

Axions

I. What are axions?

These lectures are about axions and why they represent (possibly) the most promising way to connect string compactifications to observational physics.

What are they? I use the word 'axion' for ^{scalar} fields a that have an exact periodicity

$$|a(\theta)\rangle \equiv |a(\theta + 2\pi)\rangle.$$

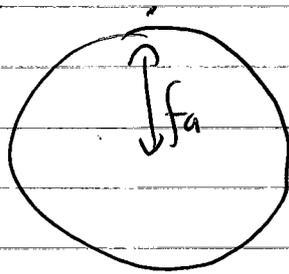
That is, the axion field a is an angular field, and is periodic: the state of the Hilbert space \mathcal{H} with angular vev θ is identical to that with angular vev $\theta + 2\pi$. (* cf axion monodromy, different usage).

As a scalar field has canonical mass dimension 1, we can also write this as

$$|a_0\rangle \quad \text{and} \quad |a_0 + 2\pi f_a\rangle$$

are the same.

The moduli space of the axion field
can then be viewed as a circle



with circumference $2\pi f_a$.

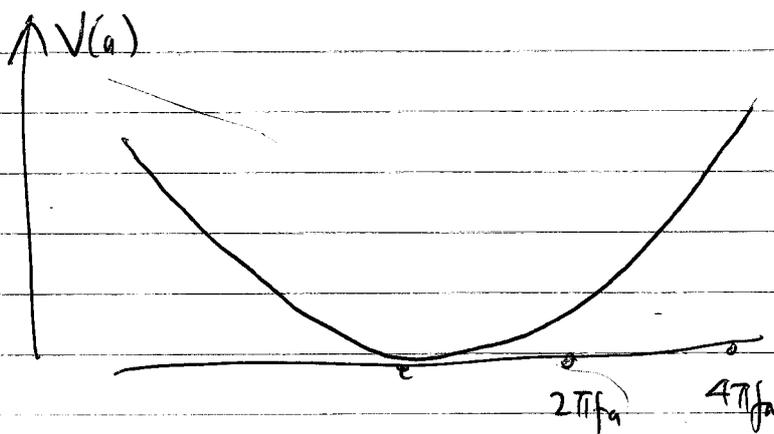
The field range of the axion is then $2\pi f_a$.

f_a is also called the axion decay constant

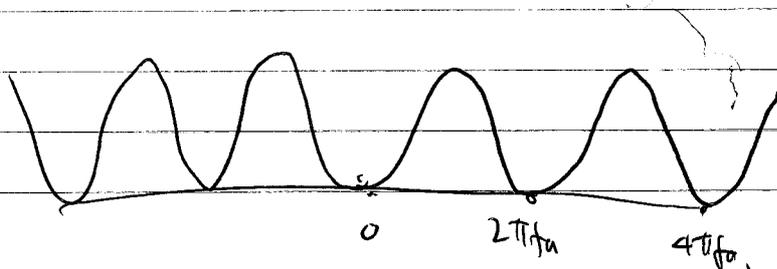
(This language is historical; also definitions may vary
by factors of 2π etc)

The exact symmetry under $a \rightarrow a + 2\pi f_a$

also restricts the form of the potential



not allowed



allowed

As we shall see, this has important consequences for axion masses:

Axions are naturally extremely (exponentially) light. Unlike e.g. the Higgs field, perturbative quantum loops do not tend to renormalise the mass in a quadratically divergent fashion.

Axions

II. Why care about axions?

Why am I giving these lectures?

