Non-geometric backgrounds and axion-monodromy inflation

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Overview and outline

The main topics of these lectures are non-geometric backgrounds in string theory, and their application to axion-monodromy inflation. The latter is a mechanism proposed by Silverstein and Westphal in 2008, which realizes large-field inflation within string theory.

The lecture is split into two parts, developing first the framework of non-geometric backgrounds in string theory, and afterwards applying them to axion-monodromy inflation. The outline for is the following:

- A. Non-geometric backgrouds in string theory
 - 1. Introduction
 - 2. T-duality
 - 3. The three-torus with H-flux
 - 4. Supergravity description
- B. Axion-monodromy inflation
 - 1. Main idea
 - 2. Realization

A Non-geometric backgrounds in string theory

A.1 Introduction

We begin with a brief introduction to and motivation for non-geometric backgrounds, and how such backgrounds can be constructed.

Non-geometry

• Let us consider the closed string, and observe that it can be decomposed into a left-moving and into a right-moving sector, which are independent of each other. Both of these sectors have to be a conformal field theory (CFT) with central charge $c_L = c_R = 15$, and restrictions from modular invariance can arise at one-loop;

closed string =
$$\operatorname{CFT}_L \otimes \operatorname{CFT}_R$$
. (A.1)

• A general closed-string solution has left- and right-moving sectors different from each other

$$\operatorname{CFT}_L \neq \operatorname{CFT}_R.$$
 (A.2)

In this case, roughly-speaking, the string sees a different background in the left- and in the right-moving sector.

- For the closed string this is a perfectly natural situation, however, from a point-particles point of view this is not possible.
- Since we cannot associate a geometric point-particle interpretation to the above backgrounds, they are also called *non-geometric*. (However, this term is not rigorously defined, and is used differently depending on the context.)

Duality transformation

• Even though some non-geometric backgrounds are known explicitly (Gepner models, asymmetric orbifolds, ...), these are usually CFT constructions valid only at particular points in moduli space. But, if one wants to learn more about these configurations and perturb away from the CFT points, supergravity formulations are more useful.



- One way to obtain non-geometric backgrounds in a supergravity framework is to start from a geometric background, and perform a duality transformation leading to a non-geometric one. (Duality transformations map solutions of a theory to new solutions of a potentially different theory.)
- In order to follow this idea, one needs a duality transformation which acts differently in the left- and right-moving sector. Such a duality is *T*-duality.
- Indeed, T-duality changes the sign of the right-moving sector of the closed string but leaves the sign of the left-moving invariant

$$X(z,\overline{z}) = X_L(z) + X_R(\overline{z}) \xrightarrow{\text{T-duality}} \tilde{X}(z,\overline{z}) = X_L(z) - X_R(\overline{z}).$$
(A.3)

A.2 T-duality

Having identified T-duality as a potential tool to construct non-geometric backgrounds, let us briefly review its basic features.

World-sheet perspective

• Roughly speaking, T-duality means that a string theory on a circle of radius R describes the same physics as a theory on a circle with radius 1/R.



• More concretely, let us consider string-theory (a free boson) compactified on a circle of radius R. The partition function reads (with $q = e^{2\pi i \tau}$)

$$\mathcal{Z}(\tau,\overline{\tau};R) = \frac{1}{|\eta|^2} \sum_{m,n\in\mathbb{Z}} q^{\frac{\alpha'}{4} \left[\frac{m}{R} + \frac{nR}{\alpha'}\right]^2} \overline{q}^{\frac{\alpha'}{4} \left[\frac{m}{R} - \frac{nR}{\alpha'}\right]^2}.$$
 (A.4)

• This partition function is invariant under $R \to \frac{\alpha'}{R}$, that is

$$\mathcal{Z}(R) = \mathcal{Z}(\alpha'/R) . \tag{A.5}$$

• This symmetry of the partition function is called T-duality. It means that the spectrum stays invariant under the above replacement. (Note that our discussion involved the one-loop partition function. However, the symmetry of $R \to \alpha'/R$ is true for all genera.)

