$X(4140)$, $X(4272)$, $X(4500)$, and $X(4700)$ in the relativized quark model

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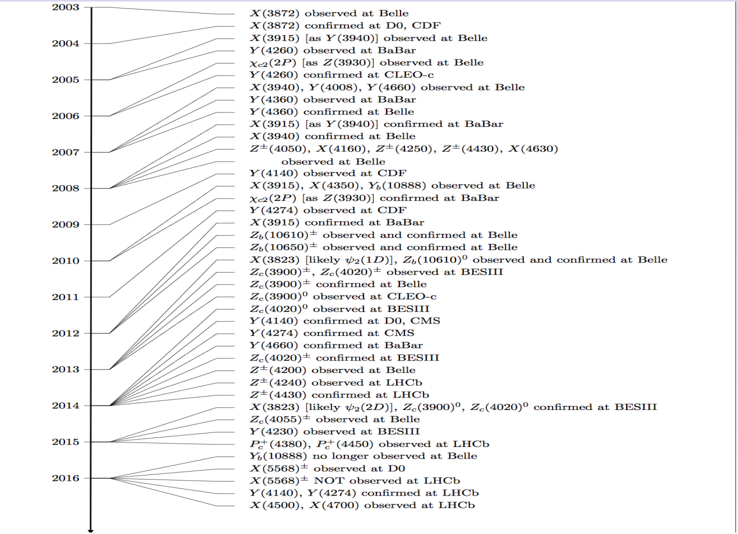
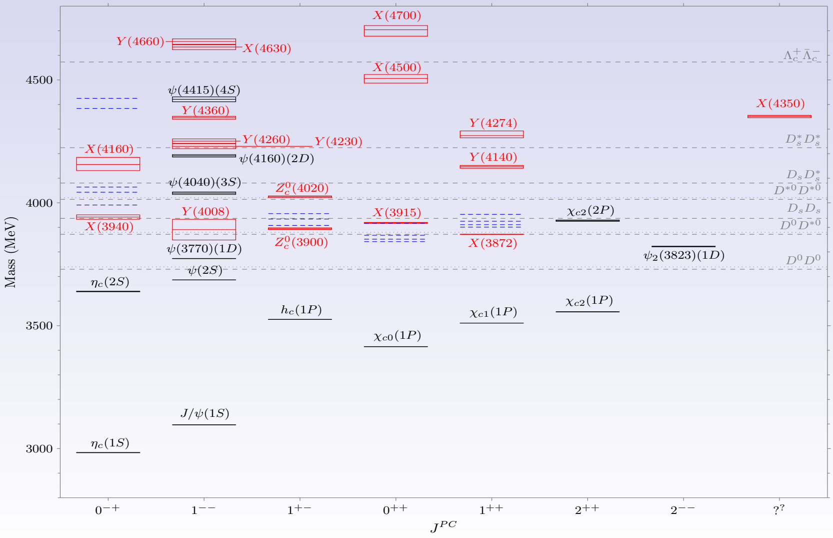
Outline

Time line of discoveries of XYZ states



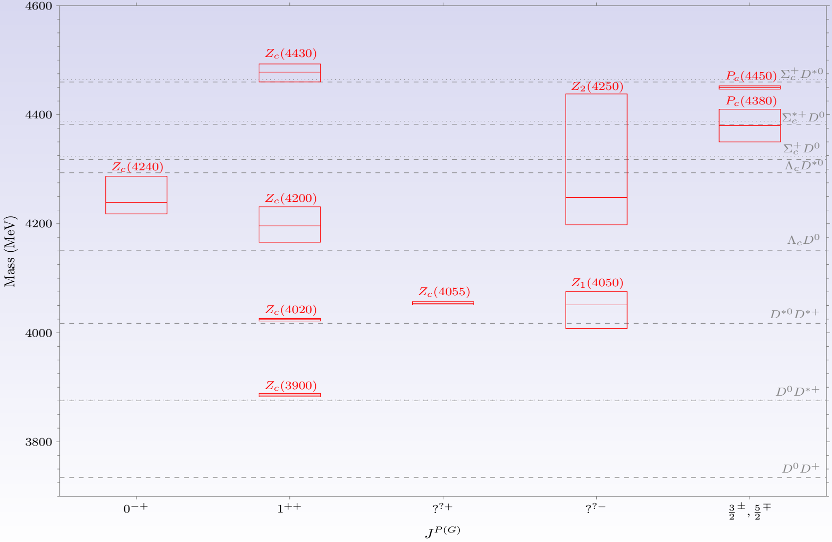
R. Lebed, R. Mitchell, E. S. Swanson, arXiv: 1610.04528(2016).

