Importance of a Model Independent Measurement of BR(H→BSM)

Tim Barklow (SLAC)

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Systematic Error on $\sigma(ZH)_{hadronic}$ Measurement from Model Dependence of Analysis

$$\left(\frac{\Delta\sigma(ZH)_{hadronic}}{\sigma(ZH)_{hadronic}}\right)^2 = \frac{S+B}{S^2}\left\{1+\frac{(L\,\sigma_{ZH})^2}{S+B}\sum_i BR_i^{\,2}\left[\left(\frac{\Delta\sigma\bullet BR_i}{\sigma\bullet BR_i}\right)^2(\xi_i-<\xi>)^2+\Delta\xi_i^2\right]\right\}$$

Penalty for non-uniform efficiency MC calculation

$\xi_i =$ efficiency for events from H decay i to pass $\sigma(ZH)_{hadronic}$ analysis

S = Number of signal events in $\sigma(ZH)_{hadronic}$ analysis

B = Number of background events in $\sigma(ZH)_{hadronic}$ analysis

For the sys error from unknown BSM decays we must assume $BR_{BSM} \geq \Delta BR_{BSM}$.
From the range of efficiencies in Mark Thomson's ILC analysis
at $\sqrt{s}$=350 GeV we get $\Delta\xi_{BSM} = .035$.

$\Delta BR_{BSM} = 0.04$ gives a sys error of 11% of the stat error for the $\sqrt{s}$=350 GeV $\sigma(ZH)_{hadronic}$ measurement.

Systematic Error on $\sigma \bullet BR_i$ from ΔBR_{BSM}

Neglecting non-Higgs background, the number of events N_i passing Higgs decay channel i selection criteria is

$$N_i = \sum_j \sigma \bullet BR_j \, \varepsilon_{ij} \, L$$

ε_{ij} = efficiency for Higgs decay mode j to pass Higgs decay channel i selection

For SM decays the efficiencies ε_{ij} can be calculated with MC. But what if decay mode j is a BSM decay? To account for this possibility a conservative systematic error can be assigned assuming $\varepsilon_{ij} = 1$. This leads to a systematic error of $\Delta N_i = L\sigma \, \Delta BR_{BSM}$

Improvement in Higgs Coupling Errors if ΔBR_{BSM} is small.

Further improvement in the Higgs coupling measurements can be obtained using the constraint $\sum_i BR_i = 1$ as first noted by Michael Peskin.

This constraint is model independent so long as the error in $BR(H \to BSM)$ is included in the fit. What error in $BR(H \to BSM)$ is required to produce an improvement in Higgs coupling measurements ?

In the following the ILC H-20 scenario is a 20 year run plan with operation at 250+350+500 GeV with 2000+200+4000 fb^{-1}

Perform coupling fit with $\sum_i BR_i = 1$ including $\Delta BR(H \to BSM)$

(the constraint $\sum_i BR_i = 1$ is model independent if $\Delta BR(H \to BSM)$ is included in the fit)

ILC Higgs Coupling Precision assuming 20 year H20 scenario

$\dfrac{\Delta BR(H \to BSM)}{\Delta BR(H \to Invis)_0}$	∞	8	4	2	1	0.1
ZZ	0.31%	0.29%	0.26%	0.22%	0.20%	0.19%
WW	0.38%	0.36%	0.31%	0.25%	0.21%	0.19%
bb	0.60%	0.57%	0.52%	0.46%	0.42%	0.40%
$\tau^+\tau^-$	0.88%	0.86%	0.83%	0.79%	0.77%	0.76%
gg	0.92%	0.91%	0.88%	0.86%	0.85%	0.84%
cc	1.1%	1.1%	1.1%	1.1%	1.1%	1.0%
$\gamma\gamma$	3.1%	3.1%	3.1%	3.1%	3.1%	3.1%
Γ_{tot}	1.7%	1.6%	1.3%	1.0%	0.84%	0.74%

$\Delta BR(H \to Invis)_0$ corresponds to the H-20 ILC 95% C.L. limit of 0.34%

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Perform coupling fit with $\sum_i BR_i = 1$ including $\Delta BR(H \to BSM)$

(the constraint $\sum_i BR_i = 1$ is model independent if $\Delta BR(H \to BSM)$ is included in the fit)

