$Z_c(3900)$ as a $D\bar{D}^*$ Molecule from Pole Counting Rule

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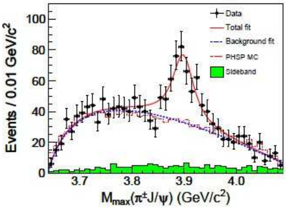
Contents

DISCOVERY OF $Z_c(3900)$, $J/\Psi\pi$ SPECTRUM

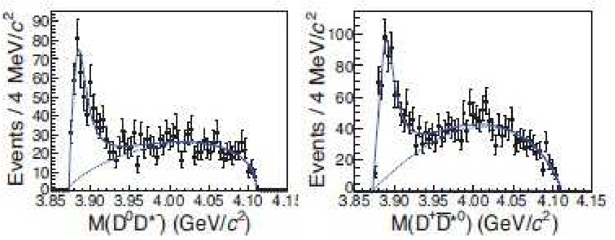
- The hadron exotic state $Z_c(3900)$ was first observed at BESIII in $e^+e^- \to X(4260) \to J/\Psi\pi^+\pi^-$ process, $J/\Psi\pi$ spectrum Ablikim et al. (BESIII), PRL 110,252001(2013)



- Mass: $3899.0 \pm 3.6 \pm 4.9$ MeV. Width: $46 \pm 10 \pm 20$ MeV

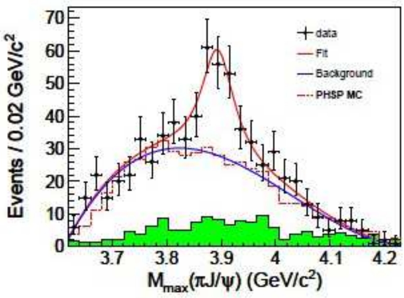
DISCOVERY OF $Z_c(3900)$, $D\bar{D}^*$ SPECTRUM

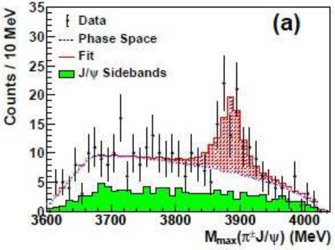
- Ablikim et al. (BESIII), PRL 112,022001(2014)



- Quantum numbers: $I(J^P) = 1(1^+)$

CONFIRM OF Z_c BY BELLE AND CLEO





Possible Theoretical Interpretations

- Z_c can not be embedded into conventional $c\bar{c}$ quark model, while it's close to $D\bar{D}^*$ threshold.
- $D\bar{D}^*$ molecular state
  - Phenomenological Lagrangian: [Dong et al, PRD 88,014030(2013)]
  - Heavy quark flavor symmetry: [Guo et al, PRD 88,054007(2013)]
  - Electromagnetic structure: [Wilbring, Hammer, and Meissner, PLB 726,326(2013)]
  - $\cdots$

Possible Theoretical Interpretations

- Elementary tetraquark state
  - QCD sum rule:
    [Wang and Huang, PRD 89,054019(2014)]
  - tetraquark model:
    [Maiani et al., PRD 87,111102(2013)]
  - QCD sum rule:
    [Dias et al, Phys. Rev. D 88,016004(2013)]
  - ...
- Non-resonance
  - cusp effect:
    [Chen, Liu, and Matsuki, PRD 88,036008(2013)]
  - triangle diagram singularity:
    [Liu, Oka, and Zhao, PLB 753,297(2016)]
  - Pang's talk.

SUMMARY

- $Z_c(3900)$:
  - $M \sim 3900$ MeV, width tens of MeV, close to $D\bar{D}^*$ threshold.
  - Significantly coupled to $D\bar{D}^*$ and $J/\Psi\pi$ channels ($h_c\pi$ less marked)
  - $I(J^P) = 1(1^+)$
- Possible theoretical interpretations:
  - $D\bar{D}^*$ molecule
  - compact tetra-quark state (i.e. elementary state)
  - non-resonance (cusp effect, ATS, etc.)

