} \]

Natural Inflation [Freese, Frieman, Olinto ’90]:

\[ V(\phi) = 1 - \Lambda^{(1)} \cos \left( \frac{\phi}{f} \right) + \sum_{k>1} \Lambda^{(k)} \left[ 1 - \cos \left( \frac{k\phi}{f} \right) \right] \]
Axions in String Theory

String theory has many higher-dimensional form-fields:

\[ F = \mathrm{d}A \]

3-form flux \( \uparrow \) \hspace{1cm} 2-form gauge potential:\n
\begin{align*}
\text{gauge symmetry:} & \quad A \rightarrow A + \mathrm{d}\Lambda \\
\end{align*}

Integrating the 2-form over a 2-cycle gives an axion:

\[ a(x) \equiv \int_{\Sigma_2} A \]

The gauge symmetry becomes a shift symmetry.

Axions with super-Planckian decay constants don’t seem to exist in controlled limits of string theory.

Svrcek and Witten
Banks et al.
Two Broad Classes of Models

Axion Monodromy

Silverstein, Westphal, ‘08;
McAllister, Silverstein, Westphal, 08;
F-term axion monodromy
(embeddable in SUGRA of string theory)
Marchesano, GS, Uranga ’14;
Blumenhagen, Plauschinn ’14;
Hebecker, Kraus, Witowski, ’14;
McAllister, Silverstein, Westphal, Wrase ’14

Multiple Axions

Alignment
Kim, Nilles, Peloso, ’04

Nflation
Dimopoulos, Kachru,McGreevy,Wacker ‘05

Kinetic and Stueckelberg mixings:
GS, Staessens, Ye, ’15;
Bachlechner, Long, McAllister, ’15; ...
The Weak Gravity Conjecture
The Weak Gravity Conjecture

Arkani-Hamed, Motl, Nicolis, Vafa ‘06

- The conjecture:

“Gravity is the Weakest Force”

- For every long range gauge field there exists a particle of charge \( q \) and mass \( m \), s.t.

\[
\frac{q}{m} M_P \geq "1"
\]
Heuristic Argument

• Take a $U(1)$ and a single family with $q < m$ (WGC)

\[ MP \equiv 1 \]
Heuristic Argument

- Take a U(1) and a single family with $q < m$ (WGC)

\[
\begin{align*}
2m & > M_2 > 2q \\
3m & > M_3 > 3q \\
Nm & > M_N > Nq \\
M_\infty & \rightarrow Q_\infty
\end{align*}
\]

\[MP \equiv 1\]
Heuristic Argument

- Take a $U(1)$ and a single family with $q < m$ (WGC)

- Infinitely many bound states

\[
2m > M_2 > 2q \\
3m > M_3 > 3q \\
Nm > M_N > Nq
\]

- Postulate the existence of a state with (“mild form” of WGC)

\[
\frac{q}{m} \geq "1" \equiv \frac{Q_{\text{Ext}}}{M_{\text{Ext}}}
\]
The Weak Gravity Conjecture

- Heuristic argument suggests \( \exists \) a state w/ \( \frac{q}{m} \geq \text{“1”} \equiv \frac{Q_{Ext}}{M_{Ext}} \)

- Perfectly OK for some extremal BHs to be stable [e.g., Strominger, Vafa] as \( q \in \) central charge of SUSY algebra.
  - No \( q<m \) states possible to begin (\( \therefore \) BPS bound).
  - BPS BHs \textit{are} the WGC states.
  - Non-trivial for theories with some \( q \not\in \) central charge

- One often invokes the remnants argument [Susskind] for the WGC but the situations are different (finite vs infinite mass range).

- The WGC is a conjecture on the \textit{finiteness of the # of stable states that are not protected by a symmetry principle}.

- Recent work gave more (and independent) evidences for the WGC [Montero, GS; Soler]; [Heidenreich, Reece Rudelius]; [Harlow] (more later).
The Weak Gravity Conjecture

• Suggested generalization to p-dimensional objects charged under (p+1)-forms:

\[ \frac{Q}{T_p} \geq "1" \]

• p=-1 applies to instantons coupled to axions:

\[ e^{-S_{inst}} = e^{-m+i\phi/f} \quad \implies \quad fm \leq "1" \]

• Seems to explain difficulties in finding \( f > M_P \)

• Is there evidence for the p=-1 version of the WGC?