CEPC Higgs Coupling Precision assuming 5 ab^{-1}

$\dfrac{\Delta BR(H \to BSM)}{\Delta BR(H \to Invis)_0}$	∞	8	4	2	1	0.1
ZZ	0.26%	0.24%	0.22%	0.19%	0.18%	0.17%
WW	1.2%	1.2%	1.2%	1.2%	1.2%	1.2%
bb	1.3%	1.3%	1.2%	1.2%	1.2%	1.2%
$\tau^+\tau^-$	1.4%	1.4%	1.4%	1.4%	1.3%	1.3%
gg	1.5%	1.5%	1.5%	1.5%	1.5%	1.5%
cc	1.7%	1.7%	1.7%	1.6%	1.6%	1.6%
$\gamma\gamma$	4.7%	4.7%	4.7%	4.7%	4.7%	4.7%
Γ_{tot}	2.8%	2.7%	2.5%	2.4%	2.3%	2.3%

$\Delta BR(H \to Invis)_0$ corresponds to the 5 ab^{-1} CEPC 95% CL limit of 0.28%

40% improvement in Δg_Z if $\Delta BR(H \to BSM) \approx \Delta BR(H \to Invis)$

Perform coupling fit with $\sum_i BR_i = 1$ including $\Delta BR(H \to BSM)$ for

(the constraint $\sum_i BR_i = 1$ is model independent if $\Delta BR(H \to BSM)$ is included in the fit)

Higgs Coupling Precision 5 ab^{-1}CEPC + H-20 ILC

$\dfrac{\Delta BR(H \to BSM)}{\Delta BR(H \to Invis)_0}$	∞	8	4	2	1	0.1
ZZ	0.20%	0.19%	0.17%	0.14%	0.13%	0.12%
WW	0.26%	0.25%	0.22%	0.19%	0.17%	0.17%
bb	0.47%	0.%	0.41%	0.37%	0.34%	0.33%
$\tau^+\tau^-$	0.65%	0.63%	0.61%	0.58%	0.57%	0.56%
gg	0.70%	0.69%	0.68%	0.66%	0.66%	0.65%
cc	0.86%	0.85%	0.83%	0.82%	0.81%	0.80%
$\gamma\gamma$	2.6%	2.6%	2.6%	2.6%	2.6%	2.6%
Γ_{tot}	1.2%	1.1%	0.96%	0.76%	0.64%	0.58%

$\Delta BR(H \to Invis)_0$ corresponds to either the H-20 ILC 95% C.L. limit of 0.34%

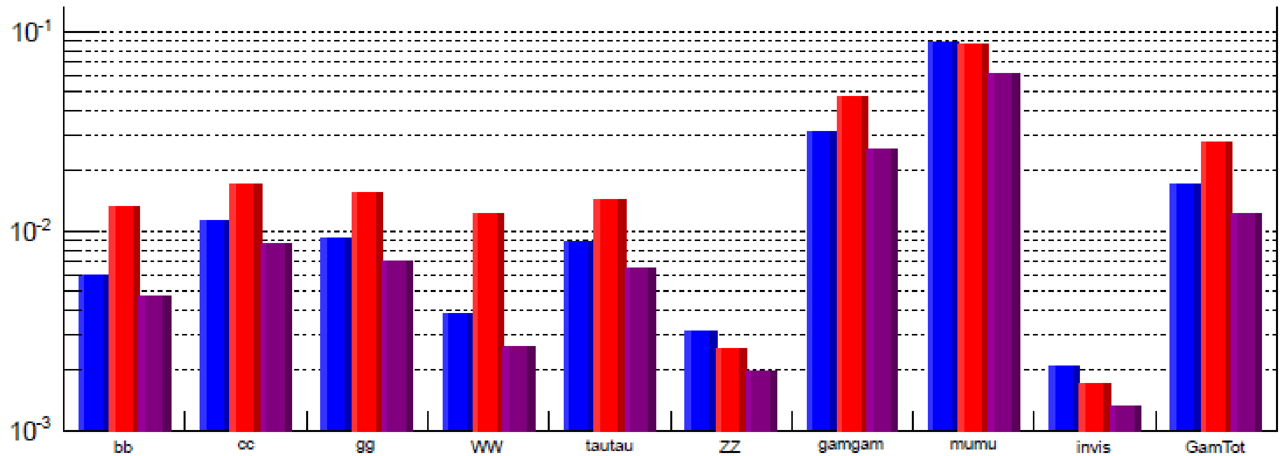
or the 5 ab^{-1} CEPC 95% CL limit of 0.28%

ILC 250+350+500 GeV with 2000+200+4000 fb^{-1} (H-20 scenario full run $\Rightarrow$ 20.2 yrs)

