1-Loop Electroweak Corrections to HZ related production through e^+e^- collision

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Outline

- 1. Why we need Electroweak Corrections?
- 2. How to obtain Electroweak Corrections?
  - Full 1-LOOP Electroweak Corrections to $e^+e^- \to WWZ/ZZZ$ as an example
- 3. Electroweak Correction to $e^+e^- \to HZ$ related production
  - Full 1-LOOP Electroweak Corrections to $e^+e^- \to HZ$
  - Full 1-LOOP Electroweak Corrections to $e^+e^- \to e^+e^-H$
  - Full 1-LOOP Electroweak Corrections to $e^+e^- \to \nu\bar{\nu}H$
  - 1-LOOP effects to $2 \to 4$ productions
- 4. Summary & Prospects

- Why we need Electroweak Corrections?

Why we need Electroweak Corrections?

The Nobel Prize in Physics 2013

Photo: A. Mahmoud
François Englert
Prize share: 1/2

Photo: A. Mahmoud
Peter W. Higgs
Prize share: 1/2

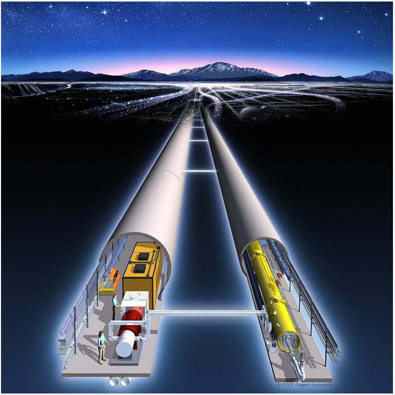
The Nobel Prize in Physics 2013 was awarded jointly to François Englert and Peter W. Higgs *"for the theoretical discovery of a mechanism that contributes to our understanding of the origin of mass of subatomic particles, and which recently was confirmed through the discovery of the predicted fundamental particle, by the ATLAS and CMS experiments at CERN's Large Hadron Collider"*

- Add the Last Piece of the Puzzle for the Standard Model
- Mass Origin
- A big step toward Understanding of Electroweak Symmetry Breaking
- Confirm LHC-Machine

Collider Physics

- New Particle Hunter
- Precise measurement

Why we need Electroweak Corrections? The LCs



The International Linear Collider:

- CM energy: $500 \div 1000$GeV.
- Luminosity: of the order $10^{34}\text{cm}^{-2}\text{s}^{-1}$.

The Compact LInear Collider:

- CM energy: $0.5 \div 3$TeV.
- Luminosity: also of the order $10^{34}\text{cm}^{-2}\text{s}^{-1}$.

e^+e^- colliders are high precision machines.

CEPC

Why we need Electroweak Corrections? per cent level needed

$$\sigma(e^+e^- \to f) = \frac{N_f^{obs} - N_{bkg}}{\epsilon_f \cdot (1+\delta) \cdot \mathcal{L}}$$

Experiment

Measurement → **percent level**

⇓

Theory

Calculation → **percent level**

Why we need Electroweak Corrections? per cent level needed

$$\sigma(e^+e^- \to f) = \frac{N_f^{obs} - N_{bkg}}{\epsilon_f \cdot (1+\delta) \cdot \mathcal{L}}$$

Experiment

Measurement → **percent level**

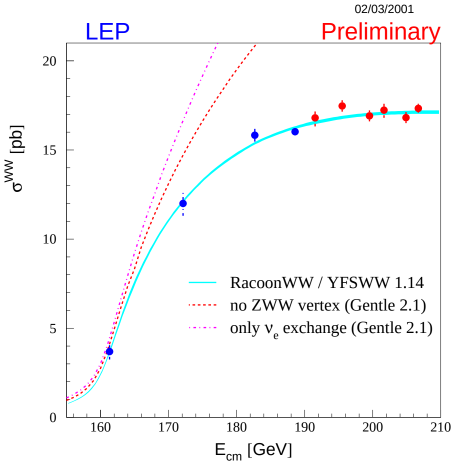
⇓

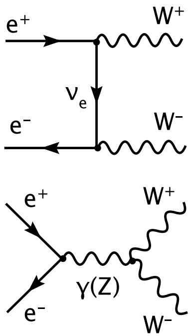
Theory

Calculation → **percent level**

threshold, resonance,
ISR, FSR, Sudakov logarithms effects, multi-body final states,
1-LOOP EW/QCD/SUSYQCD,
2-LOOP Corrections/Effects, Resummation

Why we need Electroweak Corrections? WW production at LEP





- SM trilinear couplings: well tested at LEP.
- What about the quartic gauge couplings? Not well tested.