XYZ states



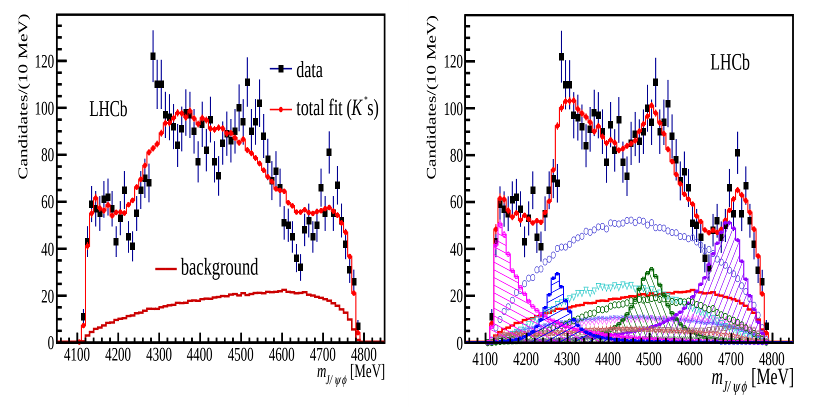
Mass (MeV)
4500
4000
3500
3000
J^{PC}
0^{-+}
1^{--}
1^{+-}
0^{++}
1^{++}
2^{++}
2^{--}
$?^?$
$Y(4660)$
$X(4630)$
$X(4700)$
$X(4500)$
$\Lambda_c^+\bar{\Lambda}_c^-$
$\psi(4415)(4S)$
$Y(4360)$
$Y(4260)$
$Y(4230)$
$\psi(4160)(2D)$
$X(4160)$
$Y(4274)$
$Y(4140)$
$X(4350)$
$D_s^* D_s^*$
$D_s D_s^*$
$D^{*0}\bar{D}^{*0}$
$\psi(4040)(3S)$
$Z_c^0(4020)$
$Y(4008)$
$X(3915)$
$\chi_{c2}(2P)$
$D_s D_s$
$D^0 \bar{D}^{*0}$
$X(3940)$
$Z_c^0(3900)$
$X(3872)$
$\psi(3770)(1D)$
$\psi_2(3823)(1D)$
$D^0 \bar{D}^0$
$\psi(2S)$
$\eta_c(2S)$
$h_c(1P)$
$\chi_{c1}(1P)$
$\chi_{c2}(1P)$
$\chi_{c0}(1P)$
$J/\psi(1S)$
$\eta_c(1S)$

XYZ states



Mass (MeV)
4600
4400
4200
4000
3800
$Z_c(4430)$
$Z_c(4240)$
$Z_c(4200)$
$Z_c(4020)$
$Z_c(3900)$
$Z_c(4055)$
$Z_2(4250)$
$Z_1(4050)$
$P_c(4450)$
$P_c(4380)$
$\Sigma_c^+ D^{*0}$
$\Sigma_c^{*+} D^0$
$\Sigma_c^+ D^0$
$\Lambda_c D^{*0}$
$\Lambda_c D^0$
$D^{*0} D^{*+}$
$D^0 D^{*+}$
$D^0 D^+$
0^{-+}
1^{++}
$?^{?+}$
$?^{?-}$
$\frac{3}{2}^\pm, \frac{5}{2}^\mp$
$J^{P(G)}$

$X(4140)$, $X(4274)$, $X(4500)$, and $X(4700)$



The $J/\psi\phi$ invariant mass distribution in $B^+ \to J/\psi\phi K^+$ decays without and with X states.

R. Aaij et al. [LHCb Collaboration], arXiv:1606.07895(2016).
R. Aaij et al. [LHCb Collaboration], arXiv:1606.07898(2016).

