“sigma” meson exchange effect in Lamb shift of muonic hydrogen

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2016.10.31, GuiLin

Outline

1. Introduction

(the energy spectrum of muonic hydrogen and the proton size)

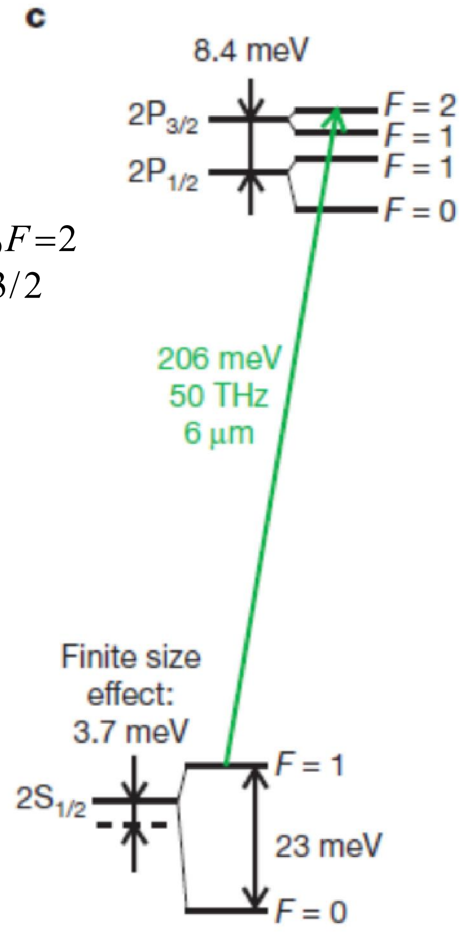
2. "meson" exchange effects between μp(ep)

3. Numerical results and discussion

Ex: the Lamb shift of muonic hydrogen

In 2010, a group in the PSI performed the first successful measurement of the μp Lamb shift (the energy difference between the 2S and 2P state) which gives[1]

