

Chengdu Lectures - String Inflation

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Plan

I. Inflation as a 4D EFT

0.1 Classical inflationary bkgd

0.2 Quantum perturbations \rightarrow CMB observations

method is going
from a model of
inflation to cosmological
observables. If we
use them for string
models too.

There lies
remarkable
success
of inflation!
predictive framework
for first batch
of a second of
the universe
which can
be tested!

I. Inflation from String Theory - Overview

I.1 Why? sensitivity of inflation to high energy defts

I.2 Method opportunity to build PT models that can be tested against observation!

I.3 Challenges

II. Inflation from String Theory - Case Studies

II.1 D \bar{D} Inflation

Very many models of inflation
from string theory ...

II.2 DBI Inflation

natural inflation
leave axion monodromy \rightarrow Erik

II.3 Fibre Inflation

III. The Future

D. Inflation as a 4D EFT

0.1 Classical inflationary bkgd

- $\int d^4x \sqrt{-g} \left[\frac{M_{\text{Pl}}^2}{2} R - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right]$

with FRW bkgd : $ds^2 = -dt^2 + a(t)^2 dx^2$
homogeneous, isotropic
expanding universe

comoving coords
- comoving distances
are const. as Universe
expands, w/ physical
distances $\propto a(t)$

and e.o.m. : $3H^2 M_{\text{Pl}}^2 H^2 = \frac{1}{2} \dot{\phi}^2 + V(\phi)$

$$\ddot{\phi} + 3H\dot{\phi} = -V'(\phi)$$

[HJ]

w/ $H \equiv \frac{\dot{a}}{a}$ Hubble parameter
 a Hubble radius H^{-1}

- Inflation = accelerated expansion $\ddot{a} > 0 \Rightarrow$

$$(\alpha H)^{-1} = (\dot{a})^{-1} \downarrow \text{shrinking comoving Hubble radius}$$

comoving distance over particles
can communicate in a given
time H^{-1}

$$\Rightarrow E \equiv -\frac{\dot{H}}{H^2} = \frac{1/2 \dot{\phi}^2}{M_{Pl}^2 H^2} \simeq \frac{3/2 \dot{\phi}^2}{V(\phi)} \ll 1$$

↑ ↗ P.E. >> K.E.

slowly varying bubble parameter

Geometry is quasi-4D: for $H \approx \text{const}$ $ds^2 = -dt^2 + e^{2Ht} dx^2$

- Energy scale of inflation $H_{\text{inf}} \sim V''^{1/4} \sim \sqrt{M_{Pl} H}$

- Amount of inflation: $N_* \equiv \int_{a_*}^{a_f} da/a$

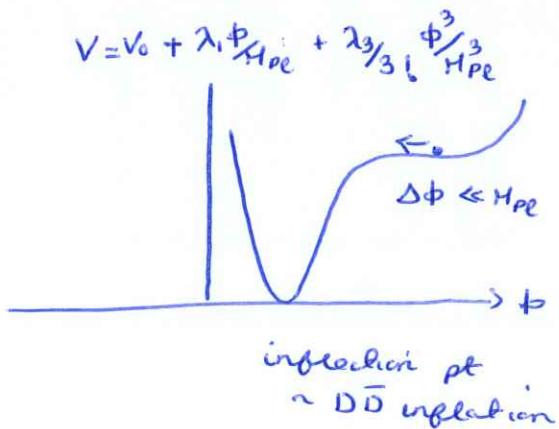
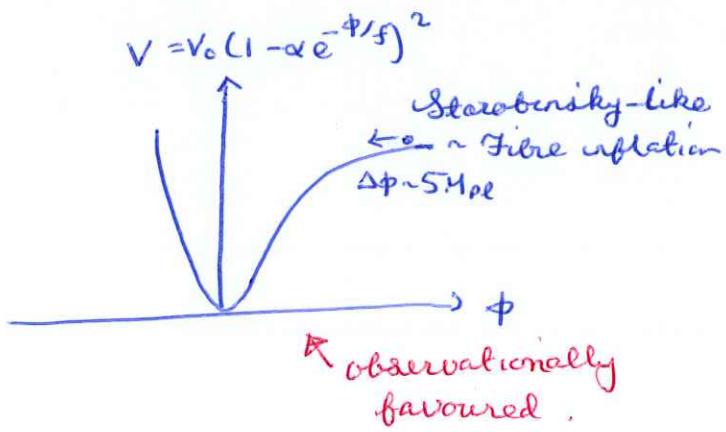
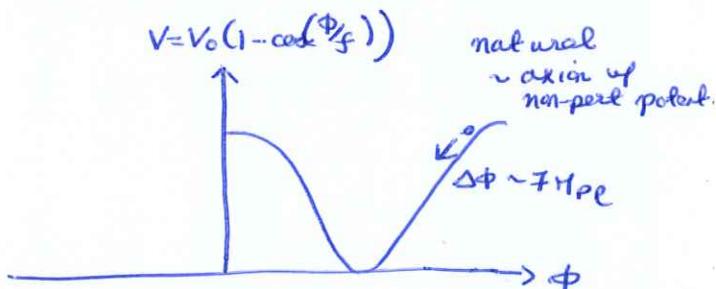
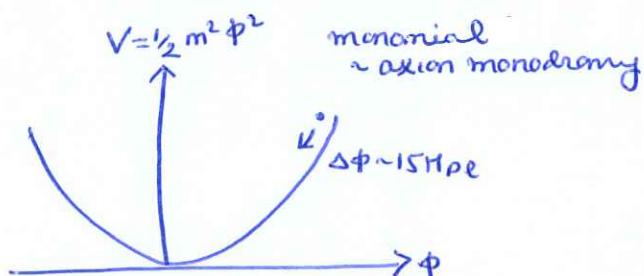
- Slow roll inflation when $\Delta\phi \ll M_{Pl}$

NOTE: other possibilities exist!

can use t , ϕ or N as a clock to measure progress of inflation.

$$E \equiv \frac{M_{Pl}^2}{2} \left(\frac{V'}{V} \right)^2 \ll 1$$

$$|\eta| \equiv M_{Pl} \frac{|V''|}{V} \ll 1$$



0.2 Quantum Perturbations \rightarrow CMB Observables

Remarkable - can relate quantum physics in first moments of Universe to observations today.

- So far classical evolution
- Treat inflationary system quantum mechanically
 - primordial density perturbation
 - CMB temperature anisotropies and LSS!

quantum fluctuations in inflaton \rightarrow slightly different $V(\phi)$ and Hinf in different regions of space \rightarrow different densities

- For single field, slow roll models, two physical modes of fluctuation:

$$\delta g_{ij} = a^2 2 R \delta_{ij} + a^2 h_{ij} \quad (\text{comoving gauge } \delta T_{0i} = 0)$$

↑
"scalar"
"curvature"/"adiabatic"
perturbation & fluctuation in $\delta g_{\mu\nu} = 4/3 \frac{\delta \rho_m}{\rho_m}$ and $\delta P = c_s^2 \delta \rho$
 $\hookrightarrow \delta \rho \propto \delta P = c_s^2 \delta \rho$ energy density speed of sound

(Note: in multifield inflation, additionally have "isocurvature" / "entropy" perturbation)

fluctuation in relative # density
 $\delta g_{\mu\nu} = -\delta \rho_m$

Fluctuations governed by perturbed Einstein's eqns $\delta g_{\mu\nu} = k \delta T_{\mu\nu}$.