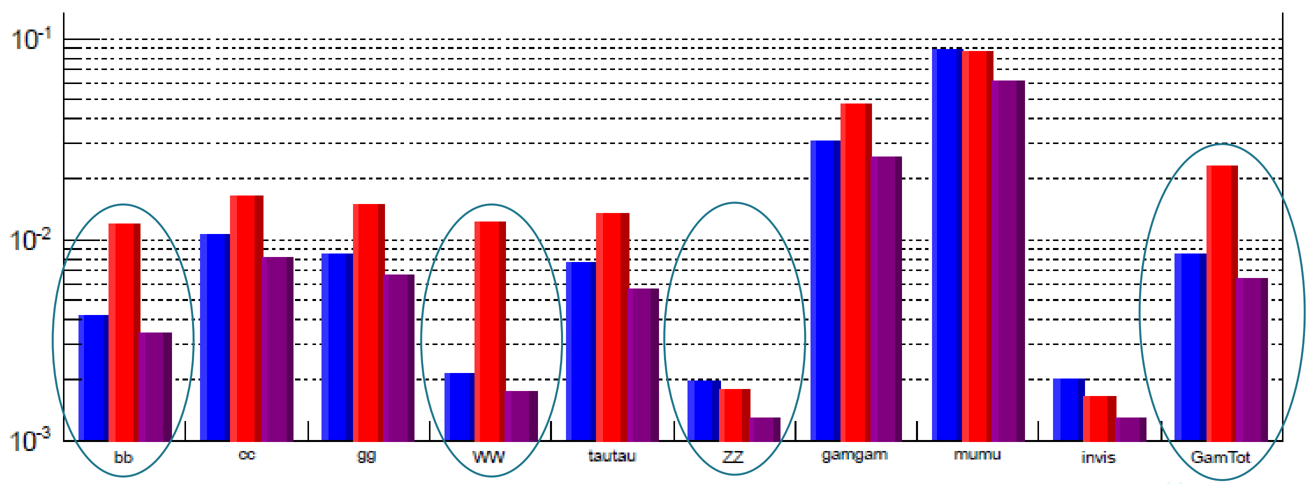
CEPC 250 GeV with 5000 fb^{-1}

ILC + CEPC under the conditions listed above

$$\frac{\Delta BR(H \to BSM)}{\Delta BR(H \to Invis)_0} = \infty$$



$$\frac{\Delta BR(H \to BSM)}{\Delta BR(H \to Invis)_0} = 1$$



How Do You Measure $\sigma \bullet BR_{BSM}$?

- Use $BR_{BSM} = 1 - \sum_{\text{SM decays i}} BR_i$

This can be used for estimating the systemtatic errors for σ(ZH) and the SM σ•BR's.

It can't of course be used to improve Higgs couplings through the constraint $\sum_{\text{i}} BR_i = 1$.

This approach assumes negligible contimination of SM σ•BR analyses by BSM events, and so is model dependent.

The error in this case is

$$(\Delta BR_{BSM})^2 = \sum_{\text{SM decays i}} \left[\left(\frac{\Delta\sigma \bullet BR_i}{\sigma \bullet BR_i} \right)^2 + \left(\frac{\Delta\sigma_{ZH}}{\sigma_{ZH}} \right)^2 \right] (BR_i)^2$$

which can be pretty good given that $\sigma \bullet BR_i$ is measured well for decay channels with large BR's. This technique was used to obtain $\Delta BR_{BSM} = 0.04$ in the discussion of the $\sqrt{s} = 350$ GeV $\sigma(\text{ZH})_{hadronic}$ systematic error.

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How Do You Measure $\sigma \bullet BR_{BSM}$?

- Include a BSM contribution in each of the SM $\sigma \bullet BR_i$ analyses when doing the correlated global fit of all SM $\sigma \bullet BR_i$

The problem here is the unknown efficiency for BSM decays to pass the selection for each decay channel. One might also gain additional information by performing the ZH leptonic and hadronic recoil analysis on an event sample from which all events passing SM $\sigma \bullet BR_i$ analyses have been removed. Work is ongoing to develop this kind of procedure.

- Go through a long list of BSM decay topologies, perform a dedicated search for each, and then convince yourself that all possible BSM decay topologies have been covered.

Sort of a brute force approach, but it might be the only way. Tricky part is proving that all topologies have been accounted for.