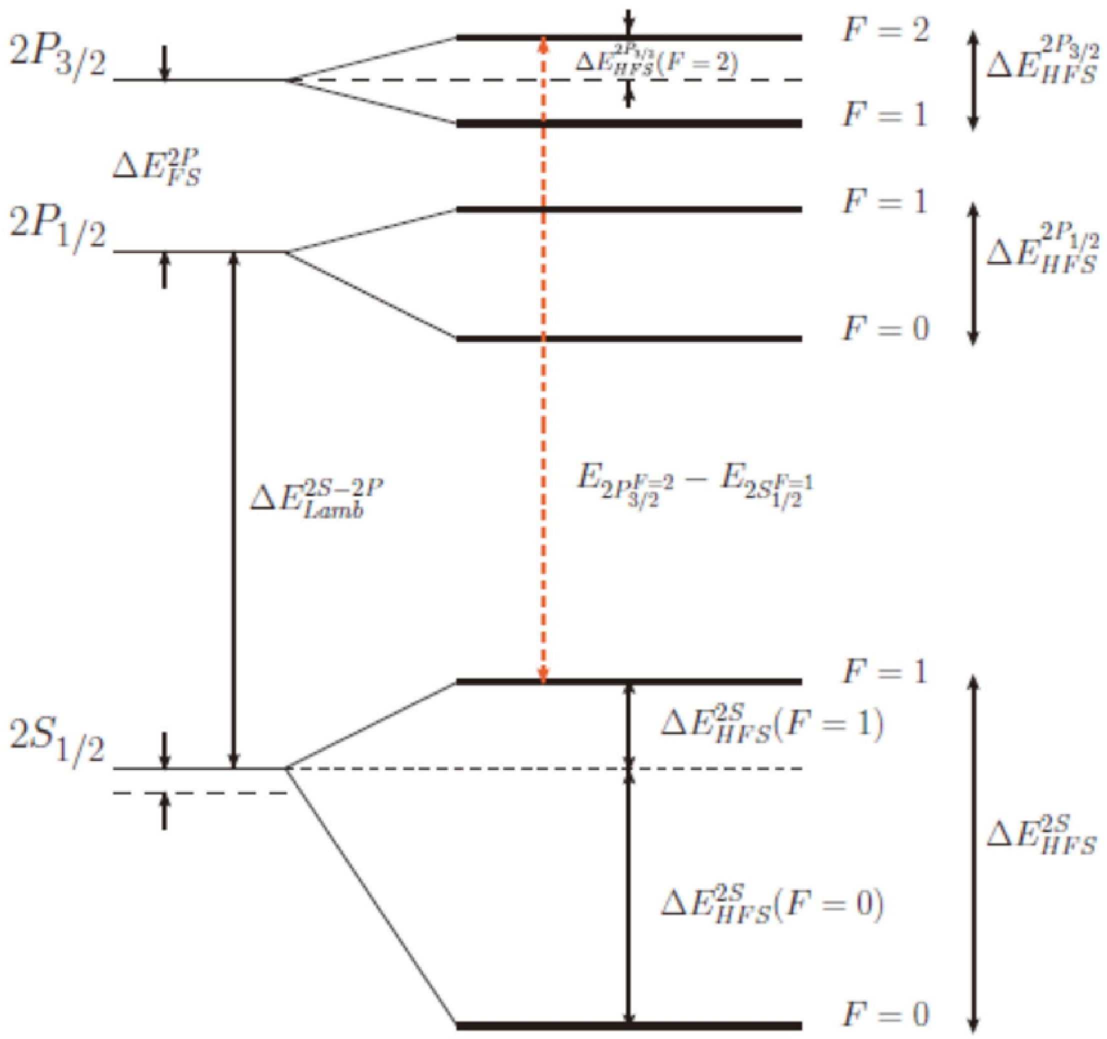
$$2S_{1/2}^{F=1} - 2P_{3/2}^{F=2}$$



$$E^{ex}_{2P_{3/2}^{F=2}} - E^{ex}_{2S_{1/2}^{F=1}} = 206.2949 \text{ meV}$$

[1] R. Pohl et al., Nature (London) 466, 213 (2010)

the energy levels of muonic hydrogen



Th: the Lamb shift and the proton size

The theoretical prediction of this energy difference can be expressed as [2]

$$E^{th}_{2P^{F=2}_{3/2}} - E^{th}_{2S^{F=1}_{1/2}} = 209.9779 - 5.2262 r_p^2 + 0.0347 r_p^3 \qquad (1)$$

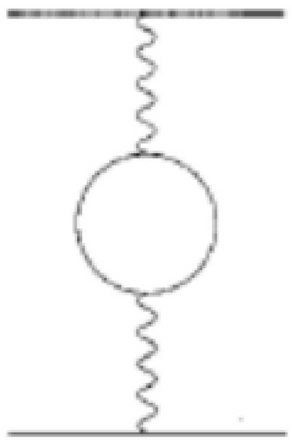
combining these two results,one can extract the size of proton, which gives

$$r_p = 0.84184(67)\ \text{fm}$$

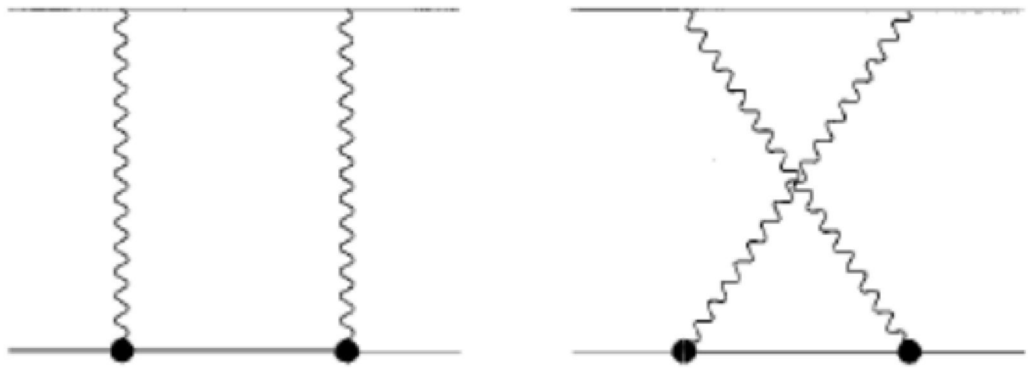
[2] Supplementary information for Nature doi: 10.1038/nature09250 and its references

Th: the Lamb shift and the proton size

some examples of the terms in the theoretical expression Eq.(1)



a



a b

FIG. 2. Proton structure corrections of order $(Z\alpha)^5$. Bold circle in the diagram represents the proton vertex operator.

Ex: the proton size from other methods

On another hand, from the Lamb shift of the hydrogen and the experimental data of the *ep* scattering, CODATA2006 gives [1]

$$r_p = 0.8768(69)\ \mathrm{fm}$$

vs. μp(2010)

$$r_p = 0.84184(67)\ \mathrm{fm}$$

Property: about 4% difference, much larger than the uncertainty.

[1] P. J. Mohr, B. N. Taylor, and D. B. Newell, Rev. Mod. Phys. **80**,633 (2008).

why the difference? – where is wrong?

the measurement/analyze of the *ep* scattering data?

the calculation of the energy shift?

the measurements of the energy shift?

....

why the difference? - wrong in the measurement?

“The most trivial explanation would be an unidentified error in the existing measurements.

However, both spectroscopic and nuclear scattering measurements have been reexamined and no problem has yet been found. Furthermore, all measurements except the muonic-hydrogen measurement were done by different groups and they all mutually agree.”

EPJ Web of Conferences 81,01009(2014)

why the difference? – wrong in the calculation?

"A mistake in the existing calculations or an overlooked higher order term do not appear to be a plausible scenario.

The calculations were made by several theoretical groups using different approaches and they all get very similar results . "

EPJ Web of Conferences 81,01009(2014)

CODATA2010 [1]

Based on the ep scattering data, CODATA2010 gives

$$r_p = 0.895(18)\ \text{fm}\ \ (\text{Sick})$$

$$r_p = 0.8791(79)\ \text{fm}\ \ (\text{Mainz})$$

also consistent with the results from the Lamb shift of the hydrogen.

*Based on a reanalysis of selected nucleon form-factor data, Adamuscin,Dubnicka, and Dubnickova (2012) find

$$r_p = 0.849(7)\ \text{fm}$$

[1] P. J. Mohr, B. N. Taylor, and D. B. Newell, Rev. Mod. Phys. **84**,1527 (2012).

