Infinitesimal moduli of G2 structure manifolds with instanton bundles

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X. de la Ossa, ML, E. Svanes (1607.03473 & work in progress),
See also X. de la Ossa, ML, E. Svanes (1409.7539) and J. Gray, ML, D. Lüst (1205.6208)

Motivation and summary

Long history of heterotic string compactifications

- Minkowski 4D $\mathcal{N} = 1$ vacua with good particle physics from Calabi–Yau manifolds with vector bundles.
- Minkowski 4D $\mathcal{N} = 1$ vacua also from Strominger–Hull compactifications with flux and bundles.

Stabilising moduli is an open problem.

This talk: heterotic compactifications on G_2 structure manifolds

- Heterotic 4D $\mathcal{N} = 1/2$ domain wall solutions.
  May "uplift" non-perturbatively to non-SUSY *AdS* solutions.
- Moduli of G_2 holonomy manifolds with bundles.
- Some comments on deformations of integrable G_2 structures (relevant also for non-SUSY M-theory compact.)

Outline

Heterotic supersymmetric vacua

Heterotic supergravity

- Bosonic fields: Metric G, B-field B, dilaton ϕ, gauge field A
- Fermionic fields: Gravitino, dilatino, gaugino

Fermionic SUSY variations vanish $\iff$

$$\left(\nabla_M + \frac{1}{8} \not{H}_M\right) \epsilon = 0$$

$$\left(\not{\nabla} \hat{\phi} + \frac{1}{12} \not{H}\right) \epsilon = 0$$

$$\not{F} \epsilon = 0$$

where $\not{\nabla} = \Gamma^M \nabla_M$, etc.

Compactifications

$\mathcal{M}_{10} = \mathcal{M}_E \times X$: SUSY $\iff$ nowhere vanishing spinor η on X: $\epsilon = \rho_E \otimes \eta$
$\iff$ X has reduced structure group

Hitchin:02, Gualtieri:04, Grana et al:05, ...

4D Heterotic $\mathcal{N} = 1$ Minkowski vacua

Geometry: 6D manifold X with $SU(3)$ structure

Candelas, *et.al.*:85, Hull:86; Strominger:86, Ivanov, Papadopoulos:00; Gauntlett, *et.al.*:03,...

SUSY $\Rightarrow$ globally defined spinor η on X: $\nabla_H \eta = 0$

$\iff$ complex decomposable $(3,0)$-form Ψ and real $(1,1)$ form ω such that

$$\omega \wedge \Psi = 0\ , \qquad \omega \wedge \omega \wedge \omega \sim \Psi \wedge \bar{\Psi}$$

- No H-flux $\iff$ X is Calabi–Yau $\qquad \mathrm{d}\Psi = 0 = \mathrm{d}\omega.$
- $H \neq 0$ * $\iff$ X is complex and conformally balanced
$$\mathrm{d}(e^{-2\phi}\Psi) = 0 = \mathrm{d}(e^{-2\phi}\omega \wedge \omega).$$

* Need α' corrections to avoid no-go theorem for flux if X is compact without boundary

4D Heterotic $\mathcal{N} = 1$ Minkowski vacua

Gauge fields $\rightarrow$ vector bundle V

Candelas, *et.al.*:85,Donaldson:85, Uhlenbeck, Yau:86, Li, Yau:87,...

SUSY $\Longrightarrow$

- F holomorphic $F^{(0,2)} = F^{(2,0)} = 0$
- F satisfies Hermitian Yang-Mills equation $F \lrcorner \omega = 0$

polystable holomorphic V: $\exists!$ A satisfying HYM.

4D Heterotic $\mathcal{N} = 1/2$ DW vacua

Lukas *et al*:10, 11; Gray, ML, Lüst:12, de la Ossa, ML, Svanes:14

Geometry

4D domain wall vacuum: $\mathcal{M}_{10} = \mathcal{M}_4 \times_W X(r) \equiv \mathcal{M}_3 \times Y$
$\mathcal{M}_4 = \mathcal{M}_3 \times \mathbb{R}$, $\mathcal{M}_3$ AdS or Minkowski

SUSY and BI at $\mathcal{O}(\alpha'^0)$

Restricts torsion, DW flow of the SU(3) structure, and the flux.