Scalar perturbations $R(t, x)$

- Convertional to Fourier transform into momentum space

$$R(t, x) = \int d^3k [R_k(t) a_k e^{ik \cdot x} + c.c.]$$

- Quantize: \hat{R}_k promote Fourier mode of classical field to quantum op.

$R(t, x)$ a quantum field in classical inflationary bkgd spacetime

How fields evolve depends on whether its scale is bigger or smaller than the Hubble radius.



- Compute "2-pt fn" or "primordial power spectrum"

$$\langle 0 | \hat{R}_k \hat{R}_{k'} | 0 \rangle = P_R S(k+k')$$

directly relate correlation func at horizon exit to observables at late times

Result: $\Delta_R^2(k) = \frac{k^3}{2\pi^2} P_R = \frac{1}{8\pi^2} \frac{H^2}{e} \quad |_{k=aH}$

For dS space $\Delta_R^2(k) = \text{const.}$, slowly varying H

evaluated at h. exit

\Rightarrow almost scale-invariant power spectrum: \rightarrow scale dependence of power spectrum

parameterise scale dependence of \approx power law $\Delta_R^2(k) = A_S \left(\frac{k}{k_*}\right)^{n_S - 1 + \frac{1}{2}\alpha_S \ln(k/k_*)} + \dots$

chosen ref scale

"pivot scale"
among scales probed by CMB

time that this scale exited the horizon depends on whole history of universe
 $N_* = 50-60$ depending on preheating

$$(\xi = H_{\text{pre}} \frac{V'''V}{V^2})$$

where

$$n_S = 1 - 6\epsilon + 2\eta$$

$$\alpha_S = -24\epsilon^2 + 16\epsilon\eta - 2\xi$$

For single field, slow roll inflation, fluctuations are Gaussian - fully characterised by 2-pt fn

phases of
inflation
associated with scalar field
are independent
and random

With non-trivial interactions or derivative terms, higher order n-pt fn contain new info:

Non-Gaussianities are measured by "3-pt fn" "bispectrum"

$$\langle 2 | \hat{R}_k \hat{R}_{k_2} \hat{R}_{k_3} | 2 \rangle = (2\pi)^3 B_R(k_1, k_2, k_3) S(k_1 + k_2 + k_3)$$

↑
vacuum of interacting theory

momentum dependence
 \Rightarrow amount of NPs
associated w/ Δ 's
of different shapes.

↑
3-momentum vectors
form closed Δ

Different models give different templates which can be searched for in data.

Useful measure is $f_{NL} = 5/18 \frac{B_R(k, k, k)}{P_R^2(k)}$ ↪ a number.

EFT Case Study:

Starobinsky Inflation

$$V = \frac{H_{\text{Pl}}^4}{4\alpha} \left(1 - e^{-\sqrt{3}\beta \frac{\phi}{H_{\text{Pl}}}} \right)^2$$

$$N_* = \int_{a_{\text{end}}}^{a_0} da n_a = \int_{\phi_{\text{end}}}^{\phi_*} \frac{d\phi}{\sqrt{2\epsilon}}$$

$$\Rightarrow \eta = H_{\text{Pl}}^2 \frac{V''}{V} = -\frac{4}{3} e^{-\sqrt{3}\beta \frac{\phi}{H_{\text{Pl}}}} \quad \epsilon = \frac{H_{\text{Pl}}^2}{2} \left(\frac{V'}{V} \right)^2 = \frac{3}{4} \eta^2$$

$$N_* = -1/\eta$$

$$\Rightarrow n_s - 1 = -6\epsilon + 2\eta \approx 2\eta = -2/N_*$$

$$N_* = 50 \Rightarrow n_s = 0.96$$

$$N_* = 60 \Rightarrow n_s = 0.97$$

$$r = 16\epsilon \approx 12\eta^2 = 12/N_*^2$$

$$N_* = 50 \Rightarrow r = 0.005$$

$$N_* = 60 \Rightarrow r = 0.003$$

$$\Delta\phi \sim 5 H_{\text{Pl}}$$

$$M_{\text{inf}} \sim 10^{16} \text{ GeV}$$

α_S negligible

N_{G_A} negligible

Tensor perturbations - can be computed in analogous way

$$\Delta_R^2(k) = \frac{2}{\pi^2} \frac{H^2}{M_{\text{Pl}}^2} \Big|_{k=aH}$$

\Rightarrow tensor-to-scalar ratio

$$r \equiv \frac{\Delta_R^2}{\Delta_s^2} = 16 \epsilon$$

and $\frac{H}{M_{\text{Pl}}} = \pi \Delta_R(k_*) \sqrt{\frac{r}{2}}$
 $k_* \text{ measured } \sim 4.7 \times 10^{-5}$

$$\Rightarrow M_{\text{inf}} \equiv V_{\text{inf}}^{1/4} \simeq (3 H^2 M_{\text{Pl}}^2)^{1/4} = 1.8 \times 10^{16} \text{ GeV} \left(\frac{r}{0.1}\right)^{1/4}$$

measuring r
would fix M_{inf} !

Moreover:

$$N_e = \int \frac{da}{a} = \int \frac{\dot{a}}{a} dt = \int \frac{H}{\dot{\phi}} d\phi = \int \frac{H M_{\text{Pl}}}{\dot{\phi}} \frac{d\phi}{M_{\text{Pl}}} \simeq \sqrt{8} r^{1/2} \frac{\Delta \phi}{M_{\text{Pl}}}$$

$$\Rightarrow \frac{\Delta \phi}{M_{\text{Pl}}} \gtrsim G \times \left(\frac{r}{0.1}\right)^{1/2}$$

Current bound $r \lesssim 0.07 \Rightarrow$ observable tensors require super-Planckian field range and $M_{\text{inf}} \sim M_{\text{GUT}}$!

Summary

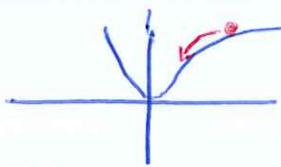
- * $\phi \propto V_{SR}(t)$ gives $\ddot{a} > 0$ and solves horizon flatness and monopole puzzle
- * Quantum fluctuations need CMB temp fluctuations & LSS

IN EXCELLENT AGREEMENT w/ EXPT

! = bounds are getting tight - should be seen soon if model is correct.

Physical models Observables	Single field slow roll	Single field w/ features (non-slow roll)	Single field w/ non-canonical kinetic terms	Multifield
Scalar power spec. $N_S \approx 0.968 \pm 0.006$ $dS \approx 0$	✓	!	✓	!
entropic & adiabatic perts $I \ll R$	✓	✓	✓	!
gravity waves $r \lesssim 0.07$	✓	✓	✓	✓
Non-gaussianities ~ 0	✓	!	!	!