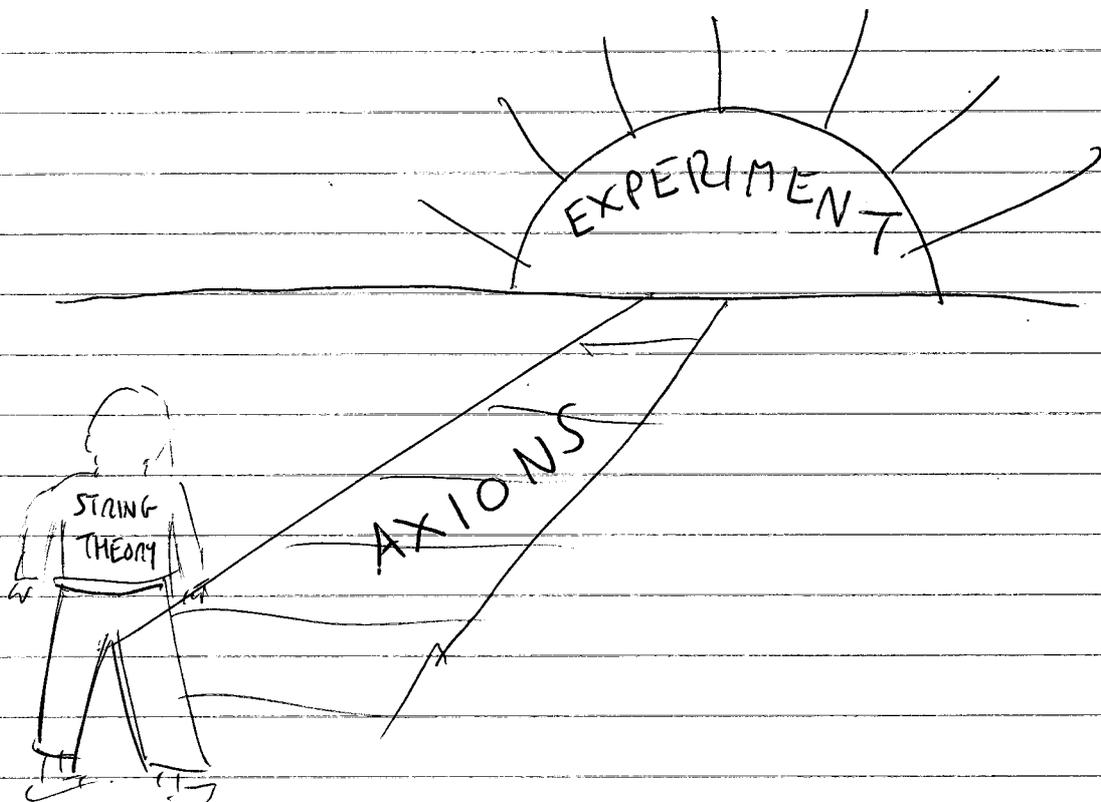
If string theory is to be a theory of physics, it must connect to the world of observations and experiment.

Axions provide one of the best ways to connect Planck-scale physics to observations, and to probe the deep UV through low-energy physics.

Several reasons:

- * Axions are generic consequences of string compactifications: almost all compactifications contain axions in the low-energy spectrum.
- * Searches for axions can probe couplings suppressed by scales $M \gtrsim 10^{16}$ GeV: far higher than those accessible to colliders such as the LHC.
- * The technology is orthogonal to those at the LHC: if the LHC does not find anything, there is no 20-year delay.

* Axions are light - there is no energetic obstruction to producing axions.



Axions

III. Axions in field theory (Peccei/Quinn 1977 Weinberg/Wilczek 1978)

Axions originate in the strong CP problem:
why does the neutron have no electric
dipole moment?

The gauge part of the QCD Lagrangian is

$$-\frac{1}{4g^2} \text{Tr}(F_{\mu\nu} F^{\mu\nu}) + \frac{\theta}{g} \epsilon_{\alpha\beta\gamma\delta} F^{\alpha\beta} F^{\gamma\delta}$$

The strong CP problem can be restated as:

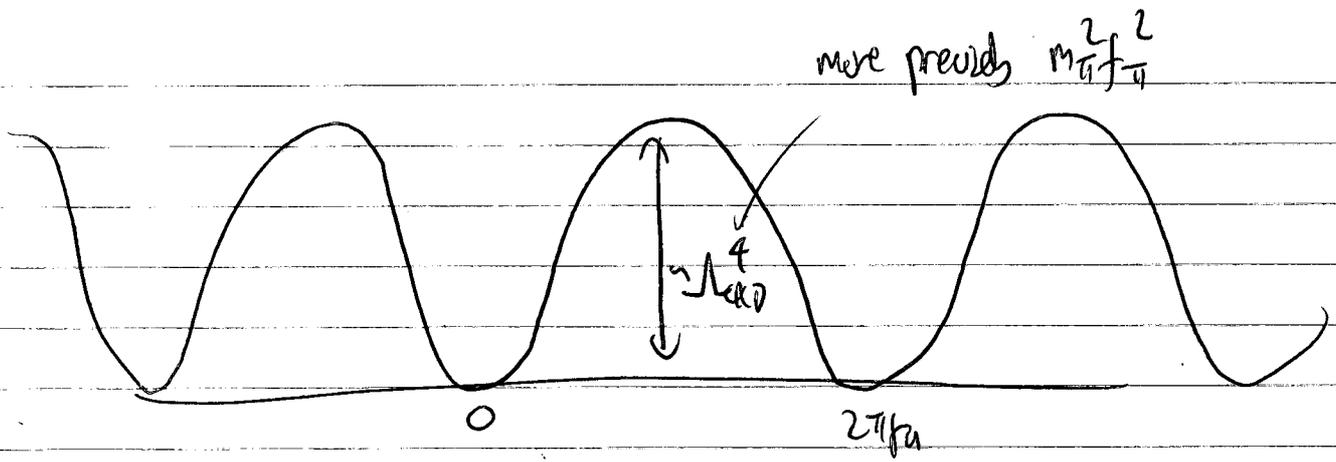
$$\text{Why is } \frac{\theta}{2\pi} \lesssim 10^{-10} \text{ ?}$$

Such a fine-tuning of the θ angle of the
strong force seems rather awkward to explain.

However if θ is promoted to a dynamical field,
so we have

$$-\frac{1}{2} \partial_\mu \theta \partial^\mu \theta - \frac{1}{4g^2} \text{Tr}(F_{\mu\nu} F^{\mu\nu}) + \frac{\theta}{2\pi f_a} \epsilon_{\alpha\beta\gamma\delta} F^{\alpha\beta} F^{\gamma\delta}$$

Then non-perturbative QCD instanton effects
generate a potential for θ , minimized at $\langle \theta \rangle = a$



Potential is approximately

$$\Lambda_{\text{QCD}}^4 \left(1 - \cos\left(\frac{a}{f_a}\right)\right)$$

$$\Rightarrow m_a^2 = V'' = \frac{\Lambda_{\text{QCD}}^4}{f_a^2}$$

$$\Rightarrow m_a \approx 10^{-3} \text{ eV} \left(\frac{10^{11} \text{ GeV}}{f_a}\right)$$

Small mass is because mass only arises from non-perturbative effects

This QCD axion is the original example of the axion.

A natural generalisation is to axion-like particles

These have a topological coupling to electromagnetism but not to the strong force.

$$-\frac{1}{2} \partial_\mu a \partial^\mu a + \frac{g}{4\pi} F_{\mu\nu} \tilde{F}^{\mu\nu} \quad \left(\equiv \frac{g}{M} \underline{E} \cdot \underline{B} \right)$$

↓ gives $a \rightarrow 2\gamma$ decays.

IV. Axions in String Theory

If string theory is true, what is the right low-energy Lagrangian of the world?

This depends on the compactifications; but under general circumstances, axions are present

(although the coupling to electromagnetism or QCD are much more model-dependent)

Heterotic: World-sheet action is

$$S_{\text{world-sheet}} = \frac{1}{2\pi\alpha'} \int \sqrt{g} + i B_2.$$

∴ In path integral

$$\int \mathcal{D}\Phi e^{-S[\Phi]}.$$

$\int B_2$ appear as a phase \Rightarrow physics is identical

for $\int B_2 = b_2$ and $\int B_2 = b_2 + 2\pi$.

Dimensional reduction of $B_2 \rightarrow \sum_i b_{2,i} \omega^i$

(where $\omega^i \in h^{2,1}$ are basis of 2-forms)

\Rightarrow each non-trivial 2-cycle gives rise to an axion in the 4d effective field theory.