CODAT2014 [1]

Fundamental Physical Constants

proton rms charge radius r_p	
Value	0.8751×10^{-15} m
Standard uncertainty	0.0061×10^{-15} m
Relative standard uncertainty	7.0×10^{-3}
Concise form	$0.8751(61) \times 10^{-15}$ m

[1] http://physics.nist.gov/cgi-bin/cuu/Value?rp|search_for=proton

Ex: Lamb shift of muonic hydrogen in 2013 [1]

the new measurements of the energy shift [1] , confirm the experimental result in 2010 and gives

$$r_E = 0.84087(39)\ \text{fm}$$

[1] A. Antognini et al., Science 339, 417 (2013).

why the difference? – wrong analysis in scattering?

the analyze the *ep* scattering data

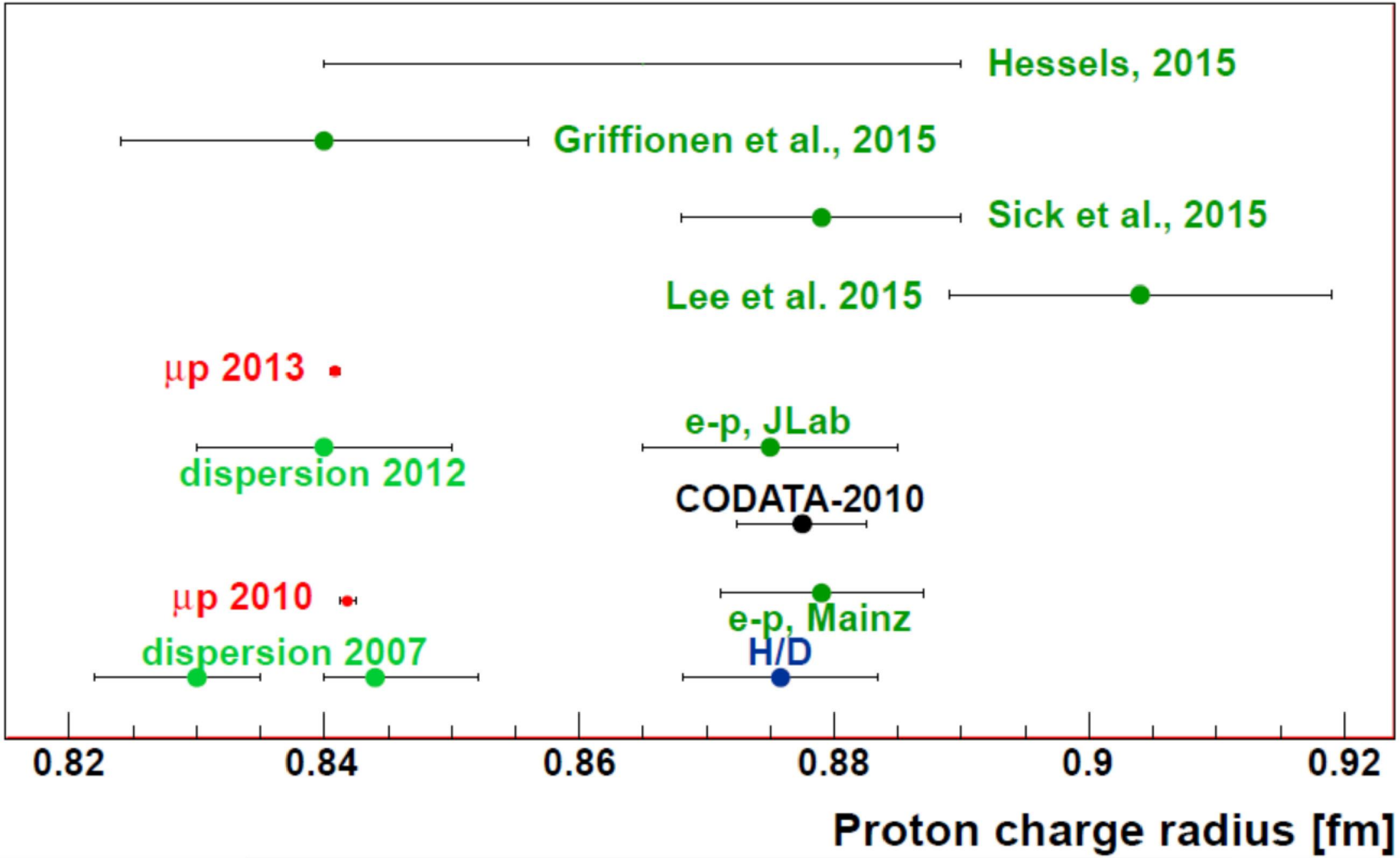
- many different results

the new low energy *ep* (*μp*) scattering experiments

- performing and plan

[1] A. Antognini et al., Science 339, 417 (2013).

r_{pm} from spectroscopy of muonic atoms (red),from electron scattering (green) and from H/D spectroscopy (blue).