Brown, Cottrell, GS, Soler
T-duality provides a subtle connection between instantons and particles.

- **Type IIA**: $\mathbb{R}^d \leftrightarrow \tilde{S}^1 \leftrightarrow \mathbb{R}^{d-1} \times \tilde{S}^1$
  - $D(p+1)$-Particle (Gauge bosons)

- **Type IIB**: $\mathbb{R}^d \leftrightarrow S^1 \leftrightarrow \mathbb{R}^{d-1} \times S^1$
  - D$p$-Instanton (Axions)
WGC and Axions

**Type IIA**

Gauge fields: \( A_i \sim \int \Sigma_2^{(i)} C_3 \)

Particles: D2 on \( \Sigma_2^{(i)} \)

WGC

\[ \tilde{m}_k = m_k \frac{\sqrt{g_{33}}}{2\pi l_s} \]

\[ \tilde{q}_k^i = (f_k^i)^{-1} \frac{\sqrt{2}}{4\pi l_s} \]

“Couplings”:

\[ \tilde{g}_s = \frac{g_s}{\sqrt{g_{33}}} \]

\[ \tilde{M}_P = M_P \sqrt{g_{33}} \]

**Type IIB**

Axions: \( \phi_i \sim \int \Sigma_2^{(i)} C_2 \)

Instantons: D1 on \( \Sigma_2^{(i)} \)

\[ S_{\text{inst}_k} \sim -m_k + i(f_k^i)^{-1} \phi_i \]

“Couplings”:

\[ g_s \]

\[ \tilde{M}_P = M_P \sqrt{g_{33}} \]
Apply the WGC to 5d particles:

<table>
<thead>
<tr>
<th>4d Type IIB</th>
<th>4d Type IIA</th>
<th>5d M-theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1-instantons</td>
<td>D2-particles</td>
<td>M2-particles</td>
</tr>
<tr>
<td>$m_i$</td>
<td>$\tilde{m}_i \sim m_i$</td>
<td>$M_i^{(5d)} \sim m_i$</td>
</tr>
<tr>
<td>$f_i$</td>
<td>$\tilde{q}_i \sim f_i^{-1}$</td>
<td>$Q_i^{(5d)} \sim f_i^{-1}$</td>
</tr>
<tr>
<td>$g_s \ll 1$</td>
<td>$\tilde{g}_s \gg 1$</td>
<td>$R_M \to \infty$</td>
</tr>
</tbody>
</table>

- Apply the WGC to 5d particles:

$$\frac{Q_i^{(5d)}}{M_i^{(5d)}} M_P^{(5d)} = \frac{M_P^{(IIB)}}{\sqrt{2 f_i m_i}} \geq "1" \equiv \left( \frac{Q}{M} M_P \right)_{\text{Ext}^{5d}} = \sqrt{\frac{2}{3}}$$
WGC and Axions

• For each axion (gauge U(1)) there must be an instanton (particle) with

\[ e^{-S_{inst}} = e^{-m + i\phi/f} \]

\[ f \cdot m \leq \frac{\sqrt{3}}{2} M_P \]

Brown, Cottrell, GS, Soler

For a RR 2-form in IIB string theory. Similar bounds for axions from other p-forms in other string theories have also been obtained.
Multiple Axions/
Multiple U(1)’s
Consider two U(1) bosons (axions): there must be 2 particles (instantons) \( i=1,2 \), so that BH’s can decay.

\[
\bar{z}_i \equiv \frac{M_P}{M_i} \begin{pmatrix} Q_i^1 & Q_i^2 \end{pmatrix} \left( = \frac{M_P}{\sqrt{2} m_i} \begin{pmatrix} 1/f_i^1 & 1/f_i^2 \end{pmatrix} \right)
\]

\[|\bar{z}_{EBH}| \equiv "1"\]
WGC and Axions

Multiple axions/U(1)s

- Consider two U(1) bosons (axions): there must be 2 particles (instantons) $i=1,2$, so that BH’s can decay.

