哈密顿有效理论对强子共振态的研究

刘占伟　　兰州大学物理科学与技术学院

Collaborators:
Johnathan M. M. Hall, Waseem Kamleh, Derek B. Leinweber,
Finn M. Stokes, Anthony W. Thomas, and Jia-Jun Wu

Department of Physics, University of Adelaide, SA5005

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CONTENTS

Introduction

Hadron Physics

Hadron physics is mainly focused on hadron scatterings, spectra, structures, interactions, etc.

- Hadron spectra are obtained from experimental
  Hadron scattering.
- Hadron structures and interactions $\rightleftharpoons$
  Hadron spectra and scattering.

Two main data sources:

- Scattering data from experiment
- Spectra simulated by lattice QCD (LQCD)
  - From the first principle of QCD, LQCD gives at finite volume hadron spectra and quark distribution functions.

Connection between Scattering Data and Lattice QCD Data

Lattice QCD

- large pion mass: extrapolation
- finite volume
- discrete space

Lattice QCD Data → Physical Data

- Lüscher Formalisms and extensions:
  Model independent; efficient in single-channel problems
    Spectrum → Phaseshifts; $m_{K_L} - m_{K_S}$ etc.
- Effective Field Theory (EFT), Models, etc
  with low-energy constants fitted by Lattice QCD data

Physical Data → Lattice QCD Data

- EFT: discretization, analytic extension, Lagrangian modification
- various discretization: eg. discretize the momentum in the loop

Hamiltonian Effective Field Theory (HEFT)

analyses both experimental data at infinite volume

and lattice QCD results at finite volume at the same time.

- at infinite volume

  Lagrangian (via 2-particle irreducible diagrams) $\longrightarrow$

  potentials (via Betha-Salpeter Equation) $\longrightarrow$

  phaseshifts and inelasticities

- at finite volume

  potentials discretized (via Hamiltonian Equation)$\longrightarrow$ spectra

- finite-volume and infinite-volume results are connected by the coupling constants etc.

This Work

We use Hamiltonian effective field theory to analyse the scatterings data at experiment and spectra of lattice QCD which are related to

- $N^*(1535)$
- $N^*(1440)$
- $\Lambda(1405)$

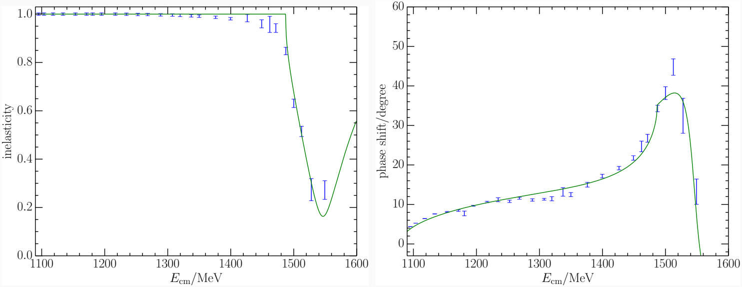
By our analyses, we try to better understand the structures of those resonances and relevant interactions.

Hamiltonian effective field theory study of the $N^*(1535)$ resonance in lattice QCD

$N^*(1535)$ with πN Scattering

$N^*(1535)$ is the lowest resonance with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$.

- One needs to consider the interactions
  among the bare baryon N_0^*, πN channel, and ηN channel.
- Phase shifts and inelasticities
  are obtained by solving Bethe-Salpeter equation with the interactions.



πN Scattering with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$.

- Pole position for $N^*(1535)$: $1531 \pm 29 - \mathrm{i}\ 88 \pm 2$ MeV.

Particle Data Group (PDG): $1510{\pm}20 - \mathrm{i}\ 85 \pm 40$ MeV.