It also follows that such axions are irrelevant in perturbation theory:

* String perturbation theory involves an expansion about the trivial embedding of the worldsheet



As $\int B_2$ is topological, on all embeddings contractible to zero it vanishes.

Only ~~for~~ non-trivial embeddings weighted by e^{-t} are sensitive to $\int B_2$.

Type IIA / IIB:

Both theories contain D-branes, as well as R-R forms. ($C_0, C_1, C_2, C_3, \dots$)

The R-R forms only couple directly to D-branes.

The D-brane action is

$$\frac{2\pi}{(2\pi\alpha')^2} \int d^4x \sqrt{g} e^{-\phi} + i C_n$$

Take a D7-brane wrapped on a 4-cycle Σ . Then the dimensionally reduced action is (omitting 27 factors)

$$\frac{g_2}{\text{Vol}(\Sigma_4)} \int d^4x \sqrt{g} F_{\mu\nu} F^{\mu\nu} + \underbrace{(C_n)}_{\int \Sigma_4} F \wedge F$$

As for the heterotic string, reduction of RR form fields (in this case RR form fields C_n) along non-trivial extra-dimensional cycles produces axions.

$\int \sum_n C_n \rightarrow$ axions, one for each independent cycle.

String perturbation theory is insensitive to the exact value of this.

It is only non-perturbatively (with the inclusion of D -instantons) gaugeless condensates

that dependence on the absolute value of $\int C_n$ can be obtained.

Generally: (most of the time)

Axions are associated with non-contractible cycles in the extra-dimensional geometry. Such cycles are common (easily $O(100)$ for CY compactifications) and can lead to $O(100)$ axions in low-energy theory.

Axion potentials are generated by effects that are non-perturbative in g_s , α' expansion: $e^{-\frac{Vol}{g_s}}$

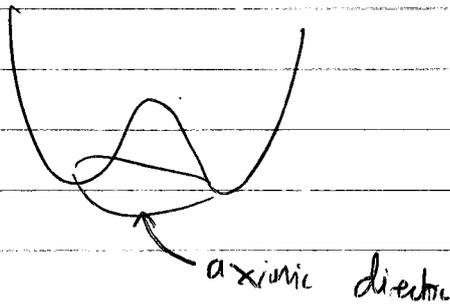
\Rightarrow produces very small masses

nb also open string axions.

These are analogous to field theoretic axions

If you view a U(1) charged field (where U(1) may be anomalous, and charge may be non-linearly realized)

then the residual symmetry contains an axion.



In this case the field range relates to the
vev of the charged field $f_a \sim \langle \phi \rangle$

V. Consequences for Low Energy Theory

Many string compactifications fit into structure of $N=1$ supersymmetric 4d effective field theory.

Described by Kähler potential and superpotential

$$\mathcal{L} = \int d^4x d^2\theta d^2\bar{\theta} K(\Phi, \bar{\Phi}) + \int d^4x d^2\theta W(\Phi)$$

Axions appear as imaginary parts of chiral multiplets

$$\tau = \tau + i a \quad a = a + 1$$

↑
axion.

Perturbative symmetry $\ln(\tau) \rightarrow \ln(\tau) + \epsilon$ implies that in in perturbation theory

$$K(\tau, \bar{\tau}) = K(\tau + \bar{\tau})$$

$$W(\tau) = W_0 \quad (\text{constant})$$

The metric on moduli space is given by $K_{\tau\bar{\tau}}$ and the field range can be found by $\int_{a=0}^{1 \text{ or } \infty} \sqrt{g}$.

example

$$K = -3 M_p^2 \ln \left(\frac{T + \bar{T}}{2 M_p} \right) \quad (\text{no-scale})$$

$$K_{T\bar{T}} = \frac{+3 M_p^2}{(T + \bar{T})^2} = \frac{3 M_p^2}{4 \tau^2}$$

∴ Metric for axion is

$$\int_0^1 \sqrt{g} da = \int_0^1 da$$

$$\int d^4x \frac{3 M_p^2}{4 \tau^2} \partial_\mu a \partial^\mu a = \int d^4x \left(\frac{3 M_p^2}{2 \tau^2} \right) \frac{1}{2} \partial_\mu a \partial^\mu a$$

