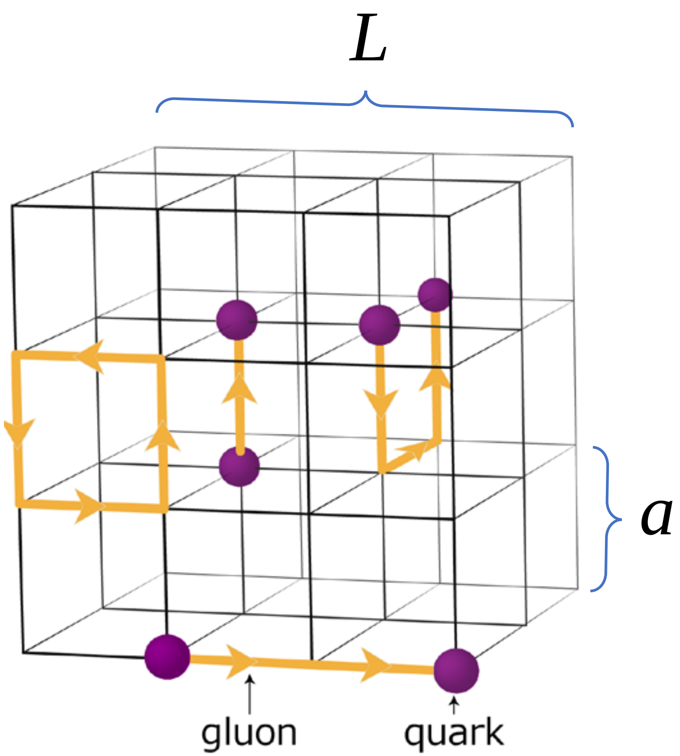
Lattice QCD: Flavor Physics and Spectroscopy

Takashi Kaneko (KEK)

XXVIII International Symposium on Lepton Photon Interactions
at High Energies, 7 – 11 Aug. 2017 @ SYSU, Guangzhou, China

lattice regularization of QCD



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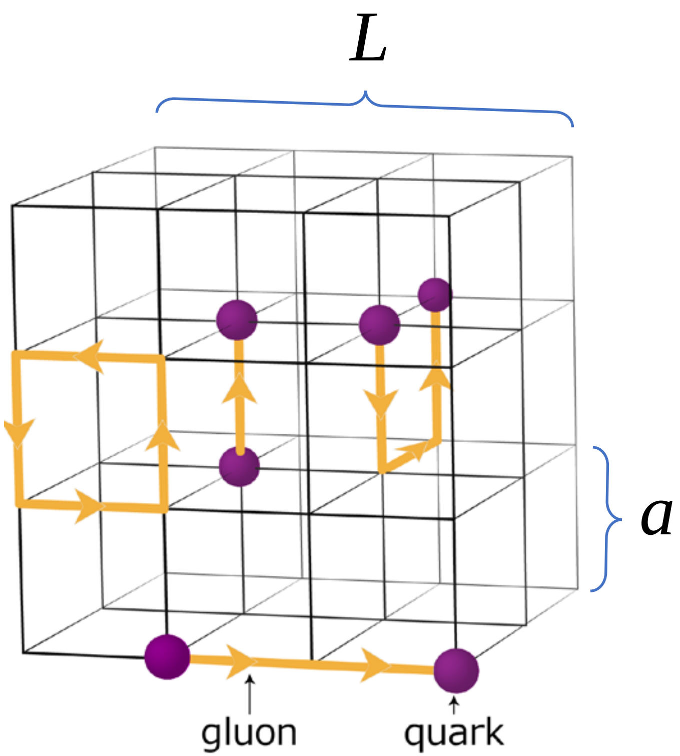

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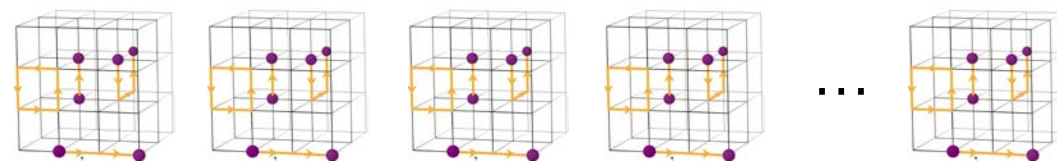

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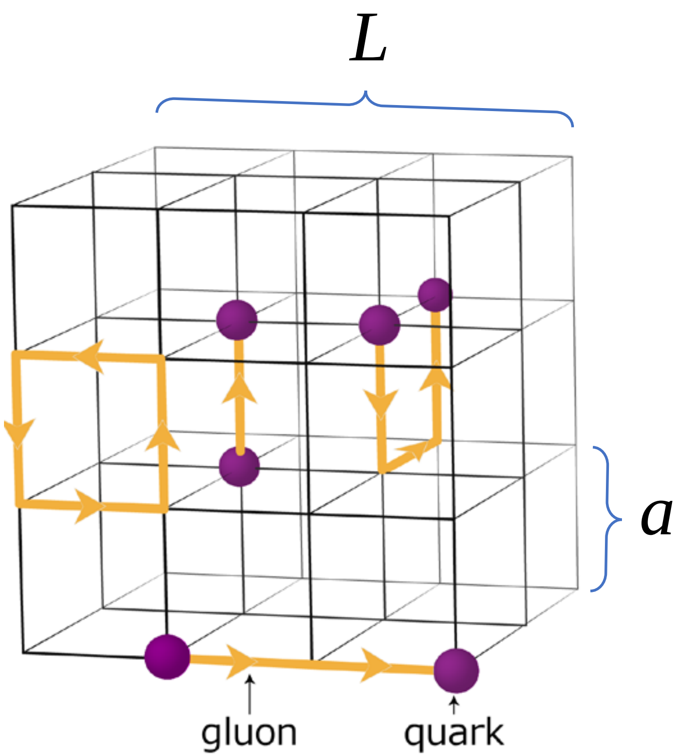

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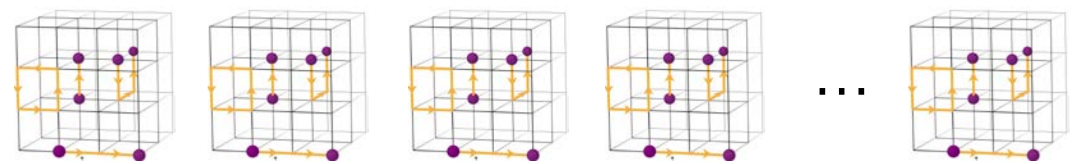

control of stat. and sys. uncertainties

finite a, L, unphysical m_u=m_d, m_b, ⋯

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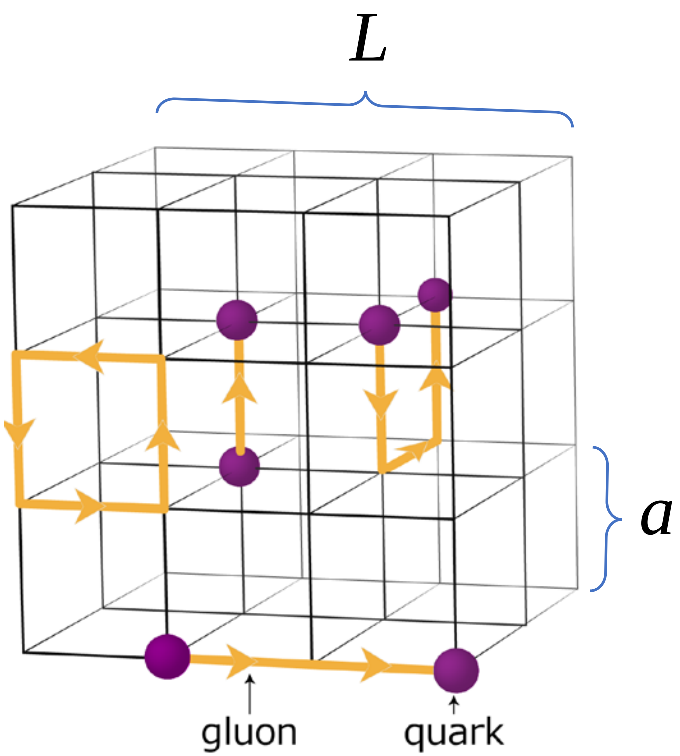

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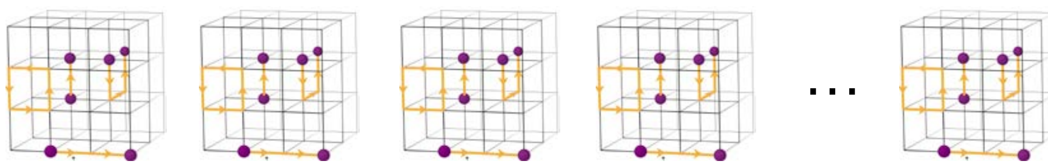

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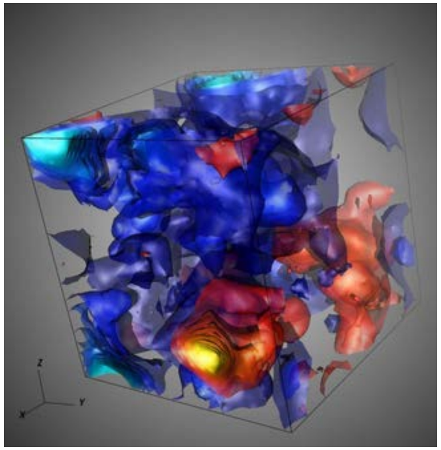

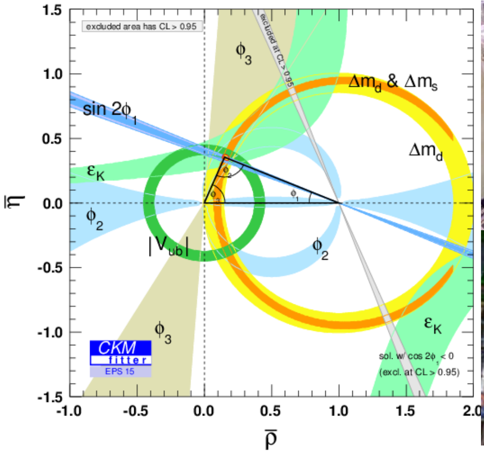

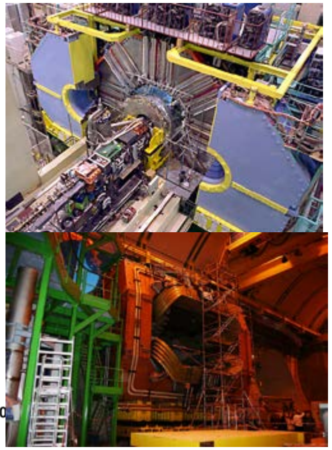

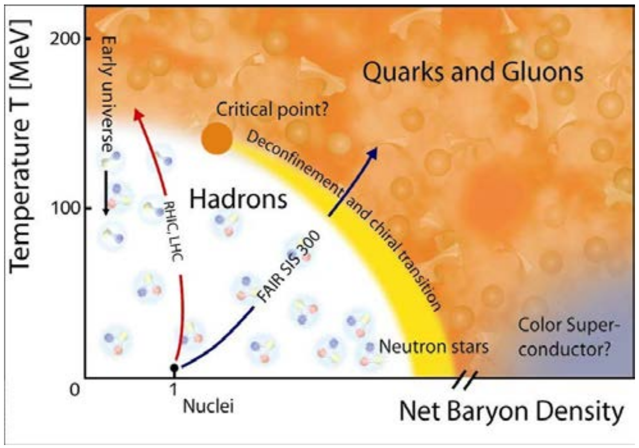

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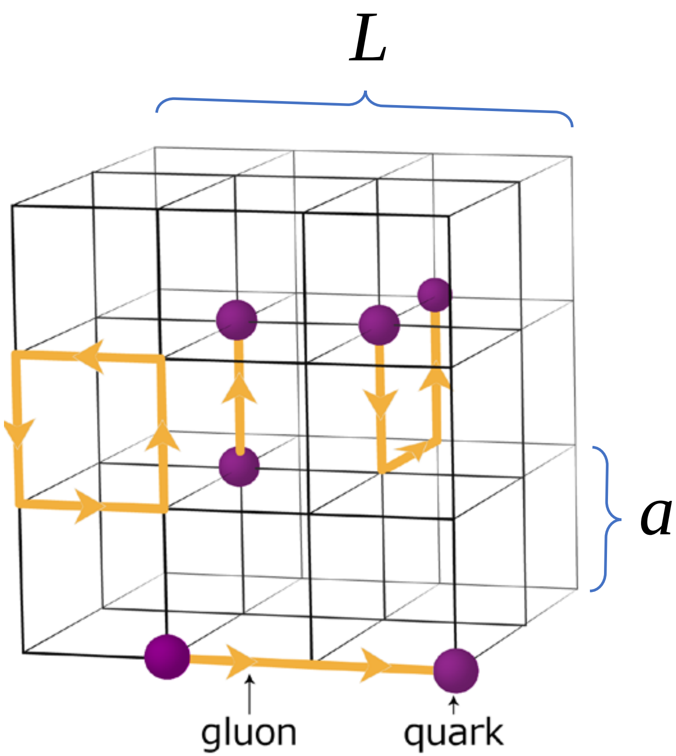

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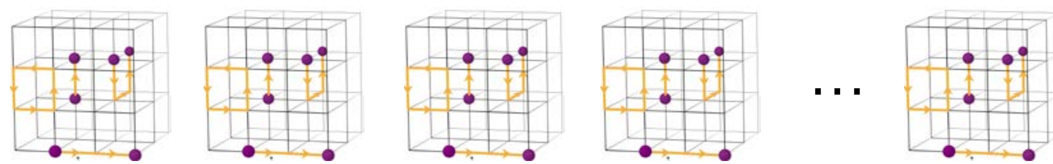

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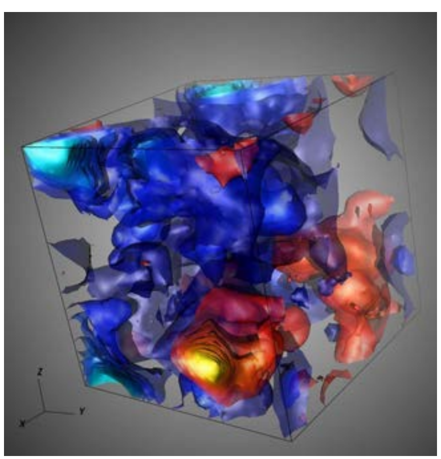

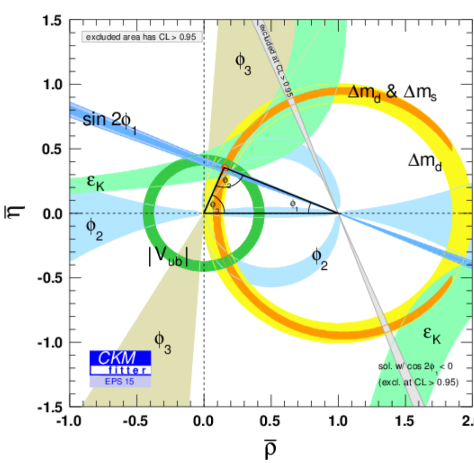

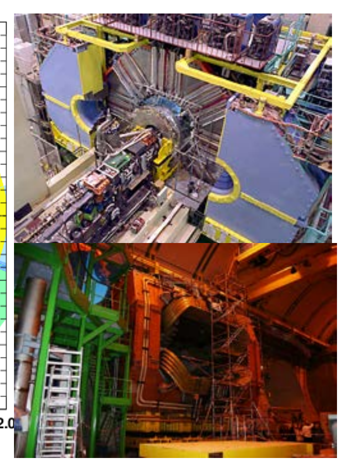

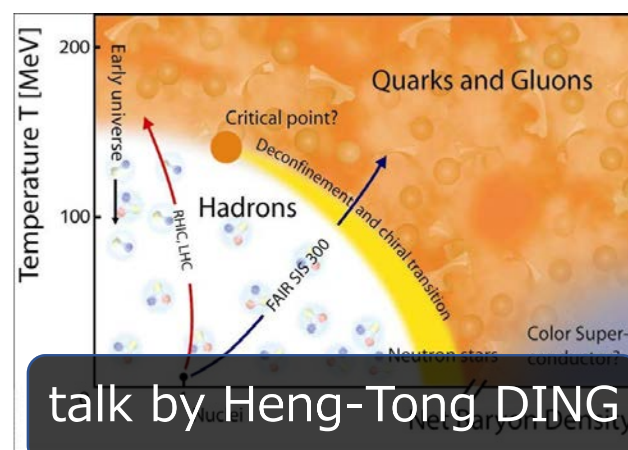

talk by Heng-Tong DING

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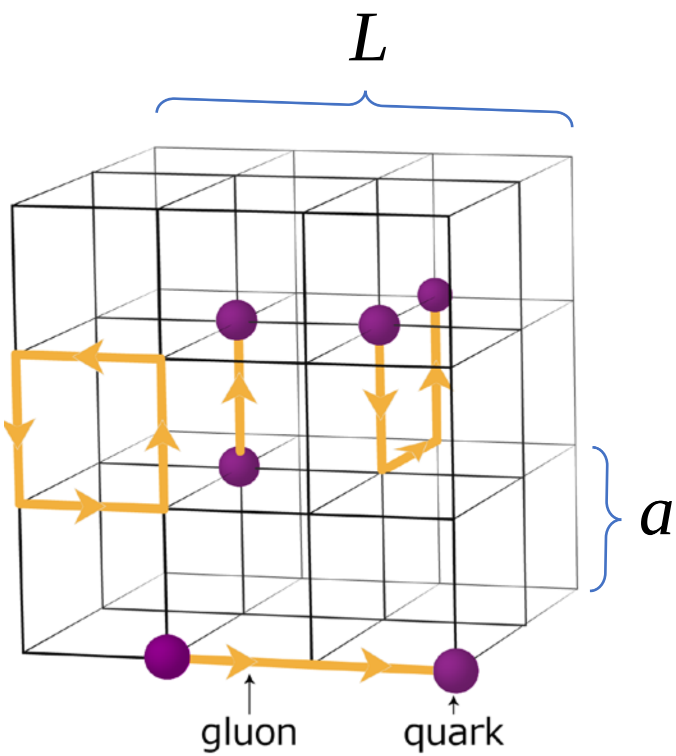