O. Inflation in 4D EFT - Brief version



$$\int d^4x \sqrt{-g} \left[\frac{M_{Pl}^2}{2} R - \frac{1}{2} (\partial\phi)^2 - V(\phi) \right]^2 \quad \text{eq. Starobinsky potential}$$

give mass dims → in FRW bkgd: $ds^2 = -dt^2 + a(t)^2 dx^2$ (100) (101)

$$3H_{Pl}^2 H^2 = \frac{1}{2} \dot{\phi}^2 + V(\phi) \quad 1+ \equiv \frac{\dot{a}}{a}$$

$$\ddot{\phi} + 3H\dot{\phi} = -V'(\phi)$$

arising scales
distances are
const as Universe
expands, if
physical distance
× alt)

Inflation $\ddot{a} > 0$, $(aH)^{-1} = (\dot{a})^{-1} \downarrow$ slowly varying
accelerated expansion \downarrow shrinking comoving radius
 $\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{k\dot{\phi}^2}{M_{Pl}^2 H^2} < 1$ other possibilities

$$\epsilon \equiv \frac{M_{Pl}^2}{2} \left(\frac{V'}{V} \right)^2 = 3/4\eta^2 \ll 1 \quad \text{and}$$

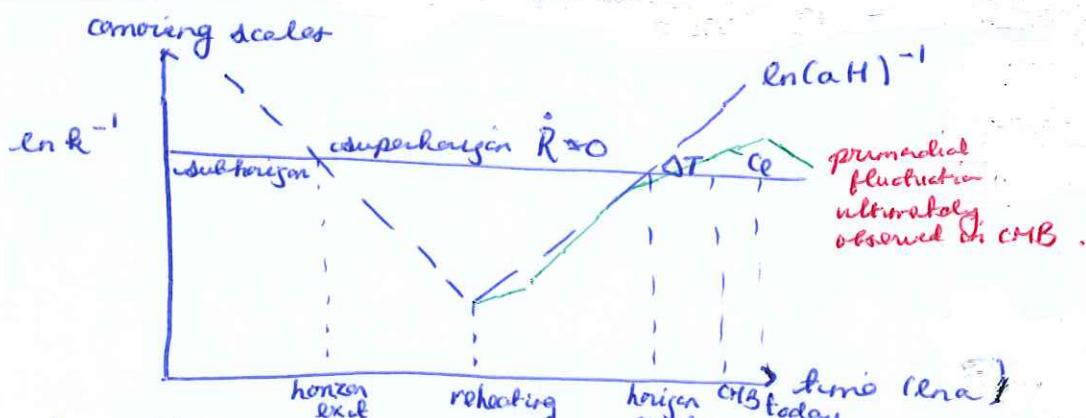
Inflation ends when
 $\epsilon \sim 1$

$$\eta = M_{Pl}^2 \frac{V''}{V} = -\frac{4}{3} e^{-\sqrt{Y_3} \phi / M_{Pl}} \ll 1$$

classical bkgd - explains large scale anisotropy

big
sites of
inflation
scale anisotropies
inflationary fluctuations
different regions of space-time, different amounts of inflat.
single field, slow-roll inflation - $\delta g_{ij} = a^2(t) R(x,t) \eta_{ij} + a^2(t) h_{ij}$ scalar "adiabatic"
in CMB. LSS "curvature" pert.

$$R(x,t) = \int d^3k R(k) a_k e^{ikx} + c.c. \rightarrow \text{quantize } R_k$$



Given $\phi, V(\phi) \rightarrow$ compute power spectrum (and higher n-correlation functions)
of fluctuations at time they exited horizon and compare to CMB observations.

Generic Inflation \Rightarrow almost scale invariant power spectrum:

$$\Delta_R^2(k) = A_s \left(\frac{k}{k_*} \right)^{n_s - 1 + \frac{1}{2} \alpha_s \ln(k/k_*)} + \dots$$

$$n_s = 1 - 6\epsilon + 2\eta$$

$$\alpha_s = -24\epsilon^2 + 16\epsilon\eta - 2\zeta$$

$$r \equiv \frac{\Delta_h^2(k)}{\Delta_R^2(k)} = 16\epsilon$$

probed by CMB among these

$$M_{Pl} = 1.8 \times 10^{16} \text{ GeV} \quad \left(\frac{r}{0.1} \right)^{1/4}$$

$$\frac{\Delta\phi}{M_{Pl}} \gg 6 \times \left(\frac{r}{0.1} \right)^{1/2} \approx$$

Time scales observed in the CMB today depends on
whole history of Universe, in particular including reheating
50~60 efolds before end of inflation.

$$N_* \equiv \int_{\text{end}}^{\phi_*} d\phi = \int_{\phi_{\text{end}}}^{\phi_*} \frac{d\phi}{\sqrt{2E}}$$

epoch
CMB probes around 10 efolds
50-60 efolds before end
of inflation.

*amount of time or
use $\dot{\phi}$ & $d\phi$ to
get ϕ vs. time*

" $\int \frac{\dot{\phi}}{\dot{a}} dt = \int \frac{H}{\dot{\phi}} d\phi$ "

e.g. for Starobinsky inflation

$$N_* = -\gamma_\eta$$

$$n_s - 1 = 2/N_*$$

$$r \propto 1/N_*^2$$

$$N_* = 50 \quad n_s = 0.96, \quad r = 0.005$$

$$N_* = 60 \quad n_s = 0.97, \quad r = 0.003$$

α_s negligible

$N G \alpha_s$ negligible

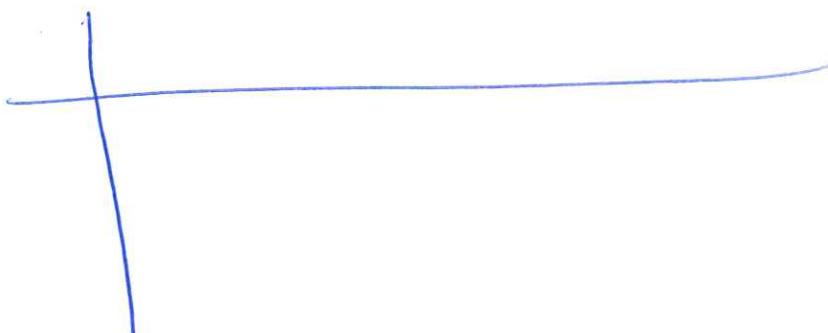
$$\Delta\phi \sim 5 \text{ Mpc}$$

$$M_{\text{inf}} \sim 10^{16} \text{ GeV}$$

large field inflation

high scale inflation

Summary of Constants



I. Inflation from String Theory - Overview

I.1 Why?

Cosmology described v. well by 4D LEEFT

as high energy theory we know
this is only the low energy
limit of something more
fundamental - gravity is not
renormalizable

BUT slow roll potential is v. sensitive to corrections from high energy dofs:

$$\mathcal{L}_{\text{eff}}[\phi] = \mathcal{L}_L[\phi] + \sum_i c_i \frac{G_i[\phi]}{\Lambda^{5i-4}}$$

light fields
 $m_e < \Lambda$

Planck suppressed higher
dim ops due to heavy
fields easily spoil
SR inflation.

Radiative corrections generate all terms allowed by symmetries!

$$\eta\text{-problem: } \eta = H_{\text{pl}}^2 \frac{|V''|}{V} \sim \frac{m_\phi^2}{H^2} > 1 \quad \text{unless there is a symmetry protecting } m_\phi$$

c.b. Higgs hierarchy problem $\rightarrow m_\phi^2 \sim \Lambda_{\text{uv}}^2 \gg H^2$

Is inflation robust against quantum gravity corrections?

Note: Inflaton has a shift symmetry $\phi \rightarrow \phi + \text{const}$ broken mildly by almost flat potential \Rightarrow potential is radiatively stable against loop corrections "technically natural" naively. BUT how do high energy dofs contribute to $V(\phi)$? be these protected by any symmetry?

String theory - a well-developed, precise framework to answer this question.

Opportunities - connect quantum gravity to expt!

- * What is inflaton ϕ and how is $V(\phi)$ protected?
have also
seen other
string inspired
mechanisms of inflation \rightarrow new patterns in data
to search for
- * Falsifiable models from string theory cf. QFT,

II.2 Method

- Ideally:

$$\frac{1}{\ell_S} \ll M_S \ll M_{Pl}$$

10d string theory

$$M_{Pl} \\ M_{KK} \\ M_{mod} \\ H_{inf}$$



choose topological data describing compactification
(compact mfd, fluxes, localised sources
→ quantum effects)
pert & non-pert.