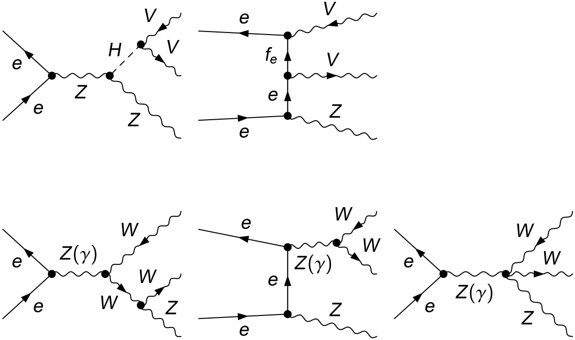
1-loop multi-leg calculation: theoretical challenges

- 1-loop multi-leg calculation: electroweak and QCD corrections.
- In some respects, electroweak radiative corrections difficult than QCD corrections (many different mass scales $\rightarrow$ different mass configurations).

- How to obtain Electroweak Corrections?

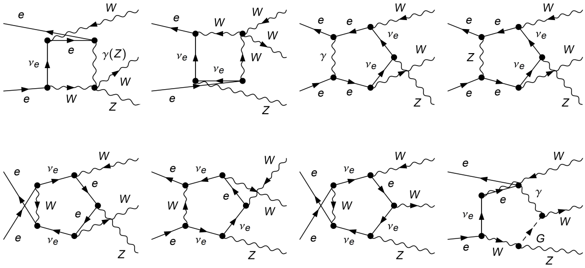
$e^+e^- \to VVZ$: tree diagrams

- ZZZ: 9 diagrams, no trilinear and quartic couplings in SM
- WWZ: 20 diagrams, trilinear and quartic couplings contribute in SM



$e^+e^- \to VVZ$: one-loop diagrams

neglecting $< eeS >$ couplings:



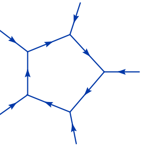
Topology	ZZZ(1767)	WWZ(2736)
Loop Amp. (FormCalc)	6.4MB	6.9MB
4-point	384	396
5-point	64	109

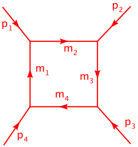
calculation framework

$$d\sigma_{1-loop}^{e^+e^-\to VVZ} = d\sigma_{virt}^{e^+e^-\to VVZ} + d\sigma_{real}^{e^+e^-\to VVZ\gamma}$$

- UV-divergence is regularised by using on-shell renormalisation.
- IR-divergences cancel out.
- FeynArts , FormCalc (Mathematica + FORM) to generate Feynman diagrams and to get the helicity amplitude expressions.
- SloopS (Baro, Boudjema and Semenov) to make sure that the amplitudes are correct by checking gauge invariance (using NLG Feynman rules).
- Improved LoopTools (FF + Tensor reduction + ...) to calculate all one-loop integrals. [Hahn, van Oldenborgh and Vermaseren]
- BASES (Kawabata) to do phase space integration and to get distributions.

LoopTools integrals





One loop tensor N-point integrals have the general form:

$$T^{N,\mu_1\cdots\mu_P}(k_1,\ldots,k_{N-1},m_0,\ldots,m_{N-1}) = \frac{(2\pi\mu)^{4-D}}{i\pi^2}\int d^Dq\,\frac{q^{\mu_1}\ldots q^{\mu_P}}{N_0N_1\ldots N_{N-1}}$$

$k_i = \sum_{j=1}^{i-1} p_j,\ i = 1,2,3,\ldots,\qquad N_k = (q+k_k)^2 - m_k^2$

- 1-point and 2-point integrals, explicit numerically stable results are used.
- 3-point and 4-point scalar integrals, expecially the scalar 4-point integrals: tricky (different mass scales, in practise the log- and Spence- arguments can be very close to 0, leading to numerical problems (observed in WWZ).
- 3-point and 4-point tensor integrals are reduced to scalar integrals with the PV reduction (in most points in phase space, small Gram problem).
- 5-point integrals are reduced in terms of five 4pt functions.