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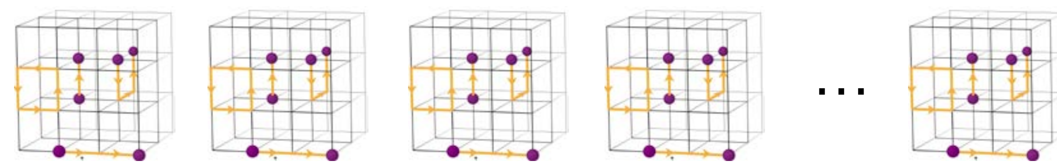

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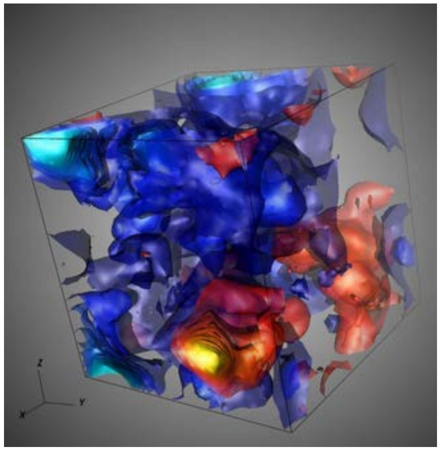
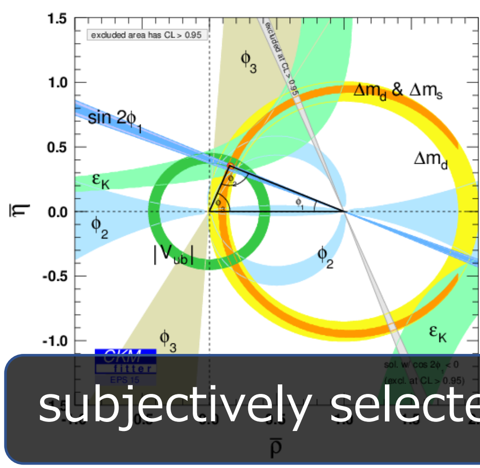
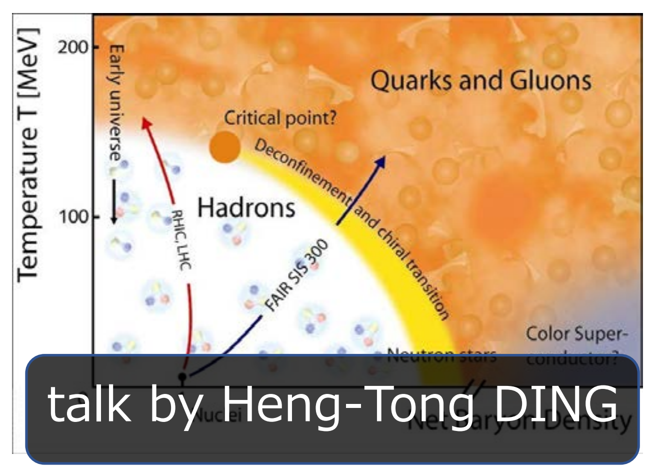




subjectively selected highlights



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1. Hadron Spectroscopy

stable / ignoring decays

interpolating fields w/ given quantum #'s $e.g.\, O_\pi = \bar{u}\gamma_5 d$

$$C_{\alpha\beta}(t) = \left\langle O_\alpha(t) O_\beta^\dagger(0) \right\rangle = \sum_n Z_{n\beta}^* Z_{n\alpha} \exp\left[-E_n t\right], \qquad Z_{n\alpha} = \left\langle n \middle| O_\alpha^\dagger \middle| 0 \right\rangle$$

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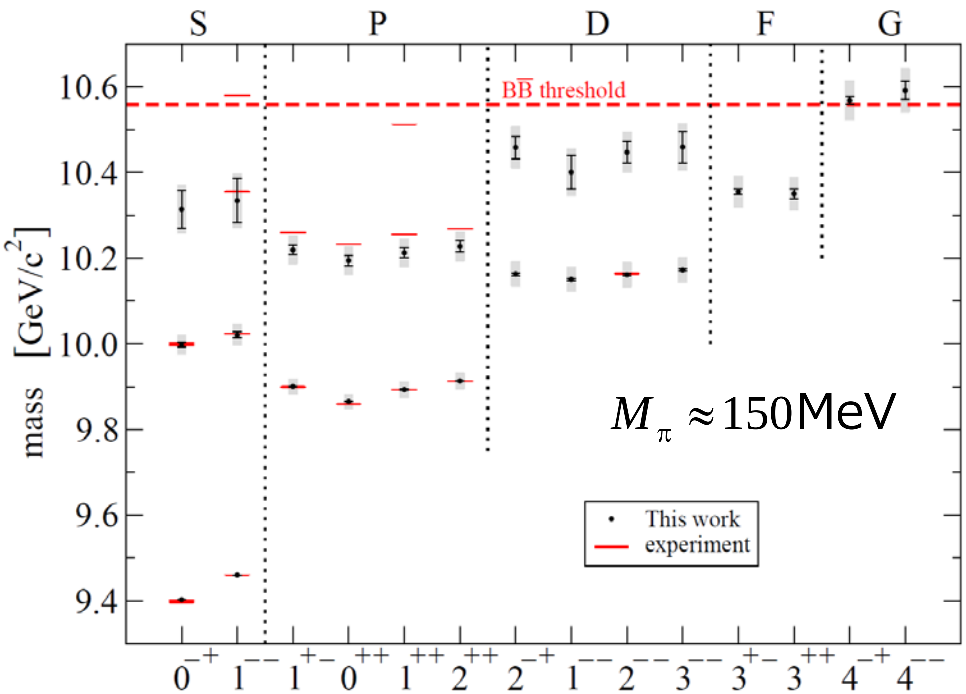
$\bar{b}b$: Wurtz et al. '15



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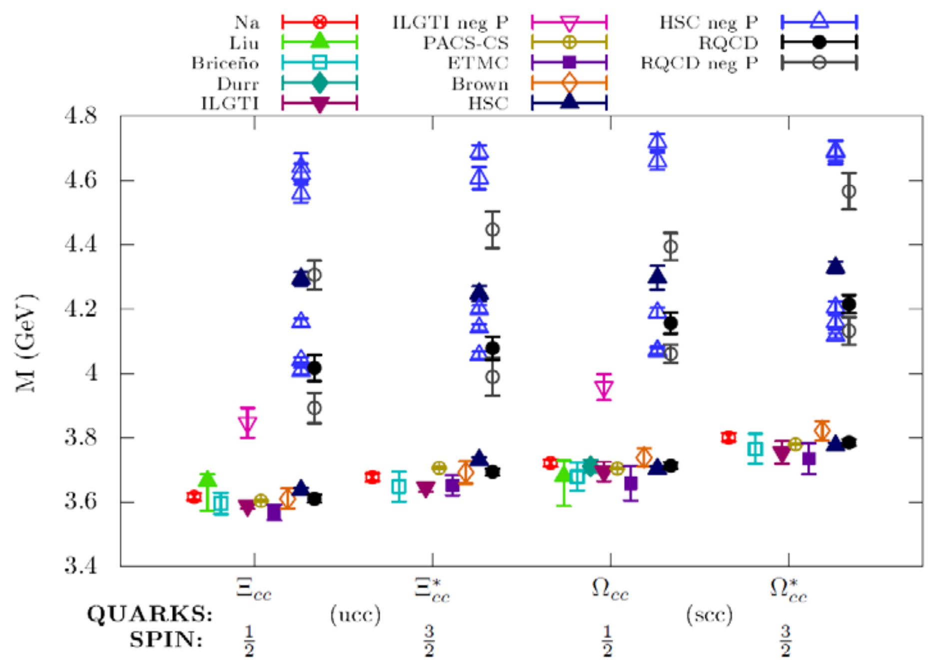

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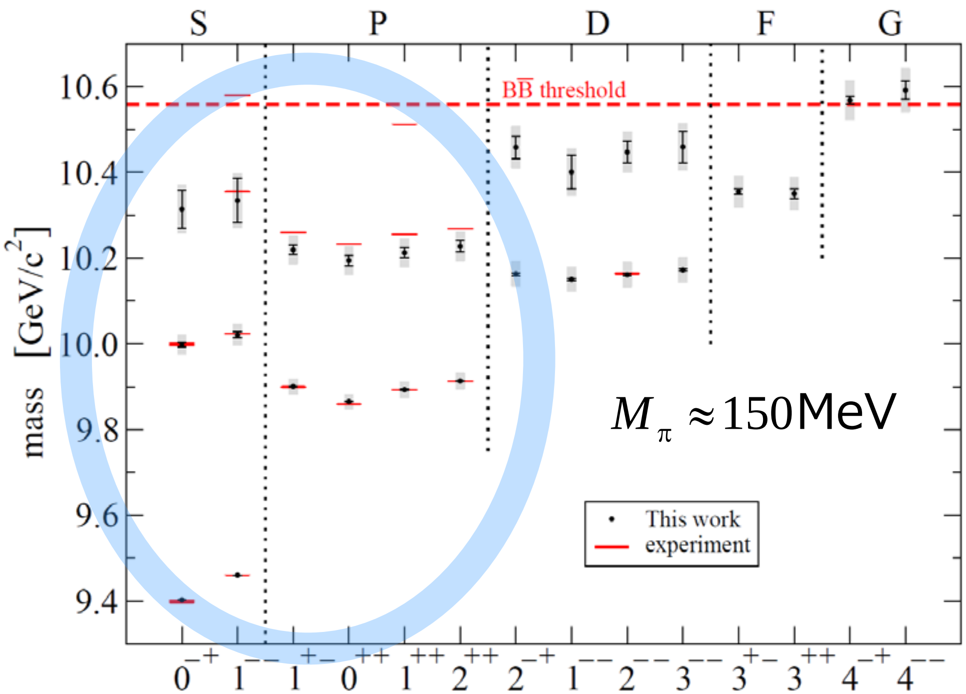

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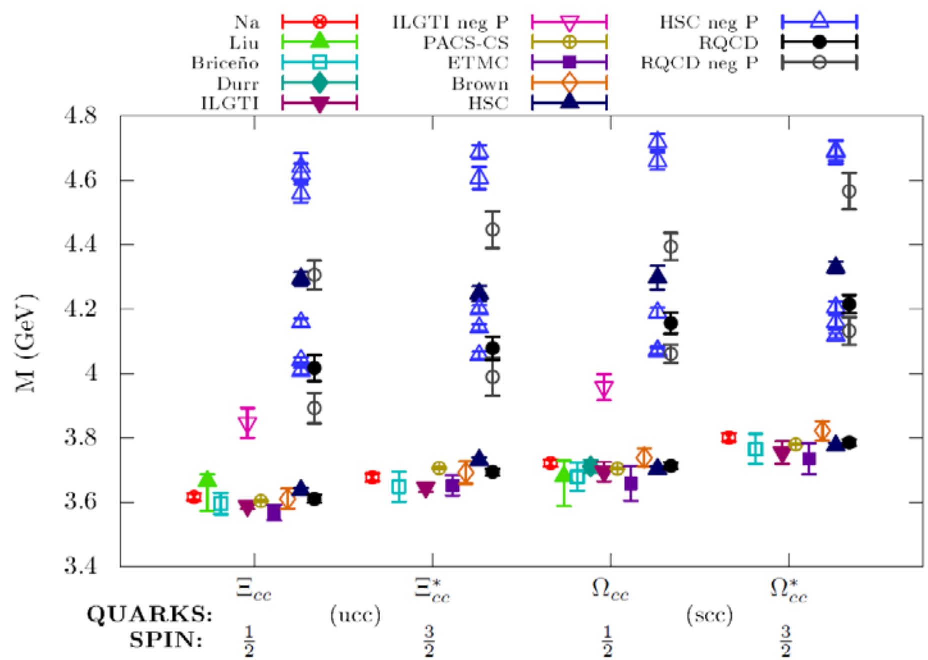

impressive agreement w/ expr't / guide for "yet un-observed"

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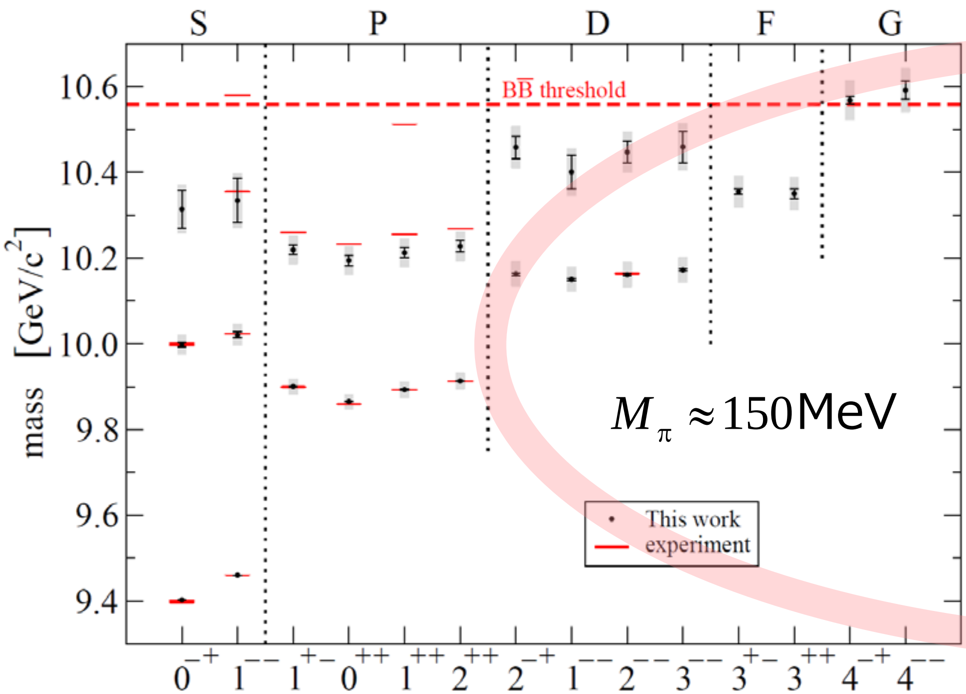
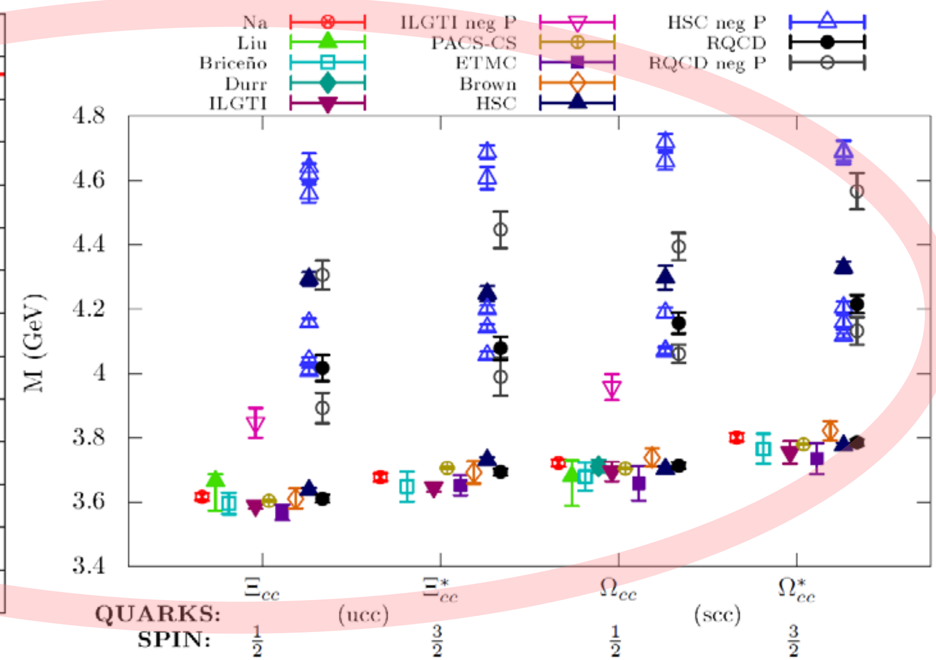


