Infrared enhancement in single-baryon systems

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Outline

1. Introduction
2. Power Counting
    - Standard counting
    - Infared enchancement
3. Numerical
    - Loop integration
    - Scaling of $\gamma(t)$
4. Summary

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Heavy baryon VS. Covariant in single-baryon system

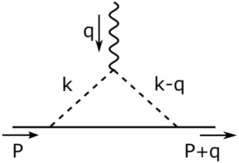
Heavy Baryon Chiral Perturbation Theory(HBChPT)

- Baryons have large masses compare to momentum(Q), it can be approximated as static object at leading order.
- The recoil terms are treated as subleading corrections.

Covariant Chiral Perturbation Theory

- Relaticistic Lagrangian of ChPT is manifestly Lorentz invariant, recoil corrections are in effect resummed.
- Phonomenological successes in several processes: Magnetic moments, baryon mass.

Triangle diagram



- Becher(1999) argued that we should use relativistic kinematics: analyticity problem of triangle diagram using static approximation.
- In covariant treament triangle diagram has a branch point in second Riemann sheet of $t(q^2)$, but it not appear in static approximation.
- Power counting must reflect the necessity of resummation: recoil terms are equal importance.
- Power counting change of two-nucleon reducible loops($\sim m_N/Q$).

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Standard counting

The loop integral of triangle diagram is:

$$\gamma(t) \equiv i\int \frac{d^4k}{(2\pi)^4}\frac{1}{k_0-\frac{\vec{k}^2}{2m_N}+i\epsilon}\frac{1}{k^2-m_\pi^2+i\epsilon}\frac{1}{(k-q)^2-m_\pi^2+i\epsilon}$$

When both pion propagators are on-shell, loop momentum

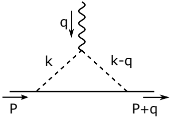
$$\vec{k}=\frac{\vec{q}}{2}+\mathcal{O}(\frac{\vec{q}^2}{m_N}), k_0=\sqrt{\vec{q}^2+m_\pi^2}[1+\mathcal{O}(\frac{\vec{q}^2}{m_N})]$$

External three-monenta($\vec{q}$) is of same order as pion mass:$Q \sim m_\pi$.

$k_0 \sim Q, \vec{k} \sim Q$, recoil term $\vec{k}^2/2m_N \sim Q^2/m_N$ is subleading.

The standard counting of loop integral $\gamma(t) \sim Q^4/Q^5 \sim Q^{-1}$

Infared enchancement



Consider a particular unphysical region:

$$\vec{q}^{\,2} = -4m_\pi^2 + \mathcal{O}(\xi^2 m_\pi^2), \qquad \xi = m_\pi / m_N$$

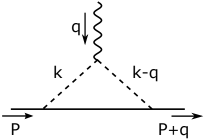
$\vec{k}$ and $\vec{q}$ may take complex value. There exist a region of $\vec{k}$:

$$k_0 \sim \sqrt{\vec{k}^2 + m_\pi^2} \sim \xi m_\pi$$

Cancellation between $\vec{k}^2$ and m_π^2 leads a small loop energy.

Recoil term $\vec{k}^2/2m_N \sim \xi m_\pi \sim k_0$.

Recoil correction is necessary!



In this small region of $\vec{k}$:

$$|\vec{k}-\frac{\vec{q}}{2}|\sim\xi m_\pi,\sqrt{(\vec{k}-\vec{q}\,)^2+m_\pi^2}\sim\xi m_\pi$$

Loop intagral scales:

$$\gamma(t)\sim(\xi m_\pi)^4\frac{1}{\xi m_\pi}\frac{1}{(\xi m_\pi)^2}\frac{1}{(\xi m_\pi)^2}\sim\frac{1}{\xi m_\pi}$$

Although phase space of $\vec{k}$ is small, simultaneously on-shell pion and baryon prapagators provide a small denominator.

Enchanment of order $1/\xi$!

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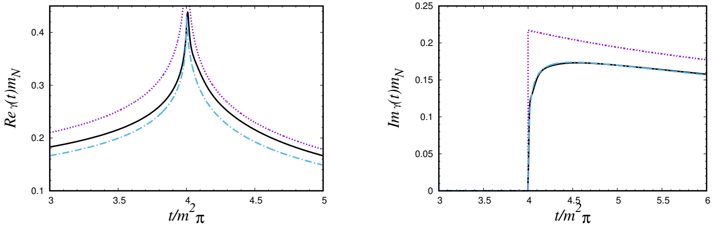
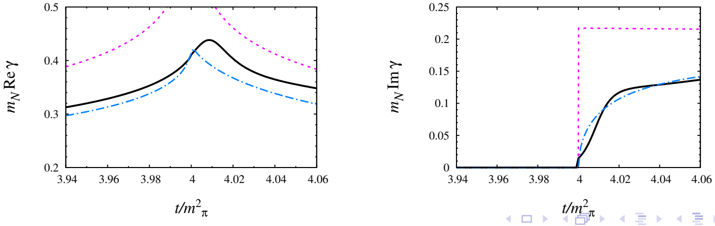
Integration result

Difference between relativistic and recoil one is tiny.



Zoom in the region around $t = 4m_\pi^2$, the enhancement is clear:

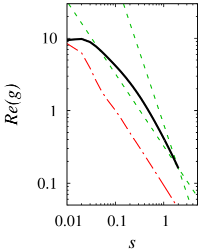


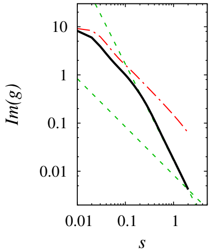
Scaling of $\gamma(t)$

$\gamma(t, m_\pi^2) \sim m_\pi^{-2}, m_\pi^{-1}$ near and far away $t = 4m_\pi^2$ respectively.

Scale m_π^2 and t with a factor s, define function $g\left(s; t/m_\pi^2\right)$

$$g\left(s; t/m_\pi^2\right) \equiv m_N \gamma\left(s^2 t^\star, s^2 m_\pi^{\star 2}\right)$$





Real part of $g(s)$ scale between $\sim s^{-1}$ and $\sim s^{-2}$.

Imaginary part of $g(s)$ really scales as $\sim s^{-2}$ near $t = 4m_\pi^2$.

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Summary

- In a small region near two pion threshold, triangle diagram is enchanced by a factor m_N/m_π.
- In this small region, power counting changed and the recoil correction should be considered.
- Loop intagral scales as m_π^{-2} inside the enhancement window.

Thank you!