SU(3) torsion: $X(r)$ conformally balanced, but otherwise generic

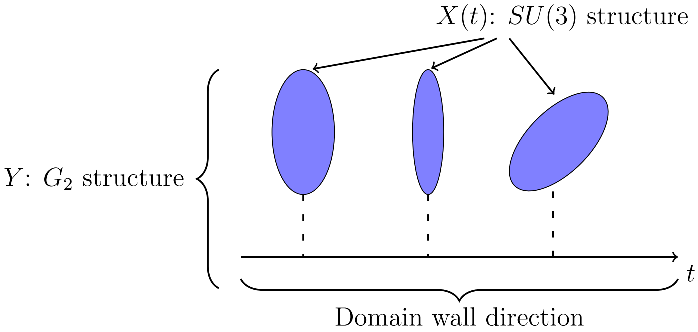
SU(3) flow: $\partial_r \omega$ fixed in terms of ϕ and SU(3) torsion
$\partial_r \Psi$ fixed up to primitive (2,1)+(1,2)-form γ.

Flux $\widehat{H}$: fixed by SUSY in terms of ϕ and SU(3) torsion up to γ
$\Rightarrow$ can check Bianchi idenitity.

4D Heterotic $\mathcal{N} = 1/2$ DW vacua

Lukas *et al*:10, 11, 12, 13; Gray, ML, Lüst:12, de la Ossa, ML, Svanes:14

Embed $SU(3)$ in G_2: $\varphi = \mathrm{d}r \wedge \omega(r) + \mathrm{Re}(\Psi(r))$.



- H-flux components allowed by symmetry: $H_{\alpha\beta\gamma}$, H_{tmn} and H_{mnp}
- $H_{tmn} = 0 = H_{\alpha\beta\gamma}$ allows non-perturbative "uplift" to 4D AdS.

4D Heterotic $\mathcal{N} = 1/2$ DW vacua

Gray, ML, Lüst:12, de la Ossa, ML, Svanes:14, Fernandez–Gray:82, Chiossi–Salamon:02

SUSY $\iff$ Y has G_2 structure determined by 3-form φ ($\psi = *\varphi$)

$$\mathrm{d}\varphi = \tau_0\, \psi + 3\, \tau_1 \wedge \varphi + *\tau_3 \ ,$$
$$\mathrm{d}\psi = 4\, \tau_1 \wedge \psi + *\tau_2 \ .$$

with torsion classes (in G_2 reps)

$$\tau_0 = \mathrm{dvol}_{\mathcal{M}_3} \lrcorner H \ , \quad \tau_1 = \tfrac{1}{2}\, d\phi \ ,$$
$$\tau_2 = 0 \qquad , \qquad \tau_3 = -H + \tfrac{1}{6}\, \tau_0\, \varphi - \tau_1 \phi \lrcorner \psi$$

This is an integrable G_2 structure.

4D Heterotic $\mathcal{N} = 1/2$ DW vacua

Gray, ML, Lüst:12, de la Ossa, ML, Svanes:14, Fernandez–Gray:82, Chiossi–Salamon:02

Gauge vector bundle

SUSY
- $F \wedge \psi = 0 \implies F$ is a G_2 instanton
- Embed $SU(3) \implies$ recover $F^{(2,0)} = 0 = F \lrcorner \omega$

Manifolds with G_2 structure

Fernandez–Gray:82, Chiossi–Salamon:02

Decomposition of forms

$\Lambda^k(Y)$ decomposes into $\Lambda^k_p(Y)$, p denotes G_2 irrep. Find these using φ:

Example: $\Lambda^1 = \Lambda^1_7 = T^*Y \cong TY$
$\implies$ *any* $\beta \in \Lambda^2$ decomposes as $\beta = \alpha \lrcorner \varphi + \gamma$, where $\alpha \in \Lambda^1$ and $\gamma \lrcorner \varphi = 0$

$$\begin{aligned}
\Lambda^0 &= \Lambda^0_1 \ , \\
\Lambda^1 &= \Lambda^1_7 = T^*Y \cong TY \ , \\
\Lambda^2 &= \Lambda^2_7 \oplus \Lambda^2_{14} \ , \\
\Lambda^3 &= \Lambda^3_1 \oplus \Lambda^3_7 \oplus \Lambda^3_{27} \ .
\end{aligned}$$

Infinitesimal Moduli

Variational studies

- Moduli of heterotic $\mathcal{N} = 1$ vacua: deformations of conformally balanced complex 3-fold X with holomorphic gauge bundle V
- Moduli of heterotic $\mathcal{N} = 1/2$ vacua: deformations of 7D Y with integrable G_2 structure and an instanton gauge bundle V
- Flow of heterotic $\mathcal{N} = 1/2$ solutions at $\mathcal{O}(\alpha'^0)$: the moduli spaces of *different* $SU(3)$ structures may connect via flow along domain wall direction in Y with integrable G_2 structure.