$M_\pi \approx 150\text{MeV}$



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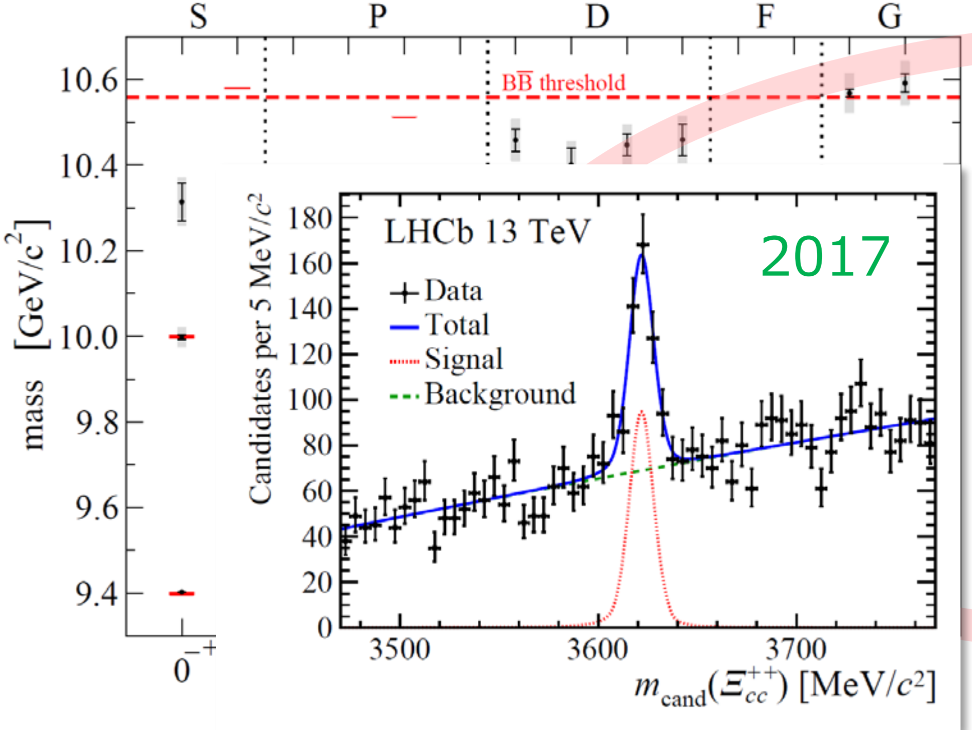

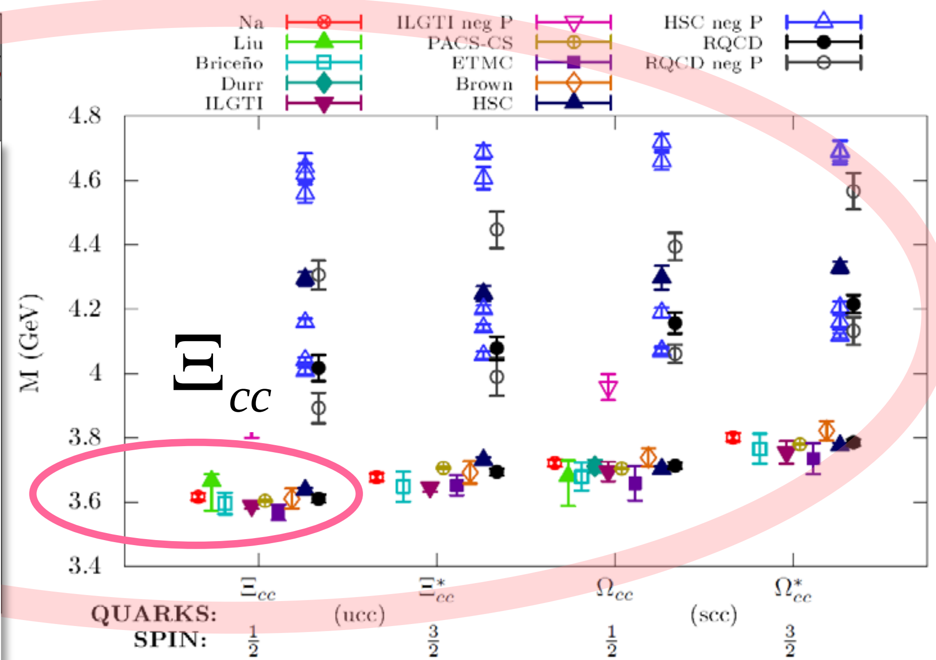

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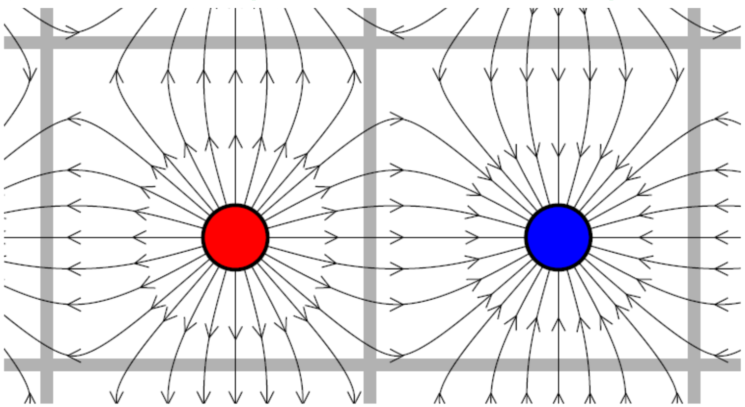
isospin splittings

- ‰ level needed ⇒ kaon, g-2 HVP
- $m_u \neq m_d$: straightforward
- EM corrections

  ⇔ QED on finite/periodic lattice

  - boundary condition / photon mass, field ⇒ Patella @ Lattice'16

C boundary condition: Polley '93



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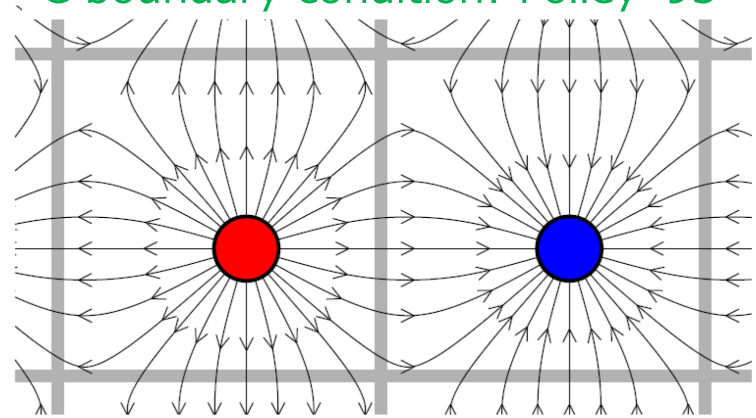

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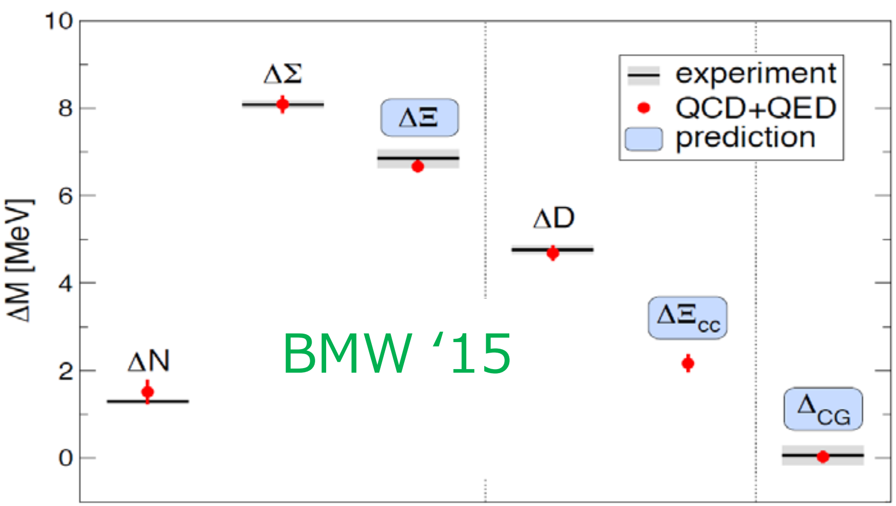

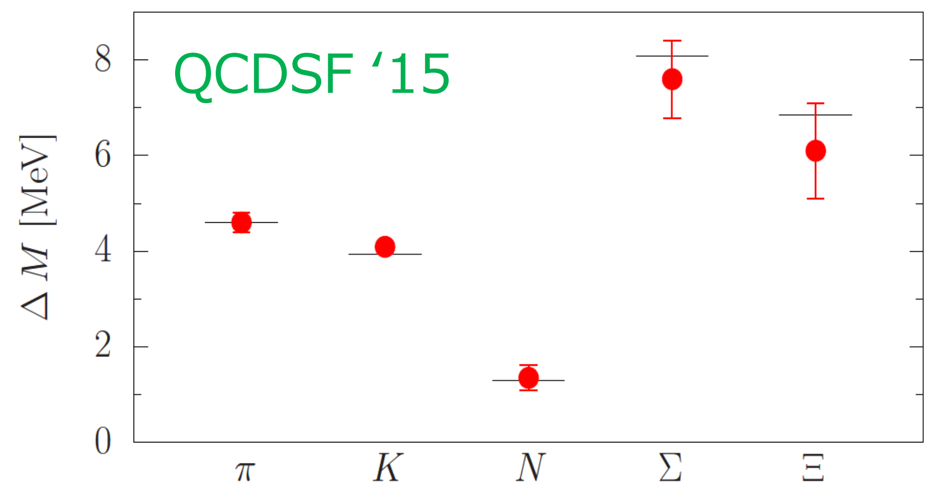

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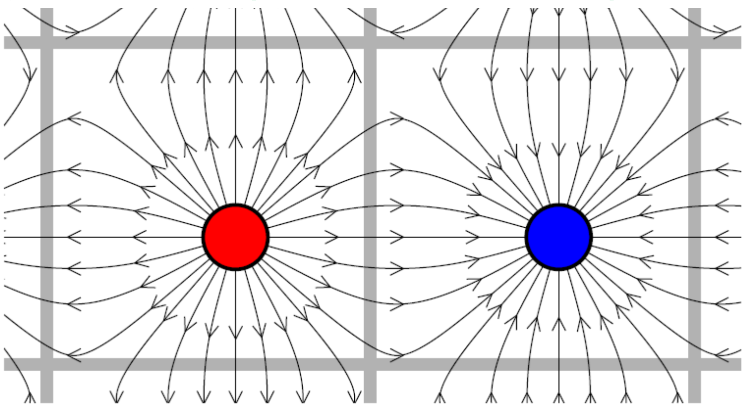

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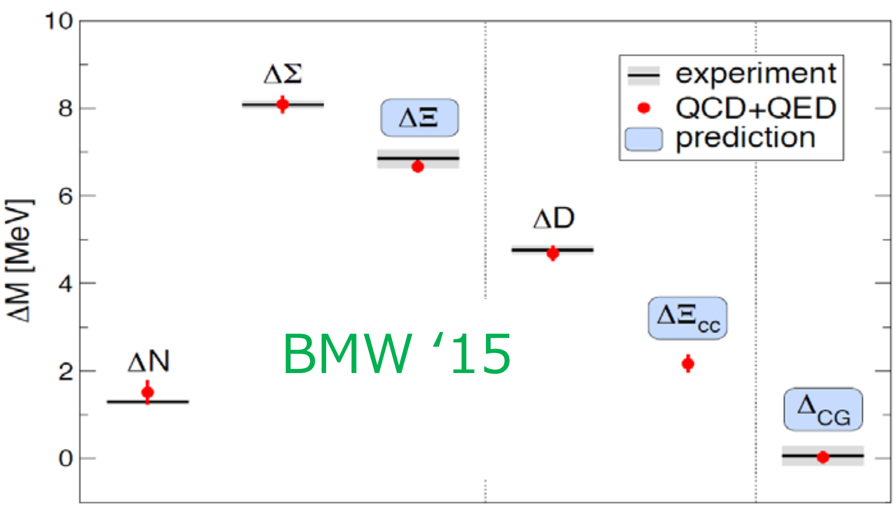

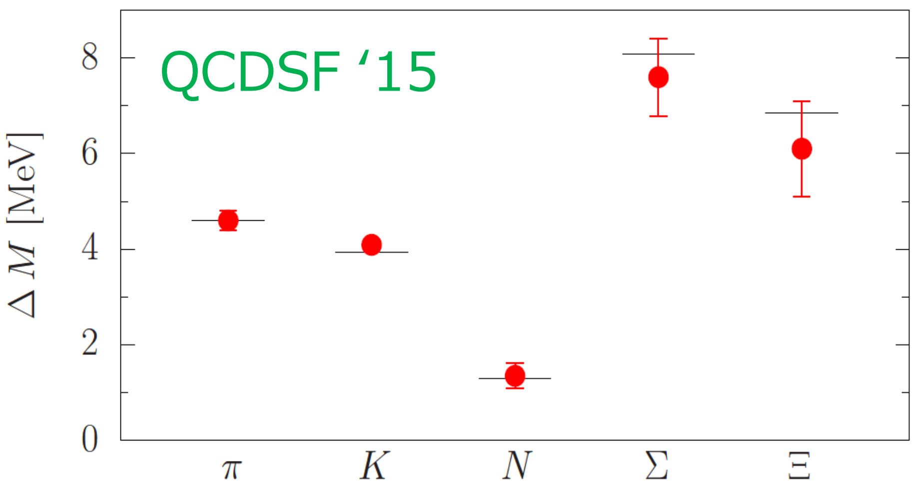

1.4 ‰ *n-p* splitting reproduced w/ different QED implementations

unstable particles

- finite V multi-particle state $\neq$ V=∞ in/out state

$$ {}_V\langle A(p)B(p')|X(q)\rangle \in \mathbb{R} $$

strong phase lost!

Miani-Testa '90

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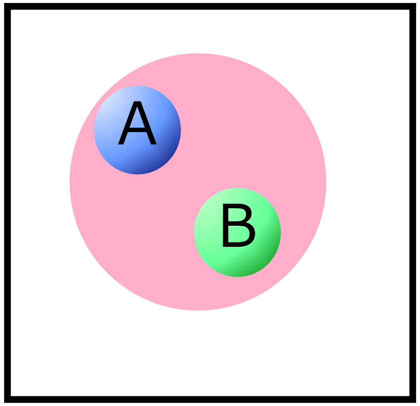
- lattice correlator does contain information on interaction!