$X(4140)$, $X(4274)$, $X(4500)$, and $X(4700)$

$$(M;\Gamma)_{X(4140)} = (4146.5 \pm 4.5^{+4.6}_{-2.8};\ 83 \pm 21^{+21}_{-14})\ \mathrm{MeV},$$

$$(M;\Gamma)_{X(4274)} = (4273.3 \pm 8.3^{+17.2}_{-3.6};\ 56 \pm 11^{+8}_{-11})\ \mathrm{MeV},$$

$$(M;\Gamma)_{X(4500)} = (4506 \pm 11^{+12}_{-15};\ 92 \pm 21^{+21}_{-20})\ \mathrm{MeV},$$

$$(M;\Gamma)_{X(4700)} = (4704 \pm 10^{+14}_{-24};\ 120 \pm 31^{+42}_{-33})\ \mathrm{MeV}.$$

$X(4140)$ and $X(4274)$: $J^{PC} = 1^{++}$.
$X(4500)$ and $X(4700)$: $J^{PC} = 0^{++}$.
Other two states observed in the $\gamma\gamma \to J/\psi\phi(\omega)$ fusion.
$X(4350)$: $J^{PC} = 0^{++}$ or 2^{++}.
$X(3915)$ (or $\chi_{c0}(2P)$): $J^{PC} = 0^{++}$ or 2^{++}.

Interpretation

1. $cs\bar{c}\bar{s}$ diquark-antidiquark or compact tetraquark states (Popular).
2. Kinematic effects or cusps (Partly).
3. Conventional charmonium (Partly).
4. Molecular states (Hardly).

Most of them are obtained with the QCD sum rule and the quark model with only spin-spin interactions. We need calculate them in a realistic potential model.

H. X. Chen, E. L. Cui, W. Chen, X. Liu and S. L. Zhu, arXiv:1606.03179.
Z. G. Wang, arXiv:1606.05872.
X. H. Liu, arXiv:1607.01385.
L. Maiani, A. D. Polosa and V. Riquer, Phys. Rev. D 94 054026 (2016).
R. Zhu, Phys. Rev. D 94 054009 (2016).
More references:
`http://inspirehep.net/search?ln=zh_CN&p=refersto%3Arecid%3A1472310`

Relativized quark model

The Hamiltonian in the relativized quark model proposed by Godfrey and Isgur.

$$\tilde{H} = H_0 + \tilde{V}(\boldsymbol{p}, \boldsymbol{r}),$$

$$H_0 = (\boldsymbol{p}^2 + m_1^2)^{1/2} + (\boldsymbol{p}^2 + m_2^2)^{1/2},$$

$$\tilde{V}(\boldsymbol{p}, \boldsymbol{r}) = G_{eff}(r) + S_{eff}(r) = \tilde{H}_{12}^{\mathrm{conf}} + \tilde{H}_{12}^{\mathrm{cont}} + \tilde{H}_{12}^{\mathrm{ten}} + \tilde{H}_{12}^{\mathrm{so}},$$

For the quark-quark interaction in a diquark, the relation $\tilde{V}_{qq}(\boldsymbol{p}, \boldsymbol{r}) = \tilde{V}_{q\bar{q}}(\boldsymbol{p}, \boldsymbol{r})/2$. We follow the route employed by Ebert, Faustov, and Galkin. The Gaussian Expansion Method are used for numerical calculations.

S. Godfrey and N. Isgur, Phys. Rev. D 32, 189 (1985).
D. Ebert, R. N. Faustov and V. O. Galkin, Phys. Lett. B **696**, 241 (2011).
E. Hiyama, Y. Kino and M. Kamimura, Prog. Part. Nucl. Phys. 51, 223 (2003).

Potential

$$
\begin{aligned}
G_{eff}(r) &= \left(1+\frac{p^2}{E_1E_2}\right)^{1/2}\tilde{G}(r)\left(1+\frac{p^2}{E_1E_2}\right)^{1/2} \\
&+\frac{\mathbf{S}_1\cdot\mathbf{L}}{2m_1^2}\frac{1}{r}\frac{\partial\tilde{G}_{11}^{so(v)}}{\partial r}+\frac{\mathbf{S}_2\cdot\mathbf{L}}{2m_2^2}\frac{1}{r}\frac{\partial\tilde{G}_{22}^{so(v)}}{\partial r} \\
&+\frac{(\mathbf{S}_1+\mathbf{S}_2)\cdot\mathbf{L}}{m_1m_2}\frac{1}{r}\frac{\partial\tilde{G}_{12}^{so(v)}}{\partial r}+\frac{2\mathbf{S}_1\cdot\mathbf{S}_2}{3m_1m_2}\tilde{\nabla}^2G_{12}^{c} \\
&-\left(\frac{\mathbf{S}_1\cdot\hat{\mathbf{r}}\mathbf{S}_2\cdot\hat{\mathbf{r}}-\frac{1}{3}\mathbf{S}_1\cdot\mathbf{S}_2}{m_1m_2}\right)\left(\frac{\partial^2}{\partial r^2}-\frac{1}{r}\frac{\partial}{\partial r}\right)\tilde{G}_{12}^{t},
\end{aligned}
$$

$$
S_{eff}(r) = \tilde{S}(r)-\frac{\mathbf{S}_1\cdot\mathbf{L}}{2m_1^2}\frac{1}{r}\frac{\partial\tilde{S}_{11}^{so(s)}}{\partial r}-\frac{\mathbf{S}_2\cdot\mathbf{L}}{2m_2^2}\frac{1}{r}\frac{\partial\tilde{S}_{22}^{so(s)}}{\partial r}.
$$

The potential is dependent with both coordinate and momentum.