\[
\tilde{z}_i \equiv \frac{M_P}{M_i} \left( Q_i^1 \quad Q_i^2 \right)
\]

\[
(= \frac{M_P}{\sqrt{2} m_i} \left( 1/f_i^1 \quad 1/f_i^2 \right))
\]

\[
|\tilde{z}_{EBH}| \equiv \text{“1”}
\]

\[
|\tilde{z}_{BH}| \leq \text{“1”}
\]
Multiple axions/U(1)s

- Consider two U(1) bosons (axions): there must be 2 particles (instantons) $i=1,2$, so that BH's can decay.

\[ z_i \equiv \frac{M_P}{M_i} \begin{pmatrix} Q_i^1 & Q_i^2 \end{pmatrix} \]

\[ = \frac{M_P}{\sqrt{2} m_i} \begin{pmatrix} 1/f_i^1 & 1/f_i^2 \end{pmatrix} \]

\[ |z_{EBH}| \equiv \text{“1”} \]

\[ |z_{BH}| \leq \text{“1”} \]

\[ z_{p1} \geq \left( \text{“1”} \ 0 \right) \]

\[ z_{p2} \geq \left( 0 \ \text{“1”} \right) \]
Consider two U(1) bosons (axions): there must be 2 particles (instantons) \( i=1,2 \), so that BH’s can decay.

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\overline{z}_i \equiv \frac{M_P}{M_i} (Q_i^1 \quad Q_i^2) \quad \left(= \frac{M_P}{\sqrt{2} m_i} \begin{pmatrix} 1/f_i^1 & 1/f_i^2 \end{pmatrix} \right)
\]

\[
|\overline{z}_{EBH}| \equiv "1"
\]

\[
|\overline{z}_{BH}| \leq "1"
\]

\[
\overline{z}_{p1} \geq ("1" \quad 0)
\]

\[
\overline{z}_{p2} \geq (0 \quad "1")
\]
WGC and Axions

Multiple axions/U(1)s

- Consider two U(1) bosons (axions): there must be 2 particles (instantons) i=1,2, so that BH’s can decay.

$$\tilde{z}_i \equiv \frac{M_P}{M_i} \left( Q_i^1 \quad Q_i^2 \right) \quad \left( = \frac{M_P}{\sqrt{2} m_i} \left( \frac{1}{f_i^1} \quad 1/f_i^2 \right) \right)$$

$$|\tilde{z}_{EBH}| \equiv \text{“1”}$$

$$|\tilde{z}_{BH}| \leq \text{“1”}$$

$$\tilde{z}_{p1} \geq \left( \text{“1”} \quad 0 \right)$$

$$\tilde{z}_{p2} \geq \left( 0 \quad \text{“1”} \right)$$
WGC and Axions

Multiple axions/U(1)s

• Consider two U(1) bosons (axions): there must be 2 particles (instantons) $i=1,2$, so that BH’s can decay.

$$
\tilde{z}_i \equiv \frac{M_P}{M_i} (Q_i^1 \ Q_i^2 ) \quad (= \frac{M_P}{\sqrt{2} m_i} (1/f_i^1 \ 1/f_i^2 ))
$$

$$
|\tilde{z}_{EBH}| \equiv "1"
$$

$$
|\tilde{z}_{BH}| \leq "1"
$$

$$
\tilde{z}_{p1} \geq ("1" \ 0)
\tilde{z}_{p2} \geq (0 \ "1")
$$

WGC

$$
\tilde{z}_{p1} \geq ("1" \ \sqrt{2} \ 0 )
\tilde{z}_{p2} \geq (0 \ "1" \ \sqrt{2} )
$$
WGC and Axions

Multiple axions/U(1)s

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\[ z_i \equiv \frac{M_P}{M_i} \left( Q_i^1 \quad Q_i^2 \right) \left( = \frac{M_P}{\sqrt{2} m_i} \left( 1/f_i^1 \quad 1/f_i^2 \right) \right) \]

\[ |z_{EBH}| \equiv "1" \]

\[ |z_{BH}| \leq "1" \]

\[ z_{p1} \geq ("1" \quad 0) \]

\[ z_{p2} \geq (0 \quad "1") \]

WGC

\[ |\tilde{z}| = "1" \]

\[ \cap \]

Convex Hull \{\tilde{z}_{p1}, \tilde{z}_{p2}\}

[Cheung, Remmen]
WGC and Axions

Multiple axions/U(1)s

- Consider two U(1) bosons (axions): there must be 2 particles (instantons) \(i=1,2\), so that BH’s can decay.

