On the classification of rank-1 four-dimensional $\mathcal{N} = 2$ SCFTs by Seiberg-Witten geometry

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July 28, 2016
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Based on work with **Philip Argyres, Matteo Lotito, Mario Martone**

arXiv:1505.04814
arXiv:1511.00669
arXiv:1601.00011
arXiv:1602.02764
and to appear

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- Introduction

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- Summary and open questions

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- Our main tool is the so called Seiberg-Witten (SW) data: the collection of low-energy observables which can be exactly predicted by $\mathcal{N}=2$ (Poincarè) supersymmetry and encoded in a geometrical object, called the Coulomb branch (CB).

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- Our main tool is the so called Seiberg-Witten (SW) data: the collection of low-energy observables which can be exactly predicted by $\mathcal{N} = 2$ (Poincarè) supersymmetry and encoded in a geometrical object, called the Coulomb branch (CB).
- The non-trivial dependence of SW data on $\mathcal{N} = 2$ SCFT data motivates us to attack the $\mathcal{N} = 2$ classification problem by directly considering the classification of SW geometries.

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- So, due to the organizing role that CFTs play in Wilson's RG picture of QFTs, it is well motivated to pursue the classification of CFTs.
- Conformal symmetry, joined with a certain number of supersymmetries, can be enlarged to superconformal symmetry, which gives further constraints on the structure of a CFT.

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- An incomplete list of pertinent SCFT data includes:
  - the flavor symmetry algebra $\mathfrak{f}$;
  - the dimensions, r, of the Coulomb branch (CB) $\mathcal{M}_V$, and the dimension, h_0, of the Higgs branch (HB) $\mathcal{M}_H$;
  - the scaling dimensions $\{\Delta(u_i)\}$ of CB operators;
  - the conformal central charges a and c;
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- Those listed observables turn out to be closely related to the Seiberg-Witten data, and can be extracted from the latter. This connection constitutes the key logical of our approach.

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$$\mathcal{M} = \bigcup_i \mathcal{M}_V^i \times \mathcal{M}_H^i, \qquad \begin{cases} \mathcal{M}_V^i = \text{Coulomb branch factor} \\ \mathcal{M}_H^i = \text{Higgs branch factor} \end{cases},$$

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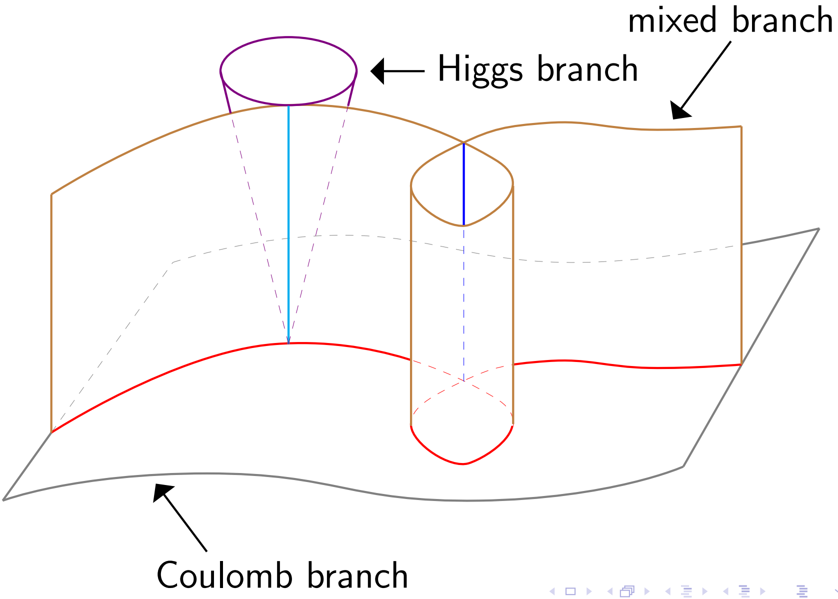
▶ These moduli spaces admit parametrization by the vevs of the so-called Coulomb branch operators and Higgs branch operators in the UV $\mathcal{N}=2$ SCFT. These vevs spontaneously break the conformal symmetry and lead to the scaling structure of the moduli spaces.