215 page "Exotic Decays of the 125 GeV Higgs Boson" arXiv:1312.4992

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more Table of Contents from 215 page "Exotic Decays of the 125 GeV Higgs Boson" arXiv:1312.4992:

Summary

- BSM decays of the Higgs are of course important in their own right. However, even if no evidence for BSM decays is found, the model independent limit that can be placed on BSM decays affects the SM coupling measurements. It is important in evaluating the systematic errors for σ(ZH) and the σ•BR's , and strong limits on BSM decays are needed to squeeze the last bit of Higgs coupling precision out of the data (the CEPC Δg_Z=0.26% improves to 0.18% if $\Delta BR(H \to BSM) \approx \Delta BR(H \to Invis)$).

- Work is ongoing to estimate $BR(H \to BSM)$ at CEPC using several techniques.

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Backup Slides

Model Independence of ZH Recoil Measurements

In order to use the hadronic ZH recoil measurement in our Higgs analyses we have to quantify the penalty associated with the fact that $\sigma(ZH \to q\bar{q} + X)$ is "almost model independent". By how much must we blow up $\Delta\sigma(ZH \to q\bar{q} + X)$ to account for the fact that the efficiencies differ by as much as 7%?

Model Independent?

★ **Combining visible + invisible analysis: wanted M.I.**

- **i.e. efficiency independent of Higgs decay mode**

Decay mode	$\varepsilon^{\mathrm{vis}}_{\mathscr{L}>0.65}$	$\varepsilon^{\mathrm{vis}}_{\mathscr{L}>0.60}$	$\varepsilon^{\mathrm{vis}} + \varepsilon^{\mathrm{invis}}$
$\mathrm{H} \to \mathrm{invis.}$	<0.1 %	22.0 %	22.0 %
$\mathrm{H} \to \mathrm{q\bar{q}/gg}$	22.2 %	<0.1 %	22.2 %
$\mathrm{H} \to \mathrm{WW}^*$	21.6 %	0.1 %	21.7 %
$\mathrm{H} \to \mathrm{ZZ}^*$	20.2 %	1.0 %	21.2 %
$\mathrm{H} \to \tau^+\tau^-$	24.7 %	0.3 %	24.9 %
$\mathrm{H} \to \gamma\gamma$	25.8 %	<0.1 %	25.8 %
$\mathrm{H} \to \mathrm{Z}\gamma$	18.5 %	0.3 %	18.8 %
$\mathrm{H} \to \mathrm{WW}^* \to \mathrm{q\bar{q}q\bar{q}}$	21.3 %	<0.1 %	21.3 %
$\mathrm{H} \to \mathrm{WW}^* \to \mathrm{q\bar{q}l\nu}$	21.9 %	<0.1 %	21.9 %
$\mathrm{H} \to \mathrm{WW}^* \to \mathrm{q\bar{q}\tau\nu}$	22.1 %	<0.1 %	22.1 %
$\mathrm{H} \to \mathrm{WW}^* \to \mathrm{l\nu l\nu}$	24.8 %	0.1 %	25.0 %
$\mathrm{H} \to \mathrm{WW}^* \to \mathrm{l\nu\tau\nu}$	20.5 %	0.8 %	22.1 %
$\mathrm{H} \to \mathrm{WW}^* \to \tau\nu\tau\nu$	16.4 %	2.5 %	18.9 %

Very similar efficiencies

Look at wide range of WW topologies

Mark Thomson Oshu City, September 2014 21

Model Independence of ZH Recoil Measurements

It is not sufficient to vary the SM Higgs branching ratios to estimate this systematic error. The problem is the BSM decays; they cannot be accounted for in this way.

To handle the BSM decays we have used an approach where we use all of our $\sigma \bullet BR$ measurements for SM Higgs decays to obtain an estimate of the average signal efficiency for $\sigma(ZH \to q\bar{q} + X)$. It is then straightforward to propagate the $\sigma \bullet BR_i$ errors, as well as the systematic errors on the individual decay mode efficiencies for the $\sigma(ZH \to q\bar{q} + X)$ selection, to the error on $\sigma(ZH \to q\bar{q} + X)$.