Infinitesimal Moduli

Variational studies

- Moduli of heterotic $\mathcal{N} = 1$ vacua: deformations of conformally balanced complex 3-fold X with holomorphic gauge bundle V

  Atiyah:57, Kodaira, Spencer:58,60, Candelas, de la Ossa:91, Becker,*et.al*:05,06, Anderson,*et.al*:10,11,13, Fu, Yau:11,Anderson, Gray, Sharpe:14, de la Ossa, Svanes:14, Garcia-Fernandez,*et.al*:13,15,...

- **Moduli of heterotic $\mathcal{N} = 1/2$ vacua: deformations of 7D Y with integrable G_2 structure and an instanton gauge bundle V**

  de la Ossa, ML, Svanes:16 + in progress, Clarke,*et.al*:16

- Flow of heterotic $\mathcal{N} = 1/2$ solutions at $\mathcal{O}(\alpha'^0)$: the moduli spaces of *different* $SU(3)$ structures may connect via flow along domain wall direction in Y with integrable G_2 structure.

  de la Ossa, ML, Svanes:14

Infinitesimal Moduli: $\mathcal{N} = 1$

Moduli space building blocks

X: conformally balanced complex 3-fold ($H = 0$: Calabi–Yau)
V: holomorphic gauge bundle

- $\partial_t \Psi$: Complex structure moduli $H^{(2,1)}_{\bar\partial}(X) \cong H^{(0,1)}_{\bar\partial}(X, TX)$
- $\partial_t \omega$: $H = 0$ Kähler moduli $H^{(1,1)}_{\mathrm{d}}(X) \cong H^{(0,1)}_{\bar\partial}(X, T^*X)$
  $H \neq 0$ Hermitian moduli
- $\partial_t A$: Vector bundle moduli $H^1(X, \mathrm{End}(V))$

Kodaira, Spencer: 58,60, Candelas, de la Ossa:91, Becker, *et.al*:05,06,...

Infinitesimal Moduli: $\mathcal{N} = 1$

Varying full set of $\mathcal{N} = 1$ constraints

"Atiyah class stabilization" restricts the moduli space:

- Gaugino variation $F \wedge \Psi = 0 \rightsquigarrow$ cs moduli $\in \ker(\mathcal{F})$
- Instanton connection: $R(\nabla) \wedge \Psi = 0 \rightsquigarrow$ cs moduli $\in \ker(\mathcal{R})$
  - Extra moduli for connection variations.
    Related to field redefinitions. de la Ossa, Svanes:14
    SUSY + BI $\implies$ EOM $\iff$ θ is an instanton Ivanov, Martelli–Sparks:10
- Gaugino $F \wedge \omega \wedge \omega$: Use Donaldson–Uhlenbeck–Yau and Li–Yau theorems: stable under first order deformations; no constraint.
  CY $\rightsquigarrow$ D-terms in 4D Anderson *et.al*:11
- Anomaly cancellation constraint together with $H = i(\partial - \bar{\partial})\omega$:
  (cs+bundle moduli) $\in \ker(\mathcal{H})$; Herm. moduli space: quotiented by $\mathrm{Im}\mathcal{H}$

Atiyah:57, Fu, Yau:11, Anderson,Gray, Lukas, Ovrut:10,11,13, Anderson, Gray, Sharpe:14, de la Ossa, Svanes:14, Garcia-Fernandez, *et.al*:13,15...

Infinitesimal Moduli: $\mathcal{N} = 1$

End result

Finite dimensional moduli space for all heterotic $\mathcal{N} = 1$ vacua.

Infinitesimal moduli equivalent to deformations of the holomorphic structure on an extension bundle of Atiyah type.

Atiyah:57, Fu, Yau:11, Anderson,Gray, Lukas, Ovrut:10,11,13, Anderson, Gray, Sharpe:14, de la Ossa, Svanes:14, Garcia-Fernandez, *et.al*:13,15...