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$$G^2_{\pi N; N_0^*}(k) = \frac{3 g^2_{\pi N; N_0^*}}{4\pi^2 f^2} \omega_\pi(k)$$

$$V^S_{\pi N, \pi N}(k, k') = \frac{3 g^S_{\pi N}}{4\pi^2 f^2} \frac{m_\pi + \omega_\pi(k)}{\omega_\pi(k)} \frac{m_\pi + \omega_\pi(k')}{\omega_\pi(k')} \tag{1}$$

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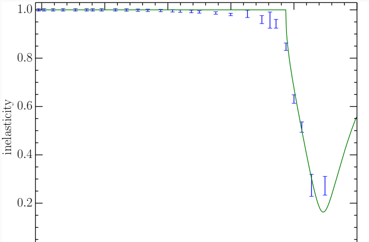

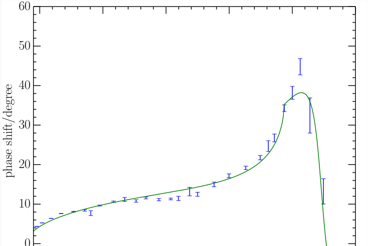

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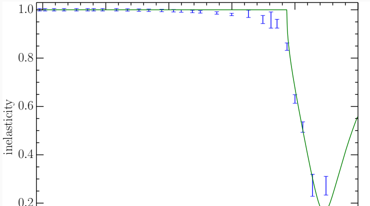
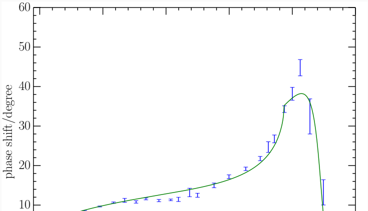
$$T_{\alpha,\beta}(k,k';E) = V_{\alpha,\beta}(k,k') + \sum_{\gamma} \int q^2 dq$$

$$V_{\alpha,\gamma}(k,q) \frac{1}{E - \sqrt{m_{\gamma_1}^2 + q^2} - \sqrt{m_{\gamma_2}^2 + q^2} + i\epsilon} T_{\gamma,\beta}(q,k';E)$$





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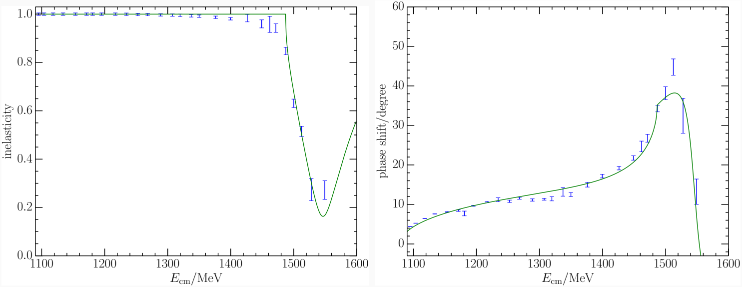

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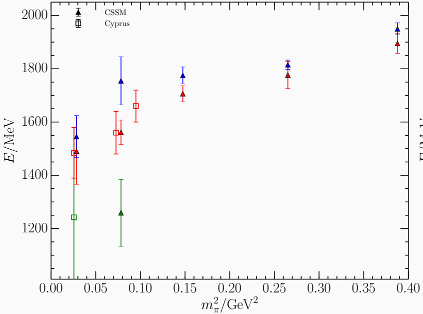
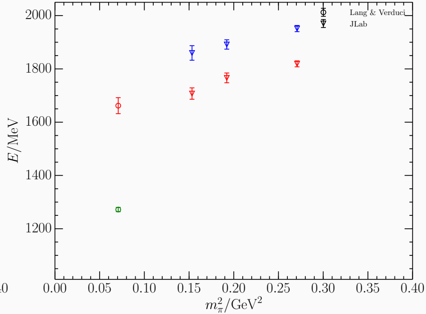
3 sets of lattice data at different pion masses and finite volumes



$L \approx 3$ fm



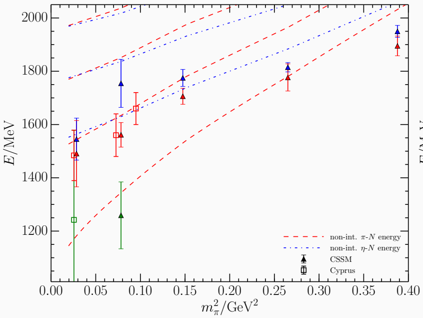
$L \approx 2$ fm

Spectra with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ at finite volumes