Seiberg-Witten data



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  - The conventional HB $\mathcal{M}_H$ admits no CB factor; $\mathcal{M}_H$ is a hyperkähler cone;
  - the branches with $h_{\dim_{\mathbb{C}}\mathcal{M}_V^i \neq 0} \neq 0$ are collectively called mixed branch;
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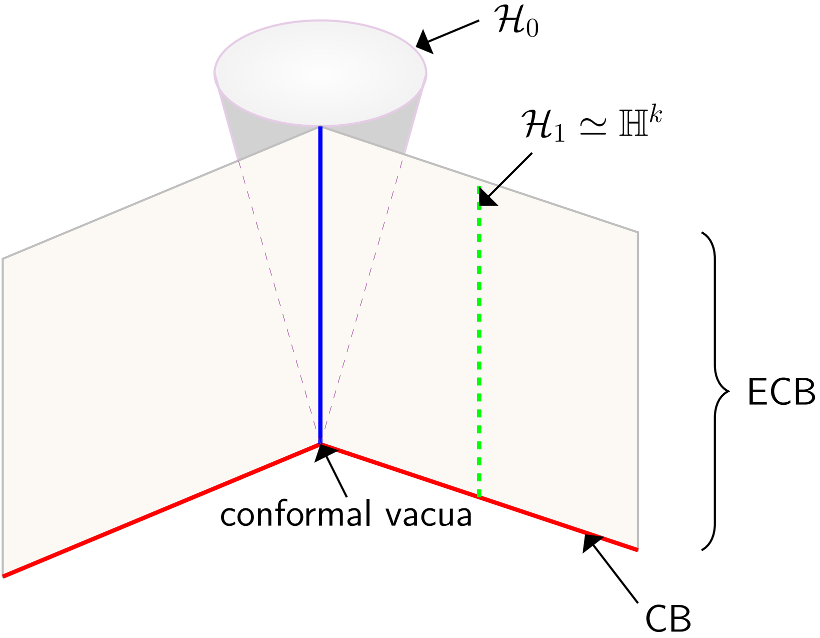
- We will be focusing our attention almost exclusively on the (rank-1) CB geometry from now on. The reason is that it is not lifted but is deformed by turning on $\mathcal{N}=2$ relevant operators in the SCFT. This means that the scale invariant CB geometry and its deformations encode detailed information (though in non-obvious ways) about the structure of the SCFT data.



$\mathcal{H}_0$
$\mathcal{H}_1 \simeq \mathbb{H}^k$
ECB
conformal vacua
CB

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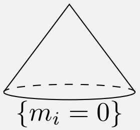
- We systematically study and classify the possible geometries of rank-1 CBs with planar topology, i.e., CB $\simeq \mathbb{C}$ globally.
- Let u be the global complex coordinate, and $\{m_i\}$ are the relevant deformation parameters of the SCFT.



- When $\{m_i\} = 0$, the theory is conformal invariant, and has a global internal symmetry $\mathfrak{u}(2)_R \oplus \mathfrak{f}$, where $\mathfrak{u}(2)_R$ is the $\mathcal{N} = 2$ R-symmetry and $\mathfrak{f}$ is the flavor symmetry algebra.
- On the CB,
  - $\{u = 0\}$ = conformal vacua.
  - $\{u \neq 0\} \Rightarrow$ scale invariance (SI) is spontaneously broken (scaling dimension $\Delta(u) > 0$).

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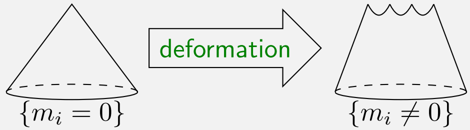

- $\{m_i\} \neq 0 \Rightarrow$ flavor symmetry explicitly broken since $\Delta(m_i) = 1$ and $m_i \in \text{adj}(F)$.
- CB is deformed.
  - $\{u = 0\} \Rightarrow \{u_i\}$: the tip of the cone is spitted to multiple tips, where sit scale-invariant vacuum with flavor subalgebras.
  - $\{u \neq u_i\} \Rightarrow$ Scale invariance is explicitly broken, and scaling structure is lost.

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- Seiberg-Witten curves in Weierstrass form:

$$\Sigma(u, \mathbf{m}) : \qquad y^2 = x^3 + f(u, \mathbf{m})\, x + g(u, \mathbf{m}),$$

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$$\partial_u \lambda = \kappa \frac{\mathrm{d}x}{y} + \mathrm{d}\phi, \qquad \text{Residues}(\lambda) \in \{\boldsymbol{\omega}(\mathbf{m}) \mid \boldsymbol{\omega} \in \Lambda_F\}.$$

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- The low energy $\mathfrak{u}(1)$ gauge coupling and BPS central charges can be determined from the curve and form.