- Strategies:
  - effective Lagrangian under C, P, χ and isospin symmetries
  - calculation of the $X(4260) \to Z_c\pi \to D\bar{D}^*\pi,\ J/\Psi\pi\pi,\ h_c\pi\pi$ amplitudes
  - combined fit to $J/\Psi\pi$ channel $J/\Psi\pi,\ \pi\pi$ data, $D\bar{D}^*$ data and $h_c\pi$ data (as background)
  - find pole position and use pole counting rule
- Pole counting rule [Morgan, NPA 543,632(1992)] :
  - based on potential scattering and effective range expansion
  - only one pole near threshold $\to$ the pole corresponds to a molecular state
  - two poles near threshold $\to$ the poles indicate an elementary state

EFFECTIVE RANGE EXPANSION

- In non-relativistic k plane, for s wave phase shift k: $k\cot\delta = -1/a + r_e/2k^2 + o(k^4)$
- a: scattering length. r_e: effective range.
- Partial wave S matrix:

$$S(k) = \frac{\frac{r_e}{2}k^2 + \mathrm{i}k - \frac{1}{a}}{\frac{r_e}{2}k^2 - \mathrm{i}k - \frac{1}{a}}$$

- Constraint on the two poles: $|k_1| + |k_2| \geq |k_1 + k_2| = 2/|r_e|$

POLE COUNTING RULE

- Potential scattering $\rightarrow r_e \sim L$ (the range of potential interaction) $\rightarrow$ AT MOST one pole lie in $|k| < 1/L$.
- Two poles near threshold $\rightarrow$ poles are generated by hidden channel other than the explicit potential $\rightarrow$ there exists an elementary state.
- In multi-channel scattering, only one pole (bound state or virtual state) coupled to the inelastic threshold may split to a pair of resonance, which also represents a potential generated molecular.

$r_e \sim L$

- Single channel case:
  - $r_e = 2\int_0^\infty dr[\varphi_0^2(r) - \psi_0^2(r)]$
  - $\psi_0^2(r)$: solution of radial Schrödinger equation at $k = 0$.
  - $\varphi_0^2(r)$: asymptotic form of $\psi_0^2(r)$ when $r \to \infty$.
  - For short-ranged and weak potential $r_e = 2\int_0^L \cdots$, and the integrand takes $\cos^2(\pi r/2L)$ form so $r_e \sim L$.
- Multi-channel case and relativistic partial wave dispersion relations give the same result.
- Two poles near threshold $\to$ CDD poles in dispersion relation: $k\cot\delta = (\nu_p m^2 - k^2)/(m^2 g^2) + \cdots$, poles $k = \pm m\sqrt{\nu_p}$.

UNCERTAINTIES

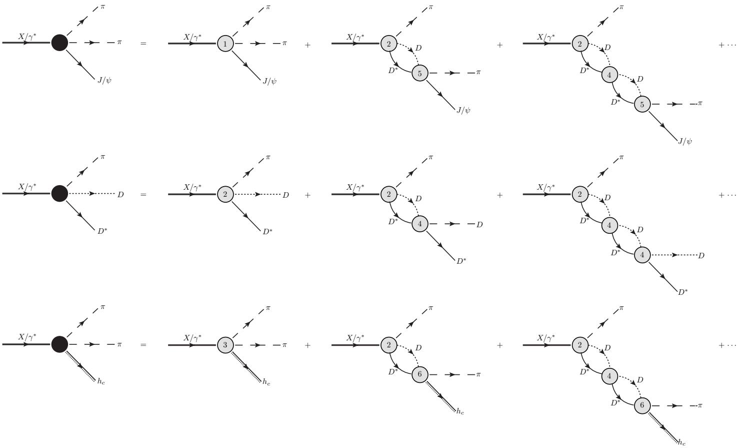
- The condition $r_e \sim L$ dose not hold for too strong binding case$/$ potentials with strange mathematical structure.
- In multi-channel case, pole counting rule is satisfied only when the pertinent poles are dominated by one channel.
- Pole counting rule is a qualitative result rather than a quantitative theorem.