4d LEEFT for
light dofs
w/ interactions
- including SM & ϕ
and $V(\phi)$

modulus
ideally just gravity & ϕ in LEEFT,
or multidim potential w/ an
effectively a single field through
the potential



choose inflationary isoln
(initial condns)

Cosmological parameters
to test against observations

Many candidate inflatons - module - scalar field w/ classically
flat scalar potential

- * open string modulus - D-brane radial or angular position
(axion)
- * closed string modulus - Kähler modulus or axion',

- Coefficients in 4D LEEFT uncomputable (depend on unknown
details of compactification)

⇒ a given string scenario gives at best a class of
inflationary lagrangians w/ symmetries and interactions
fixed but couplings unknown. \rightarrow is there a
symmetry that
predicts $V(\phi)$?

- To compute LEEFT action we use several approximations:

- * weak curvature
- * d'/L^2 expansion \rightarrow leading terms allow pert. $(d'/L^2)^n$ non-perturbative: $e^{-\frac{d'}{L}}$
- * weakly interacting
- * GS expansion
- * probe brane approxⁿ often neglect gravitational backreaction
of localized sources
- * truncation of heavy modes for 4D EFT at low energies

Must check they are under control!
especially important as
we often use subleading corrections
to these approx's for mod. stab
and inflation.
Can be computed explicitly in string theory

- Rely on mass hierarchies

$$M_{\text{heavy}} < M_{\text{inf}} < M_{\text{ek}} < M_S < M_{\text{Pl}}$$

model dependent
 unrelated - inflation SSB
 to inflation \Rightarrow partial protection
 against radiative corrections.

Hard especially for large field, high scale models!
 w/ observable pgrav!

$$n = 3.1 \times 10^8 \left(\frac{M_{\text{inf}}}{M_{\text{Pl}}} \right)^4$$

$$= 3.1 \times 10^8 \left(\frac{M_{\text{inf}}}{M_{\text{ek}}} \right)^4 \left(\frac{M_{\text{ek}}}{M_S} \right)^4 \left(\frac{M_S}{M_{\text{Pl}}} \right)^4$$

$$\text{use } \frac{1}{2k_{10}^2} \int d^{10}x \sqrt{-g_{10}} R_{10} \rightarrow \frac{M_{\text{Pl}}^2}{2} \int d^4x \sqrt{-g_4} R_4$$

$$\Rightarrow M_{\text{Pl}}^2 = \frac{V_6}{K_{10}^2} \propto \frac{1}{2(2\pi)^7 (\alpha')^4}$$

$$\text{assume } V_6 = (2\pi L)^6$$

$$M_{\text{Pl}}^2 = 1/L$$

$$\text{also } \alpha' = l_S^2 = \frac{1}{M_S^2}$$

$$= 1.6 \times 10^4 \left(\frac{M_{\text{inf}}}{M_{\text{ek}}} \right)^4 \left(\frac{M_{\text{Pl}}}{M_S} \right)^{16}$$

$$\Rightarrow \text{for } M_{\text{inf}} \lesssim 0.3 M_{\text{Pl}} \lesssim 0.3 M_S$$

$$n \lesssim 5.5 \times 10^{-7}$$

$$M_{\text{inf}} \lesssim 0.45 M_{\text{Pl}} \lesssim 0.45 M_S$$

$$n \lesssim 0.002$$

\Rightarrow large field inflation & observable pgrav at limited of validity of EFT,

I.3. Challenges

Inflaton is a modulus

Module stabilisation - moduli and inflaton dynamics cannot be decoupled.

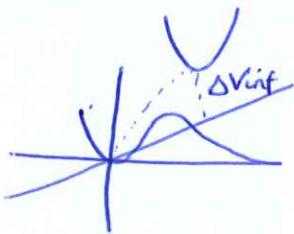
- Inflaton potential must be around metastable at vacuum other moduli must be stabilized!

NOGOs \Rightarrow achieved by balancing delicately many sources

- * p-form fluxes
- * D-branes / O-planes
- * pert / non-pert. quantum corrections

* There must be no steep runaway directions
nor multiple flat directions

* Module must not be destabilized during inflation



$$V_{\text{uplift}} = \frac{C}{\text{vol}^n} + \frac{V(\phi)}{\text{vol}^n} \quad n=2,3$$

$V_{\text{barrier}} \sim M_p^2 H^2$ $H \lesssim m_{3/2}$ w/ KLT for $m_{3/2} \sim \text{TeV}$
 $V_{\text{barrier}} \sim m_{3/2}^3 M_p^2$ $H \lesssim m_{3/2}^{3/2}$ for $m_{3/2} \sim \text{TeV}$ $H \ll \text{TeV}$

volume modulus coupled to all sources of energy

(Weyl rescaling always present when deriving EoM)

- η -problem hard to keep inflaton light $m_\phi < H$

When $M_{\text{SUSY}} < M_{\text{inf}} < M_{\text{pl}}$, susy description of K, W, f_α

$$\mathcal{L}_{\text{susy}} = -K \epsilon \bar{\epsilon} \partial_\mu \phi \partial^\mu \bar{\phi} - e^{\frac{K \bar{\epsilon} \bar{\epsilon}}{M_{\text{pl}}^2}} [K \epsilon \bar{\epsilon} D_\mu W \overline{D^\mu W} - \frac{3|W|^2}{M_{\text{pl}}^2}]$$

assume inflaton lies in complex scalar in neutral chiral supermultiplet

Expand around $\epsilon = 0$: $K = K(0) + K \epsilon \bar{\epsilon} (0) \epsilon \bar{\epsilon} + \dots$

$$\Rightarrow \mathcal{L}_{\text{susy}} \approx -\partial_\mu \phi \partial^\mu \bar{\phi} - V(0) \left(1 + \frac{\phi \bar{\phi}}{M_{\text{pl}}^2} + \dots \right)$$

note, this cannot give chaotic inflation as $\epsilon \approx 0$

where $\phi \bar{\phi} = K \epsilon \bar{\epsilon}(0) \epsilon \bar{\epsilon}$ canonically normalized ϕ

$$\Rightarrow m_\phi^2 = \frac{V(0)}{M_{\text{pl}}^2} + \dots = 3H^2 + \dots \Rightarrow \eta \approx 1$$

So a generic $W(\phi)$ has $m_\phi^2 = \mathcal{O}(H^2)$

very difficult to avoid mod slab as most schemes for mod slab involve $V_F \gg V_D$

understand in explicit string models
 can be computed explicitly in string theory

* Non-generic $W(\phi)$ eg. $K = \phi \bar{\phi}$, $W = \phi \bar{\phi}$ $\Rightarrow V = e^{\phi \bar{\phi}} \phi \bar{\phi} ((1+\phi \bar{\phi})^2 - 3\phi \bar{\phi}) = 1 + h_2(\phi \bar{\phi}) + \dots$

* Shift symmetry $\phi + \bar{\phi} \rightarrow \phi + \bar{\phi} + \text{const}$ $\Rightarrow K = (\phi - \bar{\phi})^2$, W indep. of ϕ protects $\text{Re } \phi$ only from $\mathcal{O}(1)$

- * Multi-field dynamics if ϕ is light, why are other moduli X not light? Only field w/ $m_X \lesssim H$ are classically and quantum mechanically active during inflation \therefore affect
 - * bkgd inflationary trajectory
 - * perts and observables - "isocurvature" perts & NGTs. \leftarrow so far not observed.