Scalar integrals: regulate the infra-red and collinear singularities

- Regulate the divergence by introducing a small mass parameter m_γ for the divergent lines.
  - singularities can be introduced as $\ln(m_\gamma^2)$
  - method of choice in the calculation of electroweak processes (internal massless lines are relatively rare)
  - available interface: FF package
- Regulate the divergence in dimensional regularization.
  - singularity pole can be introduced interms of $\frac{1}{\epsilon^2}a_2 + \frac{1}{\epsilon}a_1 + a_0$
  - method of choice in QCD loop processes (light fermion masses taken massless in high energy limits)
  - available interface: QCDLoop library and OneLOop library

small mass parameter scheme $\Longleftrightarrow$ dimensional regularization scheme

$$\ln(m_\gamma^2) \Longleftrightarrow \frac{1}{\epsilon}$$

Loop integrals and numerical instabilities: small Gram Problem(I)

Gram Matrix

Gram Matrix:

$$G = \begin{pmatrix} 2k_1k_1 & \dots & 2k_1k_N \\ \dots & \dots & \dots \\ 2k_Nk_1 & \dots & 2k_Nk_N \end{pmatrix}$$

determinant of Gram Matrix:

$$detG = \begin{vmatrix} 2k_1k_1 & \dots & 2k_1k_N \\ \dots & \dots & \dots \\ 2k_Nk_1 & \dots & 2k_Nk_N \end{vmatrix}$$

D_{ijkl} tensor integral

$$\boxed{D_{ijkl} = f(p_i, m_i)/detG}$$

- $\Longrightarrow$ numerical instabilities occur when $detG$ become small

Loop integrals and numerical instabilities: small Gram problem(II)

Methods to solve numerical instabilities

- drop the non-regular Phase Space points and set the integrand to zero during phase space integration (limited, QCD)
- quadruple precision (32 digits kept while double precision 16 digits kept, slow, more CPU time, computer dependent)
- QD library (higher precision used when numerical instability occurs)
- DD approach [A. Denner, S. Dittmaier, Nucl.Phys.B734:62-115,2006] (split the PS into different regions, use different expansion, $e^+e^- \to 4f$ electroweak correction, not easy to implement, slow)
- segmentation [F. Boudjema, A.Semenov, D.Temes, Phys.Rev.D72 (2005) 055024] (easy to implement, quicker)

$$\frac{1}{D_0 D_1 D_2 D_3} \to \frac{a_0}{D_1 D_2 D_3} + \frac{a_1}{D_0 D_2 D_3} + \frac{a_2}{D_0 D_1 D_3} + \frac{a_3}{D_0 D_1 D_2}$$
$$\hookrightarrow \frac{b_0}{D_1 D_2} + \frac{b_1}{D_0 D_2} + \frac{b_2}{D_0 D_1}$$

- We have Finished implementing and trying all these methods. To tri-boson production, we use QD library and segmentation method. Within integration error, we can get perfect agreements.

Real correction (I): Two Cuts

1) Two cutoff phase space slicing approach: easy to implement

$$d\sigma_{real}^{e^+e^-\to W^+W^-Z\gamma} = d\sigma_{soft}^{e^+e^-\to W^+W^-Z\gamma}(\delta_s) + d\sigma_{hard}^{e^+e^-\to W^+W^-Z\gamma}(\delta_s),$$

$$d\sigma_{hard}^{e^+e^-\to W^+W^-Z\gamma}(\delta_s) = d\sigma_{coll}^{e^+e^-\to W^+W^-Z\gamma}(\delta_s,\delta_c) + d\sigma_{fin}^{e^+e^-\to W^+W^-Z\gamma}(\delta_s,\delta_c)$$

Soft part: $E_\gamma < \delta_s\sqrt{s}/2 = \Delta E$,

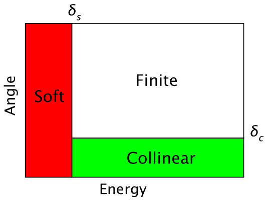
$$d\sigma_{soft} = -d\sigma_{Born}\frac{\alpha}{2\pi^2}\sum_{i,j=1}^{4}\int_{|\mathbf{k}|<\Delta E}\frac{d^3k}{2\omega_k}\frac{\pm p_i p_j Q_i Q_j}{(p_i.k)(p_j.k)}.$$

Collinear part: $\{E_\gamma \geq \Delta E, \cos\theta_{\gamma f} > 1-\delta_c\}$, $\hat{s} = xs$,