Target-space perspective

- Let us now move from the world-sheet perspective to a target-space description. That is, instead of studying the two-dimensional sigma model of the string, we consider an effective theory in ten (or 26) dimensions.
- For definiteness, let us take a ten-dimensional spacetime (corresponding to the superstring) and split it as

$$\mathbb{R}^{1,9} \to \mathbb{R}^{1,9-d} \times \mathcal{M}^d , \qquad d < 9 , \qquad (A.6)$$

where the compact *d*-dimensional space \mathcal{M}^d is assumed to have at least one direction of isometry, denoted by θ in the following.

- On the target space, the massless degrees of freedom of the string in the NS-NS sector are described by a metric G_{ab} , an anti-symmetric Kalb-Ramond two-form field B_{ab} , and a dilaton ϕ .
- Now, the T-duality transformation $R \to \alpha'/R$ along a direction of isometry θ reads as follows

$$\tilde{G}_{\theta\theta} = \frac{1}{G_{\theta\theta}}, \qquad \tilde{B}_{\theta a} = \frac{G_{\theta a}}{G_{\theta\theta}}, \qquad (A.7)$$

$$\tilde{G}_{ab} = G_{ab} - \frac{G_{\theta a}G_{\theta b} - B_{\theta a}B_{\theta b}}{G_{\theta\theta}}, \qquad \tilde{B}_{ab} = B_{ab} - \frac{G_{\theta a}B_{\theta b} - B_{\theta a}G_{\theta b}}{G_{\theta\theta}}.$$

The dilaton has to be shifted as follows

$$\Phi = \phi + \frac{1}{2} \log \sqrt{\frac{\det \tilde{G}}{\det G}}.$$
(A.8)
Buscher - 1987

• We also remark that a T-duality transformation generically mixes components of the metric G and the B-field.

A.3 Prime example: the three-torus with *H*-flux

We now want to discuss the prime example for non-geometric backgrounds. It is arises by applying T-duality transformations (Buscher rules) to a three-torus with H-flux.

• Let us recall that *H*-flux is the (vacuum expectation value of the) exterior derivative of the Kalb-Ramond *B*-field

$$H = dB . (A.9)$$

• This flux is subject to a quantization condition. In particular, for $\Sigma_3 \in H_3(\mathcal{M}^d, \mathbb{Z})$ a three-cycle in \mathcal{M}^d , the *H*-flux has to satisfy

$$\frac{1}{(2\pi\sqrt{\alpha'})^2} \int_{\Sigma_3} H \in \mathbb{Z} . \tag{A.10}$$

H-flux background

• We start with a three-dimensional flat torus $\mathcal{M}^3 = \mathbb{T}^3$ with metric

$$ds^2 = dx^2 + dy^2 + dz^2, (A.11)$$

and identifications $x \sim x + 1$, $y \sim y + 1$, $z \sim z + 1$.

• For the *B*-field we choose the following gauge

$$B_{yz} = N x$$
, \Rightarrow $H_{xyz} = N$, $N \in \mathbb{Z}$. (A.12)

• When going around the circle in say the *x*-direction, we have the following way of identifying the geometry:



Geometric flux

• The z-direction of the above configuration is a direction of isometry. We can therefore apply the Buscher rules and obtain

$$ds^{2} = dx^{2} + dy^{2} + (dz + N x dy)^{2}, \qquad B = 0.$$
 (A.13)

• This background is also called a twisted torus, since the metric describes a two-torus which is non-trivially fibered over a circle in the *x*-direction. The corresponding identifications are

$$(x, y, z) \sim (x + 1, y, z - Ny), \qquad y \sim y + 1, \qquad z \sim z + 1.$$
 (A.14)

• When going around the circle in the *x*-direction, we have the following way of identifying the geometry:



• The so-called geometric flux is then determined from a vielbein basis

$$e^x = dx$$
, $e^y = dy$, $e^z = dz + N x dy$, (A.15)

for which the spin-connection and Lie bracket are computed as

$$\omega^{z}_{xy} = N/2$$
, $[e_x, e_y] = -Ne_z$. (A.16)