$$ \langle O_{AB}(t)O_{AB}^{\dagger}(0)\rangle $$

$$ = \sum_n Z_n^{AB} \exp[-E_n t] $$



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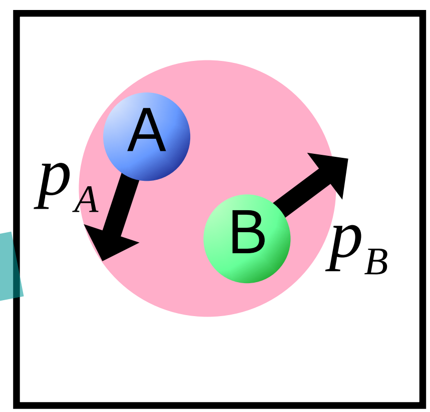

$$ E_{AB}(p_A, p_B) $$

$$ \neq E_A(p_A) + E_A(p_B) $$

scattering matrix encoded in finite V energies

⇒ Lüscher '86, '91

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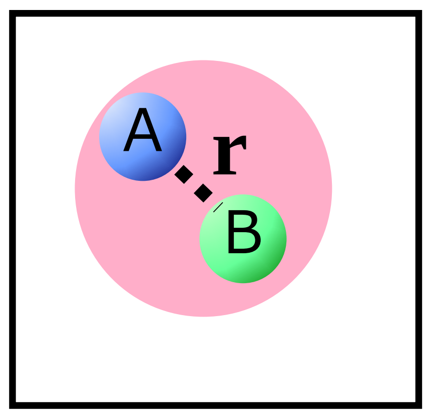
$$\langle O_{AB}(t)O^{\dagger}_{AB}(0)\rangle$$

$$=\sum_n Z_n^{AB}\exp[-E_n t]$$

$$O_{AB}(t)=O_A(0,t)O_B(\mathbf{r},t)$$



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scattering matrix encoded in finite V energies

potential ⇒ Ishii et al. '07, HALQCD '12

⇒ Lüscher '86, '91

ππ, ρ and σ

Lüscher method: successfully applied to 1 channel problems

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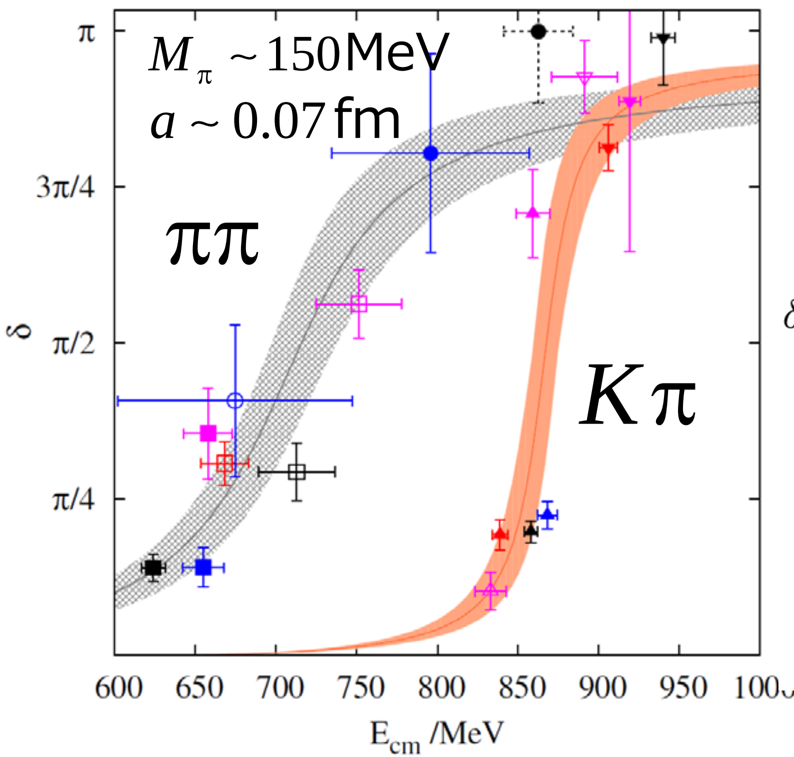
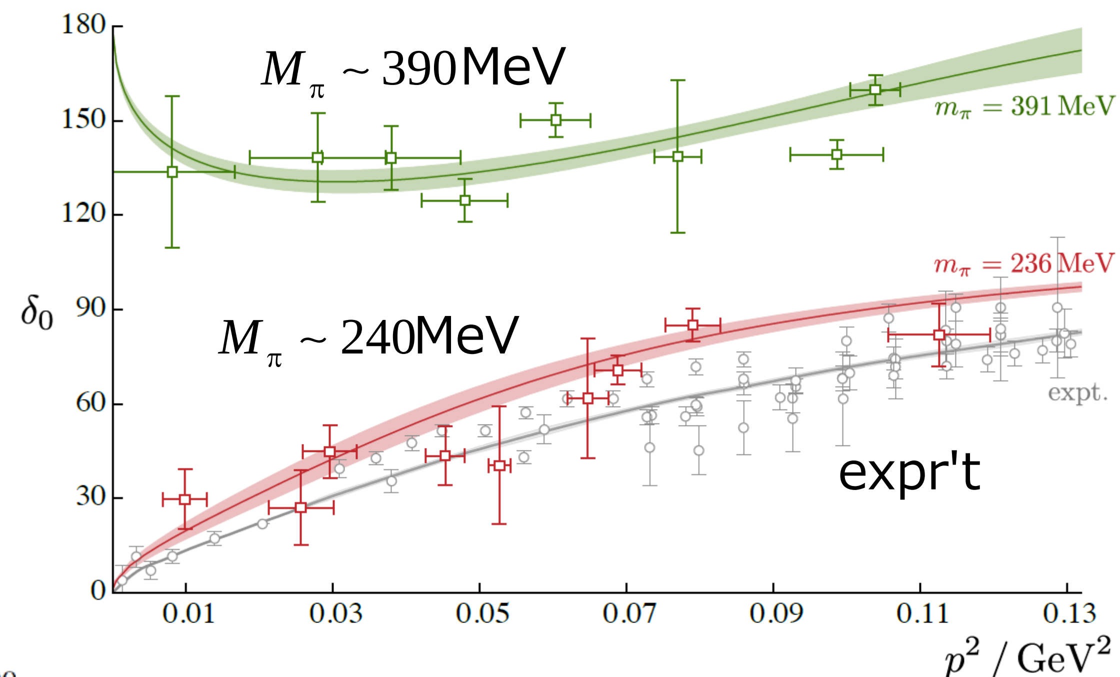
I=1 $\pi\pi$, *e.g.* RQCD 1512.08678 New!



$\delta_{I=1} \Rightarrow M_\rho, g_{\rho\pi\pi}, \Gamma_\rho$

I=0 $\pi\pi$, HS 1607.05900 New!



σ: bound state ⇒ broad resonance

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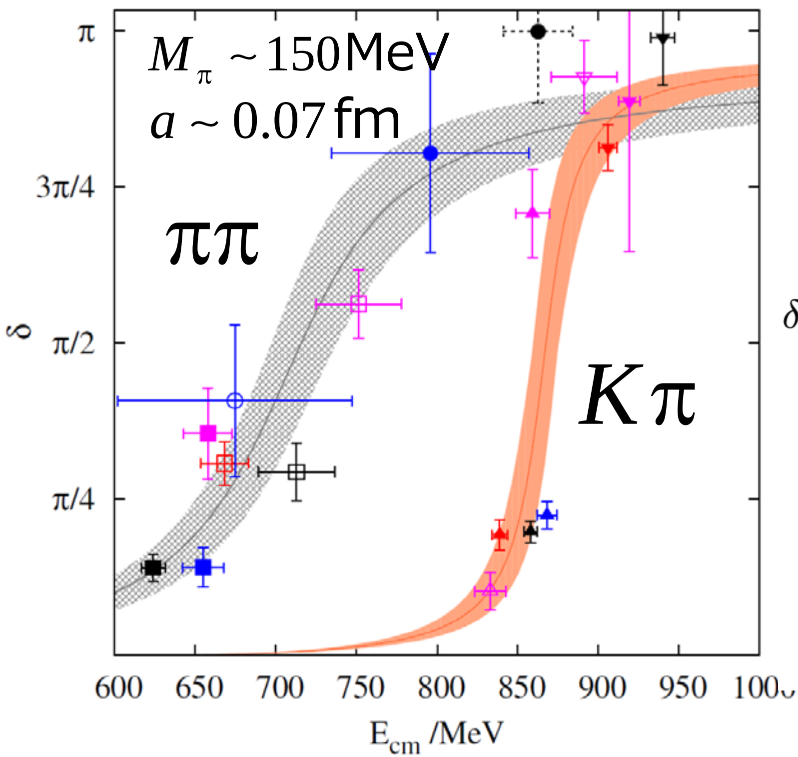

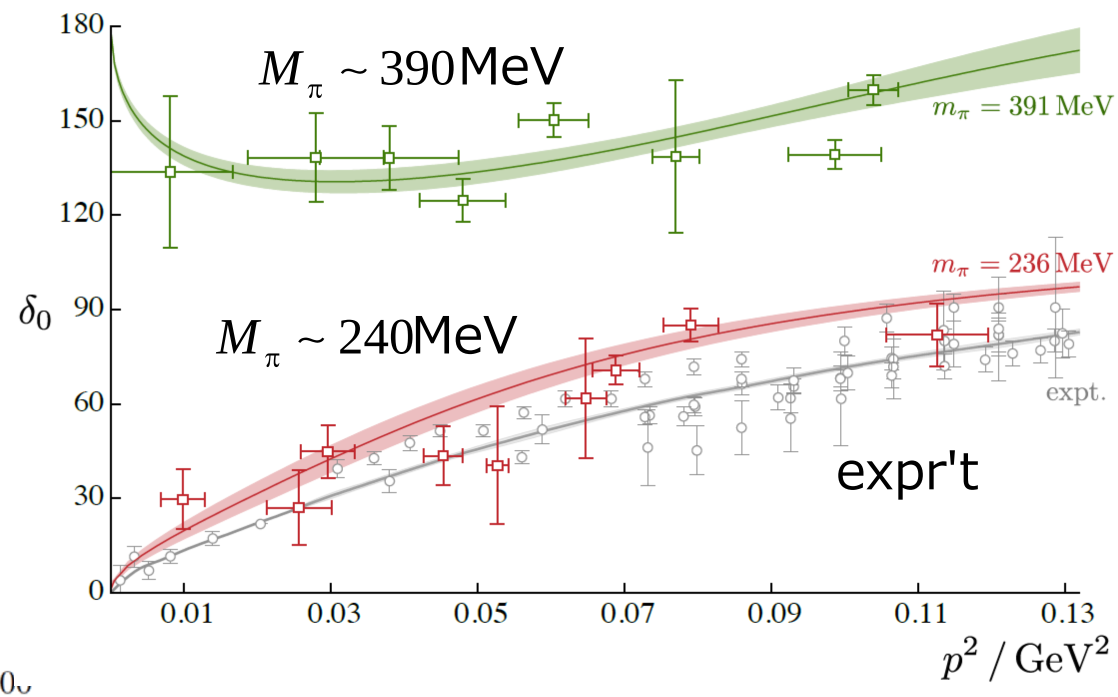

$\delta_{I=1} \Rightarrow M_\rho, g_{\rho\pi\pi}, \Gamma_\rho$

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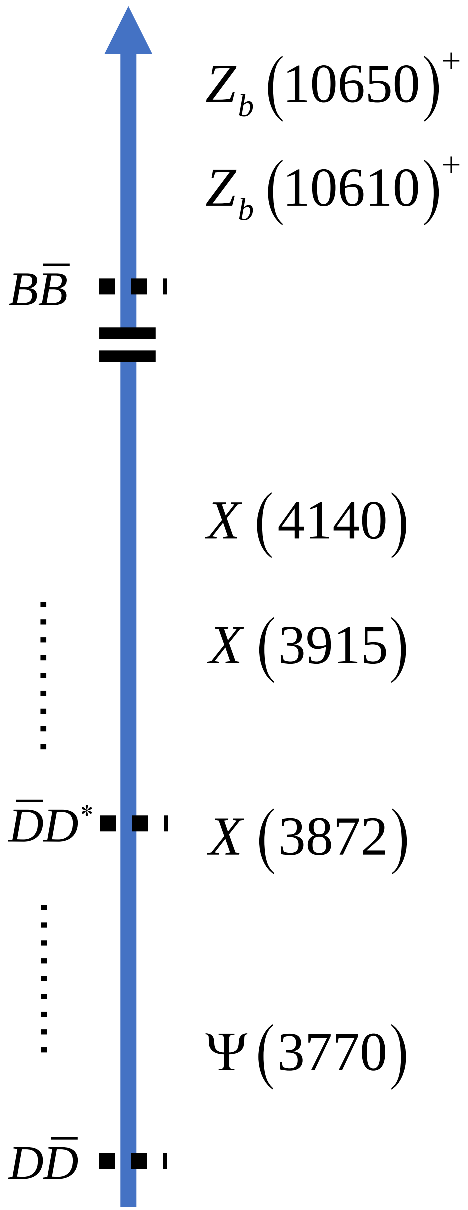
HALQCD method : on-going test on ρ (Kawai @ Lat'17)

heavy quarkonia / exotics



Lüscher method

- 2-body coupled channel: OK
- ≥3-body : active development

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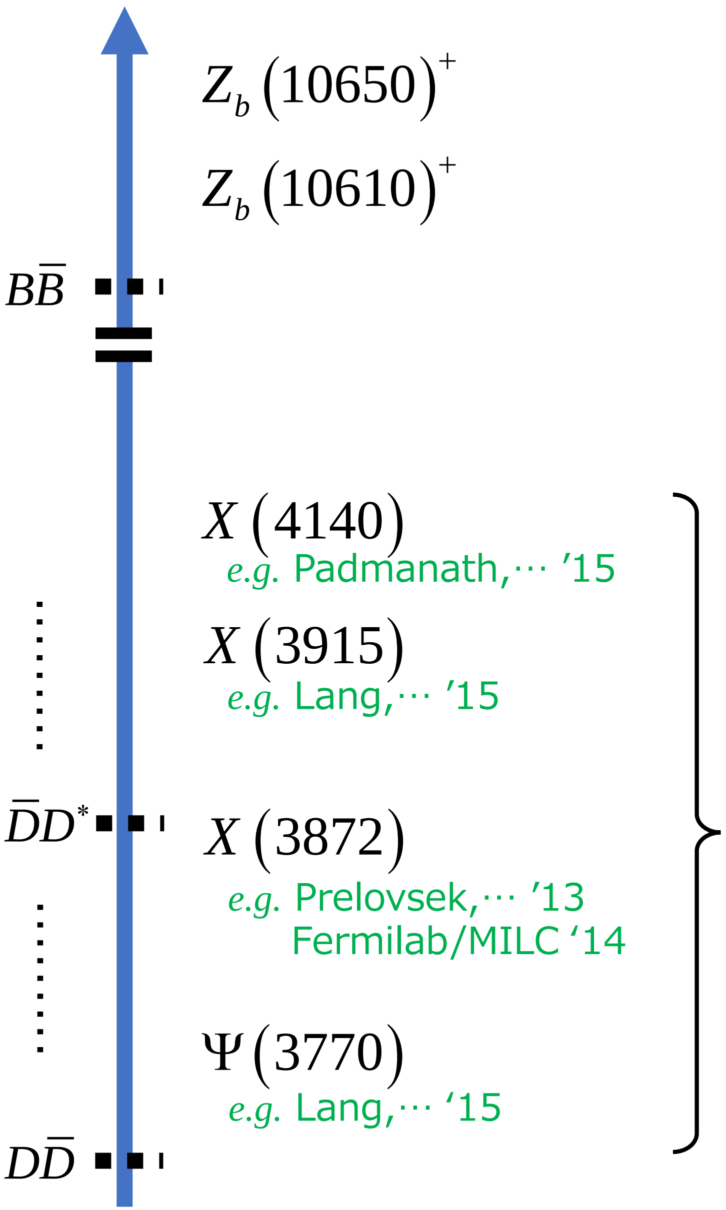

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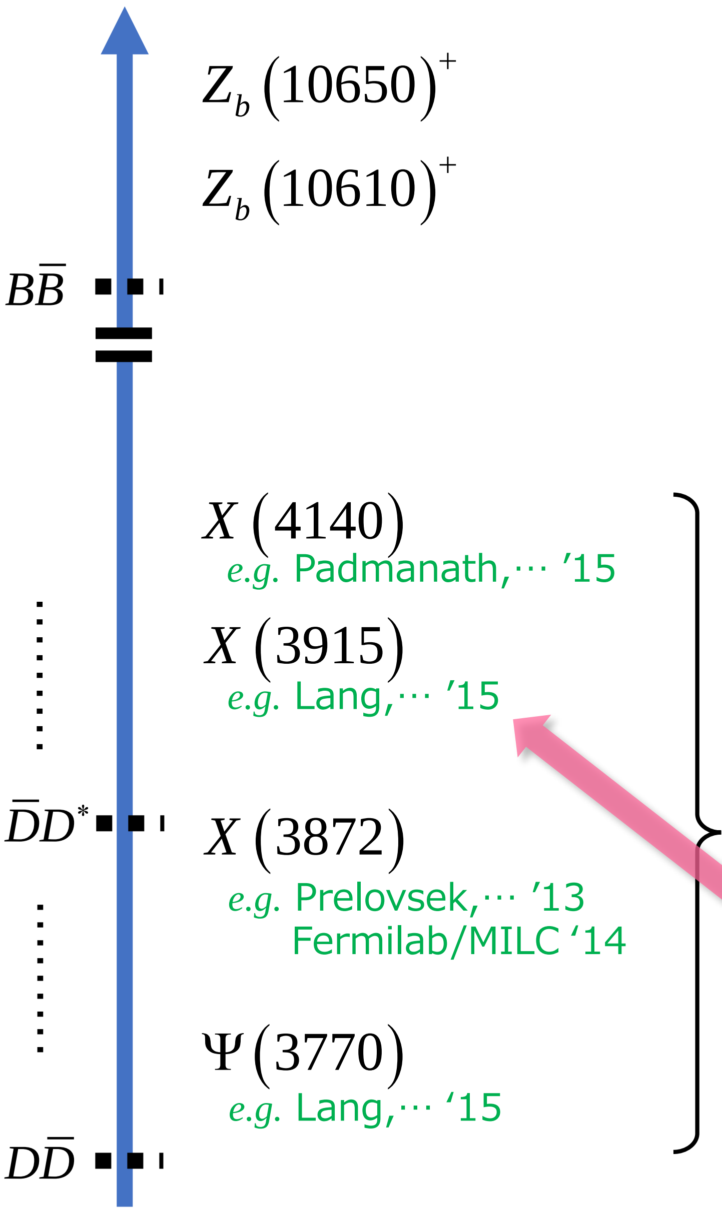

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active studies above a few thresholds