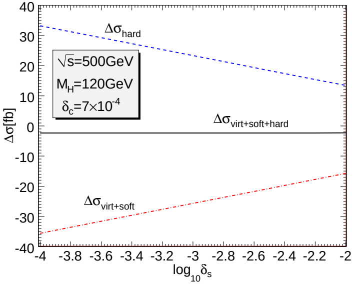
$$d\sigma_{coll} = \sum_{i=1}^{2}\frac{\alpha}{2\pi}Q_i^2\int_0^{1-\delta_s} dx d\sigma_{Born}(\hat{s})\left[\frac{1+x^2}{1-x}\ln\frac{\hat{s}\delta_c}{2m_i^2 x} - \frac{2x}{1-x}\right]$$

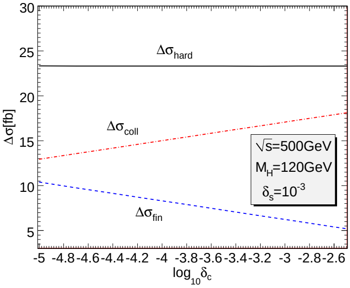
Finite part: $\{E_\gamma \geq \Delta E, \cos\theta_{\gamma f} \leq 1-\delta_c\}$, numerical integration using Monte Carlo BASES.

Real correction (II): Two Cuts



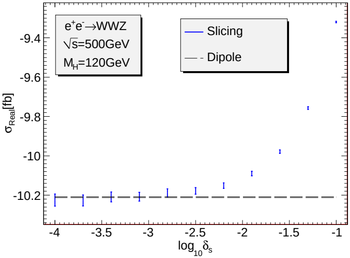
- Real correction is cutoff-independent.
- Factorization condition: δ_s and δ_c are sufficiently small. And $\delta_c \gg 2m_e^2/s$ to use the collinear integration formula.





Real correction (III): Dipole

2) Dipole subtraction approach: to cross check



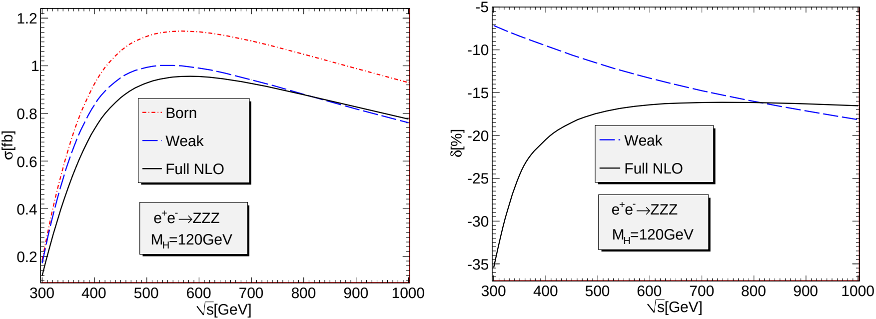
Checks on the results

- gauge invariance check: tree and one-loop squared amplitude level.
- UV and IR finiteness: one-loop squared amplitude level and for the virtual + soft corrections.

$(\tilde{\alpha}, \tilde{\beta})$	ZZZ	WWZ(1)	WWZ(2)
(0,0)	-7.8077709362570481E-4	-6.3768793214220439E-2	5.588092511112647047819820306727217E-2
(1,0)	-7.8077709362570731E-4	-6.3767676883630841E-2	5.588092511111034991142696308013526E-2
(0,1)	-7.8077709361534624E-4	-6.3772289648961160E-2	5.5880925111114608451016661052972381E-2

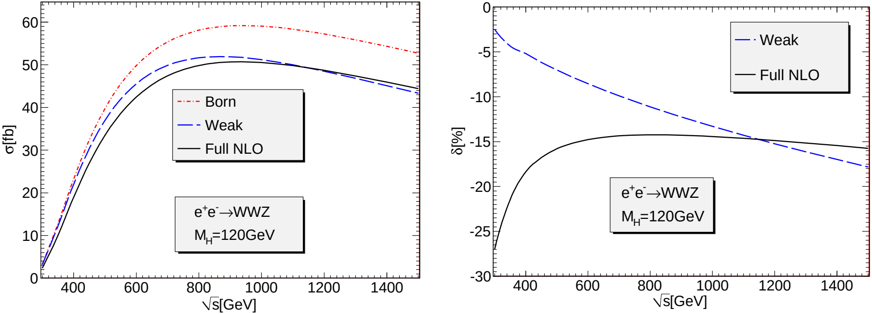
  - *ZZZ*: at least 10 digit agreement at a random point with double precision.
  - *WWZ*: for a DP random point, got only 4 digit agreement. By using QP, got 12 digits. Gauge invariance check is much worse for *WWZ*. This is an indication of numerical instability.
- Two independent calculations: different loop integral libraries/ different photon emission method