∴ Axion field range is

$$\int_0^1 da \frac{\sqrt{3} M_p}{\sqrt{2} \tau} = \sqrt{\frac{3}{2}} \frac{M_p}{\tau}$$

∴ In geometric regime ($\tau \gg 1$),
field range is sub-Planckian;

$$\sqrt{\frac{3}{2}} \frac{M_p}{\tau} \ll M_p$$

It is a big open question whether controlled models exist with trans-Planckian field range.
↑
parametrically

And if there is a limit, what is it?

Q. In a vacuum solution of string theory, what is the largest allowed axion field range?

As described in other lectures, this question is very important for models of inflation in string theory.

Trans-Planckian field excursions during inflation correspond to observable levels of tensor modes

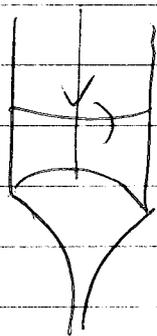
$$r \approx 0.01 \left(\frac{\Delta\phi}{M_p} \right)^2$$

As their potential is naturally flat, axions are good inflation candidates in string theory.

For this 1-modulus example, field range grows as $\tau \rightarrow 1$.

What happens to field range at small volumes?

* For cases with $N=2$ susy where we have full calculational control via mirror symmetry, field range capped at $\sim M_p$.



* String duality symmetries cut off field ranges at small radii.

(Conlon / Krippendorff)

* Three options

Max field range $\sim M_p$
Max field range finite, but $\rightarrow M_p$
Max field range unbounded.

VI. Axion Dark Matter

Axions ~~exist~~ are also a good dark matter candidate.

This is true both for the QCD axion and also for more general axions, provided they are massive and sufficiently long-lived.

The basic physics is similar in both cases - the misalignment mechanism - but easier to describe for more general axions.

How does it work?

- ① We assume in the early universe $m_a \ll H \ll f_a$ during inflation.



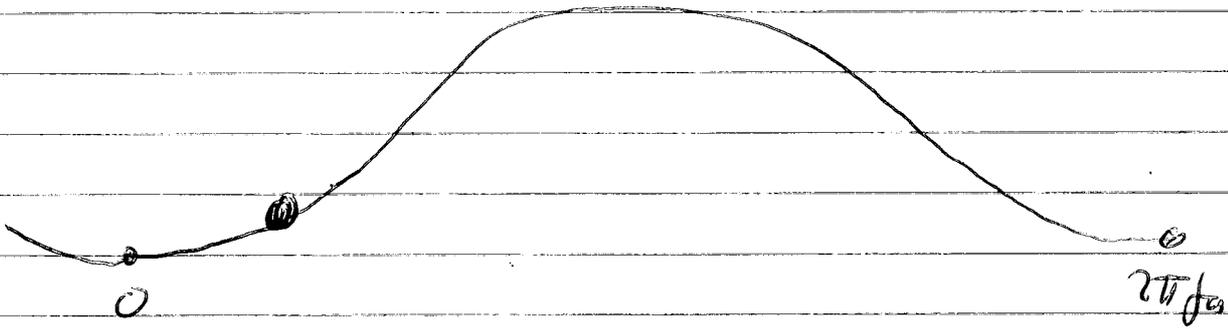
During this period axion wobbles on a Hubble scale.

- ② Dynamical equation for scalar field in de Sitter space is

$$\ddot{\phi} + 3H\dot{\phi} + m_a^2\phi = 0.$$

For $H > m_a$, Hubble friction dominates and field does not evolve.