Model Independence of ZH Recoil Measurements

Let

$$\Psi \equiv \sigma(ZH \to q\bar{q} + X)$$

$\Omega =$ Number of signal + background events in $\sigma(ZH \to q\bar{q} + X)$ analysis

$\mathrm{B} =$ Predicted number of background events in $\sigma(ZH \to q\bar{q} + X)$ analysis

Ξ = Average efficiency for signal events to pass $\sigma(ZH \to q\bar{q} + X)$ analysis

$L =$ luminosity

$$\Psi = \frac{\Omega - \mathrm{B}}{L\,\Xi} = \frac{1}{\Xi}\sum_i \psi_i \xi_i = \sum_i \psi_i \quad \text{where}$$

$$\psi_i = \sigma(ZH) \bullet BR_i$$

$\xi_i =$ efficiency for events from Higgs decay i to pass $\sigma(ZH \to q\bar{q} + X)$ analysis

$$\Xi = \frac{\sum_i \psi_i \xi_i}{\sum_i \psi_i}$$

Model Independence of ZH Recoil Measurements

$$\psi_i = \frac{\omega_i - \beta_i}{L\,\eta_i}$$

ω_i = Number of signal + background events in $\sigma(ZH)\bullet BR_i$ analysis

β_i = Predicted number of background events in $\sigma(ZH)\bullet BR_i$ analysis

η_i = efficiency for Higgs decay i to pass $\sigma\bullet BR_i$ analysis

K_i = number of signal + background events common to had Z recoil and $\sigma\bullet BR_i$ analyses

E = number of signal + background events unique to had Z recoil analysis

ε_i = number of signal + background events events unique to $\sigma\bullet BR_i$ analysis

$$\Omega = \mathrm{E} + \sum_i \mathrm{K}_i \qquad S \equiv \Omega\text{-B} \qquad \mathrm{T} \equiv \frac{\sqrt{S+\mathrm{B}}}{S}$$

$$\omega_i = \mathrm{K}_i + \varepsilon_i \qquad s_i \equiv \omega_i - \beta_i \qquad \tau_i \equiv \frac{\sqrt{s_i + \beta_i}}{s_i}$$

$$\lambda_i \equiv \frac{\mathrm{K}_i}{\omega_i} \qquad N \equiv L\,\sigma_{ZH} \qquad r_i \equiv BR_i \qquad \delta_i \equiv \xi_i - \Xi$$

$$\left(\frac{\Delta\Psi}{\Psi}\right)^2 = \mathrm{T}^2\left\{1 + \frac{N^2}{\Omega}\sum_i r_i^2\left[\tau_i^2\left(\delta_i^2 - 2\lambda_i\eta_i\delta_i\right) + \Delta\xi_i^2\right]\right\}$$

This is our result for the error on $\sigma(ZH \to q\,\bar{q} + X)$

Model Independence of ZH Recoil Measurements

$$\left(\frac{\Delta\sigma(ZH \to q\bar{q}+X)}{\sigma(ZH \to q\bar{q}+X)}\right)^2 = \mathrm{T}^2\left\{1+\frac{N^2}{\Omega}\sum_i r_i^2\left[\tau_i^2\delta_i^2+\Delta\xi_i^2\right]\right\} \quad \text{i.e. sys err} = \frac{1}{2}\frac{N^2}{\Omega}\sum_i r_i^2\left[\tau_i^2\delta_i^2+\Delta\xi_i^2\right]$$

Assume $\sqrt{s}=350$ GeV and L=500 fb^{-1}

$$\mathrm{N}=L\,\sigma_{ZH}=45383 \quad r_i=BR_i=(1-BR_{BSM})BR_i(SM) \quad \tau_i(SM)=\frac{\Delta\sigma\bullet \mathrm{BR}_i(SM)}{\sigma\bullet \mathrm{BR}_i(SM)}=\frac{\sqrt{s_i+\beta_i}}{s_i}$$

Assume $\mathrm{T}=\frac{\sqrt{S+\mathrm{B}}}{S}=0.014$ Ω=S+B $=17738$ and $\xi_i(SM)$ given in the table four pages back.

We assume that the vis+invis efficiency values in the table four pages back cover all possible BSM decays since they cover all SM decays from completely invisible to fully hadronic multi-jet decays. Assuming no knowledge of the properties of the BSM decays we can then set

$$\xi_{BSM}=0.5*[\xi_{vis+invis}(\mathrm{max})+\xi_{vis+invis}(\mathrm{min})]=0.5*[0.258+0.188]=0.22$$
$$\Delta\xi_{BSM}=0.5*[\xi_{vis+invis}(\mathrm{max})-\xi_{vis+invis}(\mathrm{min})]=.035$$

Model Independence of ZH Recoil Measurements

$$\left(\frac{\Delta\sigma(ZH \to q\,\bar{q} + X)}{\sigma(ZH \to q\,\bar{q} + X)}\right)^2 = \mathrm{T}^2 \left\{1 + \frac{N^2}{\Omega}\sum_i r_i^2 \left[\tau_i^2 \delta_i^2 + \Delta\xi_i^2\right]\right\} \quad \text{i.e. sys err} = \frac{1}{2}\frac{N^2}{\Omega}\sum_i r_i^2 \left[\tau_i^2 \delta_i^2 + \Delta\xi_i^2\right]$$