Spectra at Finite Volumes

3 sets of lattice QCD data at different pion masses and finite volumes

Non-interacting energies of the two-particle channels



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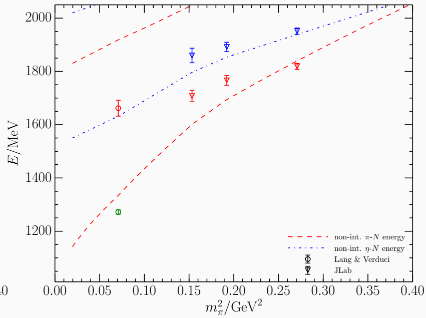

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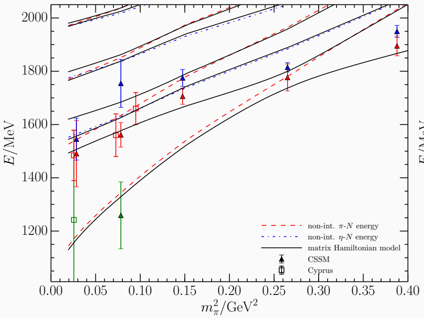
Spectra with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ at finite volumes

Spectra at Finite Volumes

3 sets of lattice QCD data at different pion masses and finite volumes
Non-interacting energies of the two-particle channels
Eigenenergies of Hamiltonian effective field theory



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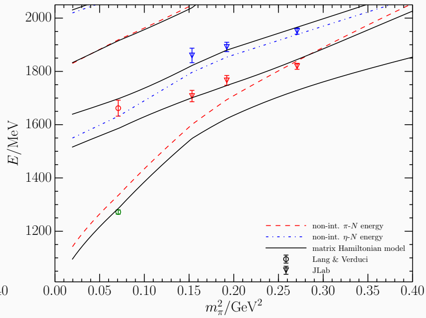

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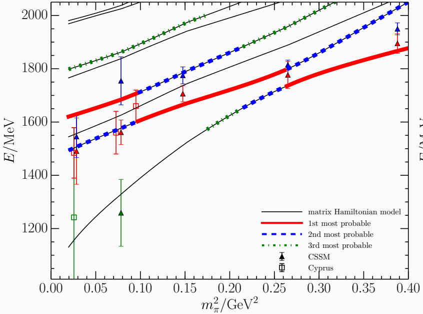
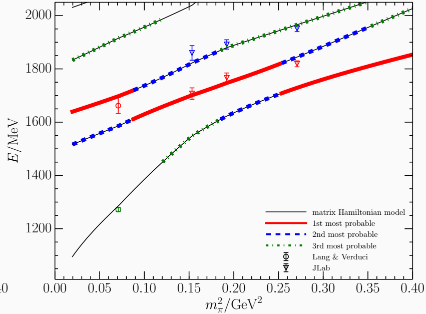
Eigenenergies of Hamiltonian effective field theory

Coloured lines indicating most probable states observed in LQCD



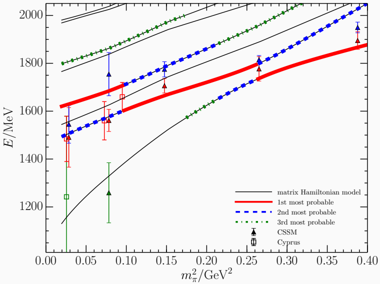
$L \approx 3$ fm



$L \approx 2$ fm

Spectra with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ at finite volumes

Components of Eigenstates with $L \approx 3$ fm

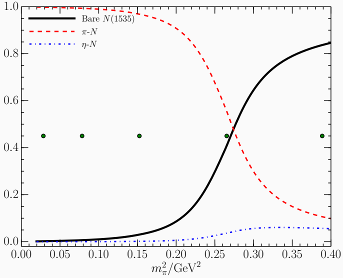


Spectra with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ and $L \approx 3$ fm

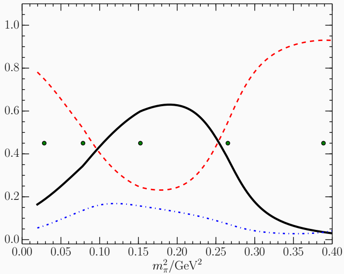
- The 1st eigenstate at light quark masses is mainly πN scattering states.
- The most probable state at physical quark mass is the 4th eigenstate.