The structure constants of the Lie algebra are then identified with the geometric flux

$$f_{xy}{}^z = -N . (A.17)$$

Scherk, Schwarz - 1979 Kachru, Schulz, Tripathy, Trivedi - 2002

Non-geometric *Q*-flux

• Since the above background has another direction of isometry, we can proceed and apply a second T-duality transformation. Using again the Buscher rules, now along the *y*-direction, we arrive at

$$ds^{2} = dx^{2} + \frac{1}{1 + N^{2}x^{2}} \left(dy^{2} + dz^{2} \right), \qquad B_{yz} = -\frac{Nx}{1 + N^{2}x^{2}}.$$
(A.18)

- Note that here the metric and *B*-field are well-defined locally, but not globally. Indeed, when going around the circle say in the *x*-direction as $x \to x + 1$, the metric at *x* and at x + 1 do *not* just differ by a diffeomorphism. Similarly, the *B*-field does not just differ by a gauge transformation.
- However, if in addition to diffeomorphisms and gauge the transformations the transition functions between charts are allowed to be T-duality transformations, one obtains a consistent picture. Indeed, in the above example (G, B) at x and x + 1 are related via a T-duality transformation which gave rise to the name T-fold for these spaces.



• The so-called non-geometric Q-flux is formally identified with

$$Q_x^{\ yz} = N \ . \tag{A.19}$$

Hellermann, McGreevy, Williams - 2002 Dabholkar, Hull - 2002 Hull - 2004

• However, through a different approach (generalized geometry) the Q-flux can be determined as the derivarive of a bi-vector field β^{ij} as $Q_i^{jk} = \partial_i \beta^{jk}$.

Non-geometric *R*-flux

• Even though there is no direction of isometry left, it has been argued that one can formally apply a third T-duality transformation. This leads to background with so-called R-flux

$$R^{xyz} (A.20)$$

- This flux can be expressed in terms of the bi-vector as $R^{ijk} = \beta^{[\underline{i}|m} \partial_m \beta^{\underline{jk}]}$.
- Here, the metric and *B*-field are not even locally well-defined. Furthermore, it has been found that this background gives rise to a non-associative structure.

Bouwknegt, Hannabuss, Mathai - 2004 Shelton, Taylor, Wecht - 2005 Ellwood, Hashimoto - 2006 ... - 2010

Summary

- To summarize, we studied T-duality transformations for a three-torus with *H*-flux. This is the prime example for non-geometric backgrounds.
- We introduced non-geometric fluxes through a chain of T-duality transformations:

 $H_{abc} \xleftarrow{T_c} f_{ab}{}^c \xleftarrow{T_b} Q_a{}^{bc} \xleftarrow{T_a} R^{abc}$. (A.21)

The corresponding backgrounds are, respectively, the torus, the twisted torus, the T-fold, and a non-associative space.

A.4 Supergravity description

We now want to embed this idea into a supergravity framework. More specifically, we consider $\mathcal{N} = 1$ supergravity.

The F-term potential

• The general form of a $\mathcal{N} = 1$ supergravity action in four space-time dimensions is as follows

$$S = S_{\text{kin.}} - \int_{\mathbb{R}^{3,1}} (V_F + V_D) \star_4 1, \qquad (A.22)$$

where \star_4 is the four-dimensional Hodge- \star operator. $S_{\text{kin.}}$ denotes the kinetic part, which will not be important here. The expressions V_F and V_D stand for the F- and D-term potential. V_F will be of interest here, while we do not consider the D-term part.

• The F-term potential V_F is computed in terms of the holomorphic superpotential W, the Kähler potential \mathcal{K} and the Kähler metric G as

$$V_F = e^{\mathcal{K}} \left(\left. G^{I\overline{J}} D_I W D_{\overline{J}} \overline{W} - 3 \left| W \right|^2 \right) , \qquad (A.23)$$

where a summation over repeated indices is understood. Here I runs over all holomorphic fields of the theory while \overline{J} labels the anti-holomorphic ones.

• The so-called Kähler covariant derivative $D_{\alpha}W$ is computed as

$$D_I W = \partial_I W + \mathcal{K}_I W , \qquad \qquad \mathcal{K}_I = \partial_I \mathcal{K} , \qquad (A.24)$$

where $\partial_I \mathcal{K}$ denotes the derivative of the Kähler potential \mathcal{K} with respect to the field labelled by I.