Effective Lagrangians

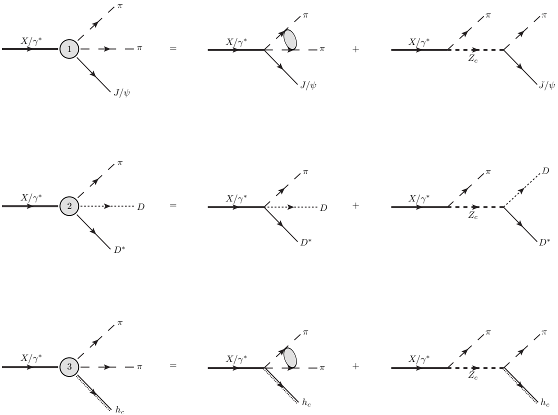
- For $X(4260) \to J/\psi(h_c)\pi\pi$ channel:
  - $\mathcal{L}_{XJ/\psi\pi\pi} = g_1 X_\mu \psi_\nu \langle u^\mu u^\nu \rangle + g_2 X_\mu \psi^\mu \langle u^\nu u_\nu \rangle + g_3 X_\mu \psi^\mu \langle \chi_+ \rangle + \cdots$
  - $\mathcal{L}_{XZ_c\pi} = g_4 \nabla_\nu X_\mu \langle Z_c^\mu u^\nu \rangle + \cdots$
  - $\mathcal{L}_{ZcJ/\Psi\pi} = g_7 \nabla_\nu \psi_\mu \langle Z_c^\mu u^\nu \rangle + \cdots$
  - $\mathcal{L}_{Xh_c\pi\pi} = f_8 \nabla^\lambda \nabla_\rho X_\mu H_\nu \langle u_\lambda u_\sigma \rangle \epsilon^{\mu\nu\rho\sigma} + \cdots$
  - $\mathcal{L}_{Z_c h_c \pi} = f_9 \nabla_\alpha H_\nu \langle Z_{c\mu} u_\beta \rangle + \cdots$
- Field operators:
  - $X : X(4260)$; $\psi : J/\psi$; $Z_c : Z_c(3900)$; $H : h_c$
  - $\nabla^\mu X = \partial^\mu X + [\Gamma^\mu, X]$
  - $\Gamma^\mu = \frac{1}{2}[u^+(\partial^\mu - ir^\mu)u + u(\partial^\mu - il^\mu)u^+]$
  - $\chi_\pm = u^+ \chi u^+ \pm u \chi^+ u$, $u_\mu = i\{u^+ \partial_\mu u - u \partial_\mu u^+\}$
  - $\chi = 2B(s + ip)$, $u = \exp(\frac{i\Phi}{\sqrt{2}F})$

- For $X(4260) \to DD^*\pi$ channel:
  - $\mathcal{L}_{XDD^*\pi} = f_1 \nabla^\nu X^\mu \langle \bar{D}^*_\mu D u_\nu \rangle + f_2 X^\mu \langle \nabla^\nu \bar{D}^*_\mu D u_\nu \rangle$ $+ f_3 \nabla^\nu X^\mu \langle \bar{D}^*_\nu D u_\mu \rangle + f_4 X^\mu \langle \nabla_\mu \bar{D}^{*\nu} D u_\nu \rangle + \cdots$
  - $\mathcal{L}_{ZcDD*} = f_7 [(\bar{D}^{*0}_\mu D^+ + D^{*+}_\mu \bar{D}^0) Z_c^{-\mu} + \text{h.c.}]$
  - $\mathcal{L}_{D\bar{D}^* D\bar{D}^*} = \lambda_1 (D^{*+\mu} \bar{D}^0 D^{*-}_\mu D^0 + D^+ \bar{D}^{*0\mu} D^- D^{*0}_\mu + \text{h.c.}) +$ $\lambda_2 (D^{*+\mu} \bar{D}^0 D^- D^{*0}_\mu + D^+ \bar{D}^{*0\mu} D^{*-}_\mu D^0 + \text{h.c.})$
  - $\mathcal{L}_{DD^* J/\psi\pi} = \lambda_3 \nabla^\nu \psi^\mu \langle \bar{D}^{*\mu} D u_\nu \rangle + \lambda_4 \psi^\mu \langle \nabla^\nu \bar{D}^{*\mu} D u_\nu \rangle$ $+ \lambda_5 \nabla^\nu \psi^\mu \langle \bar{D}^{*\nu} D u_\mu \rangle + \lambda_6 \psi^\mu \langle \nabla^\mu \bar{D}^{*\nu} D u_\nu \rangle + \cdots$
  - $\mathcal{L}_{D\bar{D}^* h_c\pi} = \lambda_9 \nabla^\alpha H^\nu \langle \bar{D}^{*\mu} D u^\beta \rangle + \lambda_{10} H^\nu \langle \nabla^\alpha \bar{D}^{*\mu} D u^\beta \rangle + \cdots$
- Heavy quark spin symmetry $\to \lambda_1 = \lambda_2$