  It contains about 60% bare $N^*(1535)$, 20% πN and 20% ηN.

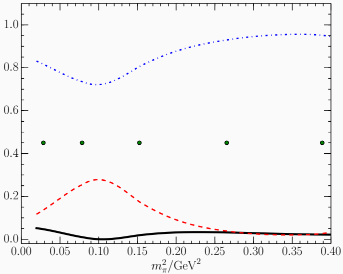
Components of Eigenstates with $L \approx 3$ fm



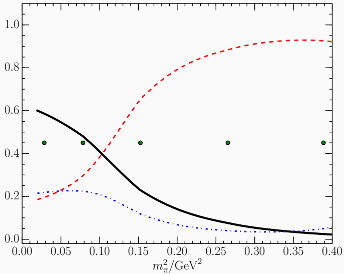
1st eigenstate



2nd eigenstate

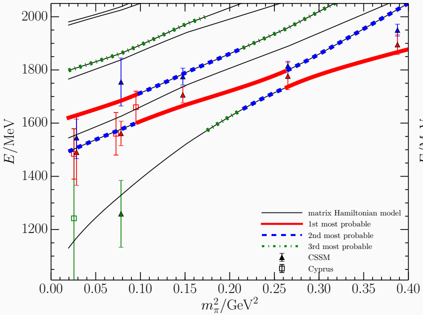


3rd eigenstate

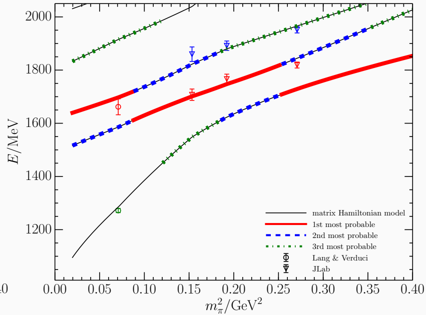


4th eigenstate

- Experimental Data → Lattice Data — We have shown that.
- Lattice Data → Experimental Data — We show it here.



$L \approx 3$ fm



$L \approx 2$ fm

Spectra with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ and the bare mass is fitted by LQCD data

By fitting lattice data, the pole position for $N^*(1535)$ at infinite volume is $1602 \pm 48 - i\ 88.6^{+0.7}_{-2.8}$ MeV. PDG: $1510{\pm}20 - i\ 85 \pm 40$.

Hamiltonian effective field theory study of the $N^*(1440)$ resonance in lattice QCD

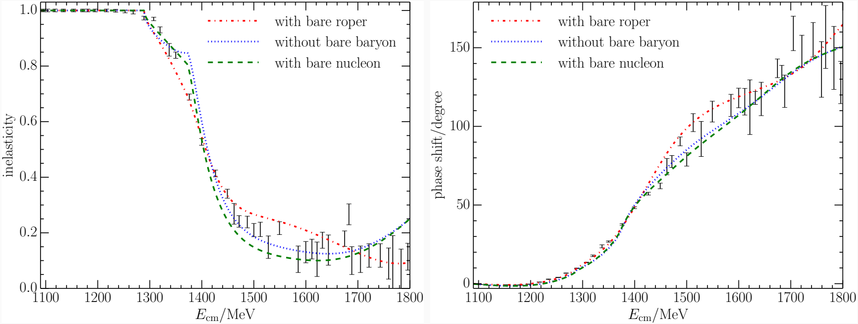
$N^*(1440)$ Resonance

- $N^*(1440)$, usually called Roper, is the excited state $I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$
- Naive quark model predicts $m_{N^*(1440)} > m_{N^*(1535)}$
  if they are both dominated by 3-quark core. But contrary to experiment.