We next obtain the error $\frac{\Delta\sigma \bullet \mathrm{BR}_{BSM}}{\sigma \bullet \mathrm{BR}_{BSM}}$ from Michael Peskin's Higgs coupling fit program. We do not use the $\sum_i BR_i = 1$ constraint, and to begin with we only use the leptonic recoil σ_{ZH} measurement. This provides a model independent measurement of g_{BSM}. For $\sqrt{s} = 350$ GeV, L=500 fb^{-1} Michael's program gives $\frac{\Delta\mathrm{g}_{BSM}}{\mathrm{g}_{BSM}} = 0.032$ which we multiply by two to get $\frac{\Delta\sigma \bullet \mathrm{BR}_{BSM}}{\sigma \bullet \mathrm{BR}_{BSM}} = 0.064$. We take this error to mean that $0 < BR(H \to BSM) < 2 \times 0.064$, and set the measured $BR(H \to BSM) = 0.064$. This gives a model independent hadronic recoil cross section error of $\frac{\Delta\sigma(ZH \to q\,\bar{q} + X)}{\sigma(ZH \to q\,\bar{q} + X)} = 0.014 * 1.27 = 0.018$.

We then add this new model indepdendent hadronic recoil σ_{ZH} measurement as input to Michael's program to obtain a new error $\frac{\Delta\sigma \bullet \mathrm{BR}_{BSM}}{\sigma \bullet \mathrm{BR}_{BSM}} = 0.041$. Setting $BR(H \to BSM) = 0.041$ we then obtain a new model independent hadronic recoil σ_{ZH} error of $\frac{\Delta\sigma(ZH \to q\,\bar{q} + X)}{\sigma(ZH \to q\,\bar{q} + X)} = 0.014 * 1.12 = 0.016$.

Iterating again we arrive at $BR(H \to BSM) = 0.039$ and $\frac{\Delta\sigma(ZH \to q\,\bar{q} + X)}{\sigma(ZH \to q\,\bar{q} + X)} = 0.014 * 1.11 = 0.016$. Further interations don't give any improvement.

Model Independence of ZH Recoil Measurements

$$\left(\frac{\Delta\sigma(ZH \to q\,\bar{q} + X)}{\sigma(ZH \to q\,\bar{q} + X)}\right)^2 = \mathrm{T}^2 \left\{1 + \frac{N^2}{\Omega}\sum_i r_i^2 \left[\tau_i^2\delta_i^2 + \Delta\xi_i^2\right]\right\} \quad \text{i.e. sys err} = \frac{1}{2}\frac{N^2}{\Omega}\sum_i r_i^2 \left[\tau_i^2\delta_i^2 + \Delta\xi_i^2\right]$$

We have shown that $\frac{1}{2}\frac{N^2}{\Omega}\sum_i r_i^2 \left[\tau_i^2\delta_i^2 + \Delta\xi_i^2\right] = 0.11$ for $\sqrt{s} = 350$ GeV, L=500 fb^{-1}.

How does this scale with luminosity?

$\frac{N^2}{\Omega} \propto L \quad \tau_i^2 \propto L^{-1} \quad r_i^2$ is independent of lumi except $r_{BSM}^2 = \tau_{BSM}^2 \propto L^{-1}$.

If we assume $\Delta\xi_i = 0$ except $\Delta\xi_{BSM} = 0.035$ then

$\frac{1}{2}\frac{N^2}{\Omega}\sum_i r_i^2 \left[\tau_i^2\delta_i^2 + \Delta\xi_i^2\right] = 0.11$ independent of the luminosity at $\sqrt{s} = 350$ GeV.

Model Independence of ZH Recoil Measurements

Caveats for hadronic recoil systematic error calculation :

(1) This systematic error analysis was only done at $\sqrt{s} = 350$ GeV; it has not yet been done for $\sqrt{s} = 250$ & 500 GeV

(2) These results assume that the true $r_{BSM} = BR(H \to \text{ BSM})$ is small. As the true r_{BSM} grows we need to keep the product $r_{BSM}\Delta\xi_{BSM}$ constant to maintain the same systematic error, where ξ_{BSM} is the effieciency for BSM Higgs decays to pass the hadronic recoil analysis. For example

true r_{BSM}	required $\Delta\xi_{BSM}$
.05	0.027
.10	0.014
.15	0.0091
.20	0.0068

These $\Delta\xi_{BSM}$ requirements may seem stringent for the larger values of true r_{BSM} . However as r_{BSM} grows we will have more *BSM* decays to analyze and the required improvement in Monte Carlo modelling of the *BSM* decays should follow.