Seiberg-Witten data

- In scale-invariant case the curve degenerates as one of Kodaira's possibilities:

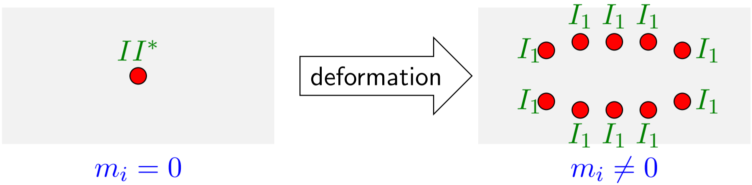
Name	singular SW curve	deg(Disc$_x$)	$\Delta(u)$
II^*	$y^2 = x^3 + u^5$	10	6
III^*	$y^2 = x^3 + u^3 x$	9	4
IV^*	$y^2 = x^3 + u^4$	8	3
I_0^*	$y^2 = \prod_{i=1}^3 (x - e_i(\tau)\, u)$	6	2
IV	$y^2 = x^3 + u^2$	4	3/2
III	$y^2 = x^3 + ux$	3	4/3
II	$y^2 = x^3 + u$	2	6/5
$I_n^* \quad (n>0)$	$y^2 = x^3 + ux^2 + \Lambda^{-2n} u^{n+3}$	$n+6$	2
$I_n \quad (n>0)$	$y^2 = (x-1)(x^2 + \Lambda^{-n} u^n)$	n	1

- Degree of the discriminant is an invariant under deformation.
- $I_n / I_n^* \Leftrightarrow$ IR-free $\mathfrak{u}(1)/\mathfrak{su}(2)$ gauge theories with scale Λ.

▶ Mass deformations split SI singularity into multiple singularities such that total degree of discriminant remains the same.

▶ There are different deformation patterns for each SI singularity.

▶ E.g., maximal deformation of II^* singularity splits $II^* \to \{{I_1}^{10}\}$ (ten I_1 singulariities). It corresponds to CFT with $\mathfrak{f} = E_8$:



▶ Discriminant has 10 distinct zeros:

$$\mathrm{Disc}_x = u^{10} + \cdots .$$

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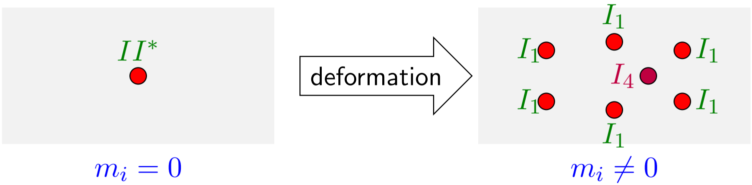
- E.g., sub-maximal deformation of II^* singularity splits $II^* \to \{I_1{}^6, I_4\}$. It corresponds to CFT with $\mathfrak{f} = C_5$:



- Discriminant has 7 distinct zeros:

$$\mathrm{Disc}_x = (u^6 + \cdots)(u + \cdots)^4.$$

Seiberg-Witten data

- An ansatz for one-form satisfying RSK condition given by Minahan-Nemeschansky:

$$\lambda(u, \mathbf{m}) = \Bigg[2\Delta(u)\, a\, u + 6b\, \mu\, x + 2W(M_d) + \sum_i r_i \sum_{\boldsymbol{\omega}_i \text{ orbit}} \frac{y_{\boldsymbol{\omega}_i}(u, \mathbf{m})}{\boldsymbol{\omega}_i(\mathbf{m})^2\, x - x_{\boldsymbol{\omega}_i}(u, \mathbf{m})} \Bigg] \frac{dx}{y}.$$

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- $a, b, W, r_i, x_{\boldsymbol{\omega}_i}$ are unknowns. Most difficult are $x_{\boldsymbol{\omega}_i}$ determined by factorization of curve.
- Sum over Weyl orbits of flavor algebra weights $\boldsymbol{\omega}_i$. Weyl group is determined by curve.
- Which weights appear and their coefficients r_i determine together with Weyl group the flavor symmetry.