\[
\tilde{z}_i \equiv \frac{M_P}{M_i} \begin{pmatrix} Q_i^1 & Q_i^2 \end{pmatrix} = \frac{M_P}{\sqrt{2} m_i} \begin{pmatrix} 1/f_i^1 & 1/f_i^2 \end{pmatrix}
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Convex Hull \{\tilde{z}_{p1}, \tilde{z}_{p2}\}
WGC and Axions

Multiple axions/U(1)s

- Consider two U(1) bosons (axions): there must be 2 particles (instantons) $i=1,2$, so that BH’s can decay.

$$\bar{z}_i \equiv \frac{M_P}{M_i} \left( Q^1_i \quad Q^2_i \right) \left( = \frac{M_P}{\sqrt{2} m_i} \left( \frac{1}{f^1_i} \quad 1/f^2_i \right) \right)$$

N-flation

$$z^k_i \geq \sqrt{N} \delta^k_i$$

WGC

$$|\vec{z}'| = \text{“1”} \cap \text{Convex Hull } \{ \vec{z}_{p1}, \vec{z}_{p2} \}$$
Axion Monodromy

- Axion is mapped to a **massive** gauge field.
- In F-term axion monodromy [Marchesano, GS, Uranga], axion mass is generated by fluxes or compactifications on torsion cycles.
- Shift symmetry is *spontaneously* broken in the 4D EFT via:
  \[
  \int d^4x |F_4|^2 + |d\phi|^2 + \phi F_4
  \]
- Gauge symmetry \(\Rightarrow\) UV corrections only depend on \(F_4\)

\[
\sum_n c_n \frac{F^{2n}}{\Lambda^{4n}} \quad \rightarrow \quad \mu^2 \phi^2 \sum_n c_n \left(\frac{\mu^2 \phi^2}{\Lambda^4}\right)^n
\]

- Multi-branched potential:
Axion Monodromy

• Possible tunneling to different branches of the potential:

\[ V(\phi) = \frac{1}{2} (ne + \mu\phi)^2 \]

• Suppressing this tunneling can lead to a bound on field range (hence r).

• Subtleties vs Coleman’s vacuum decay (e.g, tunneling between non-metastable states)  
  Brown, Cottrell, GS, and Soler, 1607.00037 [hep-th]
Evidences for the Weak Gravity Conjecture
Evidences for the Weak Gravity Conjecture

• Lots of work in using the WGC to constrain axion inflation [De la Fuente, Saraswat, Sundrum ’14]; [Rudelius ’14,’15]; [Montero, Uranga, Valenzuela ’15]; [Brown, Cottrell, GS, Soler ’15] (x2); [Bachlechner, Long, McAllister ’15]; [Hebecker, Mangat, Rompineve, Witkowski ’15]; [Junghans ’15]; [Heidenreich, Reece, Rudelius ’15] (x2), [Palti ’15]; [Kooner, Parameswaran, Zavala ’15]; ....

• Loopholes were suggested, e.g., by exploiting the “mild form”.

• But string theory seems to satisfy stronger versions of the WGC [Brown, Cottrell, GS, Soler ’15]; [Heidenreich, Reece, Rudelius, ’15]

• The WGC is suggestive based on analyticity of amplitudes [Cheung, Remmen] and holography [Nakayama, Nomura]; [Harlow]; [Benjamin, Dyer, Fitzpatrick, Kachru] but no formal proof is given.

• [Montero, GS, Soler ‘16], is a modest step in this direction. We found modular invariance + charge quantization play a key role in this conjecture (see [Heidenreich, Reece, Rudelius ’16] for similar conclusion).
The Weak Gravity Conjecture & Holography

• We will explore the WGC in AdS spacetimes, in particular in 3D.