A. Antognini, for the CREMA collaboration, arXiv:1512.01765

different r_p => different energy shifts

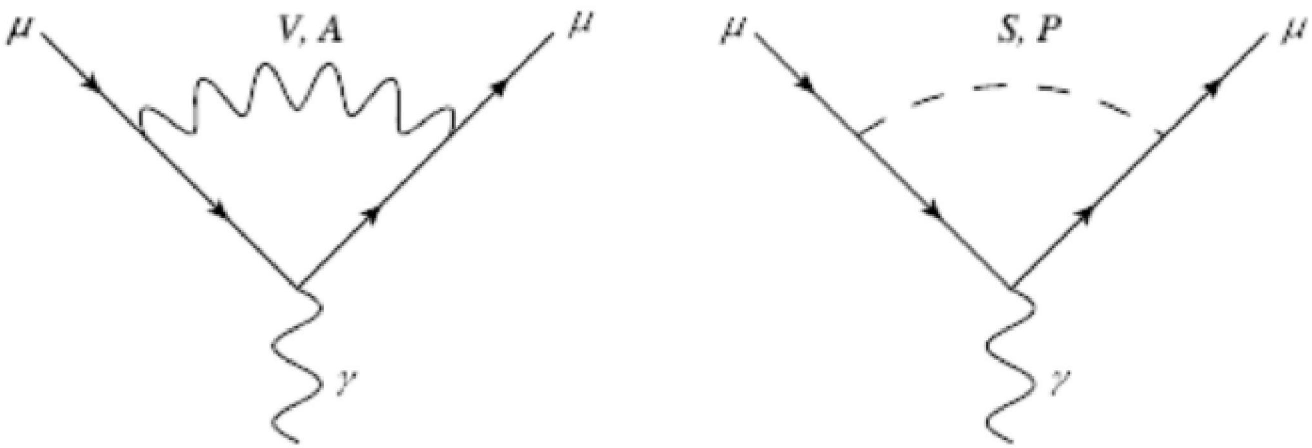
Note: if r_p=0.878 fm is taken, the theoretical prediction for the energy shift is about

$$E^{th}_{2P^{F=2}_{3/2}} - E^{th}_{2S^{F=1}_{1/2}} \,\big|_{r_p=0.87\,\mathrm{fm}} = 205.9726 \text{ meV}$$

which leads to about 0.32meV difference with the experimental result.

why the difference - new physics?

For example, the possibility that new scalar, pseudoscalar, vector, and tensor flavor-conserving nonuniversal interactions....



Corrections to the muon magnetic moment due to new particle exchange

D. Tucker-Smith and I. Yavin, Phys. Rev. D 83, 101702 (2011);
Vernon Barger *etc*, Phys.Rre.Lett. 106, 153001 (2011);
B. Batell, D. McKeen, and M. Pospelov, Phys. Rev. Lett. 107,011803 (2011);
C. E. Carlson and B. C. Rislow, Phys. Rev. D 86, 035013 (2012).

why the difference?

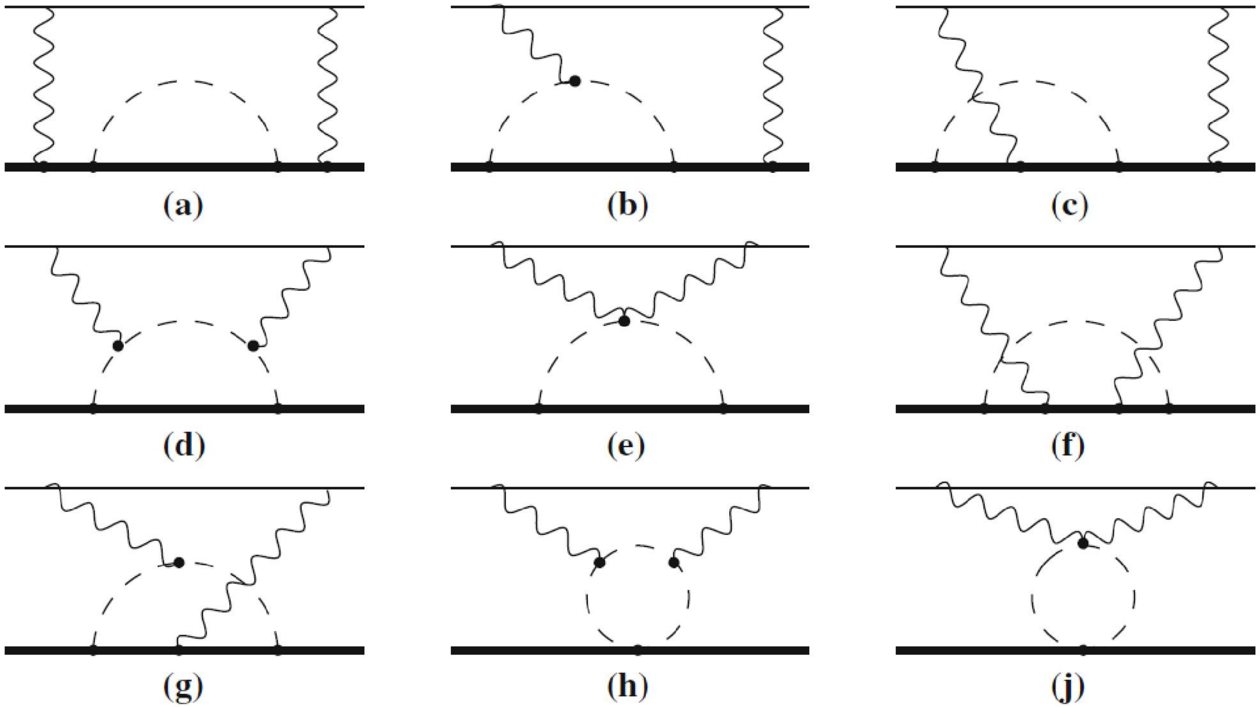
?

new contributions?