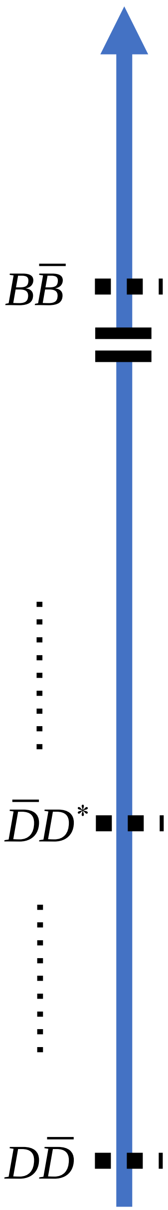
e.g. Lang et al. '15

- unobserved M=3966(20), Γ=67(18) MeV

  ⇔ Belle 1704.01872

  "alternative $\chi_{c0}(2P)$ candidate"

heavy quarkonia / exotics



$Z_b(10650)^+$
$Z_b(10610)^+$
} can decay to various final states
$\Rightarrow$ rigorous treatment is still challenging

see also, Francis et al., 1607.05214 New!
"strongly-stable" *udbb*

$X(4140)$
e.g. Padmanath,⋯ '15

$X(3915)$
e.g. Lang,⋯ '15

$X(3872)$
e.g. Prelovsek,⋯ '13
Fermilab/MILC '14

$\Psi(3770)$
e.g. Lang,⋯ '15

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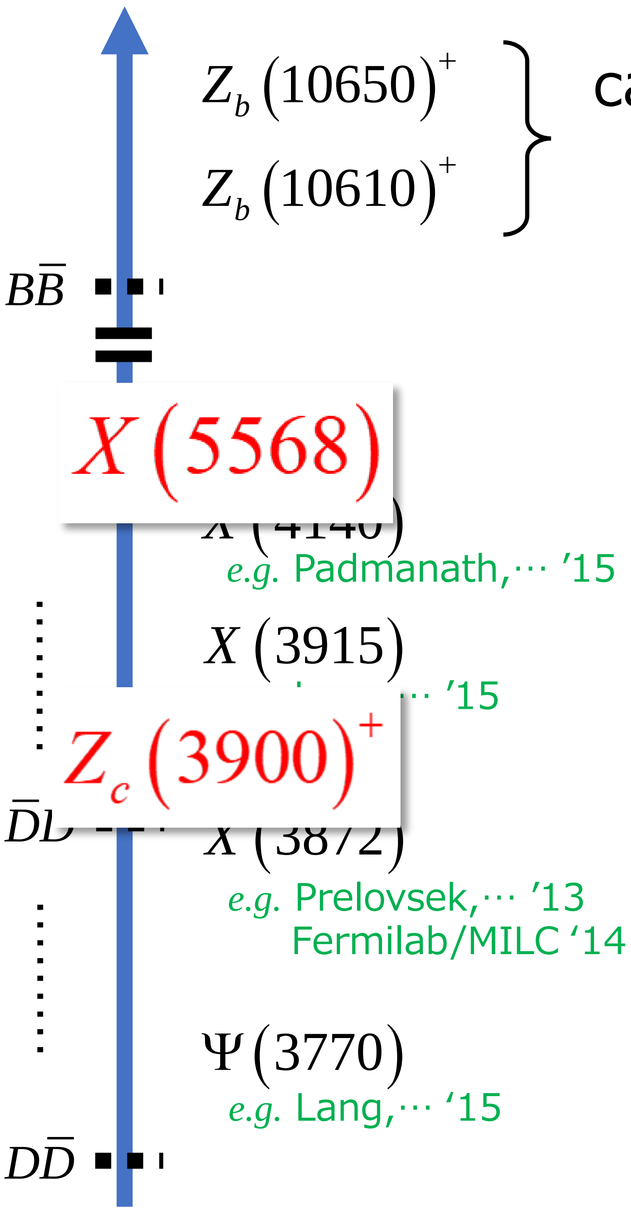

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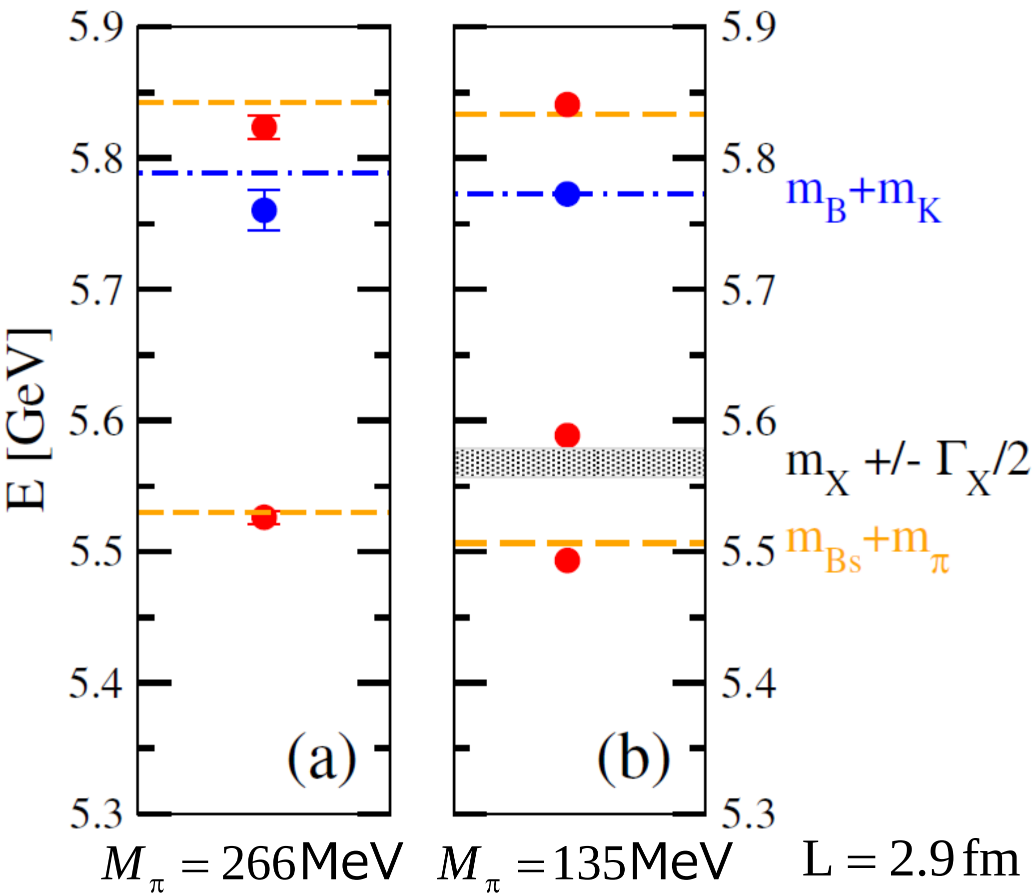
$X(5568)$

- Γ = 22(8) MeV, J^P unknown
- in $B_s\pi^+$ D0'16, not LHCb'16
- if 4 flavors $\bar{b}s\bar{d}u \oplus J^P = 0^+$

  $\Rightarrow$ decay only into $B_s\pi^+$

$X(5568)$



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Lang et al. 1607.03185 New!

- map out finite V energies
- M, Γ, Lüscher formula

  $\Rightarrow$ energy not found
- ~~deep BK bound state~~

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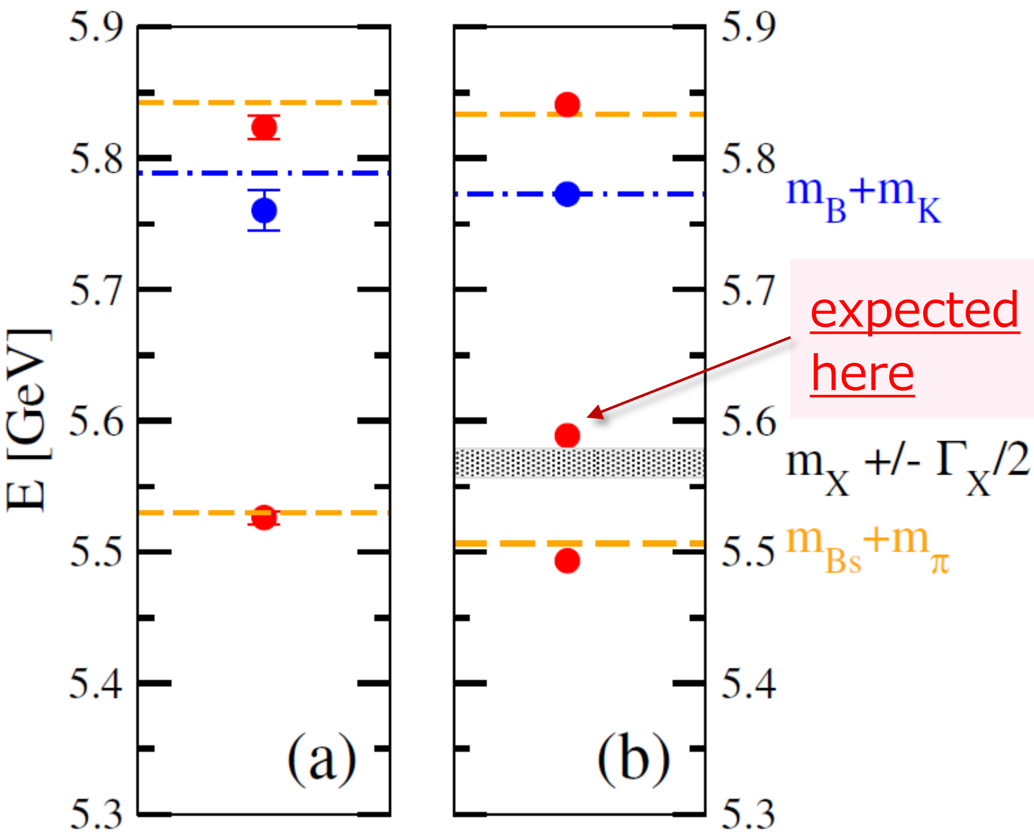

$M_\pi = 266\,\text{MeV}$ $M_\pi = 135\,\text{MeV}$ $L = 2.9\,\text{fm}$

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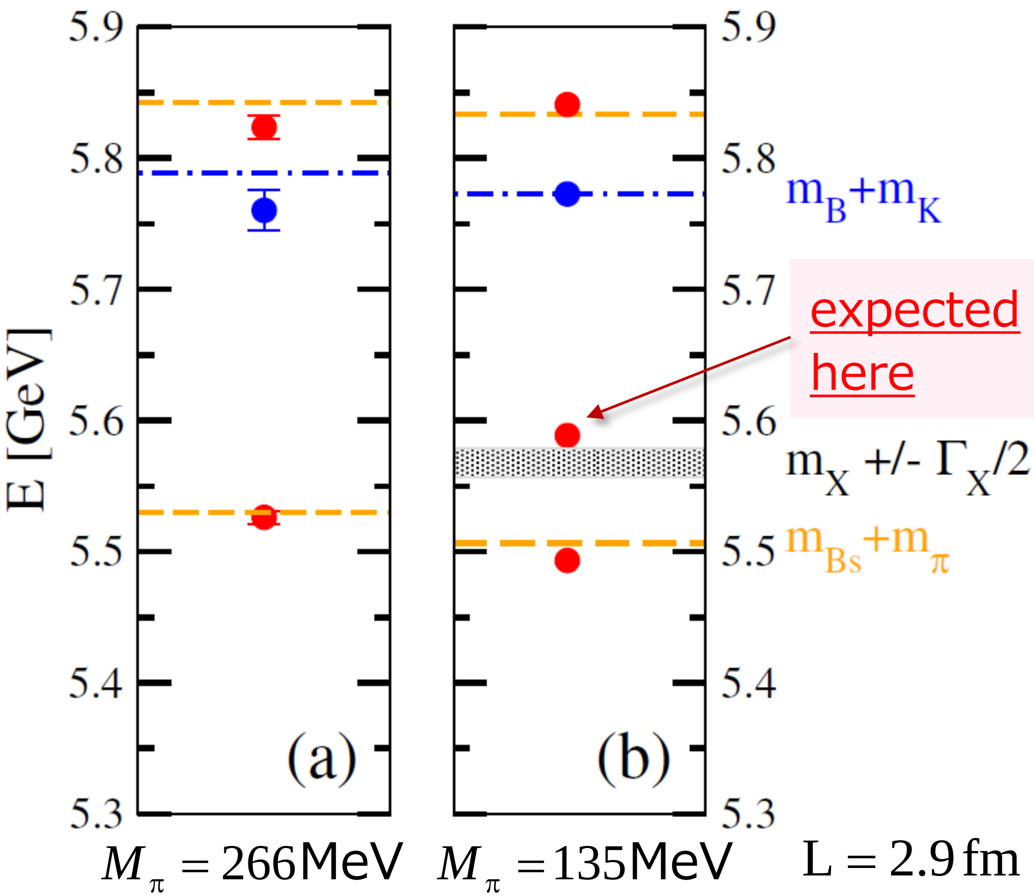

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do not support the existence of X(5568) w/ J^P=0^+

$Z_c^+(3900)$

- $M_{\mathrm{Zc}} \sim M(\bar{D}D^*)$ + 20 MeV
- Γ = 40(8) MeV
- $\Upsilon(4260) \to \pi\,\{J/\psi\pi,\, \bar{D}D^*\}$

  BESIII '13, Belle '13, Xiao et al. '13
- charged, 4 quarks $\bar{c}cu\bar{d}$

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Prelovsek et al. '13, '15

Cheng et al. '14

- Lüscher approach
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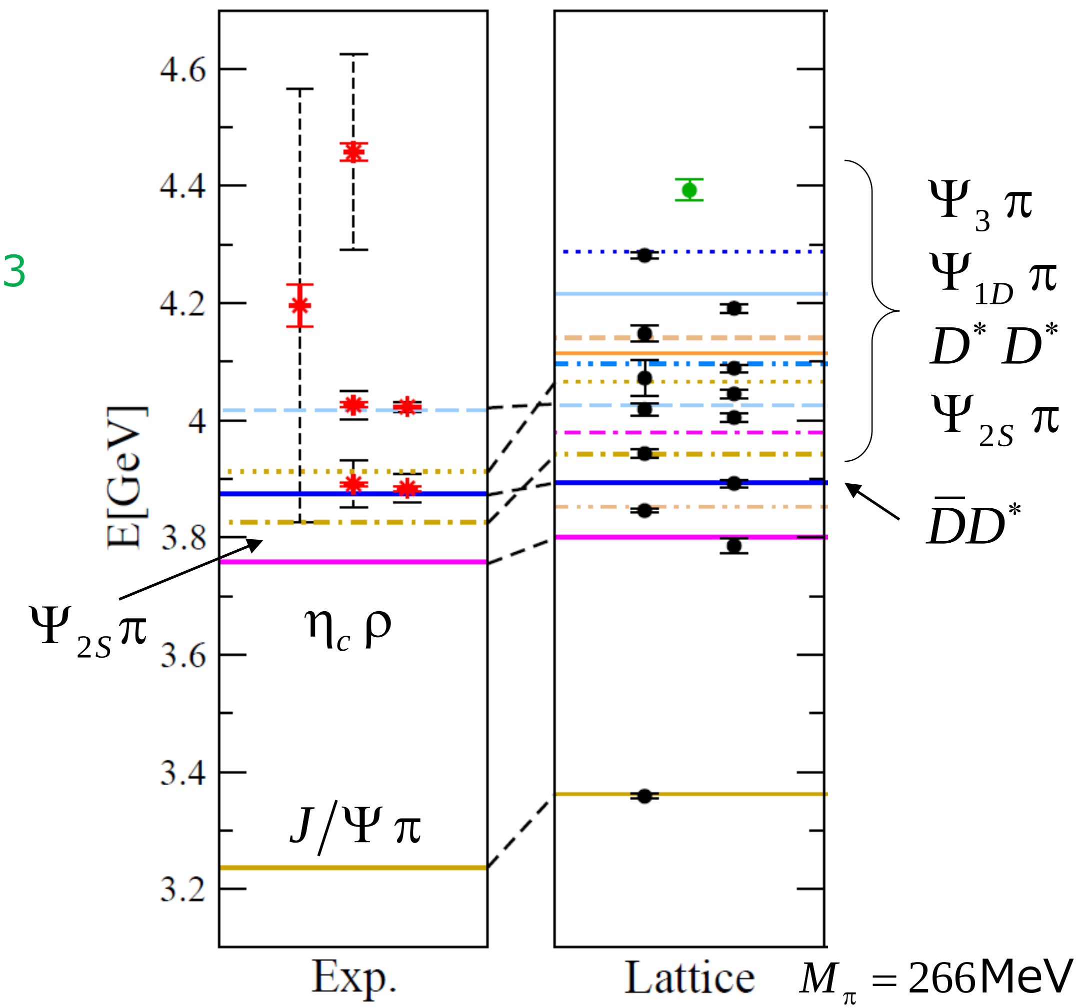
Prelovsek et al. '15