Infinitesimal Moduli: $\mathcal{N} = 1/2$

Moduli space building blocks

Y: integrable G_2 structure manifold ($H = 0$: G_2 holonomy)
V: instanton gauge bundle

- $\partial_t\psi$,$\partial_t\varphi$: geometric moduli

  Joyce:96, Dai–Wang–Wei:03, de Boer–Naqvi–Shomer:05,...
- $\partial_t A$: Vector bundle moduli $H^1(Y, \mathrm{End}(V))$

Infinitesimal Moduli: $\mathcal{N} = 1/2$

Comment on connections on G_2 holonomy manifolds

- Unique G_2 invariant connection: the Levi-Civita connection
- Variations of G_2 structure $\sim \mathrm{d}_\theta$
  - $\mathrm{d}_\theta V^a = \mathrm{d} V^a + \theta_b{}^a V^b \,, \qquad \theta_b{}^a = \Gamma_{bc}{}^a \, \mathrm{d}x^c$
  - $\Gamma_{bc}{}^a$: connection symbols of Levi-Civita connection
  - d_θ is G_2 metric compatible.
  - $R(\theta)$ is an instanton: $R(\theta) \wedge \psi = 0$

Comment on connections on G_2 structure manifolds

Can generalise to G_2 structure manifolds, but then

- Two-parameter family of metric compatible connections. Bryant:03
- Connection with totally antisymmetric torsion $\iff$ $\tau_2 = 0$ Friedrich:06
- Unique connection $\nabla^S = \nabla_{LC} + H$ with $Hol(\nabla^S) = G_2$ and totally antisymmetric torsion Bryant:03

Infinitesimal Moduli: $\mathcal{N} = 1/2$

Decomposition of de Rham cohomology

Reyes-Carrion:93,Fernandez–Ugarte:98

Analogue of Dolbeault operator on a complex manifold: project d onto G_2 irreps.

- The differential operator $\check{\mathrm{d}}$ is defined by

$$\check{\mathrm{d}}_0 = \mathrm{d}\ , \quad \check{\mathrm{d}}_1 = \pi_7 \circ \mathrm{d}\ , \quad \check{\mathrm{d}}_2 = \pi_1 \circ \mathrm{d}\ .$$

- $\tau_2 = 0 \iff \check{\mathrm{d}}^2 = 0$, so can construct differential, elliptic complex

$$0 \to \Lambda^0(Y) \xrightarrow{\check{\mathrm{d}}} \Lambda^1(Y) \xrightarrow{\check{\mathrm{d}}} \Lambda^2_7(Y) \xrightarrow{\check{\mathrm{d}}} \Lambda^3_1(Y) \to 0$$

- $H^*_{\check{\mathrm{d}}}(Y)$ is "canonical G_2-cohomology of Y".

This generalizes to TY-valued forms: Elliptic complex

$$0 \to \Lambda^0(TY) \xrightarrow{\check{\mathrm{d}}_\theta} \Lambda^1(TY) \xrightarrow{\check{\mathrm{d}}_\theta} \Lambda^2_7(TY) \xrightarrow{\check{\mathrm{d}}_\theta} \Lambda^3_1(TY) \to 0$$

with finite-dim cohomology groups $H^p_{\check{\mathrm{d}}_\theta}(Y, TY)$, if $R(\theta) \wedge \psi = 0$

Infinitesimal Moduli: $\mathcal{N} = 1/2$

Geometric moduli for G_2 holonomy

$$\partial_t \psi = \frac{1}{3!} M_t^a \wedge \psi_{bcda} \, \mathrm{d}x^{bcd} \ , \qquad M_t^a = M_{t\,b}{}^a \, \mathrm{d}x^b$$

$$\partial_t \varphi = -\frac{1}{2} M_t{}^a \wedge \varphi_{bca} \, \mathrm{d}x^{bc}$$

- Diffeomorphisms: $\boxed{\mathcal{L}_V \psi = -\frac{1}{3!} (\mathrm{d}_\theta V^a) \wedge \psi_{bcda} \, \mathrm{d}x^{bcd}}$

  where d_θ is a connection for TY-valued forms.

- Preserve $\mathrm{d}\psi = 0 = \mathrm{d}\varphi$:

$$\mathrm{d}_\theta \Delta_t^a \wedge \psi_{bcda} \, \mathrm{d}x^{bcd} = 0 \ ,$$

$$\mathrm{d}_\theta \Delta_t^a \wedge \varphi_{bca} \, \mathrm{d}x^{bc} = 0 \ .$$

  where $\Delta_{t\,b}{}^a = M_{t\,b}{}^a - \frac{1}{7} (\mathrm{tr} M_t)$

- Compact G_2 manifold: $\boxed{\mathcal{TM}_{G_2\mathrm{hol}} \cong H^3_{\mathrm{d}}(Y) \subset H^1_{\check{\mathrm{d}}_\theta}(Y, TY)}$