Two-photon exchange by BχPT[1]

Eur. Phys. J. C (2014) 74:2852 Page 3 of 10 2852

Fig. 1 The two-photon exchange diagrams of elastic lepton–nucleon scattering calculated in this work in the zero-energy (threshold) kinematics. Diagrams obtained from these by crossing and time-reversal symmetry are included but not drawn



O(p³)

$$E^{(pol)}(2P-2S)=8^{+3}_{-1}\ \mu\text{eV}$$

much smaller than 0.32 meV

[1] J. M. Alarcón,V. Lensky, V. Pascalutsa Eur. Phys. J. C (2014) 74:2852

Interactions in BχPT[1]

$$\mathcal{L}_{\pi}^{(2)} = \frac{f^2}{4}\,\mathrm{tr}\left(\partial^\mu U \partial_\mu U^\dagger + 2B_0(UM^\dagger + MU^\dagger)\right),$$

$$\mathcal{L}'^{(1)}_{\pi N} = \overline{N}\left(i\partial\!\!\!/ - M_N - i\,\frac{g_A}{f_\pi}M_N\tau^a\pi^a\gamma_5 + \frac{g_A^2}{2f_\pi^2}M_N\pi^2 + \frac{(g_A^2-1)}{4f_\pi^2}\tau^a\varepsilon^{abc}\pi^b\,\partial\!\!\!/\,\pi^c\right)N + \mathcal{O}(\pi^3),$$

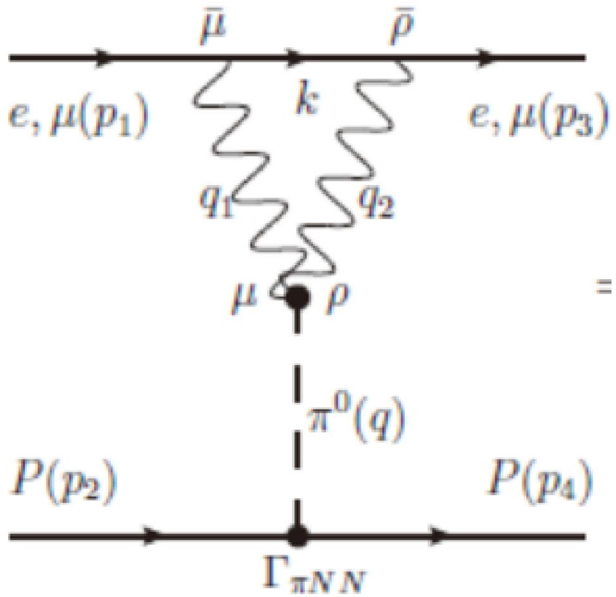
$$n = \sum_k kV_k + 4L - 2N_\pi - N_N\,. \qquad (\bigstar)$$

$$p \sim m_\pi \ll M_\Delta - M_N \ll 4\pi f_\pi$$

$$\mathcal{L}^{(1)}_{\pi N} = \overline{N}\left(i\partial\!\!\!/ - M_N + \frac{g_A}{2f_\pi}\tau^a\,\partial\!\!\!/\,\pi^a\gamma_5 - \frac{1}{4f_\pi^2}\tau^a\varepsilon^{abc}\pi^b\,\partial\!\!\!/\,\pi^c\right)N + \mathcal{O}(\pi^3),$$

Vadim Lensky, Vladimir Pascalutsa, EPJC 65 (2010) 195-209

one pion exchange between μp [1]



$$\Delta E_{HFS}^{2S}(F=1) = -0.5\ \mu\text{eV}$$

much smaller than 0.32 meV

[1] N. T. Huonga , E. Kou, B. Moussallamc, Phys.Rev. D93 (2016) no.11, 114005 , H.Q. Zhou etc. , Phys. Rev, A. 92 (2015) 032512.

“sigma” meson exchange between μp?

Phenomenologically, we ask is it possible to exchange a scalar meson between the μp by two photon coupling, this is based on the three arguments

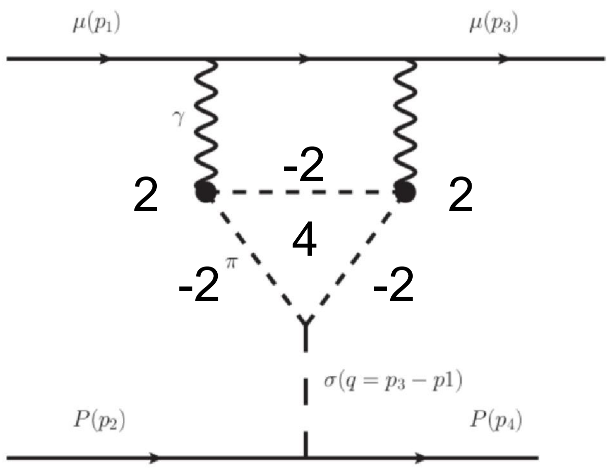
1: the “scalar” meson sigma exits.