• Advantages:
  ‣ Behavior of gravity and gauge fields much simpler
  ‣ Greatly enhanced CFT symmetry group
  ‣ Extra constraints on CFT, in particular modular invariance

• Main disadvantage:
  ‣ $d=3$ so different than $d>3$ that any relation with higher $d$ WGC is uncertain at best
Gravity and gauge theories in three dimensions
U(1) gauge theories in 3d

• U(1) gauge theories are special in 3d: electrostatic energy of charged particles is IR divergent

• Gauge coupling runs and becomes strongly coupled in IR. Electric charge confines. \[\text{[Polyakov]}\]

• Alternatively, in the presence of a Chern-Simons term, the gauge field becomes massive:

\[\frac{\mu}{2} \int F \wedge A\]

• At low energy, gauge boson behaves as scalar with mass \(\mu\)

• This term is also required by holography for the dual CFT to have non-trivial unitary representations.
U(1) gauge theories in 3d

- CS-term modifies the e.o.m: \( d \star F = \star j_e + \mu F \)

...and hence Gauss’ law: \( \int_{S^1} \star F = Q_e + \mu \int_{S^1} A \)

- Electric charge can be measured at infinity:

\[ Q_e = -\mu \int_{S^1_\infty} A \]

- Compactness of U(1) gauge group implies

  - Charge quantization: \( \mu = \frac{N g^2}{2\pi} \), quantized CS level \( N \in \mathbb{Z} \)
  
  - Aharanov-Bohm exp. measures charge mod N. Full U(1) charge is nevertheless conserved.
U(1) gauge theories in 3d

- Gravity is also special (topological) in 3d: metric has no propagating degrees of freedom

- Nevertheless, black hole solutions exist, albeit only in AdS spacetime [Bañados, Teitelboim, Zanelli]

\[ ds^2 = -\left(-8GM + \frac{r^2}{\ell^2} + 16\frac{(GJ)^2}{r^2}\right)^2 dt^2 + \frac{dr^2}{\left(-8GM + \frac{r^2}{\ell^2} + 16\frac{(GJ)^2}{r^2}\right)^2} + r^2 \left(d\phi - 4\frac{GJ}{r^2} dt\right)^2 \]

(\(\ell \equiv \ell_{AdS}\))

- Finite horizon at \(r_+ = \ell \left[4GM \left(1 + \sqrt{1 - \left(\frac{J}{M\ell}\right)^2}\right)\right]^{\frac{1}{2}}\)
U(1) gauge theories in 3d

- 3d no-hair theorem implies BHs cannot source electric field

- BTZ metric has a non-contractible one-cycle on which a flat connection can be turned:

\[ Q_e = -\mu \int_{S^1} A \]

- Although charged BHs exist:
  - No backreaction on the metric (even after including higher derivative corrections)
  - No apparent notion of extremality
  - No straightforward connection to WGC in d>3
The CFT perspective

- Weakly coupled $AdS_3$ is dual to a $CFT_2$ at large central charge

\[ c = \frac{3\ell}{2G} \]

- Bulk U(1) is dual to (holomorphic) CFT current $j(z)$ at level $N>0$:

\[ [j_m, j_n] = N\delta_{m+n,0} \quad [L_m, j_p] = -pj_{m+p} \]

- $j_0$ is proportional to $Q$ (bulk electric charge)

- $[L_0, j_0] = 0 \Rightarrow$ electric charge is exactly conserved

- $N>0$ required for non-trivial unitary representation
The CFT perspective

- In the presence of U(1) currents, the CFT stress energy tensor admits a Sugawara decomposition

\[ T(z) = T'(z) + T^S(z), \quad T^S(z) = \frac{1}{2} : j j(z) : \]

- The Virasoro generators also split

\[ L_m = L'_m + L_m^S \]

The unitarity bound arises

\[ L_0 = L'_0 + L_m^S \quad \implies \quad L_0 \geq L_0^S \geq \frac{Q^2}{2N} \]

- Eigenvalues \( h \) of \( L_0 \) measure the total energy of the bulk

- Same story holds for anti-holomorphic part \( \tilde{T} \) when \( N < 0 \)
The CFT perspective

• Both $L'_0$ and $L^S_0$ can be directly obtained from the bulk for BTZ charged BH, given the explicit solution:

$$h'_{M,J} = \frac{c}{24} + \frac{1}{2}(M\ell + J), \quad h^S = \frac{Q^2}{2N}$$

• Hence, BHs satisfy from the CFT perspective the bound

$$h_{BH} > \frac{c}{24} + \frac{Q^2}{2N}$$

• Can regard this as 3d extremality bound. A WGC could postulate the existence of charged states

$$h_{BH} > h_{WGC} \geq h_{Unit} \quad \iff \quad \frac{c}{24} + \frac{Q^2}{2N} > h_{WGC} \geq \frac{Q^2}{2N}$$