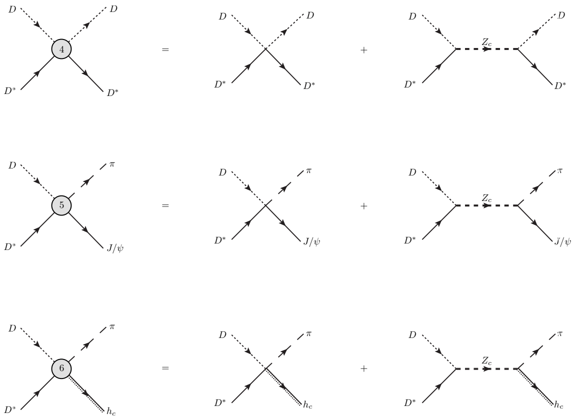
Feynman diagrams



COMPOUND 3— VERTICES



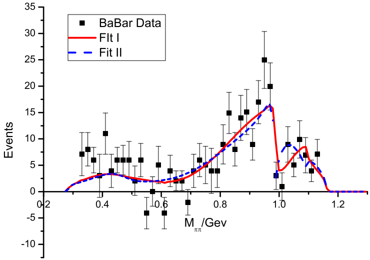
COMPOUND 4— VERTICES

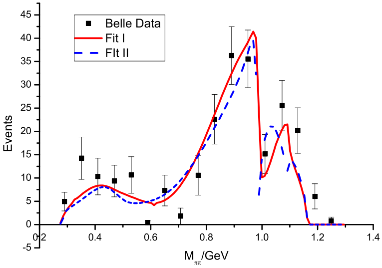


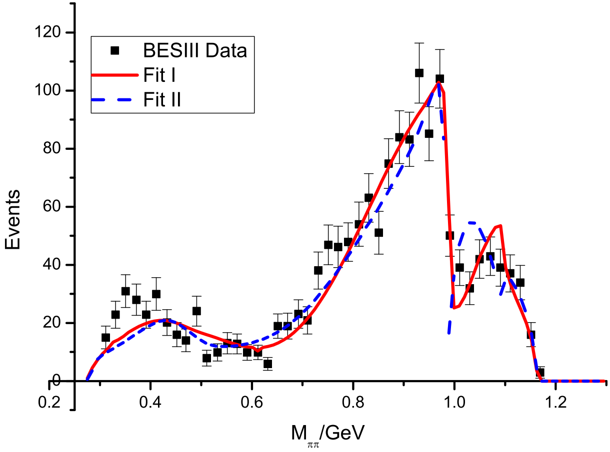
DIFFERENT MECHANISMS TO GENERATE Z_c

- Z_c: dynamically generated particle / basic d.o.f
- Different mechanisms:
  - pure dynamical (Fit I, $D, \bar{D}^*$ bubble chain) $\rightarrow g_4, g_5, f_5, f_7 = 0$
  - pure explicit Z_c field (Fit II, Breit-Wigner Z_c propagator) $\rightarrow \lambda_1 = 0$
  - a mixture $\rightarrow \lambda_1 \neq 0, f_7 \neq 0$

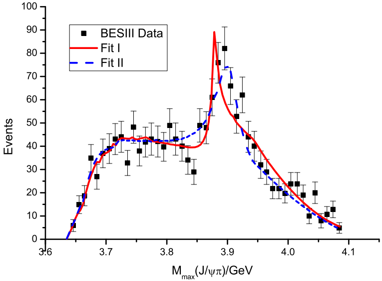
FIT TO $J/\Psi\pi\pi$ CHANNEL, $\pi\pi$ SPECTRUM