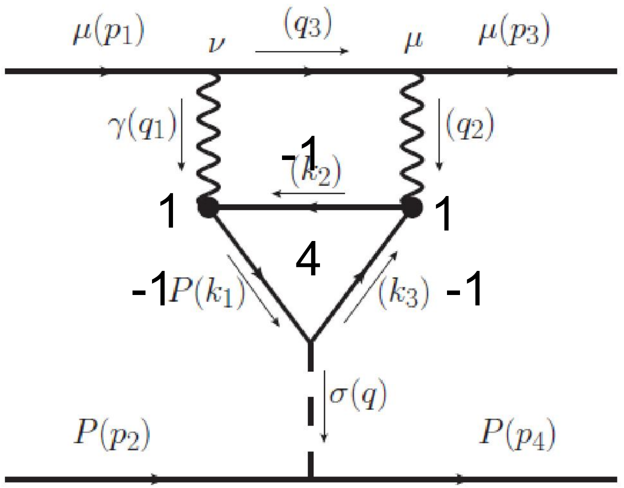
2. the “sigma” has the strong coupling to pion.

3. the “sigma” has the strong coupling to nucleon.

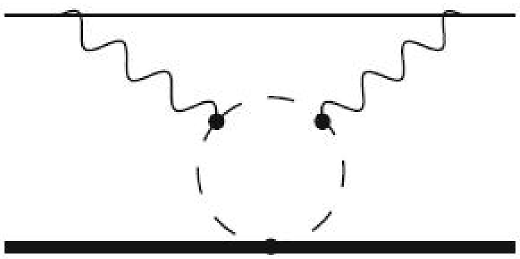
"scalar" meson exchange between μp

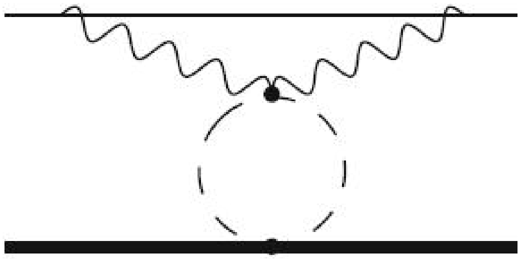
We consider the following diagram





vs.


(h)


(j)

“scalar” meson exchange between μp

We use the following effective couplings to describe the strong interactions for σπ and Nσ,the EM interaction for πγ,Nγ.

$$L_{\sigma\pi\pi} = -\frac{1}{2} g_{\sigma\pi\pi}\phi_\sigma\phi_\pi^+\phi_\pi$$

$$L_{\sigma NN} = -g_{\sigma NN}\overline{\psi}_p\psi_p\phi_\sigma$$

$$L_{\gamma\pi\pi} = (D_\mu\phi_\pi)^+ D^\mu\phi_\pi$$

$$L_{\gamma NN} = ie\overline{\psi}_p A_\mu\gamma^\mu\psi_p$$

vs.

$$L_{\pi\pi NN} = \frac{g_A^2}{2f_\pi^2} m_N\overline{\psi}_p\psi_p\phi_\pi\phi_\pi, \quad L_{NNNN} = -\frac{1}{2}C_s\overline{\psi}_p\psi_p\overline{\psi}_p\psi_p$$

"scalar" meson exchange vs. BχPT[1]

$$(-ig_{\sigma\pi\pi})\frac{i}{q^2 - m_\sigma^2}(-ig_{\sigma NN}) \xrightarrow{Q^2\to 0} \frac{ig_{\sigma\pi\pi}g_{\sigma NN}}{m_\sigma^2} = \frac{ig_A^2 m_N}{f_\pi^2},$$

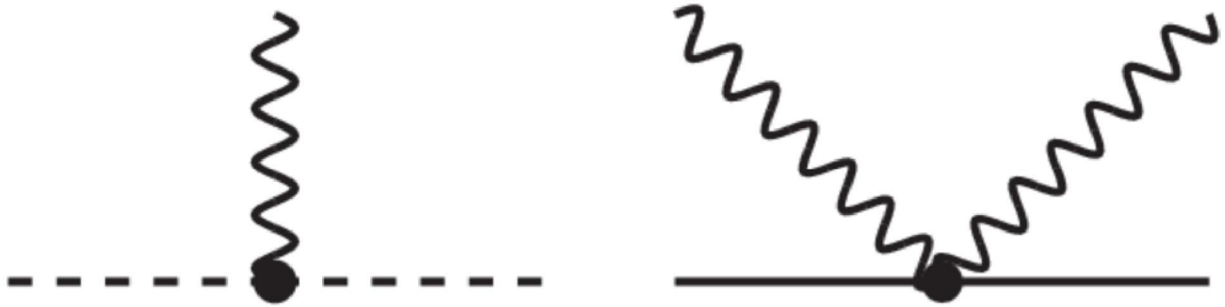
$$\frac{ig_{\sigma NN}^2}{m_\sigma^2} \sim -iC_s$$

$g_{\sigma NN}^2(Q^2 = 0)/m_\sigma^2$ (GeV^{-2})
294
282
356
343

C_s?

“scalar” meson exchange between μp

Also we add the form factors of $\pi\pi\gamma^*$,$NN\gamma^*$ coupling to describe the electromagnetic structure of π,N.