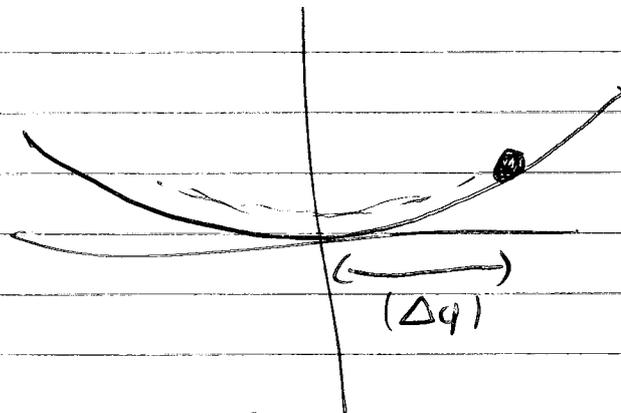
(3)



While $H > m_a$, scalar field does not move:
but is misaligned from its minimum.

(4) As universe ^{expands} ~~contracts~~ after inflation, H reduces.

When $H \sim m_a$, field starts oscillating about minimum



Initially, Energy in axion field. $m_a^2 (\Delta\phi)^2$

Overall energy $V = 3H^2 M_p^2 \sim 3m_a^2 M_p^2$

(5) Oscillating axion field now behaves as dark matter ~~condensate~~ (coherent oscillation of scalar field)

While universe's energy density is in form of radiation, its importance continues to grow.

Notes

- ① Amount of dark matter depends on initial misalignment angle.
- ② The larger m_a is, the earlier it starts oscillating.
- ③ For the QCD axion, the calculations are more subtle as $m_a = m_a(T)$

as mass comes from instantons, and their strength depends on the ambient temperature which set $\alpha_s(T)$.

The above misalignment mechanism can lead to axion dark matter

VII Axions as Dark Radiation.

Another important role for axions is as dark radiation candidates - this can place significant constraints on string compactifications.

~~String theory contains moduli massive, long-lived~~
~~weakly interacting~~

What is dark radiation? It is parametrised by N_{eff} and represents additional dark relativistic energy.

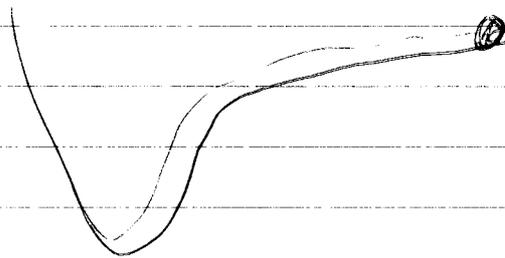
(canonically, $N_{\text{eff}} = 3.36$ (three species of neutrinos))

Additionally, $\Delta N_{\text{eff}} \lesssim 0.5$

Axions can be dark radiation as they are light (and so will propagate relativistically).

How are they produced?

After inflation, we have reheating: inflation oscillates and decays.



However, as matter $\propto \frac{\rho}{a^3}$ and radiation $\propto \frac{\rho}{a^4}$

reheating is dominated by last scalar to decay.

Last scalar to decay are those with weakest couplings

String compactifications always contain moduli with gravitational strength coupling

$$\Gamma \sim \frac{1}{g^4} \frac{m_\phi^3}{M_{pl}^2} \Rightarrow \tau \sim 15 \left(\frac{100 \text{ TeV}}{m_\phi} \right)^3$$

Moduli come to dominate the universe before decaying late.

Visible decays of moduli ($\phi \rightarrow gg, \phi \rightarrow \gamma\gamma$) lead to reheating of the universe into hot big bang

Hidden sector decays (e.g. $\phi \rightarrow aa$) lead to dark radiation.

$$e.g. \quad K = -3 \ln(T + \bar{T})$$

$$\rightarrow \frac{3}{4\pi^2} d_\tau d^\tau + \frac{3}{4\pi^2} d_a d^a$$

$$\Phi = \sqrt{\frac{3}{2}} \ln \tau \rightarrow \frac{1}{2} d_\Phi d^\Phi + \frac{3}{4} e^{-2\sqrt{\frac{2}{3}} \Phi} d_a d^a$$

After Φ get mass, leads to

$\phi \rightarrow aa$ and dark cosmic axion background.