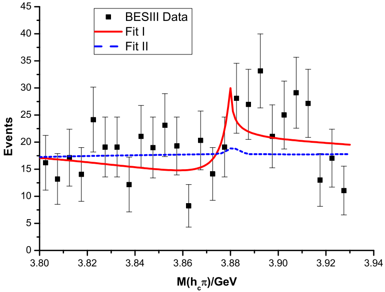


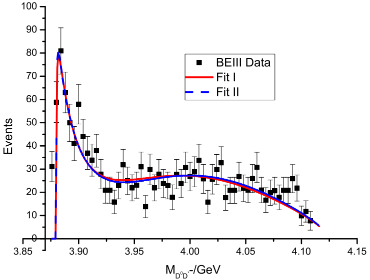


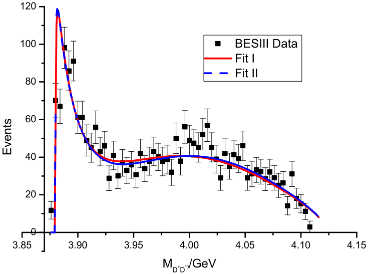

BESIII Data
Fit I
Fit II
Events
M ππ /GeV
0
20
40
60
80
100
120
0.2
0.4
0.6
0.8
1.0
1.2

Fit to $J/\Psi\pi$, $h_c\pi$, and $D\bar{D}^*$ spectrums






100
90
80
70
60
50
40
30
20
10
0
Events
3.85
3.90
3.95
4.00
4.05
4.10
4.15
$M_{D^0D^{*-}}$/GeV
BEIII Data
Fit I
Fit II


120
100
80
60
40
20
0
Events
3.85
3.90
3.95
4.00
4.05
4.10
4.15
$M_{D^+D^{*0}}$/GeV
BESIII Data
Fit I
Fit II

FIT QUALITY

- Fit quality:

Fit method	Pure bubble (Fit I)	Pure Breit-Wigner (Fit II)
$\frac{\chi^2}{\text{d.o.f}}$	$\frac{454}{291-28} = 1.726$	$\frac{497}{291-26} = 1.875$

- A mixed fit does not obviously improve the total χ^2.
- The overall quality of Fit I and Fit II are quite similar !

POLE SEARCHING

- Ignorance of $h_c\pi$ channel $\rightarrow$ a two channel system
- I $\sim$ IV Riemann sheet:
  $(\rho_{J/\Psi\pi}, \rho_{D\bar{D}^*}) \sim (+,+)\,(-,+)\,(-,-)\,(+,-)$
- Pole position (only near-threshold poles)

	Sheet I	Sheet II	Sheet III	Sheet IV
Fit I	–	$3.87988 \pm 0.00390i$ GeV	–	–
Fit II	–	–	–	$3.87909 \pm 0.00143i$ GeV

- Only one pole near threshold $\rightarrow$ Z_c is $D\bar{D}^*$ molecular state !
  (Fit II also contains another far away pole.)

Summary

- Main propose: study the nature of $Z_c(3900)$ state.
- Effective Lagrangians with C,P,χ and isospin symmetry are employed.
- Near threshold singularities are built via resummation.
- Three fits to experimental data are taken: one for pure dynamically generated Z_c, another for pure explicit Z_c field, and the last for mixture case.
- Different fits give similar χ^2, while only one pole is found in s plane. According to pole counting rule, Z_c is a $D\bar{D}^*$ molecular.

Some remarks

- A near-threshold state does not trivially correspond to a molecular state, e.g. $X(3872)$.
- An explicit Z_c field does not trivially leads to an elementary Z_c – the Breit-Wigner propagator always produces two poles, but in molecular case one of them would be far away from threshold.
- As for non-resonance case, anomalous triangle singularity (ATS) is investigated (Pang's talk).
- Alternative criterion to distinguish molecular states and elementary states besides pole counting rule?