• The matrix $G^{I\overline{J}}$ denotes the inverse of the Kähler metric $G_{I\overline{J}}$ which is computed from the Kähler potential \mathcal{K} in the following way

$$G_{I\overline{J}} = \partial_I \partial_{\overline{J}} \mathcal{K} . \tag{A.25}$$

Type IIB orientifolds

 $\langle \alpha \rangle$

• We now consider type IIB orientifold compactifications on Calabi-Yau threefolds with fluxes. The relevant fields are

 $\begin{array}{ll} \text{NS-NS sector} & g, B_2, \phi, \\ \text{R-R sector} & C_0, C_2, C_4. \end{array}$ (A.26)

• Focussing on the moduli fields in the four-dimensional effective theory, we have the axio-dilaton, *G*-moduli and Kähler moduli:

$$\tau = C^{(0)} + ie^{-\phi},$$

$$G^{a} = c^{a} + \tau b^{a},$$

$$T_{\alpha} = -\frac{i}{2} \kappa_{\alpha\beta\gamma} \hat{t}^{\beta} \hat{t}^{\gamma} + \rho_{\alpha} + \frac{1}{2} \kappa_{\alpha a b} c^{a} b^{b} - \frac{i}{4} e^{\phi} \kappa_{\alpha a b} G^{a} (G - \overline{G})^{b}.$$
Grimm - 2005

- Here we have expanded components of the fields which are purely in the internal space in bases of two-forms $\{\omega_A\}$ and four-forms $\{\sigma^A\}$ as

$$J = t^{\alpha}\omega_{\alpha}, \qquad B = b^{a}\omega_{a}, \qquad C_{2} = c^{a}\omega_{a}, \qquad C_{4} = \rho_{\alpha}\sigma^{\alpha}.$$
(A.28)

– Also, $\kappa_{\alpha\beta\gamma}$ and $\kappa_{\alpha ab}$ are the triple intersection numbers.

• Using the sum of even R-R potentials $C = C^{(0)} + C^{(2)} + C^{(4)}$, these moduli can be encoded in a complex and even multi-form Φ_c^{ev} as follows

$$\Phi_c^{\text{ev}} = e^B \mathcal{C} + i e^{-\phi} \operatorname{Re} \left(e^{B+iJ} \right)$$

= $\tau + G^a \omega_a + T_\alpha \sigma^\alpha$. (A.29)

Benmachiche, Grimm - 2006

• We also mention that the complex-structure moduli are encoded in the holomorphic three-form Ω as

$$\Omega = X^{\Lambda} \alpha_{\Lambda} - F_{\Lambda} \beta^{\Lambda}, \qquad z^{i} = \frac{X^{i}}{X^{0}}. \qquad (A.30)$$

Superpotential

- When considering pure Calabi-Yau compactifications, no scalar potential will be generated. However, this changes when turning on H- and F_3 -flux.
 - The latter are vacuum expectation values of the fields strengths H = dB and $F_3 = dC_2$.
 - The corresponding superpotential reads

$$W = \int_{\mathcal{X}} \left(F_3 - \tau H \right) \wedge \Omega \,. \tag{A.31}$$

Gukov, Vafa, Witten - 1999

- Note that W depends on τ and the complex-structure moduli z^a .
- Next, we can re-write this superpotential in two steps as follows:
 - First, we use the complex multi-form two write

$$W = \int_{\mathcal{X}} \left(F^{(3)} - H \wedge \Phi_c^{\text{ev}} \right) \wedge \Omega \,. \tag{A.32}$$

– Second, we recall that Ω is closed and so we have

$$W = \int_{\mathcal{X}} \left(F^{(3)} + (d - H \wedge) \Phi_c^{\text{ev}} \right) \wedge \Omega.$$
 (A.33)

- We can now finally come back to the non-geometric fluxes.
 - The superpotential is obtained by introducing the following twisted differential

$$d - H \land \longrightarrow \mathcal{D} = d - H \land -F \circ -Q \bullet -R \llcorner .$$
 (A.34)