- * Backreaction of heavy moduli even when $m_X \gg H$, time dependent inflationary energy can induce evolution of moduli which can then change form of inflaton potential

eg. "flattening effect" [Song, Han, Silverstein, Westphal '10]

$$V(\phi, X) = -\frac{1}{2} \phi^2 X^2 - \frac{1}{2} M_X^2 (X-m)^2$$

↑
inflaton ↑
heavy field

\downarrow
 $M_X > H$

For $\phi=0$, modulus fixed
at $X=m$
 $\Rightarrow \frac{1}{2} m^2 \phi^2$ inflaton mass m

BUT for ϕ large, X

Integrate out X , assuming it adiabatically shifts away from min and alters $V_{\text{eff}}(\phi)$
follows its min:

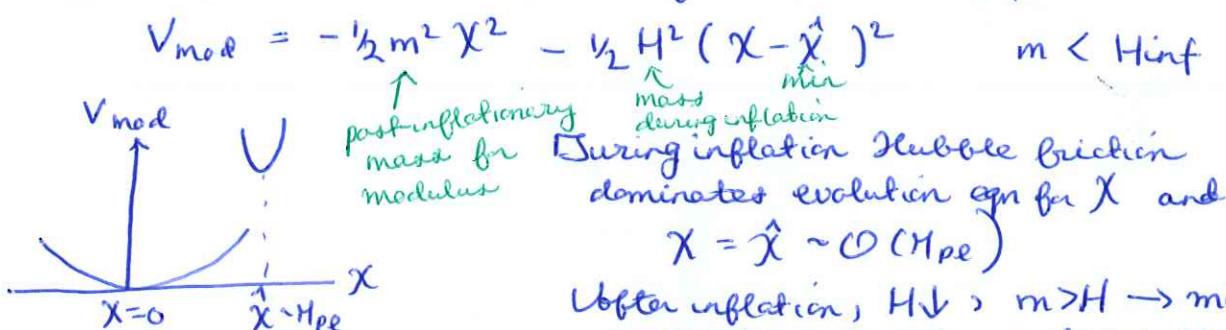
$$\partial_X V = 0 \quad \leftarrow \text{solve for } X \text{ and plug } \langle X \rangle \text{ into } V(\phi, X)$$

$$\Rightarrow V(\phi, X(\phi)) = -M_X^2 m^2 \frac{\frac{1}{2} \phi^2}{\frac{1}{2} \phi^2 + M_X^2}$$

flat for large ϕ !

- * Reheating - inflation must end and energy transferred to SM fields to initiate BB; avoid overproduction of relics from other sectors - moduli, KK modes, excited strings, gravitons, axions, hidden matter, hidden radiation.

eg. "Cosmological moduli problem" [Bie, Fisher, Rennacharya '84]
as inflaton evolves, min for module shifts



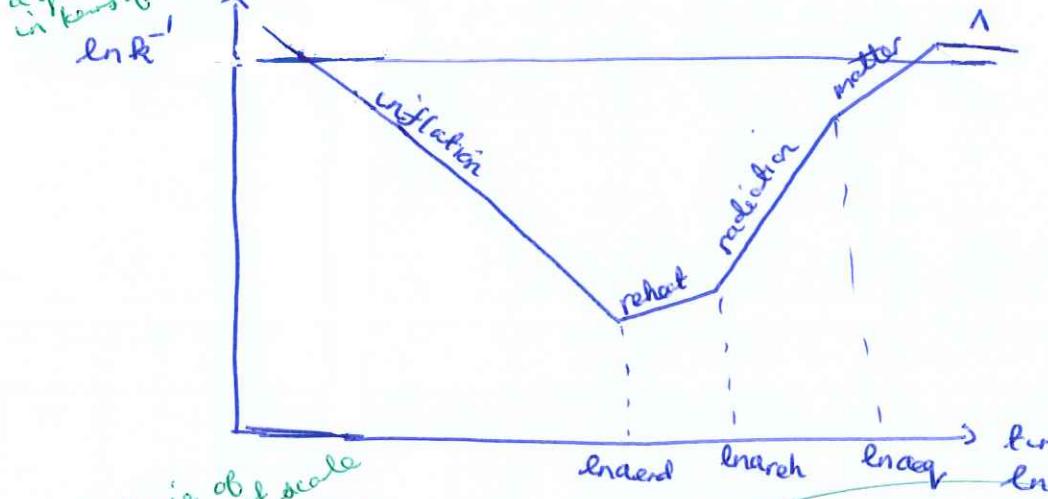
After inflation, $H \downarrow \rightarrow m > H \rightarrow$ module rolls down to min. and oscillates there \rightarrow particle production

- * Epoch of module domination - Module must decay before BBN
 $\Rightarrow m_x > 10 \text{ TeV}$

between time that CMB
is scales exit horizon and
end of inflation

- * Observables depend on N_e , N_e depends on history of

express nsids, if for
a given model Universe, including reheating.



$$n^{(1)} = \frac{a_{\text{end}}}{a_k} = \frac{\partial \ln H_0}{\partial \ln H_k} \frac{\partial \ln H_{\text{eq}}}{\partial \ln H_0} \frac{\partial \ln a_{\text{eq}}}{\partial \ln a_{\text{end}}} \frac{\partial \ln a_{\text{end}}}{\partial \ln a_{\text{rel}}} \frac{\partial \ln H_k}{\partial \ln a_{\text{eq}}}$$

Known from CMB
known - standard thermal history of Universe

Uncertainty in $\frac{a_{\text{end}}}{a_{\text{rel}}}$ and $H_k \leftarrow$ energy scale of inflation?

$$\Rightarrow N_* \approx 55 \pm 5$$

CMP \Rightarrow after inflationary reheating there is additional modulus dominated epoch and second reheating. Batra & Maharana '14

$$\Rightarrow N_* = (55 - \frac{1}{4} N_{\text{mod}}) \pm 5$$

\downarrow # of e-folds of modulus domination

$$\Rightarrow \text{alter ns at } 1\% \text{ level}$$

\downarrow lightest modulus - last to decay

given accuracy of CMB expt
this is important!

Summary

- * Use string theory to understand robustness of inflation against QG effects
- * Method - identify compactifications of 4DLEFT of interesting cosmologies - what is inflaton, what is its potential, what protects its potential?
- * Must understand dynamics of moduli - during & after inflation

Recap

Lecture II

0. Inflation: $(aH)^{-1} = (\dot{a})^{-1} \downarrow$ if $\epsilon \equiv \frac{H_P^2}{2} \left(\frac{v'}{v} \right)^2 \ll 1$

$$\eta \equiv \frac{H_P^2}{2} \frac{v''}{v} \ll 1$$

Classical evolution \Rightarrow large scale history of Universe,

Quantum fluctuations \Rightarrow small scale anisotropies in CMB.

Power spectrum of quantum fluctuations: $n_S, \eta_S \equiv \frac{dn_S}{dk} , \eta$
test against CMB observations

1. Embed in string theory: ϕ is a modulus -

- can compute $V(\phi)$

- can understand suppression of quantum corrections to $V(\phi)$

Ps $|\eta| \ll 1$ after quantum gravity corrections

4D scalar scalar field w/ leading order flat potential

- various sources fluxes & quantum corrections give a potential to the

moduli module

2. ϕ couples to other moduli, X_i - understand

- are other moduli stable during inflation and after inflation.