SPECTRAL DENSITY FUNCTION SUM RULE

- Spectral density function sum rule [Baru et. al. PLB 586,53(2003)] : based on a coupled channel non-relativistic QM: $C(E)|E_0\rangle + \int \frac{d^3\mathbf{k}}{(2\pi)^3}\chi^{\mathbf{k}}(E)|E_{\mathbf{k}}\rangle$, one elementary state $|E_0\rangle$ plus infinite continuous states.
- For resonance states with energy E and momentum k hidden in continuous state, $w(E) = \frac{\mu k}{2\pi^2}|C^{\mathbf{k}}(E)|^2$ $(E = k^2/2\mu)$. μ: two particle reduced mass.
- Sum rule: $|C(-B)|^2 + \int_0^\infty w(E)dE = 1$, with $Z = |C(-B)|^2$ the possibility for a bound state to be elementary.
- The possibility for a resonance to be elementary: $\mathcal{Z} = \int_{E_{\text{min}}}^{E_{\text{max}}} w(E)dE$. The integral interval is where the resonance locate.

SPECTRAL DENSITY FUNCTION SUM RULE

- For Z_c, two channel Flatté formula ($E = 0$: $D\bar{D}^*$ threshold)

$$w(E) = \frac{1}{2\pi} \frac{g_1\sqrt{2\mu E}\theta(E) + \Gamma_0}{|E - E_0 + \frac{i}{2}g_1\sqrt{2\mu E} + \frac{i}{2}\Gamma_0|^2}$$

- Parameters from fit and matching: $E_0 = 0.02800$ GeV, $\Gamma_0 = 0.01333$ GeV, $g_1 = 0.4910$.
- Integral intervals: $[E_c - n\Gamma, E_c + n\Gamma]$, for $E_c = 0.005000$ GeV and $\Gamma = 0.02984$ GeV; $n = 1/2, 1, 2, 3$.

SPECTRAL DENSITY FUNCTION SUM RULE

- Results

n	$1/2$	1	2	3
$\mathcal{Z}$	12.58%	20.62%	31.97%	39.67%

- This method contains some ambiguities and uncertainties since the integral interval is arbitrary to some extent.
- However, the possibility is still smaller than 50% even if the interval become as large as 6Γ. So in quality Z_c is a molecular state.

Thank you!

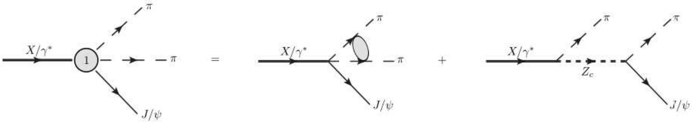
Appendix

Decomposition of Amp.s

- D, D^* propagators:
  - $\Pi_{\mu\nu} = \int \frac{\mathrm{d}^D k}{(2\pi)^D} \frac{g_{\mu\nu} - \frac{k_\mu k_\nu}{m_{D^*}^2}}{(k^2 - m_{D^*}^2)[(p-k)^2 - m_D^2]} = P_{T\mu\nu}\Pi_T + P_{L\mu\nu}\Pi_L$
  - $P_{T\mu\nu} = g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}$,$P_{L\mu\nu} = \frac{p_\mu p_\nu}{p^2}$
- Transverse and longitudinal components (p: the momentum of Z_c):
  - transverse part $1 - i(\lambda_1 + \frac{f_7^2}{p^2 - m_Z^2})\Pi_T \to$ pertinent poles
  - longitudinal part $1 - i(\lambda_1 - \frac{f_7^2}{m_Z^2})\Pi_L \to$ far away poles

PARTIAL WAVE ANALYSIS & FSI

- Extract S-wave part for contact tree vertex



  - $\Gamma_{Z_c} \approx$ 40MeV $\to$ ignorable final state interaction (FSI) in the second picture
- For $X(4260) \to J/\psi(h_c)\pi\pi$ FSI:
  - $\mathcal{A} = \mathcal{A}^{\text{tree}}_{X\to J/\Psi\pi\pi}\alpha_1(s)T_{\pi\pi\to\pi\pi} + \mathcal{A}^{\text{tree}}_{X\to J/\Psi K\bar{K}}\alpha_2(s)T_{K\bar{K}\to\pi\pi} + \mathcal{A}'$
  - $\alpha_i = \frac{c_0^i}{s-s_A} + c_1^i + c_2^i s + \cdots$
    (s_A: Adler zero. $\mathcal{A}'$: amplitude w/o contact tree. T: amplitudes given by unitarized χ PT.)