To check whether a 3-quark core largely exists in Roper, we consider models

- with a bare Roper
- without any bare baryons
- including the effect of the bare nucleon

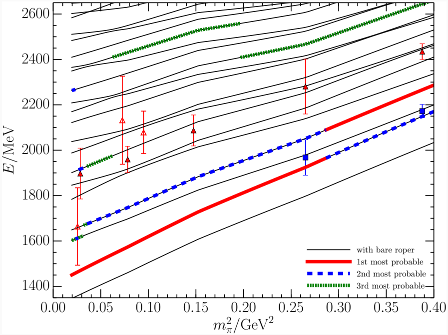
$N^*(1440)$ Resonance



πN scattering with $I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$

- with a bare Roper
- without any bare baryons
- including the effect of the bare nucleon

Results of the Model with a Bare Roper

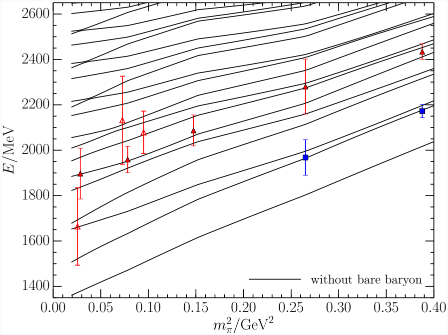


Spectrum given by the scenario with a bare Roper.
$I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$ and $L \approx 3$ fm.

At low pion masses, the 2nd state contains more than 20% bare Roper, so this state should be observed with a 3-quark interpolating operators on the lattice.

But it is not.

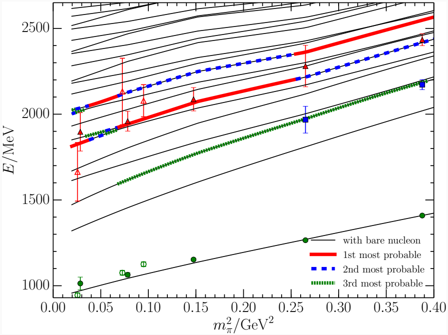
Results of the Model without Bare Baryons



Spectrum given by the scenario without any bare baryon.
$I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$ and $L \approx 3$ fm.

- The lattice data sit on the eigenenergy spectrum of this model;
- ALTHOUGH it is hard to predict which state is easier to observe on the lattice,
- we notice that lattice QCD prefers to extract eigenstates with non-trivial mixing of scattering states.

Including the Effect of the Bare Nucleon



Spectrum given by the scenario with a bare nucleon.
$I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$ and $L \approx 3$ fm.

- The bare nucleon does not affect the spectrum very much compared to the results of the model without any bare baryons;
- We can plot the probability based on the distribution of the bare nucleon;
- It can explain both the experimental data and lattice data.

Structure of the $\Lambda(1405)$ from Hamiltonian effective field theory

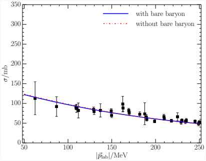
$\Lambda(1405)$ with $K^- p$ scattering

- The well-known Weinberg-Tomozawa potentials are used.

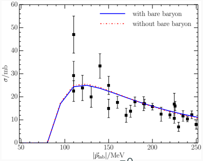
  momentum-dependent, non-separable

- We can fit the cross sections of $K^- p$ well

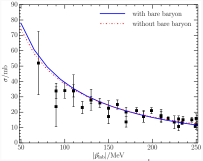
  both with and without a bare baryon.



