Wake potential calculation of a Gaussian bunch

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Background

- Beam instability analysis needs Pseudo-Green function wake (bunch wake with smaller bunch length <sigz/10).

- Impedance formula for double layer beam tube is complicated and difficult to get the wake expression directly.

- To solve the problem -> calculate the bunch wake directly from impedance formula or impedance spectrum numerically.

Compute bunch wake from impedance

$$W_{\parallel} = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega e^{i\omega z/c} Z_m^{\parallel}(\omega)$$

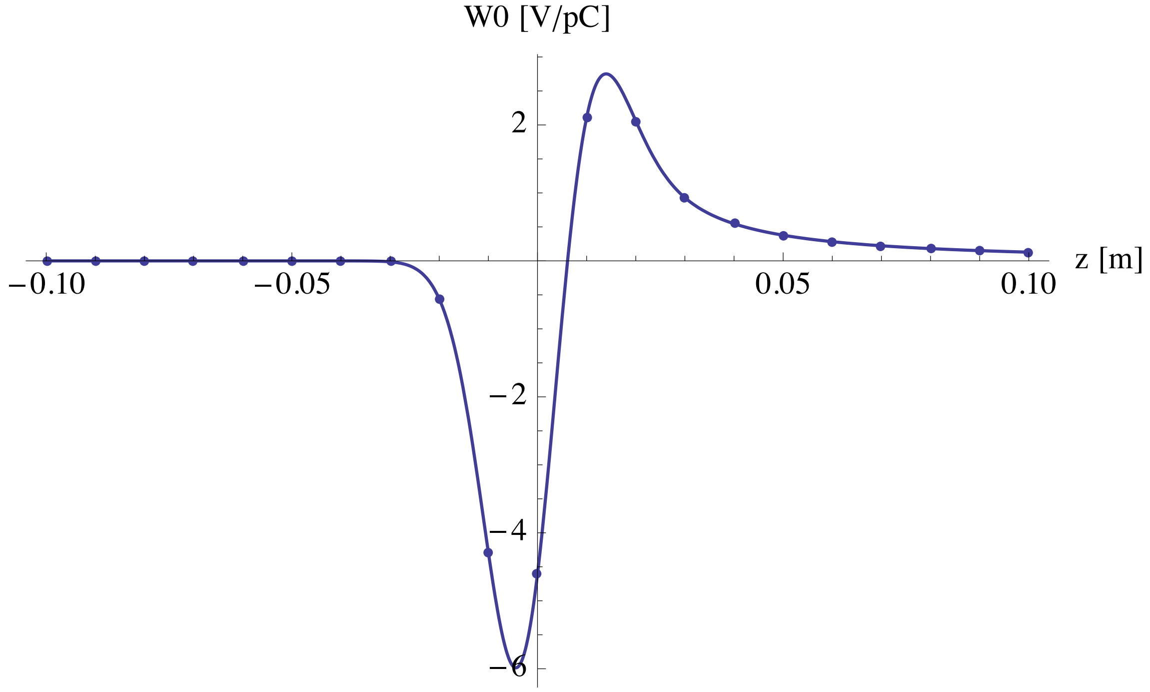
$$W_{\parallel\lambda}(\tau) = \int_{-\infty}^{\infty} dt \lambda(\tau - t) W_{\parallel}(t)$$

$$W_{\parallel\lambda}(\tau) = \int_{-\infty}^{\infty} dt \lambda(\tau - t) \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega e^{i\omega t} Z_m^{\parallel}(\omega)$$

Resistive wall - Single layer thick wall

$$Z_{\parallel} = \frac{L}{2\pi b_0}\sqrt{\frac{Z_0}{2\sigma_c c_0}}(1+sign(\omega)i)\sqrt{|\omega|} \qquad W_{\parallel} = \frac{c_0 L}{8\sqrt{2}\pi b_0}\frac{1}{\sigma_z^{1.5}}\sqrt{\frac{Z_0}{\sigma_c}}f_{sc}$$

$$f_{sc} = \sqrt{|x|^3}e^{-y}(I_{1/4}(y) - I_{-3/4}(y) + I_{-1/4}(y) - I_{3/4}(y))\Big|_{y=(s/\sigma_z)^2/4,\, x=s/\sigma_z}$$