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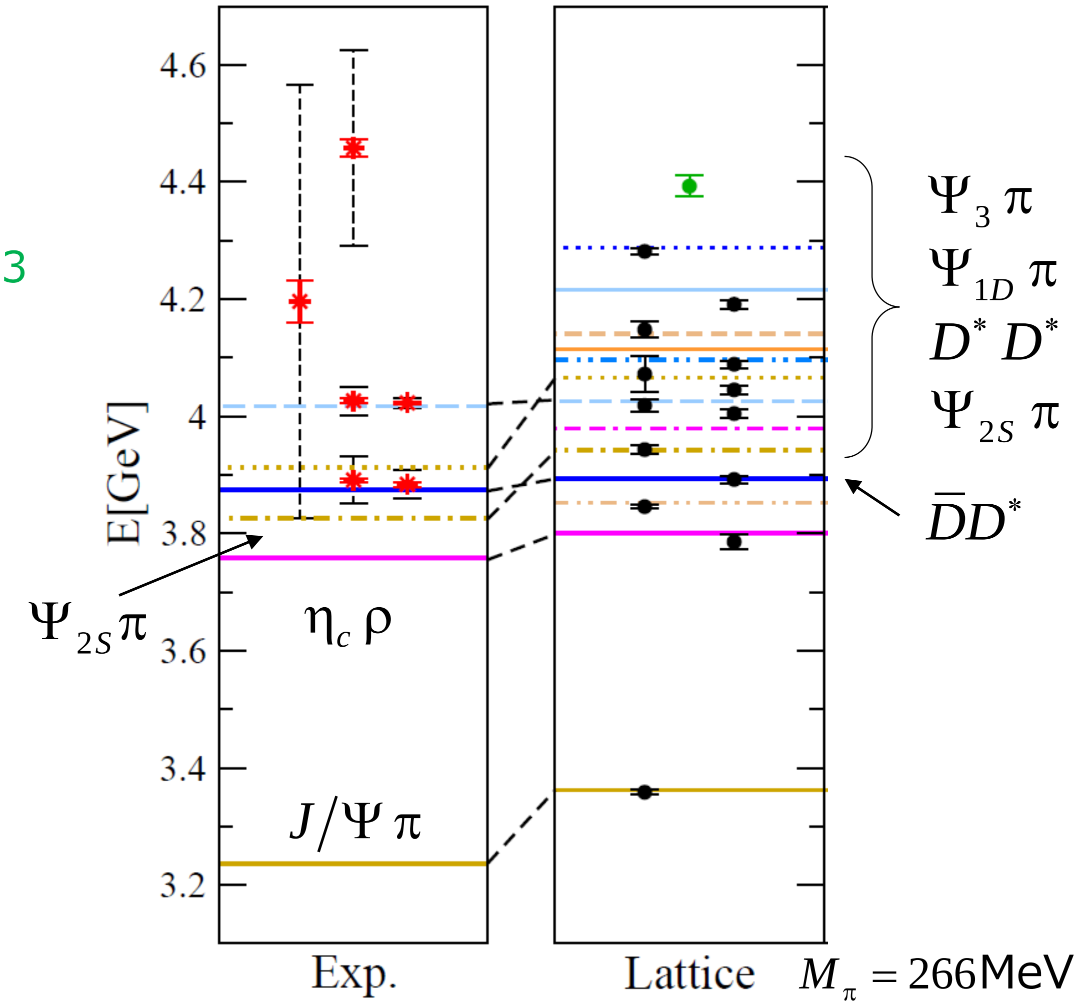

Prelovsek et al. '13, '15

Cheng et al. '14

- Lüscher approach
- map out finite V energies

no extra energy level $\sim Zc(3900)$, kinematical origin?

$Z_c^+(3900)$

HALQCD 1602.03465 New!

- HALQCD method
- weak couplings among

$$J/\Psi\,\pi - J/\Psi\,\pi \quad \rho\eta_c - \rho\eta_c$$

$$\bar{D}D^* - \bar{D}D^* \quad J/\Psi\,\pi - \rho\eta_c$$

strong ⇒ peaks in J/ψπ, DD*

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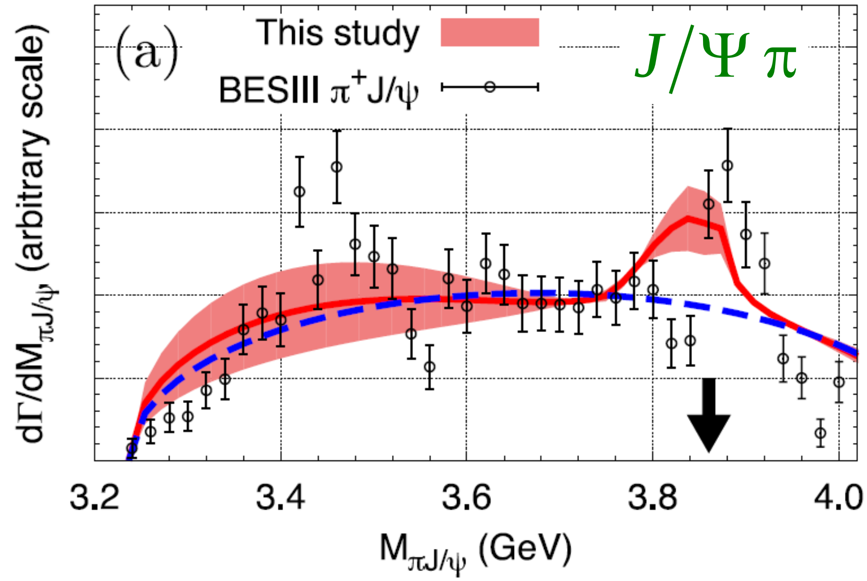

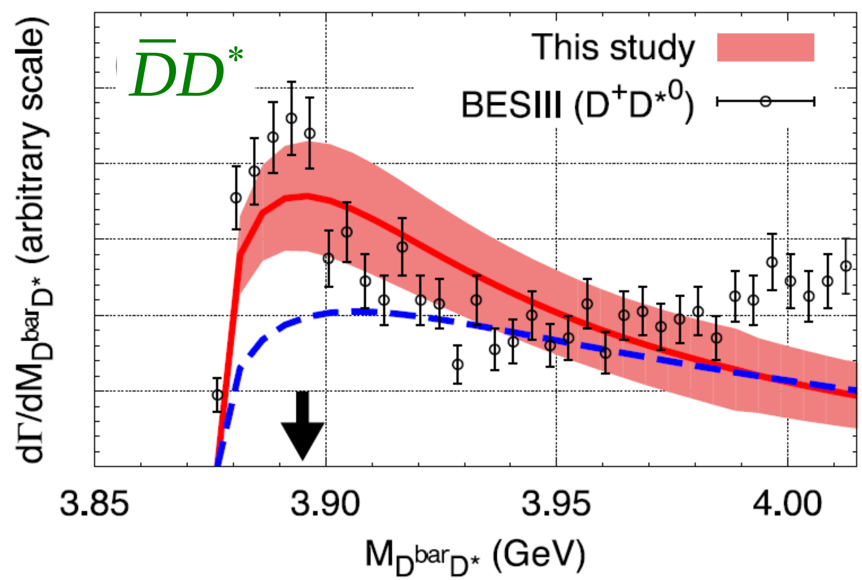

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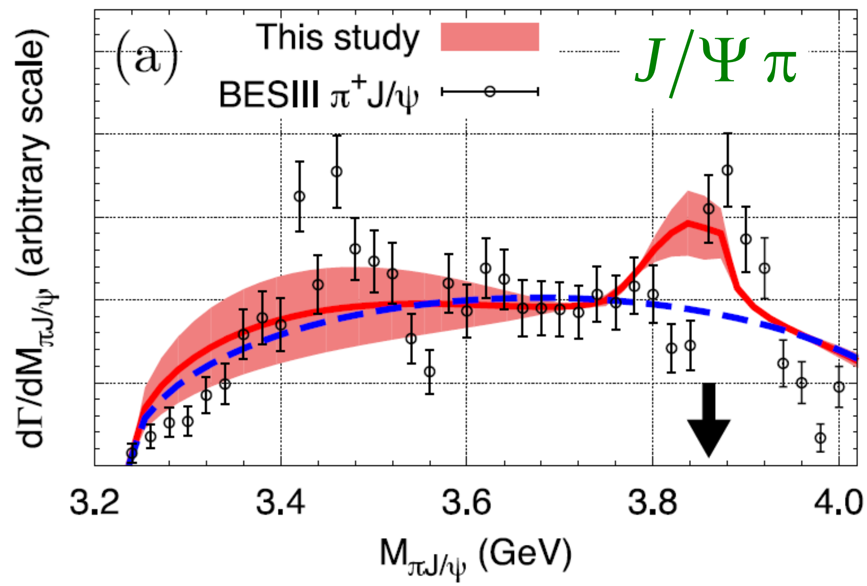

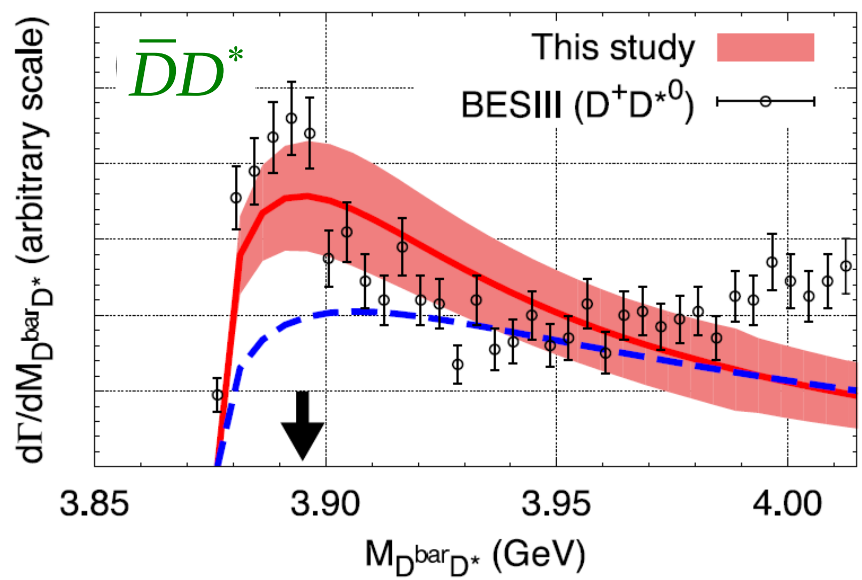

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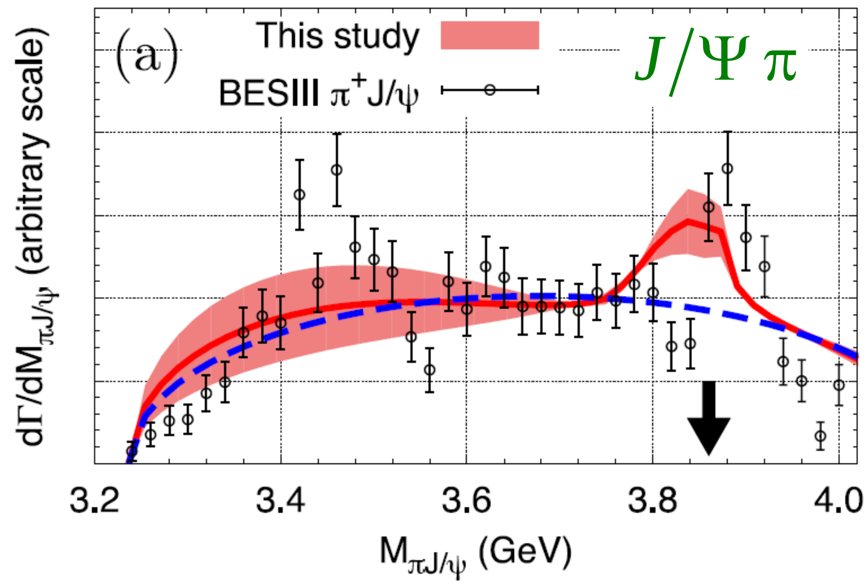
strong ⇒ peaks in J/ψπ, DD*

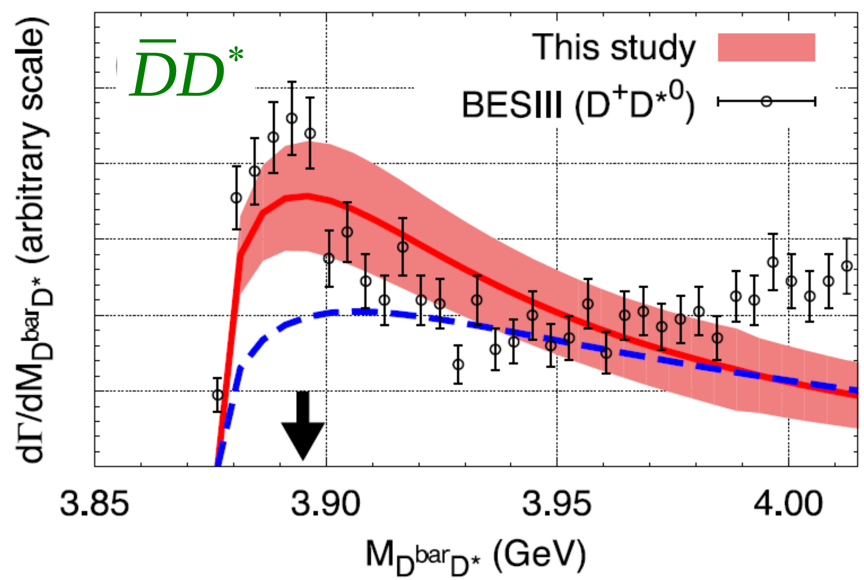
$$\bar{D}D^* - J/\Psi\,\pi \quad \bar{D}D^* - \rho\eta_c$$

- no S-matrix pole ~ Z_c^{+}

suggesting Zc(3900), as a threshold cusp

need physical M_π simulation





summary on spectroscopy

- spectrum below thresholds
  - fully realistic simulations, impressive agreement w/ expr’t

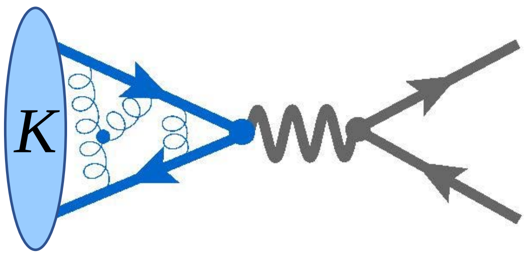
- finer structure
  - QED on the lattice, ‰ isospin splittings becoming accessible

- heavy quarkonia / exotics
  - framework ready for coupled 2-body channels
  - shed light on states above a few thresholds
  - simulations are still unphysical set-up, single a, large M_π
  - states above more thresholds technically challenging
  - general framework for 3-body under active construction

2. Flavor Physics

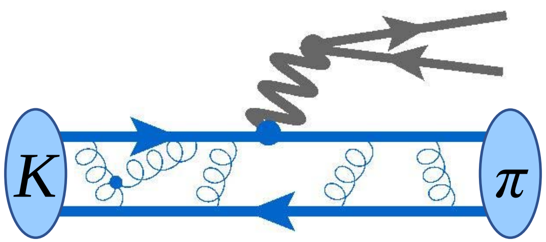
kaon physics

"gold-plated" : w/o initial/final state interaction : 0.3-1.3%



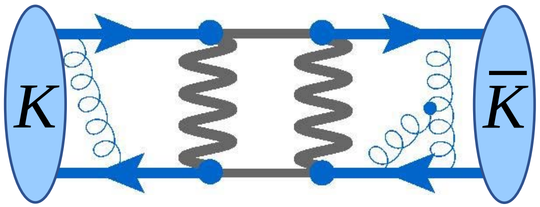
$K \to \ell\nu$

f_K



$K \to \pi\ell\nu$

$f_{+,0}\left(q^2\right)$



$K^0 \to \bar{K}^0$

$B_K \to \varepsilon_K,$

kaon physics

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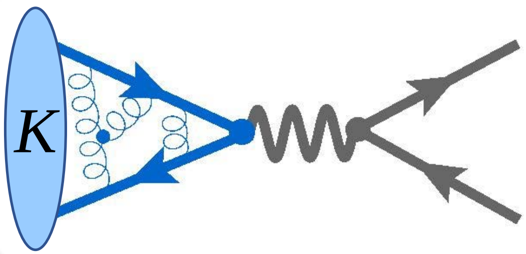
Moulson @ CKM'16



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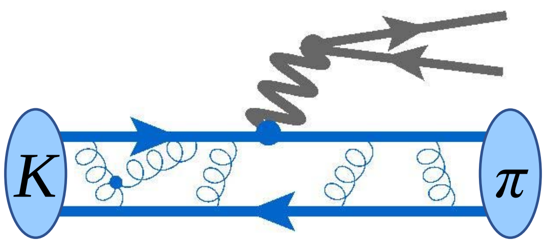

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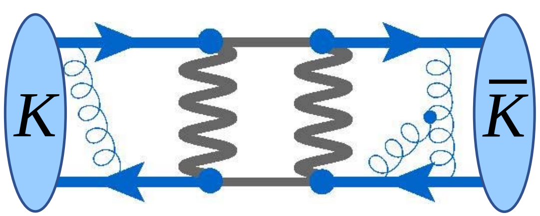