Let

$\Psi \equiv \sigma(ZH)\bullet BR(visible)$

$\Omega =$ Number of signal + background events in $\sigma(ZH)\bullet BR(visible)$ analysis

$\mathrm{B} =$ Predicted number of background events in $\sigma(ZH)\bullet BR(visible)$ analysis

Ξ = Average efficiency for signal events to pass $\sigma(ZH)\bullet BR(visible)$ analysis

$L =$ luminosity

$$\Psi = \frac{\Omega - \mathrm{B}}{L\,\Xi} = \frac{1}{\Xi}\sum_i \psi_i \xi_i = \sum_i \psi_i \quad \text{where}$$

$\psi_i = \sigma(ZH)\bullet BR_i$

$\xi_i =$ efficiency for events from Higgs decay i to pass $\sigma(ZH)\bullet BR(visible)$ analysis

$$\Xi = \frac{\sum_i \psi_i \xi_i}{\sum_i \psi_i}$$

$$\psi_i = \frac{\omega_i - \beta_i}{L\,\eta_i}$$

$\omega_i =$ Number of signal + background events in $\sigma(ZH)\bullet BR_i$ analysis

$\beta_i =$ Predicted number of background events in $\sigma(ZH)\bullet BR_i$ analysis

η_i = efficiency for Higgs decay i to pass $\sigma\bullet BR_i$ analysis

K_i = number of signal + background events common to had Z recoil and $\sigma\bullet BR_i$ analyses

E = number of signal + background events unique to had Z recoil analysis

ε_i = number of signal + background events events unique to $\sigma\bullet BR_i$ analysis

$$\Omega = \mathrm{E} + \sum_i \mathrm{K}_i \qquad S \equiv \Omega\text{-}\mathrm{B} \qquad \mathrm{T} \equiv \frac{\sqrt{S + \mathrm{B}}}{S}$$

$$\omega_i = \mathrm{K}_i + \varepsilon_i \qquad s_i \equiv \omega_i - \beta_i \qquad \tau_i \equiv \frac{\sqrt{s_i + \beta_i}}{s_i}$$

$$\lambda_i \equiv \frac{\mathrm{K}_i}{\omega_i} \qquad N \equiv L\,\sigma_{ZH} \qquad r_i \equiv BR_i \qquad \delta_i \equiv \xi_i - \Xi$$

$$(\Delta\Psi)^2 = \left(\frac{\partial\Psi}{\partial\Omega}\right)^2 V_{\Omega\Omega} + \left(\frac{\partial\Psi}{\partial\Xi}\right)^2 V_{\Xi\Xi} + 2\frac{\partial\Psi}{\partial\Omega}\frac{\partial\Psi}{\partial\Xi}V_{\Omega\Xi}$$

$$\frac{\partial\Psi}{\partial\Omega} = \frac{1}{L\,\Xi} = \frac{\Psi}{\Omega}\left(1-\frac{\mathrm{B}}{\Omega}\right)^{-1} \qquad \frac{\partial\Psi}{\partial\Xi} = -\frac{\Omega-\mathrm{B}}{L\,\Xi^2} = -\frac{\Psi}{\Xi}$$

$$V_{\Omega\Omega} = \mathrm{E} + \sum_i \mathrm{K}_i = \Omega$$

$$V_{\Xi\Xi} = \frac{1}{L^2\Psi^2}\sum_i \frac{(\xi_i - \Xi)^2}{(\eta_i)^2}(\varepsilon_i + \mathrm{K}_i)$$

$$V_{\Omega\Xi} = \frac{1}{L\,\Psi}\sum_i \frac{\xi_i - \Xi}{\eta_i}\mathrm{K}_i$$

$$\left(\frac{\Delta\Psi}{\Psi}\right)^2 = \frac{1}{\Omega^2}\left(1-\frac{\mathrm{B}}{\Omega}\right)^{-2} V_{\Omega\Omega} + \frac{1}{\Xi^2} V_{\Xi\Xi} - \frac{2}{\Omega\Xi}\left(1-\frac{\mathrm{B}}{\Omega}\right)^{-1} V_{\Omega\Xi}$$