- if $m_X < H$ - multi-field

- if $m_X > H$ - cannot just truncate them

must compute $V(\phi, X_i(\phi))$

Reheating

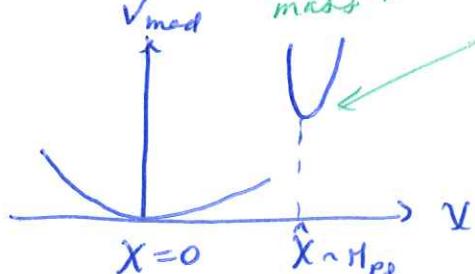
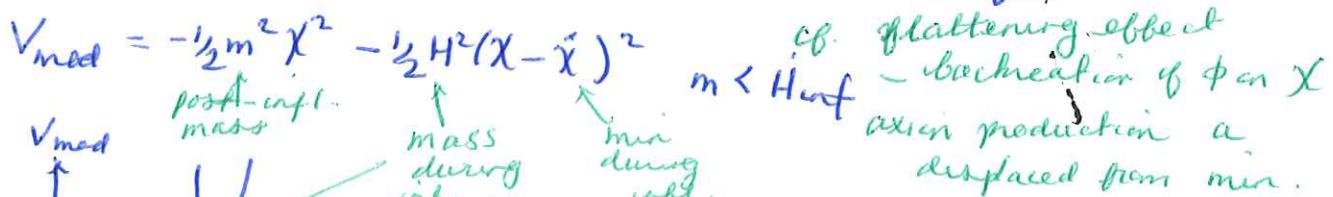
Inflation must end and energy transferred to

SIM dofs to initiate standard Big Bang cosmology.

- avoid overproduction of relics from other sectors which would overclose Universe. modules, KK modes, strings, gravitons, axions, hidden sectors.

e.g. Cosmological module problem Baez, Fischer, Nemeschansky '84

as inflaton evolves, min for module shift



after inflation $H \downarrow$, $m > H$

- \rightarrow modulus rolls down to min at $X=0$ and oscillates
- \rightarrow moduli particle production

and era of module domination.

\Rightarrow lowers the # of e-folds before end of inflation that produces observed in the CMB excited horizon $\Rightarrow n_S \approx 0.01$

II. Inflation from String Theory - Case Studies

II.1 D3 Inflation

Dvali & Tye '98

KKLT '03

Baumann, Dymarsky, Klebanov, Matekalo '07

Setup: IIB KKLT flux compactification w/ "warped throat"

ϕ : ^{open string scalar field in 4D} position modulus of ^{neglect} slowly moving probe D3-brane

$V(\phi)$: Interaction of D3-brane and moduli stabilizing effects

* Prelude: in string th. we have localized dir. spacetime $X^M(0^+, \dots, 0^P)$ higher diml fundamental objects.

$$S_{D3} \approx -T_3 \int d^4\sigma \sqrt{-\det G_{ab}} + \mu_3 \int C_4$$

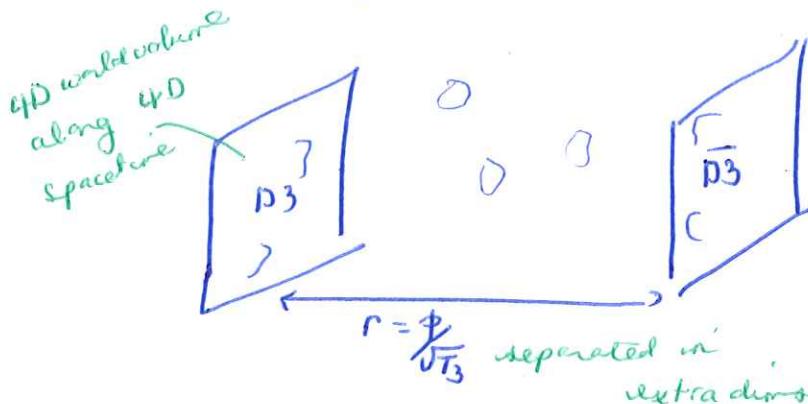
$\left\{ \begin{array}{l} \text{higher diml} \\ \text{analogue of} \\ \text{EM A}_\mu \end{array} \right. \quad \begin{array}{l} \text{electric} \\ \text{charge of} \\ \text{P+ particle} \end{array}$

$$G_{ab} = \frac{\partial X^M}{\partial \sigma^a} \frac{\partial X^N}{\partial \sigma^b} g_{MN} \quad q \int A_\mu d\sigma^M$$

$$T_3 = \frac{1}{(2\pi)^3 \alpha'^2} = \mu_3$$

$$\frac{1}{T_3} \propto \alpha'^2 \propto -\mu_3$$

Take D3 + $\bar{D3}$ in flat space. $Mink_4 \times T^6$



$$r = \frac{C M^{1/2}}{C M^{1/2} \phi / \sqrt{T_3}}$$

Coulomb force due to exchange of gravitons, dilaton and C_4

$$V(\phi) = 2T_3 \left(1 - \frac{1}{2\pi^3} \frac{T_3^3}{H_S^8 \phi^4} \right)$$

$\phi = \sqrt{T_3} r$

ϕ

when $r \ll$
tachyon produced
 $\propto D3 \times \bar{D3}$ amplitude $\propto 1/r^2$

? η ?

$$S_{10D} = \int d^4x \int d^6y \sqrt{g_{10}} \left[\frac{M_p^8}{2} R^{(10)} + \dots \right]$$

$$= \int d^4x \sqrt{-g_{4D}} \left[\frac{M_p^2}{2} R^{(4)} + \dots \right]$$

$$\Rightarrow M_{pl}^2 = M_S^8 (2\pi)^6$$

$$\Rightarrow |\eta| \equiv M_{pl}^2 \frac{V''}{V} \simeq 0.3 \left(\frac{2\pi}{r}\right)^6$$

\Rightarrow need $r \gg \pi L$ impossible!

Compactification effects spoil slow roll, although string th. is very rich, it is also highly constrained.

* Bkgd geometry - warped throat

Consider a stack of N D3-branes in 10D Mink.

\hookrightarrow spacetime curved to

$$ds^2 = e^{2A(r)} \eta_{\mu\nu} dx^\mu dx^\nu + e^{-2A(r)} (dr^2 + r^2 dS_5^2)$$

$$e^{-4A(r)} = 1 + L^4/r^4 \quad w/ \underbrace{\frac{4}{\alpha'^2} = 4\pi g_S N}_{\text{cone over } S^5} \quad \overbrace{\text{5-sphere}}$$

$$\Phi = \text{const} \quad x(r) \equiv (c_4)_{+x_1 x_2 x_3} = e^{4A(r)}$$

$$\text{for } r \ll L \quad ds^2 = \underbrace{r^2/L^2 \eta_{\mu\nu} dx^\mu dx^\nu}_{AdS_5} + \underbrace{L^2 dr^2}_{x} + \underbrace{L^2 dS_5^2}_{S^5}$$

Other solns possible:

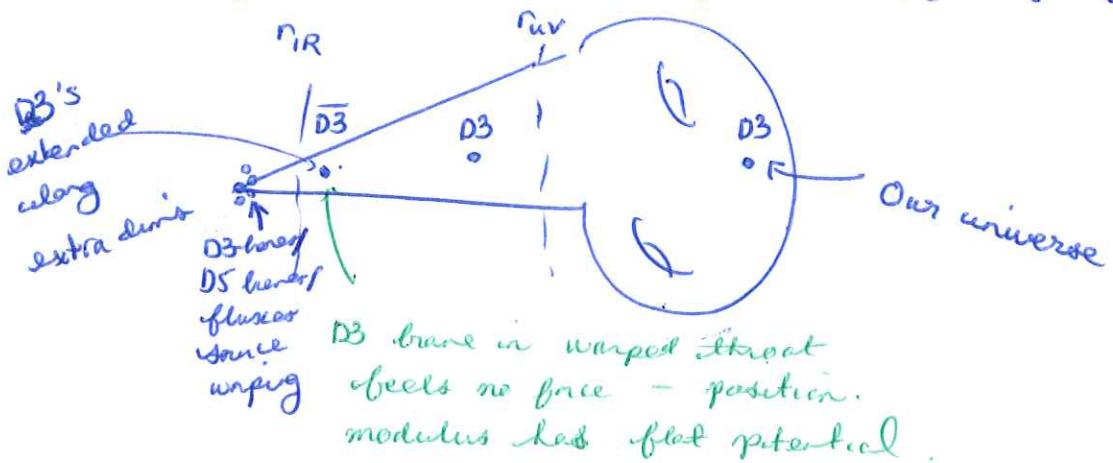
replace S^5 w/ other Einstein mflds ($R_{\mu\nu\rho\sigma} g_{\mu\nu}$)

e.g. T^{11} (topologically $S^2 \times S^3$) \rightarrow warped manifold.

smoothen singularity at $r=0$ \rightarrow warped deformed manifold (KS)

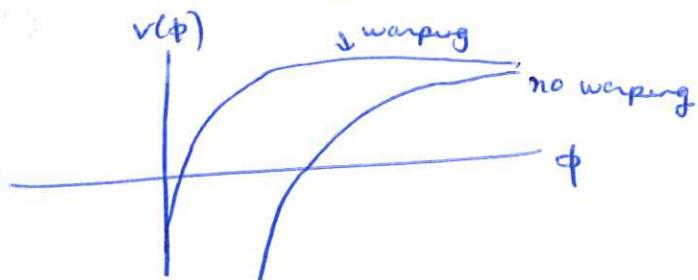
So far 6D internal space is non-compact. (∞ Vol)

need finite volume for finite $H_{pe}^2 = H_5^8 V_6$



- $D\bar{3}$ -brane minimizes energy near tip at IR cutoff $r_b = r_{IR}$
- Coulomb force between brane and anti-brane: ∇ neglect angular motion

$$V = \frac{4\pi^2 \phi_0^4}{N} \left(1 - \gamma_N \frac{\phi_0^4}{\phi^4} \right) \quad \phi_0^2 \equiv T_3 r_b^2$$



Warping \Rightarrow potential if flat even for small ϕ !

BUT Moduli stabilising effects give extra contribution to $V(\phi)$
 see also lecture 1.

Volume modulus coupling -

$$R(p, \bar{p}; e, \bar{e}) = -3 \log(p + \bar{p} - R(e, \bar{e})) \quad \begin{cases} \rightarrow V(p) + D_3 \\ \text{sugra} \\ \text{uplift} \end{cases}$$

$$W(p) = W_0 + A e^{-ap}$$

Assume p stabilized at $p = p_c$ if $e, \bar{e} = 0$ and dS vacuum.

\Rightarrow additional contribution to $V(\phi)$

$$V_{mod}(\phi) = \frac{V_0(p_c)}{(1 - \phi \bar{p}/p_c)^2} \approx V_0(p_c) (1 + 2/3 \phi \bar{p})$$

$$- \Phi^- \Rightarrow m_\Phi^2 = 2V_0 = 2H^2 \Rightarrow \eta^2/3$$

where $\Phi = \sqrt{3} \frac{p - \bar{p}}{p + \bar{p}}$

- * ~~from compactification~~
 ultimately from compactification
Fine-tune other contributions to $V(\phi)$ that can cancel
 K -contribution and allow flat potential :

e.g. NP effects on E D3-branes or D7 branes wrapping 4-cycles in CY w/ $\text{Vol}_+(\phi)$

t ~~quarks subbd~~
 w/ no bdy
 (equivalence if
 they differ by
 bdy)

$$\Rightarrow |W| \sim A e^{-b (\text{Vol}_+(\phi))}$$

Fine-tuning $|y_1| \ll 1$ near $\hat{\phi}$ \leftarrow need to also fine-tune initial zero

$$V(\phi) = V_0 + \frac{\lambda_1}{M_{\text{Pl}}} (\phi - \hat{\phi}) + \frac{1}{3!} \frac{\lambda_3}{M_{\text{Pl}}^3} (\phi - \hat{\phi})^3 + \dots$$

Class of potential w/ coefficients unknown

- try to match to data suitably choosing coefficients
- understand suppression of quantum corrections
 - NR theory for W .
 - large volume, weak coupling expansion is suppressed contribution $\rightarrow K$

at least in principle !

* Observables

$$\eta_S = 1 - \frac{1}{N_{\text{CMB}}} \approx 0.93 \quad (\text{for } N_{\text{TOT}} \gg N_{\text{CMB}})$$

$\eta_S \approx -10^{-3}$

\uparrow out of favour w/ Planck.

Γ bounded by field range : $(\Delta\phi)^2 \leq T_3 r v^2$

$$\Rightarrow \Delta\phi^2 / M_{\text{Pl}}^2 \leq 2/\sqrt{N} \quad M_{\text{Pl}}^2 = \frac{V_0}{E_{10}^2}$$

Summary & compactification effects limit field range and slopes

* Warping factors $D\bar{D}$, vol coupling $\Rightarrow \eta \approx 0(1) \Rightarrow$ fine-tune against other effects.

$$\eta \approx 10^{-7}, M_{\text{int}} \approx 10^{12} \text{ GeV}$$

II.2 DBI Inflation Silverstein & Tong '03

Setup: IIB KKL.T flux compactification up "warped throat" region
 ϕ : Open string position modulus of relativistically moving D3-brane
 $V(\phi)$: Interaction of D3-brane and moduli-stabilizing effects (steep!)

- Start again from DBI action in warped bkgd:

$$\mathcal{L} = -T_3 e^{4A(\phi)} \sqrt{1 + \frac{(\partial\phi)^2}{T_3 e^{4A(\phi)}}} - V(\phi) \quad \phi \equiv \sqrt{T_3} r_b$$

where e.g. $V(\phi) = V_0 - \frac{1}{2} \beta H^2 \phi^2$ steep! from moduli stabilizing effects

Define "Lorentz-factor" $\gamma = \left(1 - \frac{\dot{\phi}^2}{T_3 e^{4A(\phi)}}\right)^{-1/2}$

\Rightarrow maximal speed for probe D3

$$\dot{\phi}^2 \ll T_3 e^{4A(\phi)}$$

$$\phi \cdot \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

- Couple to gravity \Rightarrow const $3M_{Pl}^2 H^2 = (\gamma - 1) T_3 e^{4A(\phi)} + V(\phi)$

$$\dot{\phi}^2 = -2 M_{Pl}^2 H' \frac{H'}{\gamma}$$

- Potential is steep but

$$\epsilon_H \equiv -\frac{\dot{H}}{H^2} = \frac{2M_{Pl}^2}{\gamma} \left(\frac{H'}{H}\right)^2$$

$$\eta_H \equiv \frac{\ddot{\epsilon}_H}{H\epsilon_H} = \frac{2M_{Pl}^2}{\gamma} \left[2\left(\frac{H'}{H}\right)^2 - 2\frac{H''}{H} + \frac{H'}{H} \frac{\gamma'}{\gamma} \right]$$

Hubble slow roll parameters have γ^{-1} suppression!