- The (additional) fluxes can be interpreted as operators mapping

$$H \wedge : p\text{-form} \rightarrow (p+3)\text{-form},$$

$$F \circ : p\text{-form} \rightarrow (p+1)\text{-form},$$

$$Q \bullet : p\text{-form} \rightarrow (p-1)\text{-form},$$

$$R \llcorner : p\text{-form} \rightarrow (p-3)\text{-form}.$$

(A.35)

- Employing a local basis $\{dx^i\}$ and the contraction ι_i satisfying $\iota_i dx^j = \delta_i^j$, this mapping can be implemented by

$$H \wedge = \frac{1}{3!} H_{ijk} dx^{i} \wedge dx^{j} \wedge dx^{k} ,$$

$$F \circ = \frac{1}{2!} F^{k}{}_{ij} dx^{i} \wedge dx^{j} \wedge \iota_{k} ,$$

$$Q \bullet = \frac{1}{2!} Q_{i}{}^{jk} dx^{i} \wedge \iota_{j} \wedge \iota_{k} ,$$

$$R \sqcup = \frac{1}{3!} R^{ijk} \iota_{i} \wedge \iota_{j} \wedge \iota_{k} .$$

(A.36)

- The superpotential then finally reads

$$W = \int_{\mathcal{X}} \left(F^{(3)} + \mathcal{D}\Phi_c^{\text{ev}} \right) \wedge \Omega$$

=
$$\int_{\mathcal{X}} \left(F^{(3)} + -\tau H - G^a F_a - T_\alpha Q^\alpha \right) \wedge \Omega ,$$
 (A.37)

where we denoted

$$F_{a} = F \circ \omega_{a} = F_{a}{}^{\Lambda}\alpha_{\Lambda} + F_{a\Lambda}\beta^{\Lambda},$$

$$Q^{\alpha} = Q \bullet \sigma^{\alpha} = Q^{\alpha\Lambda}\alpha_{\Lambda} + Q^{\alpha}{}_{\Lambda}\beta^{\Lambda}.$$
(A.38)

- Note that the superpotential, as it stands, contains all possible fluxes (except the *R*-flux). It is therefore the most symmetric expression in these quantities.
- We also mention that in general field strengths/fluxes are subject to Bianchi identities. These are known, and can be reproduced by requiring \mathcal{D} to be nilpotent, $\mathcal{D}^2 = 0$.

B Axion-monodromy inflation

For models of large-field inflation, the distance travelled by the inflaton has to be larger than the Planck scale. Realizing such a scenario in effective field theory – or string theory – is difficult since it not easy to maintain control over various corrections. However, so-called axion-monodromy inflation is a possible solution.

B.1 Main indea

• In order to realize inflation, the potential V for the inflation field ϕ has to be of a special form. In particular, it should be rather flat, which is described by the so-called η - and ϵ - parameters

$$\epsilon = \frac{M_{\rm Pl}^2}{2} \left(\frac{V'}{V}\right)^2 \ll 1, \qquad \eta = M_{\rm Pl}^2 \left(\frac{V''}{V}\right)^2 \ll 1. \quad (B.1)$$

- However, even if the potential satisfies the necessary conditions at tree-level, higher-order corrections easily spoil them (η -problem).
- The following approach has been proposed to circumvent this problem:
 - Consider a field ϕ with a shift symmetry $\phi \rightarrow \phi + a$ at all orders in perturbation theory. In string theory, such fields are usually axions. The shift-symmetry implies that no potential is generated for ϕ at the perturbative level, i.e.

$$V(\phi + a)\Big|_{\text{pert.}} \stackrel{!}{=} V(\phi)\Big|_{\text{pert.}} \Rightarrow V(\phi) = 0.$$
 (B.2)

– On the other hand, non-perturbative corrections $e^{2\pi i\phi}$ can induce a potential of the form

$$V(\phi) \sim 1 - \cos\phi, \qquad (B.3)$$

which breaks the continuous shift-symmetry to a discrete one as $\phi \rightarrow \phi + \mathbf{a}$ with $\mathbf{a} \in \mathbb{Z}$. The resulting theory is called natural inflation, which has been shown to not allow for trans-Planckian field ranges.