Avoiding over-production of dark radiation places strong constraints on string ^(axionic) cosmology

VIII

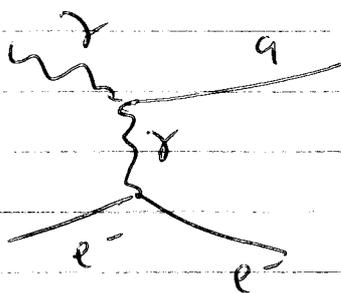
Constraints and Searches for Axions

I want to describe briefly some of the methods used to constrain (or search for) axions and axion-like particles

For both cases I use SN1987A.

These constraints arise from the coupling to electromagnetism,
 $g \frac{F \tilde{F}}{EM}$

Primakoff effect



$\gamma \rightarrow q$ scattering
can lead to an
additional cooling
channel for
white dwarfs, supernovae etc.

Axions produced in the core of stars (or supernovae) provide an additional cooling channel.

Absence of such additional cooling constrains the QCD axion decay constant

$$(f_a \gtrsim 10^9 \text{ GeV}, \quad m_a \lesssim 10^{-3} \text{ eV})$$

ALP-photon \leftrightarrow

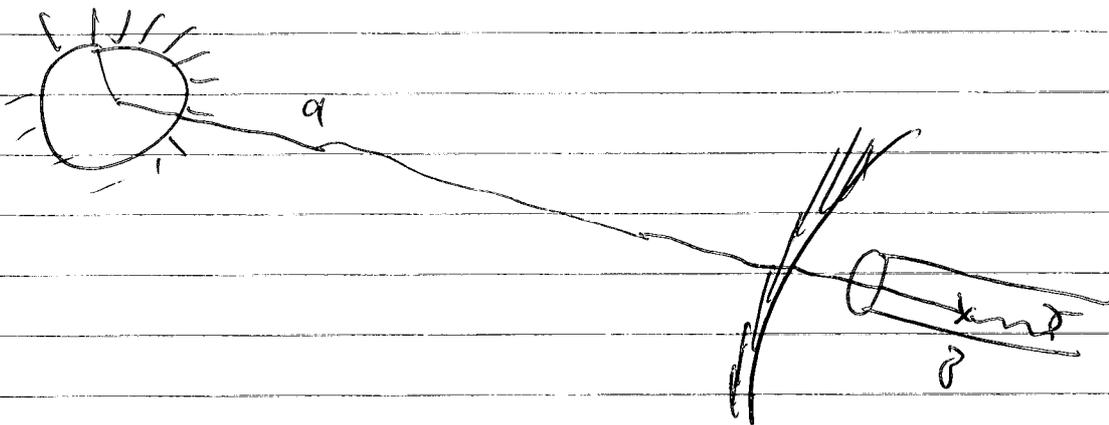
CAS T / SN1977A

For sufficiently light axions or axion-like particles, $\alpha \rightarrow \gamma$ back conversion can occur in astrophysical magnetic fields

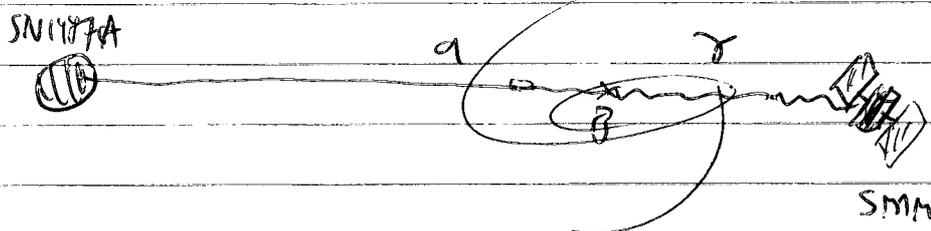
The coupling $\frac{\alpha}{M} F \tilde{F} \equiv \frac{g}{M} \underline{E} \cdot \underline{B}$

produces a 2-particle axion-photon coupling in the presence of background B fields.

Relativistic axions / ALPs can convert back into photons



CAS T



SMM

For ALP, this constrains the cutoff M to be

$$M \gtrsim 2 \times 10^4 \text{ GeV}.$$