› Our goal is to find such “super-extremal” states
Modular invariance and “super-extremal” states
Modular invariance & super-extremal states

- Take CFT partition function with chem. potential

\[ Z(\tau, z) = \text{Tr} \left( q^{L_0 - \frac{c}{24}} \bar{q}^{\bar{L}_0 - \frac{\tilde{c}}{24}} e^{2\pi izQ} \right) \]

- **Charge quantization** implies \( Z(\tau, z) = Z(\tau, z + 1) \)

- On the other hand, **modular invariance** implies:

\[ Z(\tau', z') = \exp \left( i\pi N \frac{z'^2}{c\tau' + d} \right) Z(\tau, z), \quad \tau \rightarrow \frac{a\tau + b}{c\tau + d}, \quad z \rightarrow \frac{z}{c\tau + d} \]

- Together, these mean

\[ Z(\tau, 0) = \exp (-i\pi N\tau) Z(\tau, \tau) = \text{Tr} \left( q^{L_0 - \frac{c}{24}} + Q + \frac{N}{2} \frac{\tilde{c}}{24} \bar{q}^{\bar{L}_0 - \frac{\tilde{c}}{24}} \right) \]
Conclusion: Modular invariance and charge quantization imply invariance under spectral flow

\[ L_0 \rightarrow L_0 + Q + \frac{N}{2}, \quad Q \rightarrow Q + N, \quad \tilde{L}_0 \rightarrow \tilde{L}_0 \]

Acting k times on the vacuum \((L_0 = \tilde{L}_0 = Q = 0)\) we infer the existence of states with

\[ Q = kN \quad \text{and} \quad L_0 = k^2 \frac{N}{2} = \frac{Q^2}{2N} = h_{\text{Unit}} < h_{BH} \]

These states saturate the unitarity bound and lie below the BH threshold.

- 3d WGC satisfied in the sector of charges \(Q = N \cdot \mathbb{Z}\)
Modular invariance & super-extremal states

• Remarks: Usual WGC heuristics do not apply in AdS in three dimensions:

  ‣ Gauge field is massive due to CS term. There is no tunable gauge coupling and no obvious $g \to 0$ limit.

  ‣ Large BHs (larger than $\ell_{AdS}$) do not evaporate, no trouble with remnants

  ‣ Small BHs are subject to large quantum corrections

• However, modular invariance + charge quantization imply a certain version of WGC for $Q = N \cdot \mathbb{Z}$

  ‣ Sub-lattice WGC
The $\mathbb{Z}_n$ charge

- Can modular invariance test WGC for $0 < Q < N$?
  - Partition function splits into $\mathbb{Z}_N$ sectors $Z(\tau) = \sum_{Q=0}^{N-1} Z_Q(\tau)$
  - In the low T limit ($\tau_2 \to \infty$), $Z_Q(\tau)$ gives the conformal weight of the lightest state with charge $Q \mod N$
  - $\mathbb{Z}_N$ - WGC: $Z_Q > e^{-\tau_2 \frac{Q^2}{N}}$, $\forall Q \neq 0 \mod N$

- Modular invariance and spectral flow can be used to constrain the spectrum of $\mathbb{Z}_N$ - charged states
  - Modular bootstrap [Benjamin, Dyer, Fitzpartrick, Kachru]

- These are however not sufficient to prove $\mathbb{Z}_N$-WGC
Conclusions
Conclusions

• Motivated by gravity waves & large field inflation, we have revisited the WGC and the “Swampland” proposal.

• We have formulated the WGC for (a large class of) axions which can be dualized to U(1) gauge fields.

• Constraints on multiple axions in terms of convex hull (bound on the “diameter” of axion space):
  • KNP, N-flation, kinetic mixing,…

• String theory examples suggest stronger versions of the WGC.
Conclusions

• Evidence of the WGC in AdS$_3$/CFT$_2$. Key ingredients are modular invariance & compactness of Abelian group.

• Exciting interface between Black holes, Inflation & String Theory.
Conclusions

• Evidence of the WGC in AdS$_3$/CFT$_2$. Key ingredients are modular invariance & compactness of Abelian group.

• Exciting interface between Black holes, Inflation & String Theory.

谢谢！

THANKS