However, when slightly but explicitly breaking the shift symmetry by a
perturbative potential, the periodicity is broken. Hence, some features
of the periodic potential are preserved while the field range is enlarged.

Silverstein, Westphal - 2008 McAllister, Silverstein, Westphal - 2008 • This idea can be visualized as follows:



B.2 Realization

Let us now describe, how axion-monodromy inflation can be realized in type IIB string theory. (For type IIA the story is similar, and will not be explained here.)

The setting

- The first question we have to address is, whether the axion is a closed-string or an open-string field. Here we choose the closed-string sector.
- Next, for the closed-string sector, we have the following complex scalars (part of $\mathcal{N} = 1$ chiral multiplets) available:

$$au, \qquad G^a, \qquad T_{\alpha}, \qquad z^i.$$
 (B.4)

Note that all of these contain an axion – in our conventions this is the real part.

- Concerning the non-perturbative contributions,
 - for τ a D(-1)-brane instanton can generate a contribution to the scalar potential.
 - For G^a a D1-brane instanton on a vanishing two-cycle would be the relevant contribution.
 - For T_{α} a D3-brane instanton on a four-cycle can generate a contribution to the scalar potential.
 - Non-perturbative correction involving the complex-structure moduli z^i can be present, but only through corrections of the so-called prepotential. They will be more difficult to analyze, and so we do not consider them here.
- Finally, we have to choose which contribution to the scalar potential should break the shift symmetry. We have at our disposal the F- and the D-term potential; here we choose the F-term potential.

- To summarize, we choose a closed-string axion field as the inflaton, which is contained in τ , G^a or T_{α} . We furthermore use D-brane instanton effects to break the continuous shift symmetry, and the F-term potential to realize axion-monodromy inflation.
- We also mention that that the breaking due to the explicit breaking has to be tuned in order to be sufficiently small. On the other hand, for certain scenarios higher-order corrections to the explicit breaking only involve the value of the scalar potential itself. Hence, the latter are under control.

Biellemann, Ibanez, Valenzuela - 2015

A scenario

- We now become somewhat more concrete and develop a scenario for axion monodromy inflation. As the inflaton, we take a Kähler modulus T_{\star} , but a similar story works for τ or G^a .
- To generate a scalar F-term potential for axion-monodromy inflation, we consider the superpotential with non-geometric fluxes and D3-brane instanton corrections.

$$W = W_{\text{flux}} + W_{\text{instanton}} \,. \tag{B.5}$$

• In order to have the Kähler moduli to appear, we have to consider *Q*-flux, which leads to (recall equation (A.37))

$$W_{\text{flux}} = \int_{\mathcal{X}} \left(F^{(3)} + -T_{\star} \left[Q^{\star \Lambda} + Q^{\star \Lambda} \beta_{\Lambda} \right] \right) \wedge \Omega \,. \tag{B.6}$$

• For the non-perturbative instanton contribution, we take a D3-brane instanton (and assume that the technical details are satisfied) wich gives

$$W_{\text{instanton}} = \mathcal{A} \, e^{2\pi \, i \, T_{\star}} \,, \tag{B.7}$$

where \mathcal{A} is a constant which in principle depends on other moduli as well.

- Plugging this superpotential into the scalar F-term potential (A.23) one obtains the inflaton potential. The precise form can be rather involved, and depends on the precise data of the background space.
- Recalling that we chose the inflaton as $\phi = \operatorname{Re} T_{\star}$, the resulting scalar potential takes the schematic form

$$V \sim A Q^2 \phi^2 + B Q \phi + C \cos \phi + D, \qquad (B.8)$$

with A, B, C, D depending on the other moduli. These can be determined concretely, but they depend on the details of the model.

Summary

- From the above example we see, that indeed axion-monodromy inflation can be realized in string theory (for the Kähler axion). A similar mechanism works for the axion in τ or in G^a . The crucial point here was to consider non-geometric Q-flux and to consider D3-brane instantons.
- The main problem is to achieve a small breaking through the fluxes, which implies some amount of fine-tuning of the flux parameters.
- Furthermore, the prefactors A, B, C, D in general depend on other moduli fields than ϕ . One has to make sure that those do not interfere with inflation, which is a non-trivial task.