Infinitesimal Moduli: $\mathcal{N} = 1/2$

Geometric moduli for integrable G_2 structure

- Diffeomorphisms: $$\boxed{\mathcal{L}_V \psi = -\frac{1}{3!} \left(\mathrm{d}_\theta V^a\right) \wedge \psi_{bcda}\, \mathrm{d}x^{bcd}}$$

  where d_θ is a connection for TY-valued forms.
- Preserve $\tau_2 = 0$: $$\boxed{(\check{\mathrm{d}}_\theta \Delta_t^a) \wedge \psi_{bcda}\, \mathrm{d}x^{bcd} = 0}$$
- $\partial_t \varphi \implies$ Variational constraints on torsion

Infinitesimal Moduli: $\mathcal{N} = 1/2$

G_2 "Atiyah" class stabilization

Instanton condition $F \wedge \psi = 0$: couples bundle and geometric moduli

$$\boxed{0 = \partial_t (F \wedge \psi) \iff \check{\mathrm{d}}_A(\partial_t A) = -\check{\mathcal{F}}(\Delta_t)\ .}$$

where $\Delta_t \in \Lambda^1(Y, TY)$, d_A covariant derivative, and $\check{\mathrm{d}}_A$, $\check{\mathcal{F}}$: project to G_2 irreps

- "Atiyah" map

$$\begin{aligned} \mathcal{F}: \quad \Lambda^p(Y, TY) \quad &\longrightarrow \quad \Lambda^{p+1}(Y, \mathrm{End}(V)) \\ \Delta \quad &\mapsto \quad \mathcal{F}(\Delta) = -F_{ab}\, \mathrm{d}x^b \wedge \Delta^a\ . \end{aligned}$$

  $\check{\mathcal{F}}$ is a map in cohomology:
  Bianchi identity $\mathrm{d}_A F = 0 \implies \check{\mathcal{F}}(\check{\mathrm{d}}_\theta(\Delta)) + \check{\mathrm{d}}_A(\check{\mathcal{F}}(\Delta)) = 0.$

- In fact, $\check{\mathcal{F}}$ also maps geometric modulus of integrable G_2 structure to a $\check{\mathrm{d}}_A$-closed form

Corrected moduli space for bundle and geometric moduli:

$$\boxed{H^1_{\check{\mathrm{d}}_A}(\mathrm{End}(V)) \oplus \ker(\check{\mathcal{F}})}$$

Infinitesimal Moduli: $\mathcal{N} = 1/2$

Corrected moduli space for bundle and geometric moduli:

$$\boxed{H^1_{\check{\mathrm{d}}_A}(\mathrm{End}(V)) \oplus \ker(\check{\mathcal{F}})}$$

Remark 1: B-field deformations

- Infinitesimal moduli of G_2-holonomy metrics only spans part of $H^1_{\check{\mathrm{d}}_\theta}(Y, TY))$:
  $H^1_{\check{\mathrm{d}}_\theta}(Y, TY)) \cong \check{\mathcal{H}}^1(Y, TY) = \boxed{\check{\mathcal{S}}^1(Y, TY)} \oplus \check{\mathcal{A}}^1(Y, TY)$
- On compact G_2 manifolds, the rest is spanned by $\partial_t B$
- All $\partial_t B$ are in the kernel of $\check{\mathcal{F}}$.
- Thus easily incorporate B-field deformations in the infinitesimal moduli space.

Infinitesimal Moduli: $\mathcal{N} = 1/2$

Corrected moduli space for bundle and geometric moduli:

$$\boxed{H^1_{\check{\mathrm{d}}_A}(\mathrm{End}(V)) \oplus \ker(\check{\mathcal{F}})}$$

Remark 2: Extension bundle

- Use the G_2 Atiyah map to define a new bundle

$$0 \longrightarrow \mathrm{End}(V) \longrightarrow E \longrightarrow TY \longrightarrow 0 \ ,$$

- E has connection $\mathcal{D}_E$:

$$\mathcal{D}_E = \begin{pmatrix} \check{\mathrm{d}}_A & \check{\mathcal{F}} \\ 0 & \check{\mathrm{d}}_\theta \end{pmatrix} \ .$$

- $\mathcal{D}_E^2 = 0 \iff \check{\mathcal{F}}(\check{\mathrm{d}}_\theta(\Delta)) + \check{\mathrm{d}}_A(\check{\mathcal{F}}(\Delta)) = 0$
- $H^1_{\mathcal{D}_E}(Y, E)$ is the moduli space
  Proof: construct long exact sequence in cohomology.