Rank-1 SW geometries and $\mathcal{N}=2$ SCFT: classification

sing.	deform.	flavor symm.	$k_{\mathfrak{f}}$	$12 \cdot c$	$24 \cdot a$	$\mathbf{h}_1$	h_1	h_0
II^*	$\{I_1^{10}\}$	E_8	12	62	95	—	0	29
	$\{I_1^6, I_4\}$	C_5	7	49	82	$\mathbf{10}$	5	16
	$\{I_1^3, I_1^*\}$	$A_3 \rtimes \mathbb{Z}_2$	14	42	75	$\mathbf{4} \oplus \overline{\mathbf{4}}$	4	9
	$\{I_1^2, IV_{Q=1}^*\}$	$A_2 \rtimes \mathbb{Z}_2$	14	38	71	$\mathbf{3} \oplus \overline{\mathbf{3}}$	3	?
III^*	$\{I_1^9\}$	E_7	8	38	59	—	0	17
	$\{I_1^5, I_4\}$	$C_3 \oplus A_1$	(5,8)	29	50	$(\mathbf{6},\mathbf{1})$	3	8
	$\{I_1^2, I_1^*\}$	$A_1 \oplus (\mathrm{u}(1) \rtimes \mathbb{Z}_2)$	(10, ?)	24	45	$\mathbf{2}_+ \oplus \mathbf{2}_-$	2	?
	$\{I_1, IV_{Q=1}^*\}$	$\mathrm{u}(1) \rtimes \mathbb{Z}_2$	?	21	42	$\mathbf{1}_+ \oplus \mathbf{1}_-$	1	0
IV^*	$\{I_1^8\}$	E_6	6	26	41	–	0	11
	$\{I_1^4, I_4\}$	$C_2 \oplus \mathrm{u}(1)$	(4, ?)	19	34	$\mathbf{4}_0$	2	4
	$\{I_1, I_1^*\}$	$\mathrm{u}(1)$	?	15	30	$\mathbf{1}_+ \oplus \mathbf{1}_-$	1	0
I_0^*	$\{I_1^6\}$	D_4	4	14	23	—	0	5
	$\{I_1^2, I_4\}$	A_1	3	9	18	$\mathbf{2}$	1	1
	$\{I_2^3\}$	A_1	3	9	18	$\mathbf{2}$	1	1
IV	$\{I_1^4\}$	A_2	3	8	14	—	0	2
III	$\{I_1^3\}$	A_1	8/3	6	11	—	0	1
II	$\{I_1^2\}$	$\varnothing$	—	22/5	43/5	—	0	0

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- There are question marks where there is not enough information from the CB geometry to usefully constrain an entry.
  - Note that even the known h_0, $\mathbf{h_1}$ and h_1 are not obtained directly from CB geometry. The knowledge about them depend on other independent constructions mentioned above;
  - There is no available ways in determining $\mathfrak{u}(1)$ flavor central charge $k_{\mathfrak{u}(1)}$.

Further results on conformal and flavor central charges

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$$Z = \int [dV][dH] \; A^{\chi} \; B^{\sigma} \; \prod_i C_i^{n_i} \; e^{S_{\mathsf{IR}}[V,H]}$$

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▶ We consider the rank-1 case, while this method is applicable to a very large class of theories with arbitrary rank; one can get

$$\begin{aligned} 24a &= 5 + h_1 + 12\Delta(A) + 8\Delta(B) \\ 12c &= 2 + h_1 + 8\Delta(B) \\ k_i &= T_i(2\mathbf{h_1}) - 2\Delta(C_i) \end{aligned}$$

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- For considering generic deformations (Z counts the number of distinct zeros of discriminant of the curve):

$$
\begin{aligned}
\Delta(A) &= \frac{\Delta - 1}{2} \\
\Delta(B) &= \frac{\Delta}{8} \sum_{i=1}^{Z} \frac{12c_i - 2 - h_i}{\Delta_i}
\end{aligned}
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- In turn the conformal central charge a and c can be determined:

$$24a = 5+h_1+3\frac{\Delta-1}{2}+\Delta\sum_{i=1}^{Z}\frac{12c_i-2-h_i}{\Delta_i},$$

$$12c = 2+h_1+\Delta\sum_{i=1}^{Z}\frac{12c_i-2-h_i}{\Delta_i}.$$

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$$\Delta(C) = \sum_i \frac{\Delta}{\Delta_i d_i} (T(2\mathbf{h}_1^{\mathbf{i}}) - k_i),$$

for all i such that $\mathfrak{f}_i$ is nonabelian.

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for all i such that $\mathfrak{f}_i$ is nonabelian.

- In turn the flavor central charge k can be determined:

$$k = \sum_i \frac{\Delta}{\Delta_i d_i}(k_i - T(2\mathbf{h}_\mathbf{1}^\mathbf{i})) + T_i(2\mathbf{h}_\mathbf{1}).$$

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- Further discussion on the $\mathfrak{u}(1)$ flavor central charge $k_{\mathfrak{u}(1)}$?
- Similar story of 4d rank-1 $\mathcal{N}=2$ SCFTs with non-planar CB?
- Generalization to higher rank SCFTs?
- Possible d=5 N=1/d=6 N=(1,0) versions?