Screening effects

The screening potential $br \to b(1 - e^{-\mu r})/\mu$ are used. The screening parameter μ varies from 0 to 0.04 GeV.

1. GI model is a typical quenched quark model. The coupled channel effects or the screening effects have been ignored.
2. The lower mass puzzle of $D^*_{s0}(2317)$ and $D_{s1}(2460)$.
3. The modified formalism with a new screening parameter gives a better description of the charmed-strange meson spectra at $\mu = 0.02$ GeV.

Q. T Song, D. Y. Chen, X. Liu, and T. Matsuki, Phys. Rev. D 91, 054031 (2015).

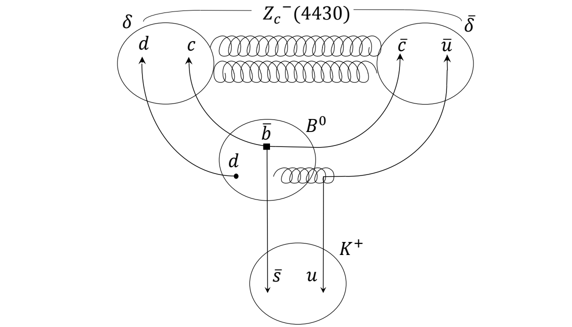
Masses of cs diquarks

Table : Obtained masses of the cs diquarks. S and A denote scalar and axial-vector diquarks in the ground states, respectively. The notation $n^{2S+1}P_J$ is used to stand for the excited diquarks. The brace and bracket correspond to symmetric and antisymmetric quark contents in flavor, respectively. The units are in MeV.

Quark content	Diquark type	Mass (GI model)	Mass ($\mu =$ 0.02 GeV)	Mass ($\mu =$ 0.04 GeV)
$[c,s]$	S	2230	2221	2212
$\{c,s\}$	A	2264	2254	2244
$[c,s]$	1^1P_1	2523	2503	2482
$\{c,s\}$	1^3P_0	2518	2496	2475
$[c,s]$	1^3P_1	2529	2508	2486
$[c,s]$	2^1S_0	2624	2593	2563
$\{c,s\}$	2^3S_1	2644	2612	2580
$\{c,s\}$	1^3D_1	2743	2708	2673

Diquark as pointlike antidiquark

The diquark is a pointlike antiquark or the distance between diquark and antidiquark is large enough. This can be partly understood by the production mechinism.



$$B^0 \to Z_c^-(4430)K^+ \to \psi(2S)\pi^- K^+.$$

S. J. Brodsky, D. S. Hwang and R. F. Lebed,

Phys. Rev. Lett. **113**, 112001 (2014).

Masses of $cs\bar{c}\bar{s}$ tetraquark states

Table : Masses of $cs\bar{c}\bar{s}$ tetraquark states composed of the S and A diquarks and antidiquarks in $1S$ and $2S$waves. In the $A\bar{S}$ case, the linear combinations together with $S\bar{A}$ are understood to form the eigenstates of charge conjugation. The units are in MeV.

J^{PC}	Diquark	Anti-diquark	$n+1$	S	L	Mass	Exotic Candidate
$\|0^{++}\rangle$	S	$\bar{S}$	1	0	0	4164	
$\|0^{++}\rangle$	A	$\bar{A}$	1	0	0	3962	$X(3915)$
$\|1^{++}\rangle$	A	$\bar{S}$	1	1	0	4195	$X(4140)$
$\|1^{+-}\rangle$	A	$\bar{S}$	1	1	0	4195	
$\|1^{+-}\rangle$	A	$\bar{A}$	1	1	0	4117	
$\|2^{++}\rangle$	A	$\bar{A}$	1	2	0	4302	
$\|0^{++}\rangle$	S	$\bar{S}$	2	0	0	4733	$X(4700)$
$\|0^{++}\rangle$	A	$\bar{A}$	2	0	0	4703	$X(4700)$
$\|1^{++}\rangle$	A	$\bar{S}$	2	1	0	4764	
$\|1^{+-}\rangle$	A	$\bar{S}$	2	1	0	4764	
$\|1^{+-}\rangle$	A	$\bar{A}$	2	1	0	4750	
$\|2^{++}\rangle$	A	$\bar{A}$	2	2	0	4833	

Masses of $cs\bar{c}\bar{s}$ tetraquark states

Table : Masses of $cs\bar{c}\bar{s}$ tetraquark states composed of of internal excited diquarks. When the diquark and antidiquark are in different types, the linear combinations are understood to form the eigenstates of charge conjugation. The units are in MeV. We only list the relevant ones.