$e^+e^- \to ZZZ$: Total Xsection



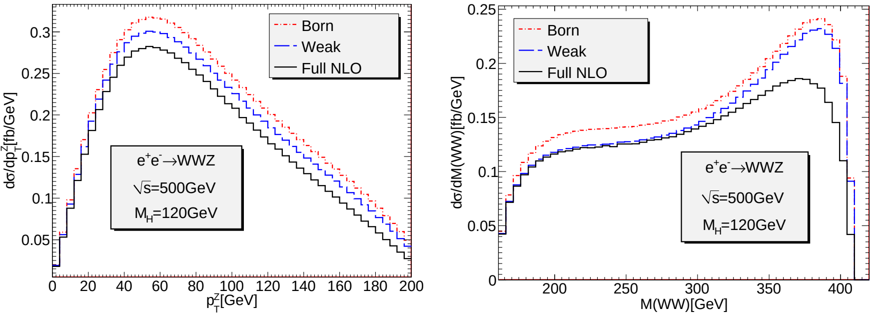
- Total Xsection peak about 1fb is at $\sqrt{s} \approx 550$GeV.
- The weak correction goes from -3.5% to -10% when $\sqrt{s}$ increases from 500GeV to 1TeV.
- The total electroweak corrections can be larger then -15%.

$e^+e^- \to W^+W^-Z$: Total Xsection



- Total Xsection peak about 50fb (50 times larger than σ_{ZZZ}) is at $\sqrt{s} \approx$ 900GeV.
- The weak correction goes from 1.6% to −8.9% when $\sqrt{s}$ increases from 500GeV to 1.5TeV.
- The total electroweak correction larger than −15%, significant and should be taken into account.

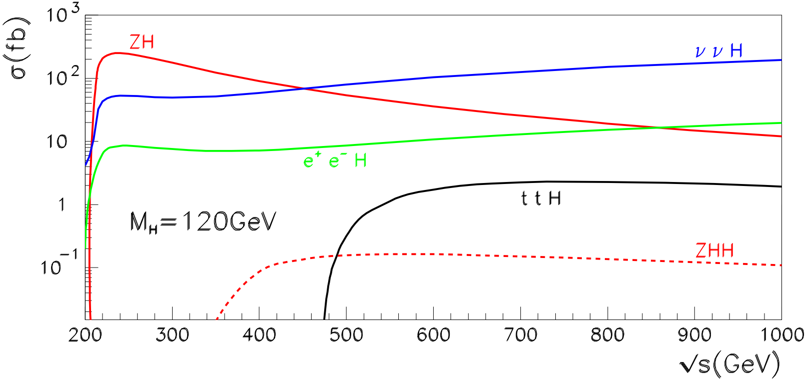
$e^+e^- \to W^+W^-Z$: Distributions



- Quite small corrections (less than -5%) at small GeV. At large GeV, large corrections (-30%) due to the hard photon effect [dominant contribution comes from the low-energy photon region (see the δ_s-plot) which corresponds to large p_T^Z and large M_{WW}.]

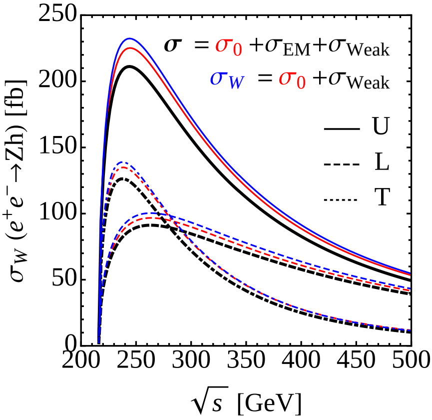
- Electroweak Correction to $e^+e^- \to HZ$ related production

Total cross section for main Higgs production channels at the linear collider



F.Boudjema, arXiv:hep-ph/0407065.

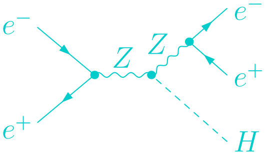
EW Corrections to $e^+e^- \to ZH$: Next to Leading Order Contributions

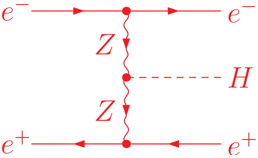


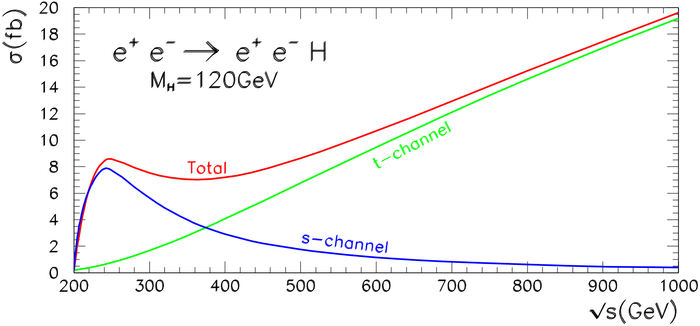
$$\sigma = \sigma_0 + \sigma_{QED} + \sigma_{Weak}$$

- Modified Higgs Sectors and NLO Associated Production C. Englert and M. McCullough, arXiv:1303.1526.