Infinitesimal Moduli: $\mathcal{N} = 1/2$ in progress

Integrable G_2 structure

Corrected moduli space for bundle and geometric moduli:

$$\boxed{H^1_{\check{\mathrm{d}}_A}(\mathrm{End}(V)) \oplus H^1_{\check{\mathrm{d}}_\theta}(\mathrm{End}(TY)) \oplus \ker(\check{\mathcal{F}} + \check{\mathcal{R}})}$$

Integrable G_2 structure

SUSY + BI $\implies$ EOM $\rightsquigarrow$ expect instanton condition: $R(\theta) \wedge \psi = 0$:
θ must vary with ψ

- Extra moduli for connection variations.
  Related to field redefinitions as in $\mathcal{N} = 1$?
- $\mathcal{R}$ map

$$\begin{aligned} \mathcal{R}: \quad \Lambda^p(Y, TY) \quad &\longrightarrow \quad \Lambda^{p+1}(Y, \mathrm{End}(TY)) \\ \Delta \quad &\mapsto \quad \mathcal{R}(\Delta) = -R_{ab}\, \mathrm{d}x^b \wedge \Delta^a \ . \end{aligned}$$

  $\check{\mathcal{R}}$: map in cohomology that maps **any** $\Delta \in \mathcal{TM}_0(Y, \varphi)$ to a $\check{\mathrm{d}}_\theta$-closed form
- $\ker(\check{\mathcal{F}} + \check{\mathcal{R}})$ may still be infinite-dimensional.

Anomaly cancellation condition: $\mathcal{N} = 1/2$

Integrable G_2 structure

Corrected moduli space for bundle, instanton and geometric moduli:

$$\boxed{H^1_{\check{\mathrm{d}}_A}(\mathrm{End}(V)) \oplus H^1_{\check{\mathrm{d}}_\theta}(\mathrm{End}(TY)) \oplus \ker(\check{\mathcal{F}} + \check{\mathcal{R}})}$$

Last equation to bring in: anomaly cancellation condition

$$H = \mathrm{d}B + \frac{\alpha'}{4}(\mathcal{CS}[A] - \mathcal{CS}[\theta]) \qquad (\star)$$

- Recall from $\mathcal{N} = 1$: Vary $(\star)$ together with $H = \mathrm{d}^c\omega \rightsquigarrow$ map $\mathcal{H} : \Lambda^p(X, E) \longrightarrow \Lambda^{p+1}(X, T^*X)$
  - well-defined in cohomology
  - finite-dim Hermitian moduli space
- $\mathcal{N} = 1/2$
  Vary $(\star)$ together with $H = \frac{1}{6}\tau_0\,\varphi - \tau_1 \lrcorner \psi - \tau_3 \implies$ map $\mathcal{H}$
  $\mathcal{H}$ well-defined in cohomology? Finiteness of moduli space? In progress...

Conclusions and outlook

Conclusions

- 4D heterotic $\mathcal{N} = 1/2$ DW solutions
  - Y Integrable G_2 structure $\supset$ G_2 holonomy
  - X Conformally balanced (non-complex) SU(3) structure
- Infinitesimal def. space of G_2 holonomy manifold w. instanton bundle V:

$$H^1_{\check{\mathrm{d}}_A}(\mathrm{End}(V)) \oplus \ker(\check{\mathcal{F}}),$$
$$\ker(\check{\mathcal{F}}) \subset H^1_{\check{\mathrm{d}}_\theta}(Y, TY))$$

  Atiyah's deformation space of holomorphic structures on extension bundles applies to (real) G_2 holonomy instanton bundles.

- Infinitesimal def. space of integrable G_2 structures w. instanton bundle V:

$$H^1_{\check{\mathrm{d}}_A}(\mathrm{End}(V)) \oplus H^1_{\check{\mathrm{d}}_\theta}(\mathrm{End}(TY)) \oplus \ker(\check{\mathcal{F}} + \check{\mathcal{R}})$$

Conclusions and outlook

Conclusions

Atiyah's deformation space of holomorphic structures on extension bundles applies to (real) G_2 holonomy instanton bundles.

Outlook

- Include constraint from Bianchi identity $\mathrm{d}H = \ldots$ (in progress).
  $\rightsquigarrow$ Determine conditions for finite-dimensional infinitesimal moduli space.
- Relation of $SU(3)$ and G_2 structure moduli spaces for domain wall solutions.
- Relevance for compactifications of M-theory and type II string theory.