Diquark	Antidiquark	$n+1$	S	L	Mass	Exotic Candidate
$\lvert \mathbf{0}^{++}\rangle$						
2^1S_0	$\bar{S}$	1	0	0	4516	$X(4500)$
2^3S_1	$\bar{A}$	1	0	0	4315	$X(4350)$
$\lvert \mathbf{1}^{++}\rangle$						
2^1S_0	$\bar{A}$	1	1	0	4547	
2^3S_1	$\bar{S}$	1	1	0	4534	
2^3S_1	$\bar{A}$	1	1	0	4461	

Character

1. The splitting coefficients are -2,-1,2 for the 0^+, 1^+, 2^+ $A\bar{A}$ states. (This is approximate, since we do not treat spin-spin interaction perturbatively.)
2. The $A\bar{A}$ with 0^+ are the lowest one rather than the $S\bar{S}$.
3. The mass gap between the $1S$ 0^{++} doublet is larger, while the gap between the $2S$ states is extremely small and the theoretical errors overlap with each other.
4. The 200 MeV mass gap prohibits the $X(4500)$ and $X(4700)$ as the same $2S$ doublet.
5. There is no room left for the $X(4274)$.

$1S$ and $2S$ $cs\bar{c}\bar{s}$ tetraquark states

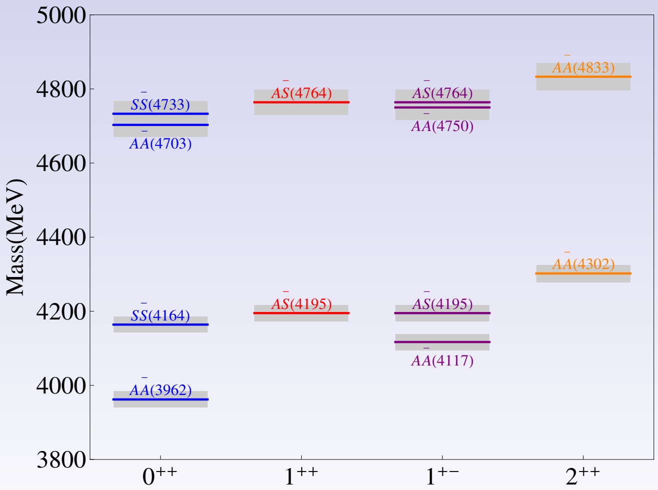


Figure : $1S$ and $2S$ $cs\bar{c}\bar{s}$ tetraquark states within S and A diquarks.

P wave charmonium up to 5 GeV

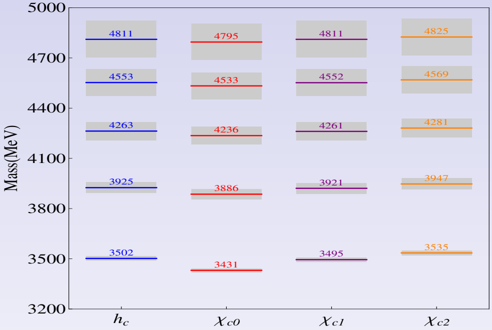


Figure : Mass of the P wave charmonium up to 5 GeV.

$X(4274)$ can be described as the conventional $\chi_{c1}(3P)$ charmonium state via mass, total width and production.

T. Barnes, S. Godfrey and E. S. Swanson, Phys. Rev. D 72, 054026 (2005).

Summary

1. We calculate the $cs\bar{c}\bar{s}$ spectra in relativized quark model.
2. $X(4140)$: 1^{++} tetraquark ground state.
3. $X(4500)$ and $X(4700)$ are the radial excited tetraquark states.
4. $X(4274)$ can be assigned as the conventional charmonium $\chi_{c1}(3P)$.

Q. F. Lü, and Y. B. Dong, Phys. Rev. D 94 074007 (2016).

Thank you for your attention!