- Despite large derivatives, quantum corrections negligible due to symmetries in bkgd spacetime!
- Inflation requires P.E \gg R.E $\Rightarrow \frac{V}{\gamma T_3 e^{4A(\phi)}} \gg 1$ \gg ^{log scale} difficult!

Difficult to realise explicitly in string th but interesting as EFT mechanism

- Distinctive signatures:

- * field dependent sound speed $c_s^2 = 1/\gamma^2(\phi)$

- * NG: $f_{NL}^{\text{equil}} = -\frac{35}{108} \gamma^2 \rightarrow$ Planck $\Rightarrow \gamma < 24$

- * $n \sim 10^{-7}$, $M_{inf} \sim 10^{12} \text{GeV}$

whatever gives rise to potential will also react on string bkgd soln.

II.3 Fibre Inflation Cicoli, Burgess, Pueyo '08

Setup: IIB Large Volume Scenario flux compactification

ϕ : Kähler modulus - closed string

$V(\phi)$: Perturbative + non-perturbative corrections

homotopically equivalent to
K3 (bending, stretching, shearing)

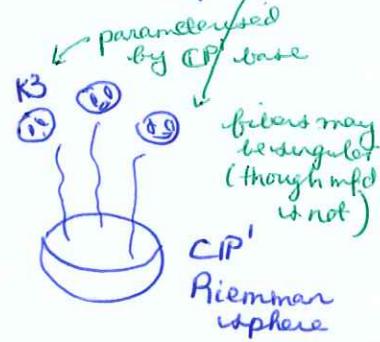
- Setting: K3 fibration: $K3 \rightarrow CY_3 \rightarrow \mathbb{CP}^1$

choose CY_3
if 3 Kähler moduli
and vol given by

$$\mathcal{J} = \alpha (\sqrt{\tau_1} \tau_2 - \lambda_S \tau_S^{3/2})$$

↑ basis chosen
st τ_1 is volume
of K3 fibre

↑ vol of
blow up cycle
(small)



- Kähler potential at leading δ' -correction

$$K = -2 \ln \mathcal{J} - \frac{\phi}{\mathcal{J}}$$

↑ no scale : $\langle V_F \rangle \sim \mathcal{J}^{2/3}$
 $\Rightarrow \mathcal{J}$ unfixed

breaks no-scale and lifts ϕ

Superpotential from fluxes and NP effects:

$$W = W_0 + v_{bs} e^{-as\tau_S}$$

\Rightarrow scalar potential fixes τ_S and \mathcal{J} leaving flat direction in (τ_1, τ_2) plane

$\mathcal{J} = \text{const}$
keeping

- Flat direction lifted by string loop corrections to Kähler potential

$$\delta K_{(gs)} \sim \sqrt{\mathcal{J}} / \mathcal{J} = \delta K_{(q)s} > \delta K_{(d)}$$

but leading gs contribution in V cancels "extended no-scale" structure

$$\rightarrow \delta V_{(gs)} = \frac{W_0^2}{\mathcal{J}^2} \left(a \frac{g_S^2}{\tau_1^2} - b \frac{1}{\sqrt{\mathcal{J}}} + c \frac{g_S^2 \tau_1}{\mathcal{J}^2} \right)$$

$$\text{fixed } \tau_1 \sim g_S^{4/3} \mathcal{J}^{2/3} \quad \text{can be related}$$

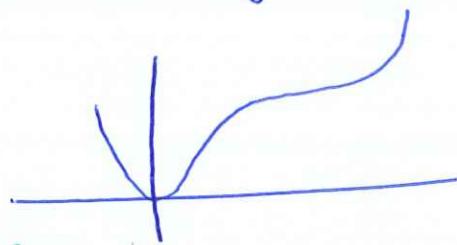
- Suppose τ_S and \mathcal{J} remain fixed at min. while τ_1 is initially displaced: $(d) \mathcal{J}^{-10/3}$

K3 fibre starts long and evolves to small values.

$$\begin{aligned} \rightarrow V(\phi) &= V_0 (1 - 4/3 e^{-\Phi/153} + 1/3 e^{4\Phi/153} + C e^{2\Phi/153}) \\ &\approx V_0 (1 - 4/3 e^{-\Phi/153}) \end{aligned}$$

where $\Phi \equiv \sqrt{3}/2 \ln \tau_1$

Starobinsky / Higgs-like potential



Balancing leading order pert effects against each other

- Higher order loop & α' corrections may be suppressed by additional (possibly fractional) powers of g_s and \mathcal{V}^{-1} and \mathcal{V} is large.

BUT to match w/ amplitude of scalar power spectrum

$$\mathcal{V} \approx 10^2 - 10^3$$

- Symmetry - in ∞ volume limit there are enhanced symmetries - 10D general covariance
 \Rightarrow inflaton has weakly broken non-compact shift symmetry $\phi \rightarrow \phi + \text{const.}$
- Cosmology:

$$V(\phi) \simeq V_0 (1 + \alpha e^{-\phi/f})$$

$$\epsilon = \frac{H_{\text{Pl}}^2}{2} \left(\frac{V'}{V} \right)^2 \simeq \frac{1}{2} \eta^2 f^2 \quad \eta \equiv H_{\text{Pl}}^2 \frac{V''}{V} = -\frac{\alpha}{f^2} e^{-\phi/f}$$

$$\Rightarrow \epsilon \ll \eta$$

$$n_s \simeq 1 + 2\eta$$

$$r = 2f^2/H_{\text{Pl}}^2 (n_s - 1)^2 \simeq 0.005 \text{ for } f = \sqrt{3} \text{ and } n_s = 0.97$$

$$\alpha_s = -\frac{1}{2}(n_s - 1)^2 \simeq 5 \times 10^{-4}$$

$$\Delta\phi \sim 8 H_{\text{Pl}} \text{ and } H_{\text{Pl}} \sim 10^{16} \text{ GeV}$$

$$\Rightarrow \text{Difficult to achieve } H_{\text{Pl}} \ll H_{\text{IR}} \ll H_S \quad \frac{H_{\text{Pl}}}{H_{\text{IR}}} \sim \frac{H_{\text{Pl}}}{H_S} \sim 0.4$$

IV The Future

- CMB & LSS raise important questions
 - Inflationary scenario provides compelling answers BUT
 - Very sensitive to high energy dofs
 - need to embed in consistent th. of quantum gravity!
 - So far - string theory has provided general mechanism's for inflation and its robustness, which can be tested against observations
 - Expt already favours some models over others
 - Expect further improvements in expt and theory!
- * Increasing precision over a wide range of scales:
- LSS surveys
 - $n_S \rightarrow$ Ns, reheating & history of Universe
 - $\Delta_S \rightarrow$ slow roll vs features in potential
 - NGs \rightarrow single field vs. multifield
slow roll vs. derivs/ interactions
 - CMB polarisation (ground based / balloon / space)
 - $n \rightarrow$ large field, high scale vs small field, low scale
 - $n_t = \frac{dH}{H^2} < 0$ and $n_t = -r/8$ for single field
slow roll
 - GWs - inflationary gw's unlikely, but other poss (e.g. cosmic shurgs could be seen)
- * Theoretical questions:
- UV completion of inflation that is well under control and explains flatness of SR potential (or gives another)
 - Effect of moduli on inflationary bgd and observables
 - End of inflation and reheating
 - Initial cndns. — 9-