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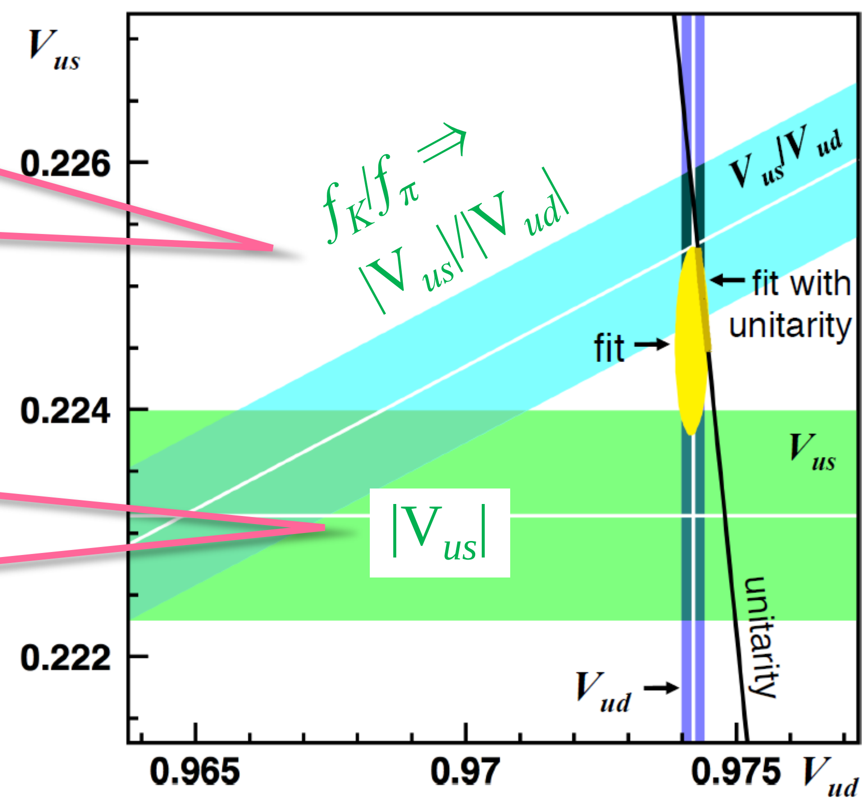

$$\sum_{q=d,s,b} \left|V_{uq}\right|^2 = 0.9995(5)$$

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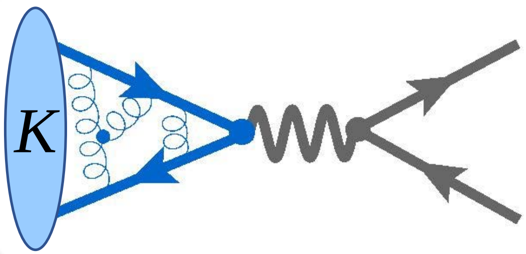

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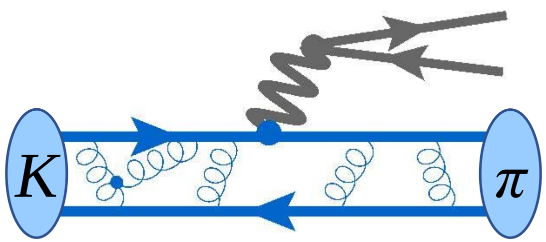

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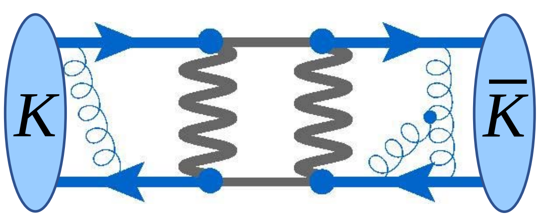

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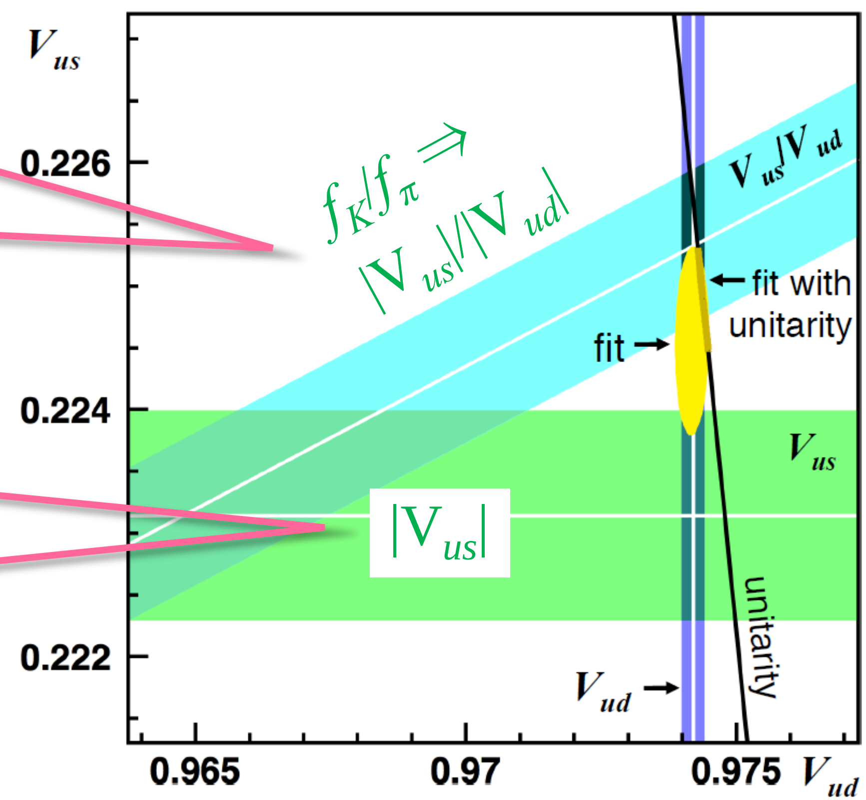


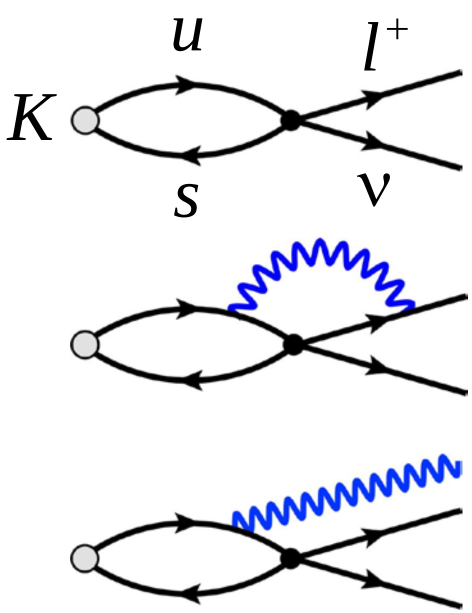
$$\sum_{q=d,s,b} \left|V_{uq}\right|^2 = 0.9995(5)$$

⇒ isospin corrections / hadronic decays

isospin corrections

- IR singularity!

Carrasco et al. '15



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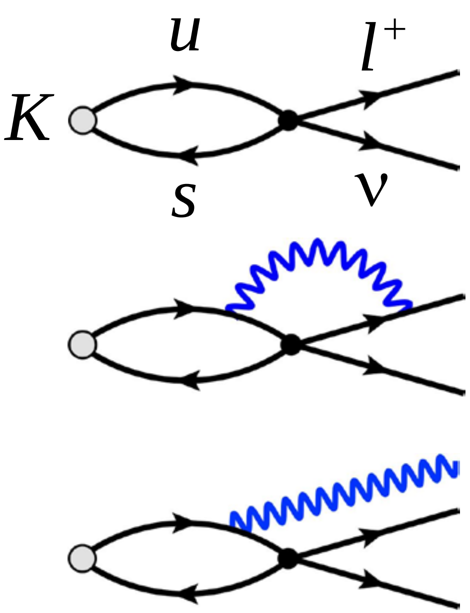
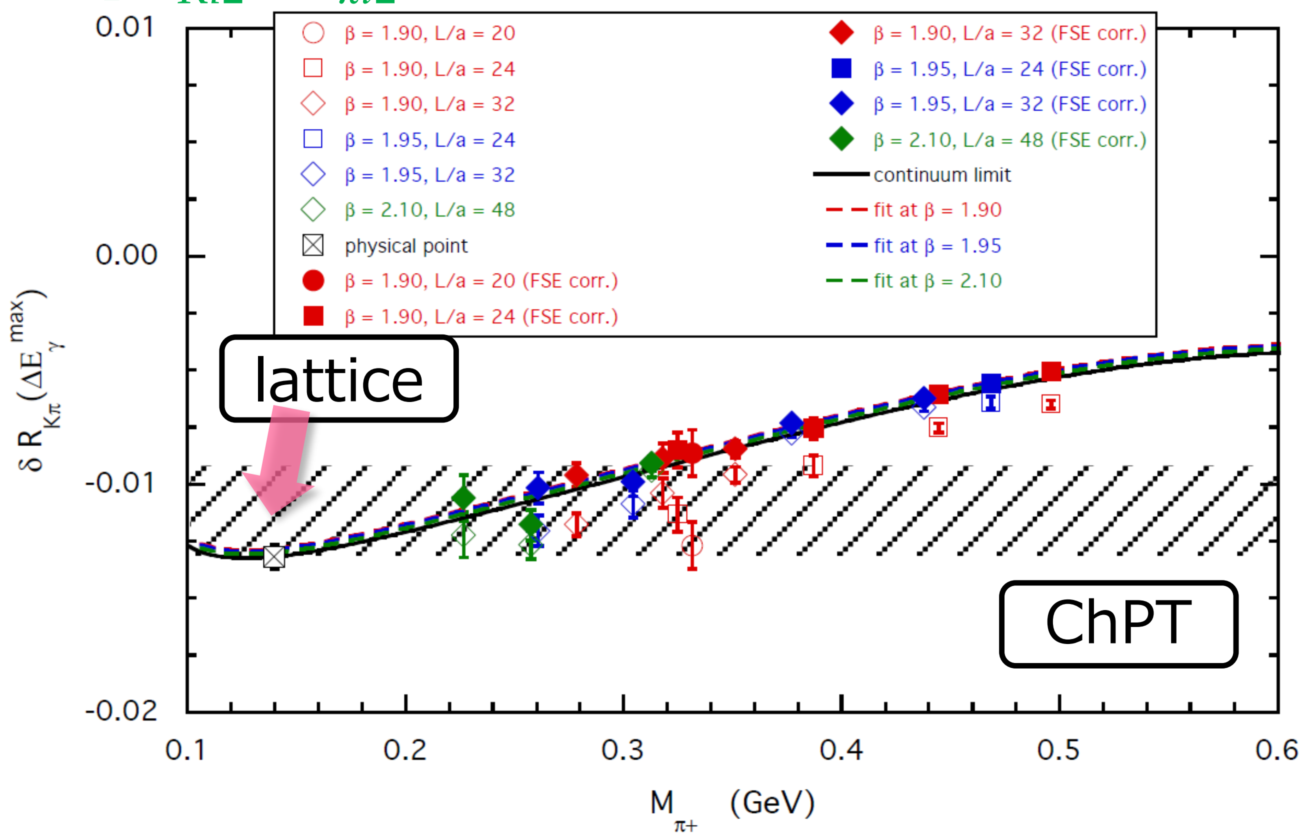


$\Delta[\, \Gamma_{Kl2} / \Gamma_{\pi l2} \,]$ Lubicz et al. 1610.09668 New!



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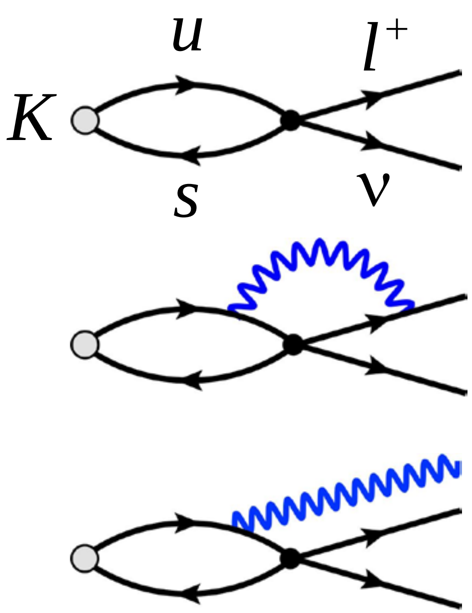

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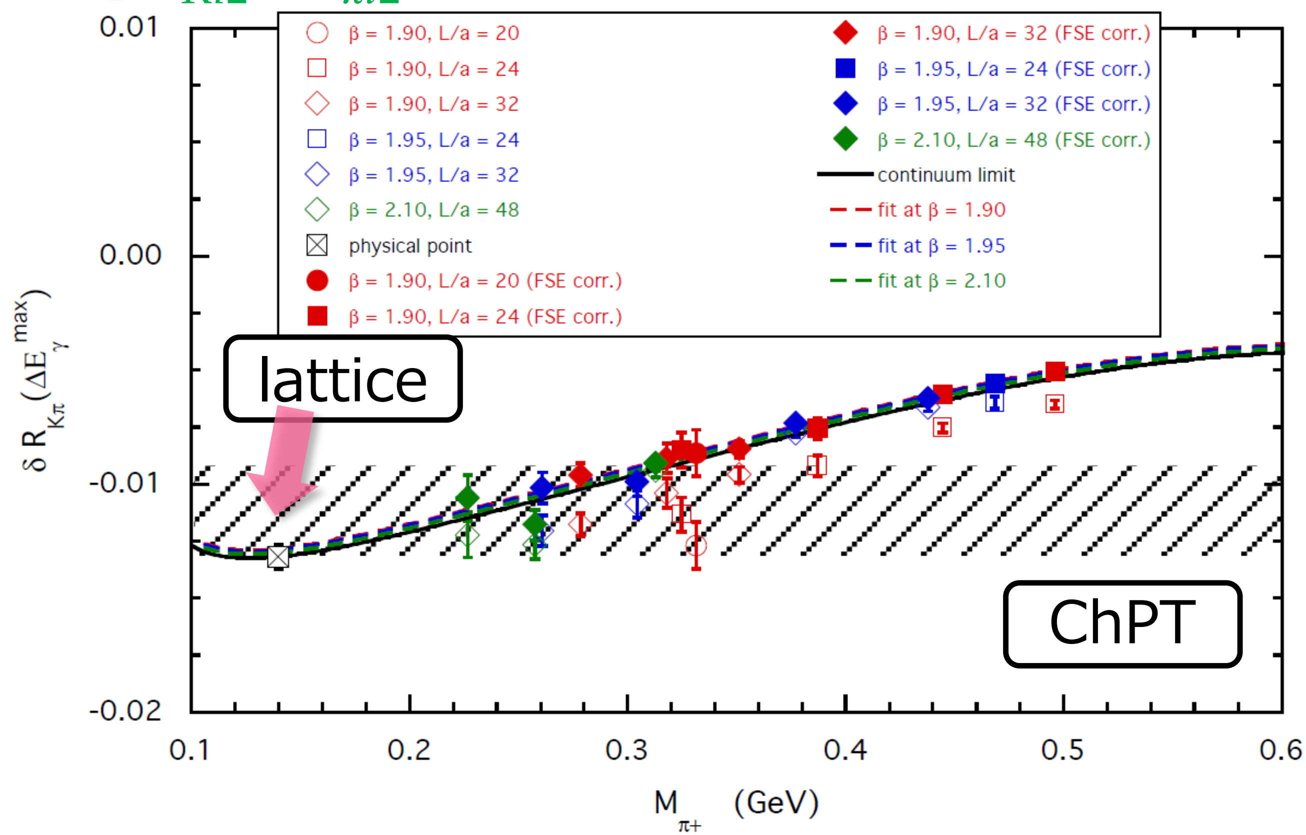

$$\Delta\left[\Gamma_{Kl2}/\Gamma_{\pi l2}\right] = -0.0137(13) \quad \Leftrightarrow \quad -0.0112(21)\,(\text{ChPT})$$

higher orders involve unknown LECs

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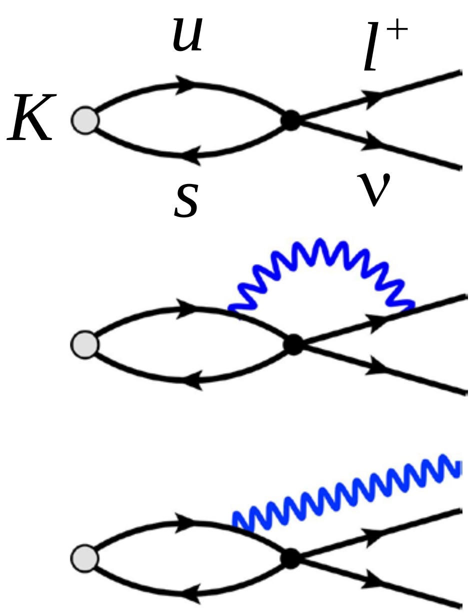