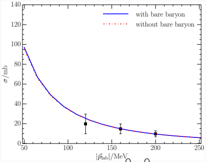
$K^- p \to K^- p$



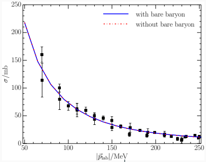
$K^- p \to \bar{K}^0 n$



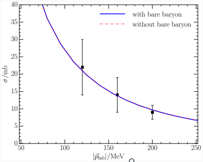
$K^- p \to \pi^- \Sigma^+$



$K^- p \to \pi^0 \Sigma^0$



$K^- p \to \pi^+ \Sigma^-$



$K^- p \to \pi^0 \Lambda$

- Two-pole structure of $\Lambda(1405)$

  $1430 - i\,22$ MeV, $\quad 1338 - i\,89$ MeV

Λ(1405) with $K^- p$ scattering

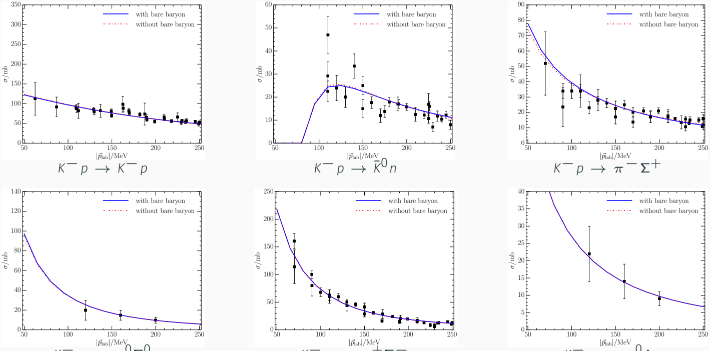
- The well-known Weinberg-Tomozawa potentials are used.

  momentum-dependent, non-separable

$$V_{\alpha,\beta}(k,k') = g_{\alpha,\beta} \frac{\omega_{\alpha_M}(k) + \omega_{\beta_M}(k')}{8\pi^2 f^2 \sqrt{2\omega_{\alpha_M}(k)}\sqrt{\omega_{\beta_M}(k')}}$$

- We can fit the cross sections of $K^- p$ well

  both with and without a bare baryon.



$\Lambda(1405)$ with $K^- p$ scattering

- The well-known **Weinberg-Tomozawa** potentials are used.

  momentum-dependent, non-separable

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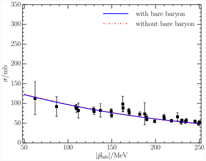
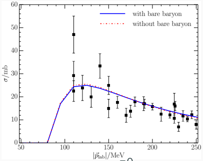
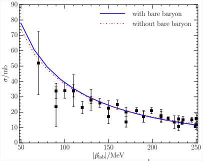


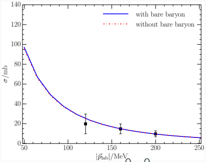
$K^- p \to K^- p$



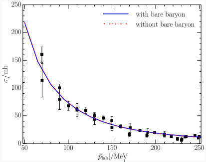
$K^- p \to \bar{K}^0 n$



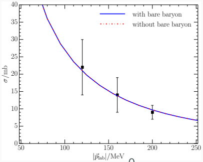
$K^- p \to \pi^- \Sigma^+$



$K^- p \to \pi^0 \Sigma^0$



$K^- p \to \pi^+ \Sigma^-$

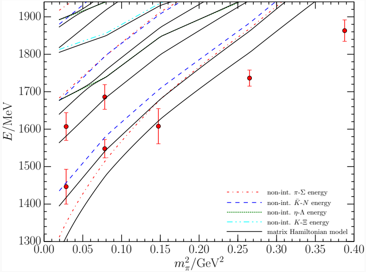


$K^- p \to \pi^0 \Lambda$

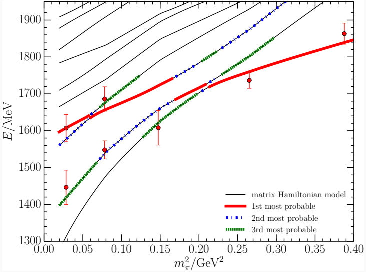
- **Two-pole structure** of $\Lambda(1405)$

  $1430 - i\,22$ MeV, $1338 - i\,89$ MeV

Spectrum on the Lattice



without a bare baryon



with a bare baryon

Spectra with $S = -1, I(J^P) = 0(\frac{1}{2}^-)$ in the finite volume.

- The bare baryon is important for interpreting the lattice QCD data at large pion masses.
- $\Lambda(1405)$ is mainly a $\bar{K}N$ molecular state containing very little of bare baryon at physical pion mass.

Summary

Summary

We have analysed the scattering data at experiment and the lattice spectra on the lattice relevant to $N^*(1440)$, $N^*(1535)$, and $\Lambda(1405)$ with Hamiltonian effective field theory

- $N^*(1535)$ contains a 3-quark core;
- $N^*(1440)$ should contain little of 3-quark consistent;
- $\Lambda(1405)$ is mainly a $\bar{K}N$ molecular state at physical quark mass, while a 3-quark core dominates at large quark masses.