Review on impedance and instabilities

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1. Impedance and Wake [1-2]

1.1 Definitions of impedance and wake





The bunch wake can be fitted with the analytical model [13]

,

where *L* and *R* are effective inductance and resistance, respectively.

1.2 Loss factor and transverse kick factor

As a beam traverses an impedance, it loses a certain amount of energy to the impedance. This energy loss is referred to as the parasitic loss of the beam.

Consider a beam whose *m*th moment has a longitudinal distribution *ρ*(*s*−*ct*), normalized so that ∫*dzρ*(*z*)=*Im*, the total *m*th moment of the beam. As this beam travels down the pipe for a distance *L*, its energy changes by

 

or

 

where

  and .

Only the real part of the impedance contributes to the energy loss of the beam.

The energy deposited by the point charge into a given mode after its passage through the impedance is given by

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The loss factor , usually stated in units V/pC=1012V/C, is

 ,

where  is the spectral power density of the bunch of rms length σ.

For a line spectrum, the integral becomes an infinite sum. For a single bunch in a circular accelerator,

 

For short bunches in a large machines (*ω*0 << 1/*σ*), the sum can be replaced by an integral.

The transverse loss factor for the dipole (m=1) mode, also called the kick factor, with dimensions V/(pC-m), is defined by

 ,

in terms of the transverse impedance *Z*⊥(*ω*), wake potential *W*⊥*λ*(*τ*), and wake function *W*⊥(*τ*).

For a resonator, the longitudinal loss factor for a Gaussian bunch with rms bunch length of *σ* is (V/pC or Ω⋅s−1)

 

The transverse loss factor or kick factor is (V/(pC-m) or Ω⋅s−1⋅m−1)

 

其中. For short bunch length, the transverse loss factor becomes

 

2. Collective instabilities

2.1 Single bunch instabilities

**2.2.1 Microwave instability**

The average threshold current for the longitudinal microwave instability can be estimated according to the Boussard or Keil-Schnell criterion [9, 10]



with *R* is the radius of the ring, *αp* is the momentum compaction factor, *σδ* is the momentum spread and *σz* is the rms bunch length.

**2.2.2 Transverse mode coupling instability**

*Mode Coupling Theory*[3, 4] :

TMCI occurs when the frequencies of two neighboring head-tail modes approach each other due to detuning with increasing beam current. For a Gaussian bunch, the threshold of the instability can be expressed with the transverse loss factor [3, 4]

 

where *I*0th is the threshold of the beam current, *Qs* is the synchrotron tune, *βy,j* is the average beta function in the *j*th element, *κy,j* is its loss factor, *E* is the beam energy and Θ≈0.7.

*Eigen-value Solver* [2, 4-5]:

The threshold of transverse mode coupling instability can be found by solving the following eigen-value problem:

 .

We consider azimuthal mode coupling only for the lowest radial mode (*k*=0)

 ,

here <*β*⊥> is the average beta function in the region where the transverse impedance is important. *Ib* is bunch current and *σz* is rms bunch length. The tune of each mode (Ω−*ωβ*)/*ωs* is obtained by solving the eigen-value problem for matrix *M*. The frequency Ω=*ωβ*±*lωs* corresponds to the ±*l*th synchrotron sideband. The threshold can be found when the head-tail modes approach each other.

*Eigen-value Solver* [2]:

Let us consider a broad-band impedance and *χ*=0. The mode coupling effect can be demonstrated as follows. The eigenvalue I s determined by the condition

 

The matrix elements of *M* are

 

The matrix *M* has the following form

 

The *I*’s and *R*’s are real quantities, all different from one another. With *I* coming only from Im*Z*0|| and *R* coming only from Re*Z*0||. All *R*’s and *I*’s are proportional to the beam intensity *N*. The elements of *M* are all real.

**2.2.3 Bunch lengthening**

Interaction of the beam with broadband impedance can change the bunch length and longitudinal distribution due to potential well distortion. The longitudinal bunch density distribution is obtained by numerically solving the Haissinski equation [2, 8].



**2.2.4 Beam tilt**

When a beam passes through transverse impedance, the tail particles will receive transverse kicks and induce bunch shape distortion. The transverse kick experienced by a particle located at longitudinal position *z* is given by [2]



This will lead to a transverse displacement of the bunch tail at IP [11, 12]

,

where *βy\** and *βy* are the vertical beta function at the IP and at the location of the impedance, respectively.

**2.2.5 Transverse tune shift**

Tune shift is evaluated by the formula for effective impedance [2],

$$∆ν\_{β}=-i\frac{N\_{e}r\_{e}}{4π^{3/2}γ}\frac{L}{ν\_{β}σ\_{z}}\frac{Z\_{eff}}{Z\_{0}}$$

$$Z\_{eff}=\sum\_{}^{}Z(ω^{'})h\_{0}(ω^{'}-ω\_{ξ})/\sum\_{}^{}h\_{0}(ω^{'}-ω\_{ξ})$$

where the summation is performed for $p=\pm \infty $ with $ω^{'}=ω\_{β}+pω\_{0}$.

**2.2.6 CSR**

In the model used for the calculations the beam is assumed to be moving in a circle of radius *ρ* (in the plane *y*=0) between two parallel plates at locations *y*=±*h*. In normalized units the threshold current *S*th is given as a function of shielding parameter Π by [6,7]

 ,

with

 ,,

with *Nb* being the number of electrons per bunch and *νs* being the synchrotron tune.

2.2 Coupled bunch instabilities

**2.2.1 Transverse resistive wall instability**

One of the main origins for exciting the transverse multi-bunch instability is due to the interaction of the beam with the resistive wall impedance. Considering *nb* uniformly distributed bunches, the rise time of the transverse multi-hunch instability can be estimated by [2]

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where *ωp* = (*pnb*+*μ*+*νβ*)*ω*0.

**2.2.2 Longitudinal instability due to HOMs**

**2.2.3 Transverse instability due to HOMs**

References

[1] A. Chao, Physics of Collective Beam Instabilities in High Energy Accelerators, Wiley-Interscience Publication, John Wiley & Sons, Inc., 1992.

[2] Alexander Wu Chao and Maury Tigner, Handsbook of Accelerator Physics and Engineering, World Scientific 1998.

[3] S. Krinsky, in the proceedings of PAC07, TUPMS074, 2007.

[4] L. Wang and G. Stupakov, Transverse single bunch instability in PEP-X, SLAC-PUB-13658, 2009.

[5] Kohtaro Satoh and Y. Chin, Nucl. Instr. And Meth. V207, p309, 1983.

[6] Yunhai Cai, Karl Bane, Robert Hettel, Yuri Nosochkov, Min-Huey Wang, PEP-X: An ultimate storage ring based on fourth-order geometric achromats, SLAC-PUB-14785, 2011.

[7] K. Bane, Y. Cai, and G. Stupakov, Threshold studies of the microwave instability in electron storage rings. Phys. Rev. ST Accel. Beams, 13:104402, 2010.

[8] J. Haissinski, Nuovo Cimento 18B, 72, 1973.

[9] D. Boussard, CERN II/RF/Int. 75-2, 1975.

[10] E. Keil and W. Schnell, CERN ISR-TH-RF 69/48, 1969.

[11] A. W. Chao and S. Kheifets, SLAC-PUB-3052, 1983.

[12] D. Zhou, K. Ohmi, A. W. Chao, IPAC2011, p.601~602, 2001.

[13] K. Bane, SLAC-PUB-14151, 2010.