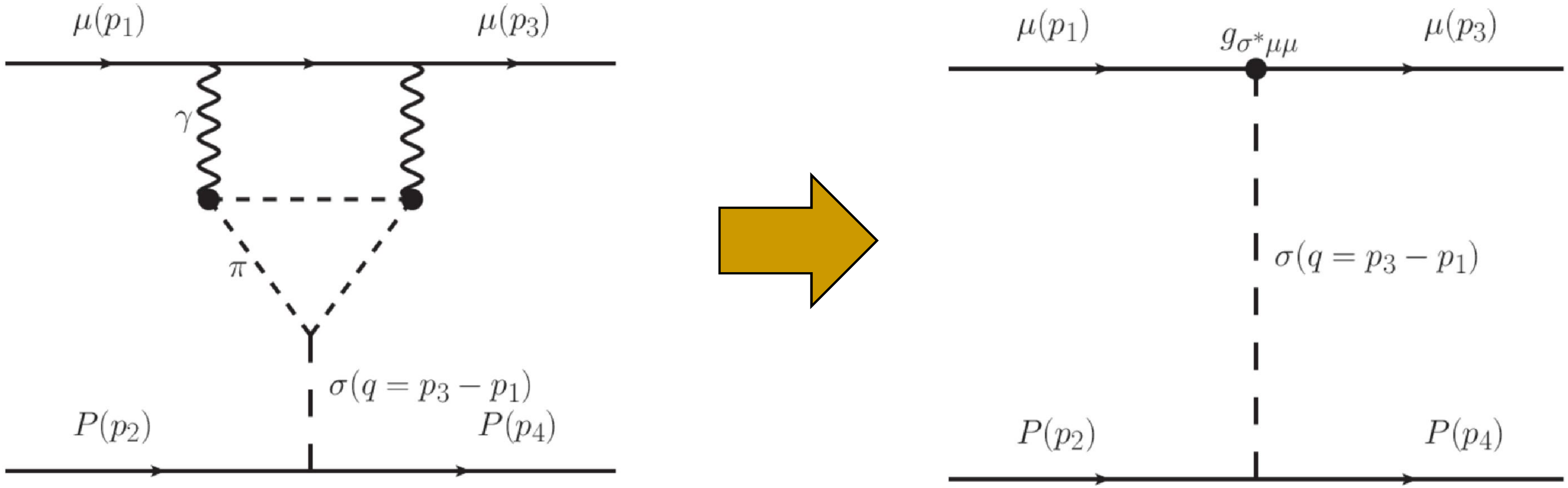
$$\frac{-\Lambda^2}{q_\gamma^2-\Lambda^2}$$

gauge invariant $\Lambda \approx 0.77\ \mathrm{GeV}$ or from $0.5-1.0\ \mathrm{GeV}$

In one poin exchange case, such FFs gives consistent results with the usual renormalized method.

numerical results

In the calculation of the effective coupling of μμσ*, we expand the result on the momentum transfer $Q=\sqrt{-q^2}$, since we are interesting its contribution to the energy shift of μp (non-relativistic bound state).



numerical results

At last, the numerical result can be expressed as

$$g_{\mu\mu\sigma^*}^{(\pi)} \approx g_{\sigma\pi\pi}[C_1^{(\pi)} + C_2^{(\pi)} Q]$$

$$g_{\mu\mu\sigma^*}^{(N)} \approx g_{\sigma NN}[C_1^{(N)} + C_2^{(N)} Q]$$

numerical results

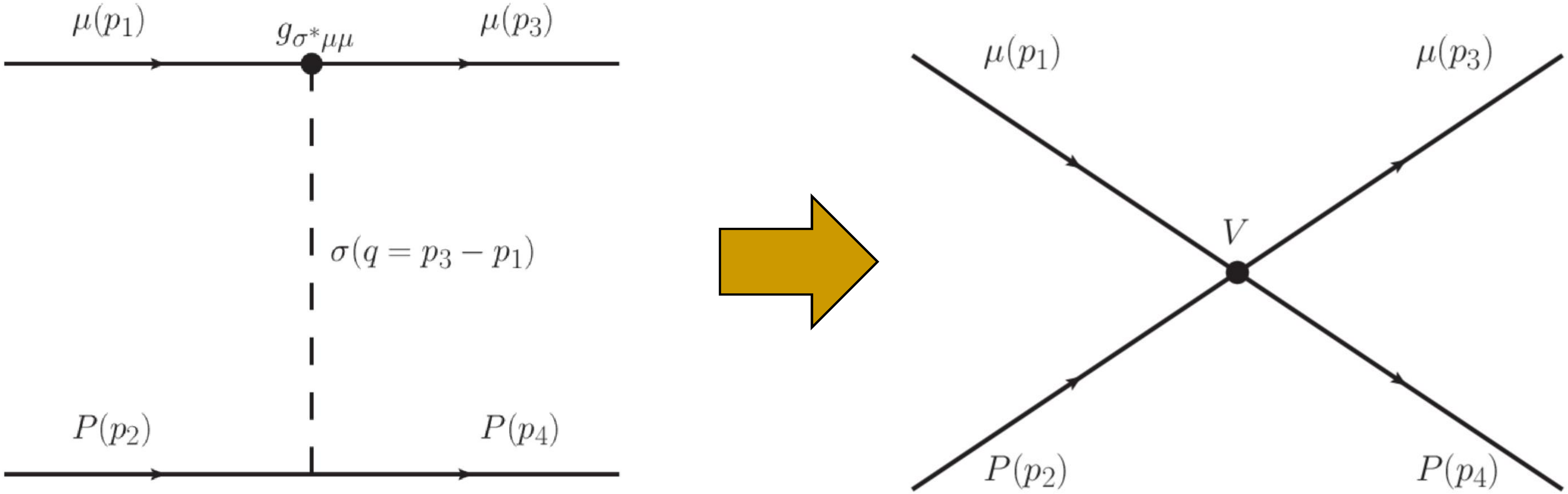
C_2 are only dependent on the mass of pion and muon, but not dependent on the cut off of the form factor, C_1 are dependent weakly on the cut off. The numerical results are