$$= \frac{1}{\Omega}\left(1-\frac{\mathrm{B}}{\Omega}\right)^{-2} + \frac{1}{L^2\Xi^2\Psi^2}\sum_i \frac{(\xi_i-\Xi)^2}{(\eta_i)^2}(\varepsilon_i + \mathrm{K}_i) - \frac{2}{L\,\Omega\Xi\Psi}\left(1-\frac{\mathrm{B}}{\Omega}\right)^{-1}\sum_i \frac{\xi_i-\Xi}{\eta_i}\mathrm{K}_i$$

$$= \frac{1}{\Omega}\left(1-\frac{\mathrm{B}}{\Omega}\right)^{-2} + \frac{1}{L^2\Xi^2\Psi^2}\sum_i \frac{(\xi_i-\Xi)^2}{(\eta_i)^2}(L\eta_i\psi_i + \beta_i) - \frac{2}{L\,\Omega\Xi\Psi}\left(1-\frac{\mathrm{B}}{\Omega}\right)^{-1}\sum_i \frac{\xi_i-\Xi}{\eta_i}\lambda_i(L\eta_i\psi_i + \beta_i)$$

$$= \frac{1}{\Omega}\left(1-\frac{\mathrm{B}}{\Omega}\right)^{-2}\left[1+\frac{L}{\Omega}\sum_i \frac{(\xi_i-\Xi)^2}{\eta_i}\psi_i\left(1+\frac{\beta_i}{s_i}\right) - \frac{2L}{\Omega}\sum_i (\xi_i-\Xi)\psi_i\lambda_i\left(1+\frac{\beta_i}{s_i}\right)\right]$$

$$= \frac{S+\mathrm{B}}{S^2}\left\{1+\frac{L}{\Omega}\sum_i (\xi_i-\Xi)\psi_i\left(\frac{s_i+\beta_i}{s_i^2}\right)\left[(\xi_i-\Xi)L\psi_i - 2\lambda_i s_i\right]\right\}$$

$$= \mathrm{T}^2\left\{1+\frac{N^2}{\Omega}\sum_i r_i^2\tau_i^2\left[\delta_i^2 - 2\lambda_i\eta_i\delta_i\right]\right\}$$

What if we don't do a hadronic Z recoil measurement and instead only use $\sigma(ZH)\bullet BR_i$ to calculate $\sigma(ZH)\bullet BR(visible) = \sum_i \sigma(ZH)\bullet BR_i$?

$$\Psi' = \sum_i \psi_i \qquad \psi_i = \frac{\omega_i - \beta_i}{L\,\xi_i}$$

$$(\Delta\Psi')^2 = \sum_i \left(\frac{\partial\Psi'}{\partial\omega_i}\right)^2 \omega_i \ , \qquad \frac{\partial\Psi'}{\partial\omega_i} = \frac{1}{L\eta'_i}$$

$$(\Delta\Psi')^2 = \frac{1}{L^2}\sum_i = \frac{1}{L^2}\sum_i \frac{s_i + \beta_i}{\xi_i^2}$$

$$\left(\frac{\Delta\Psi'}{\Psi'}\right)^2 = \left(\sum_i \frac{\omega_i - \beta_i}{L\,\xi_i}\right)^{-2} \frac{1}{L^2}\sum_i \frac{s_i + \beta_i}{\xi_i^2}$$

$$= \frac{S + \mathrm{B}}{S^2}\frac{L}{\Omega}\Xi^2 \sum_i \frac{\psi_i}{\xi_i}\left(1 + \frac{\beta_i}{s_i}\right)$$

Compare this now with our formula for $\left(\frac{\Delta\Psi}{\Psi}\right)^2$ for $\lambda_i = 1$:

$$\left(\frac{\Delta\Psi}{\Psi}\right)^2 = \frac{S + \mathrm{B}}{S^2}\left\{1 + \frac{1}{\Omega}\sum_i \omega_i \left[\left(1 - \frac{\Xi}{\xi_i}\right)^2 - 2\left(1 - \frac{\Xi}{\xi_i}\right)\right]\right\}$$

$$= \frac{S + \mathrm{B}}{S^2}\left\{1 + \frac{1}{\Omega}\sum_i \omega_i \left[1 - \frac{2\Xi}{\xi_i} + \left(\frac{\Xi}{\xi_i}\right)^2 - 2 + 2\frac{\Xi}{\xi_i}\right]\right\} = \left(\frac{\Delta\Psi'}{\Psi'}\right)^2$$