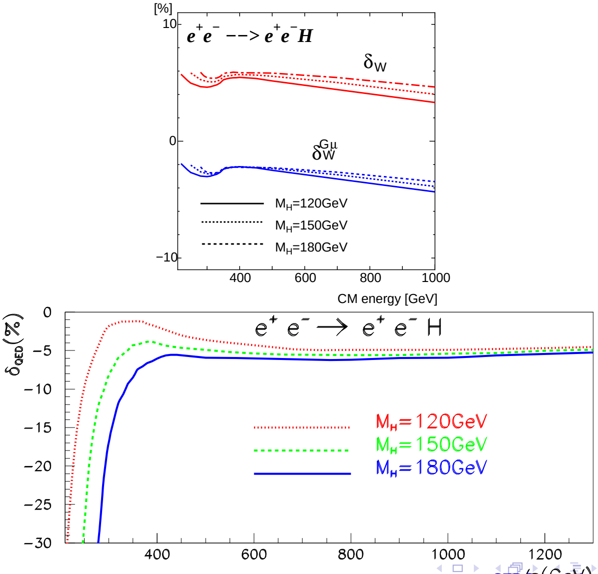
EW Correction to $e^+e^- \to e^+e^-H$: Leading Order Contributions







EW Correction to $e^+e^- \to e^+e^-H$: Next to Leading Order Contributions



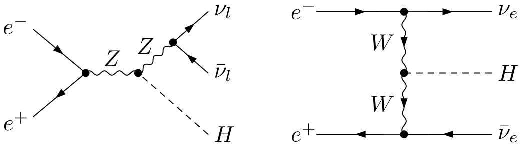
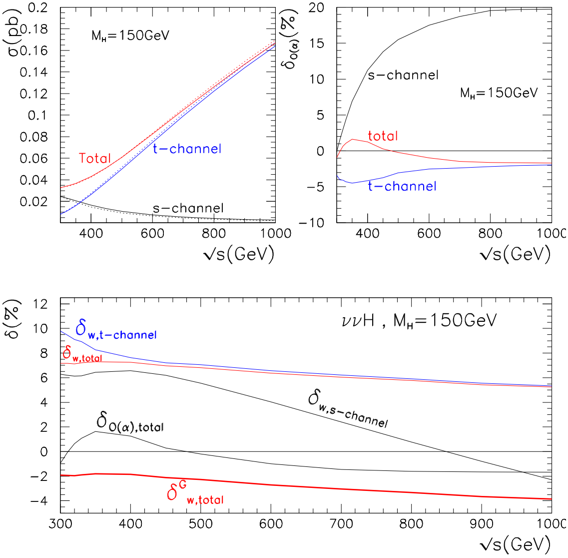
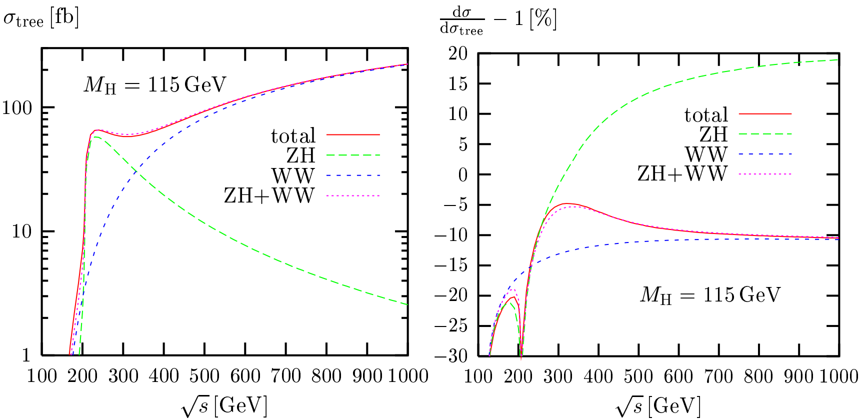


Figure : Lowest-order diagrams for $e^-e^+ \to \nu\bar{\nu}H$

EW Corrections to $e^+e^- \to \nu\nu H$: Next to Leading Order Contributions





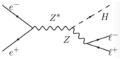
- Electroweak radiative corrections to single Higgs-boson production in e+e- annihilation A.Denner and S.Dittmaier, arXiv:hep-ph/0301189.