$$C_1^{\pi} = 5.2; \quad C_1^{\pi} = -29 \quad \times 10^{-6}$$

$$C_1^{N} = 1.08; \quad C_1^{N} = -4.74 \quad \times 10^{-6}$$

numerical results

From this effective coupling, the corresponding contribution to the Lamb shift can be calculated directly by the perturbative theory using the quasi-potential method by matching the amplitudes with QM.



numerical results

The energy shift   not sensitive on the mass of sigma within [0.3,1] GeV

$$\Delta E_{2S}^{(\pi)} = -12\mu eV, \Delta E_{2S}^{(N)} = -4\mu eV,$$
$$\Delta E_{2P}^{(\pi)} = -0.005\mu eV, \Delta E_{2P}^{(N)} = -0.0014\mu eV,$$

comparable with the results by BχPT
much smaller than 0.32 meV.

Question:
N-loop is not small, $O(p^4)$?

Discussion (personal opinions)

1. on the BχPT when the leptons are included.

if no EM FFs are used, then to cancel the UV divergence in the sub amplitudes ep/eπ scattering, counter terms such as (llNN,llπ) have to be included, which will lead to the un-predictivity of BχPT in Lamb shift .

2. on the contributions from the Δ/N* in the loop.

if the coupling constants are matched from the Compton scatting,ππ, NN, πN scattering without Δ /N*, no need to consider their contributions in lp system.

Discussion (further plan)

3. Compton scattering and the low energy constants in BχPT.

include the N-loop to study the Compton scattering and determine the low energy constants only from Compton scattering?

4. ……

Thanks!

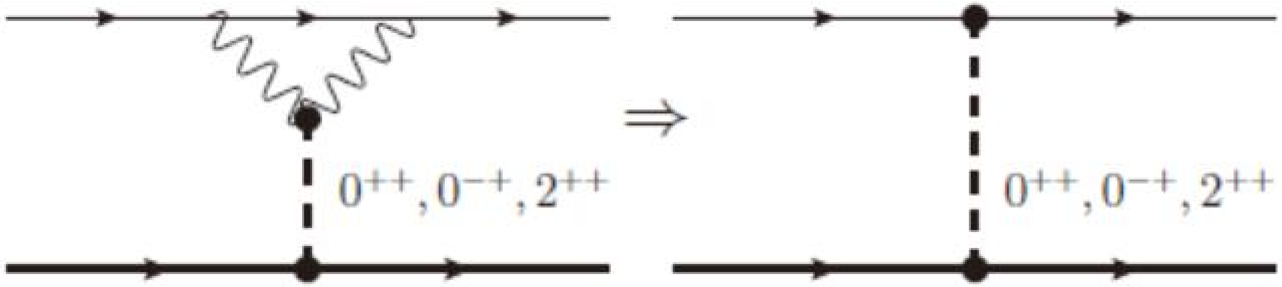
other well known contributions

Table 3
Contributions to the muonic hydrogen Lamb shift. The proton radius is taken from [15].

Contribution	Value (meV)	Uncertainty (meV)
Uehling	205.0282	
Källen–Sabry	1.5081	
VP iterations [4,28]	0.1507	
sixth order [28]	0.00752	
Total "LBL" [32]	−0.00089	0.00002
mixed mu-e VP	0.00007	
hadronic VP	0.011	0.001
recoil [3, Eq. (136)]	−0.04497	
recoil, higher order [3]	−0.0100	
recoil, finite size [36]	0.013	0.001
recoil correction to VP [1]	−0.0041	
additional recoil [51]	0.0575	
muon Lamb shift		
second order	−0.66788	
higher orders	−0.00171	
nuclear size ($R_p = 0.875$ fm)		0.007 fm
main correction $B \cdot \langle r^2 \rangle$	−4.002	0.064
Zemach moment [36]	0.0244	0.002
remaining order $(\alpha Z)^6$ [36]	−0.0014	
polarization	0.0127	0.003
correction to the $2p_{1/2}$ level	0.00004	

Annals of Physics 327(2012)733-763

meson exchange in ep scattering -- 2^{++}



We found in the ep scattering, after including the contributions from a 2^{++} meson exchange, the un-polarized and polarized ep scattering can be understood well together.

H.Q. Zhou etc., Phys. Rev, C. 90(2014)045205