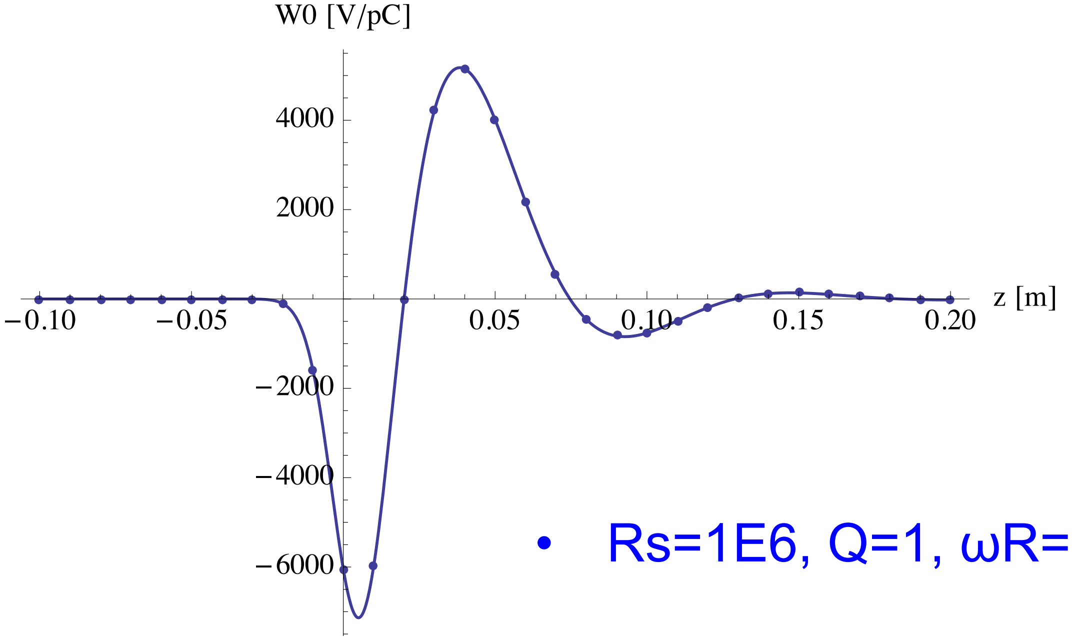
High Order modes

$$Z_{\parallel} = \frac{R_s}{1 + iQ\left(\frac{\omega_R}{\omega} - \frac{\omega}{\omega_R}\right)}$$

$$W_{\parallel} = -2\alpha R_s (f_{sc2} - \frac{\alpha}{\bar{\omega}} f_{sc1})$$

$$f_{sc1} = \frac{1}{2} e^{-\frac{\sigma_z^2}{2}(k_r^2 - \alpha_r^2) - \alpha_r \sigma_z x} \mathrm{Im}(e^{ik_r \sigma_z (x - \alpha_r \sigma_z)}(1 + Erf(\frac{(ik_r - \alpha_r)\sigma_z + x}{\sqrt{2}})))|_{x = s/\sigma_z}$$

$$f_{sc2} = \frac{1}{2} e^{-\frac{\sigma_z^2}{2}(k_r^2 - \alpha_r^2) - \alpha_r \sigma_z x} \mathrm{Re}(e^{ik_r \sigma_z (x - \alpha_r \sigma_z)}(1 + Erf(\frac{(ik_r - \alpha_r)\sigma_z + x}{\sqrt{2}})))|_{x = s/\sigma_z}$$



- Rs=1E6, Q=1, ωR=20GHz

Resistive wall – Double layer finite thickness