EW corrections to $2 \to 4$ productions: signal examples

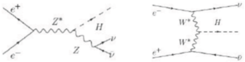
$\sqrt{s}$ = 250 GeV

- **ZH->$\mu\mu$+jj**
  - ZH production with Z decay to muon pair, H decay to bb/cc quark or gluon pair
  - Very clean signal in muon pair invariant mass and recoil mass
- **ZH->vv+jj**
  - Via ZH(~86%) or WW fusion(~14%)
  - Clean background
- **ZH->Multi-jet**
  - Both Z and Higgs decay hadronically
  - Much larger cross section than semi-leptonic channel

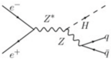
ZH->$\mu\mu$+jj



ZH->vv+jj



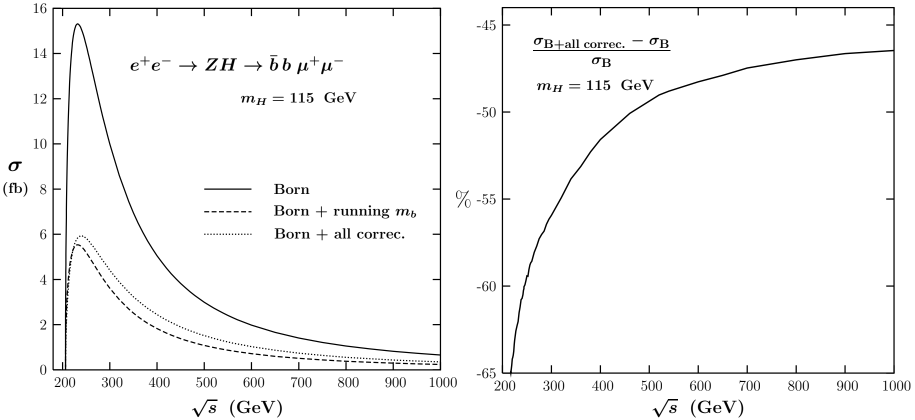
ZH->multi-jets



Slide taken from Y.B., CEPC-SPPC workshop, April 8, 2016.

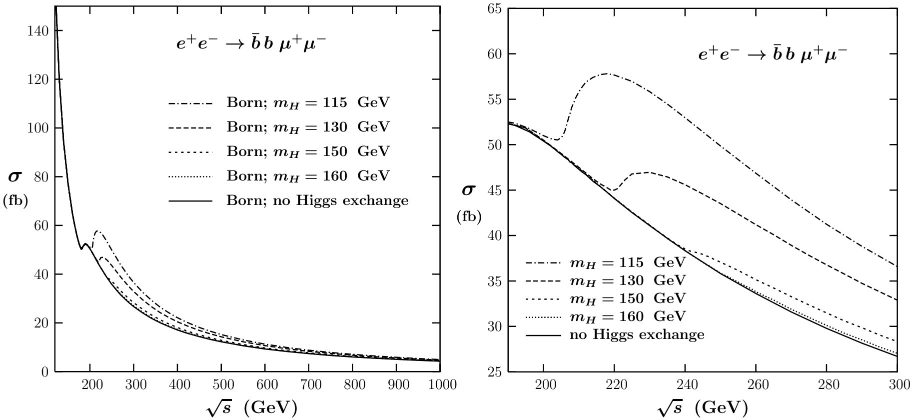
- Complete EW corrections to 4 fermion final states arising from Higgs-boson production needed?

EW corrections to $2 \to 4$ productions: $\mathrm{e^+e^-} \to \mathrm{HZ} \to \mathrm{b\bar{b}}\mu^+\mu^-$



- EW correction to ZH production, Z boson decay width, QCD and EW corrections to Higgs decay (HDECAY), ISR effects.
- One-loop electroweak factorizable corrections for the Higgsstrahlung at a linear collider F.Jegerlehner, K.Kolodziej, T.Westwanski, arXiv:hep-ph/0503169.

EW corrections to 2 $\rightarrow$ 4 productions: $e^+e^- \rightarrow \mu^+\mu^- b\bar{b}$



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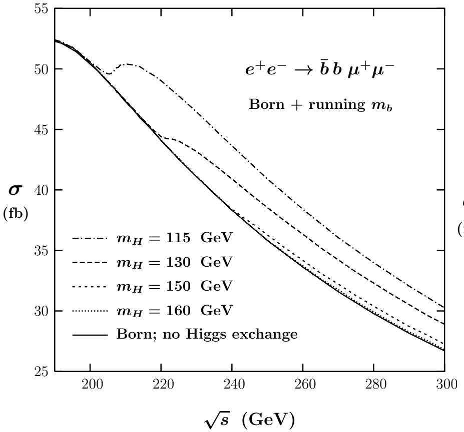

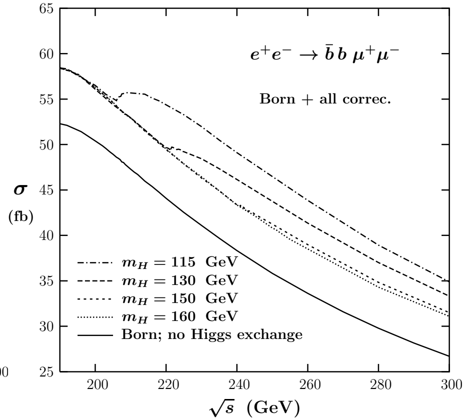

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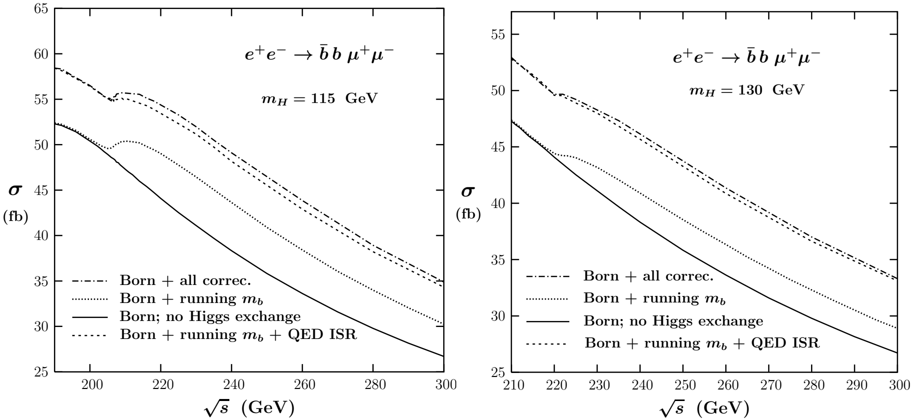

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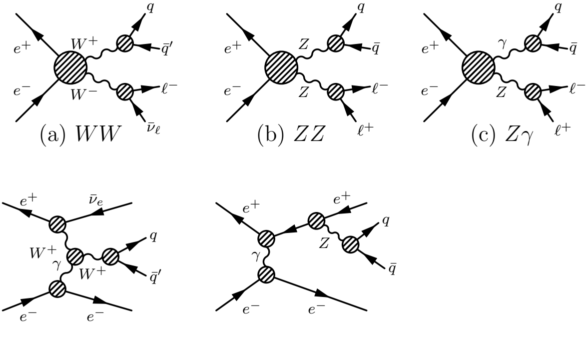
EW corrections to $2 \to 4$ productions: background examples



(a) WW (b) ZZ (c) $Z\gamma$

(d) $We\nu$ (e) Zee

- Complete EW $\mathcal{O}(\alpha)$ corrections to charged-current $\mathrm{e^+e^-} \to \mathrm{W^+W^-} \to 4\mathrm{f}$ processes A.Denner and S.Dittmaier, arXiv:hep-ph/0502063.
- EW corrections to other $2 \to 4$ productions at $\mathrm{e^+e^-}$ colliders?

Summary & Prospect

- 1. Technical development on Electroweak Corrections has been last for about more than 20 years. Until 2016, the full NLO EW corrections for the production up to any 4 final state particles are available. See Electroweak Corrections to $\mathrm{pp} \to \mu^+\mu^-\mathrm{e}^+\mathrm{e}^-$ at the LHC - a Higgs background study [Denner, etc, Phys. Rev. Lett. 116, 161803 (2016)].

- 2. Some higher order effects has been taken into account in HZ related productions (include signal and background) up to 4 final state particles. But the full NLO EW calculation seems not done yet. Might it be important at CEPC energy region?

- 3. From theoretical calculation to experimental data, a simple and automatic generator include higher order contributions might needed.


Thanks