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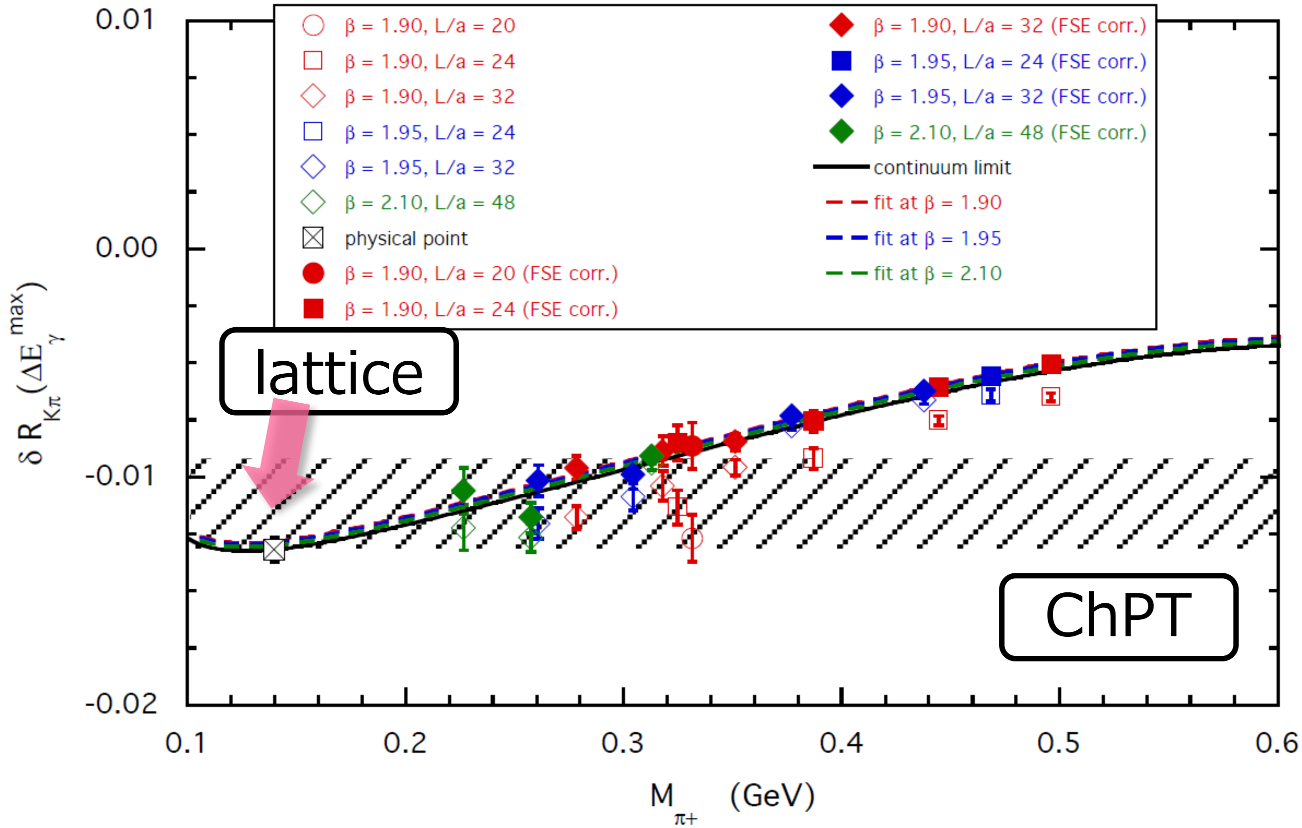

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higher orders involve unknown LECs

systematically improvable / application to semileptonic decays

$K \to \pi\pi$

Lüscher relation $\to \delta_{0,2}$ / Lellouch-Lüscher '01 $\to A_0, A_2$

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RBC/UKQCD ‘15

$\mathrm{re}\left[\varepsilon'/\varepsilon\right] \times 10^4 = 16.6(2.3)$ (ex)

$\Leftrightarrow 1.4(5.2)(4.4)$ (LQCD)

$\mathrm{im}\left[A_0\right] \times 10^{11} = -1.9(1.2)(1.1)$ GeV

$\Leftrightarrow -6.7(5.6)$ (Ishizuka et al. '15)

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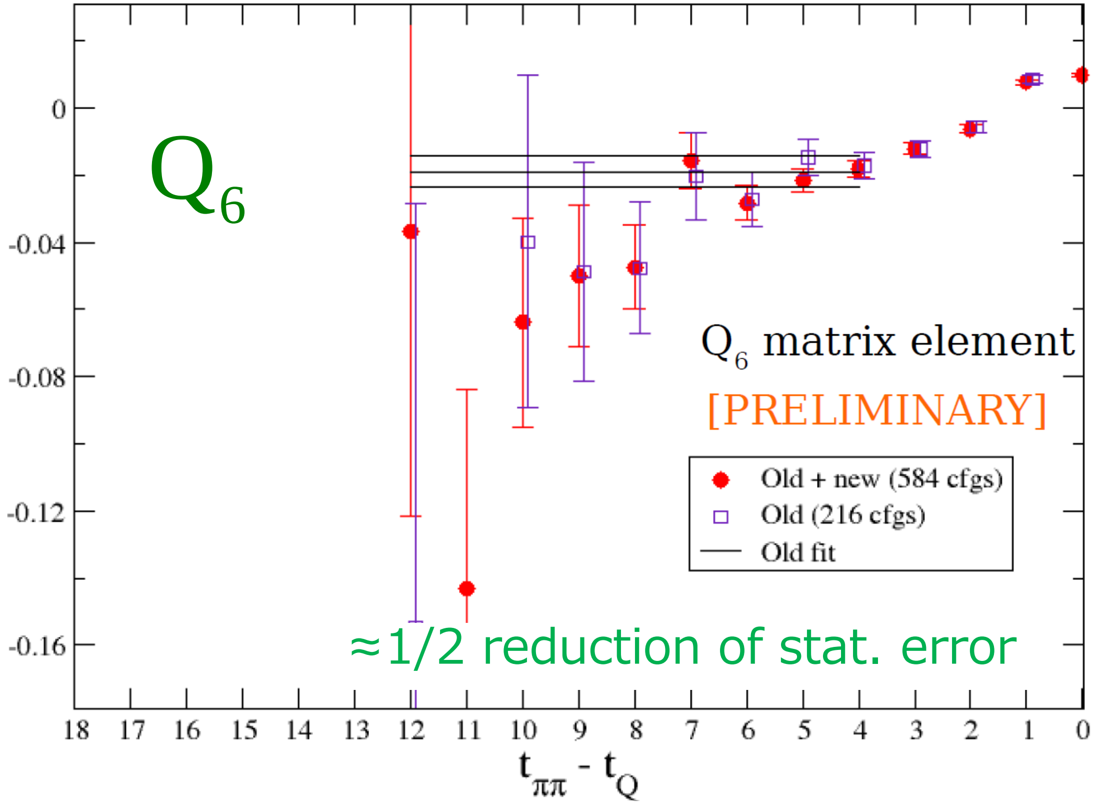
Kelly @ Lattice'17 New!



≈1/2 reduction of stat. error

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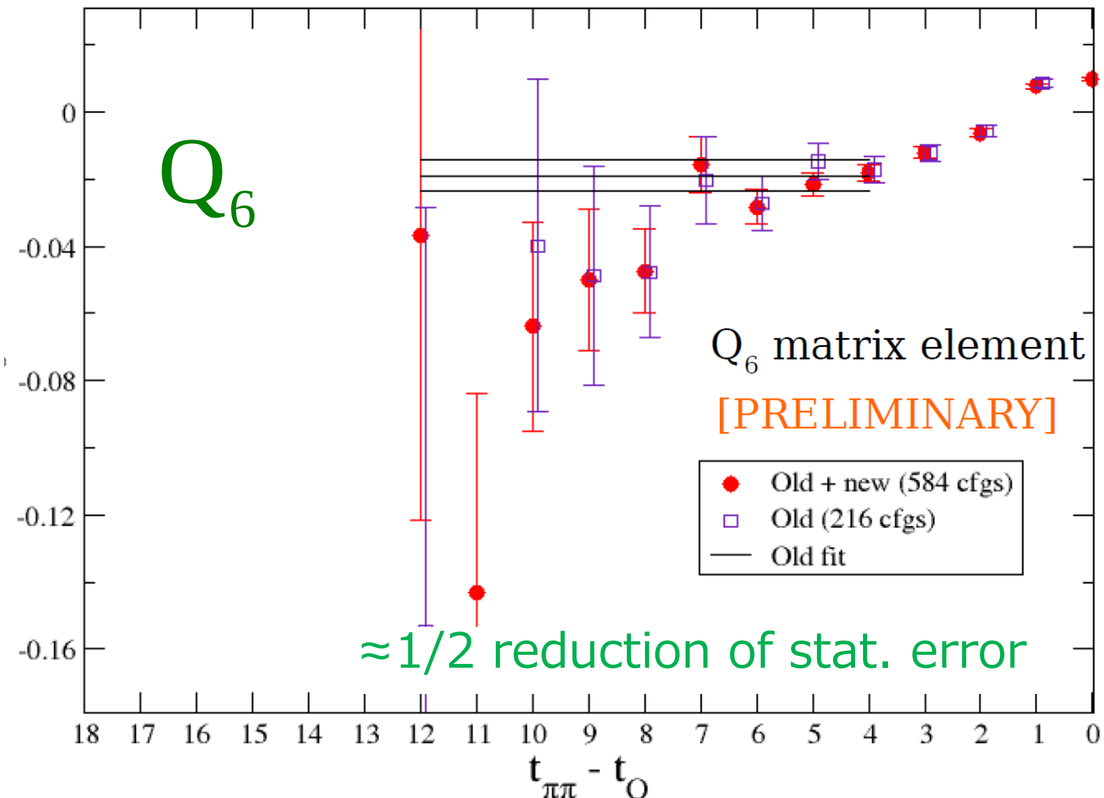

- renormalization of $Q_{6,\mathrm{lat}}$
- Wilson coefficients

  (Kelly, Bruno @ Lat'17)

⇒ significant reduction in a few years (?)

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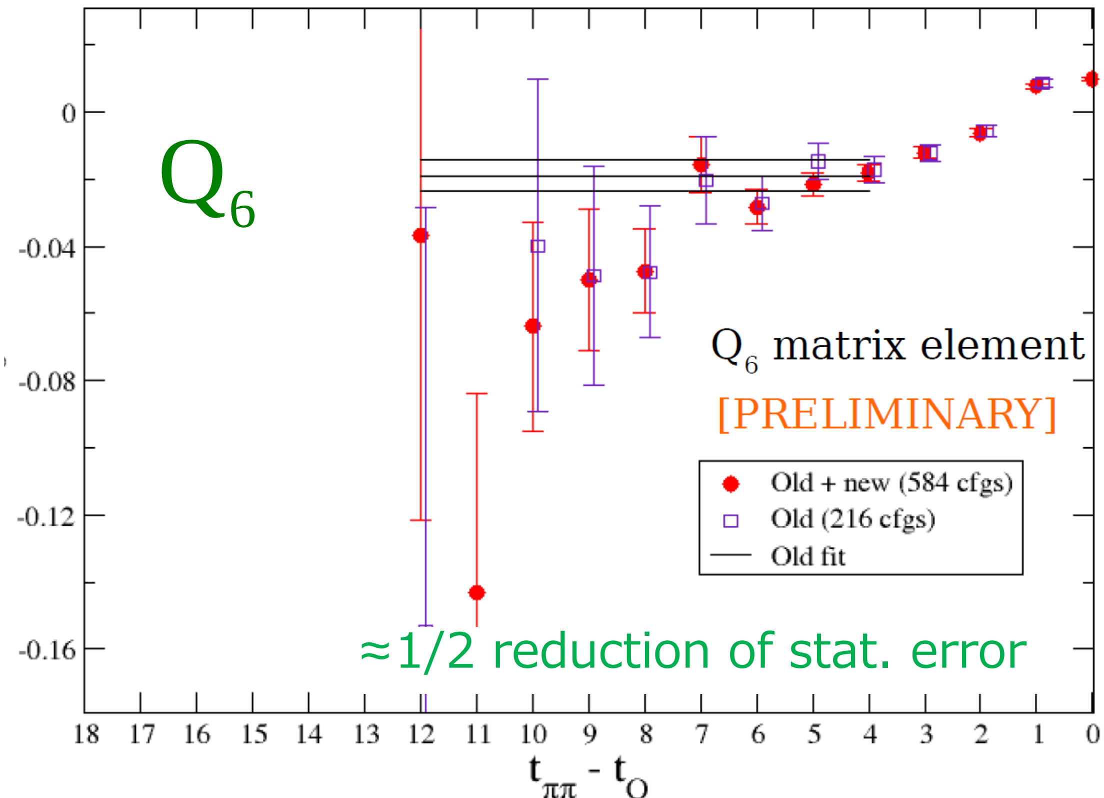

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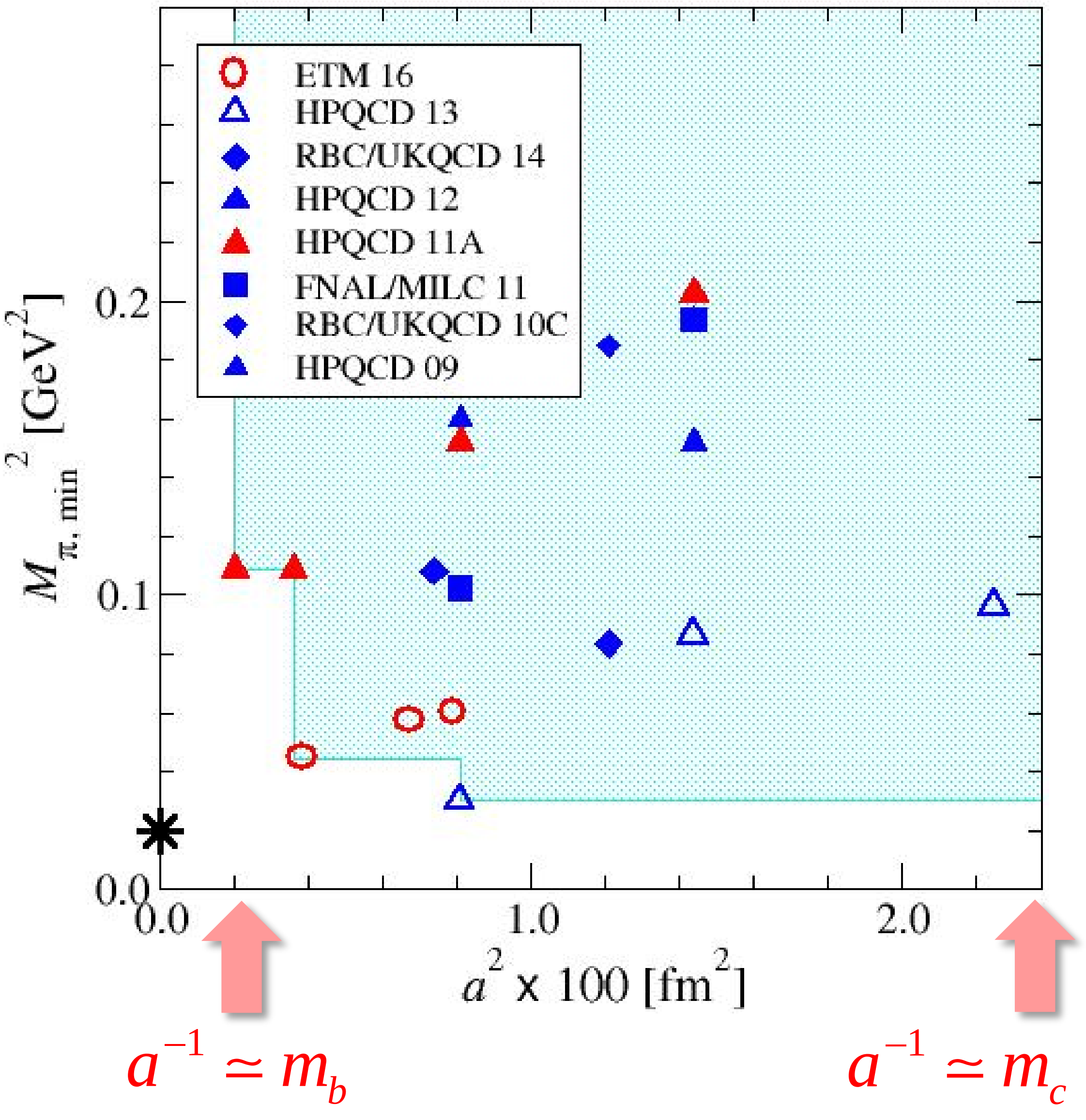
$2\mathcal{H}_w$ insertions ⇒ long distance ε_K; $K\to\pi\nu\nu$, πll RBC/UKQCD '17

heavy meson decays

FLAG'16 average for $f_{B(s)}$



heavy meson decays

"relativistic QCD action"

- $m_c \ll a^{-1} \lesssim m_b \;\Rightarrow\; m_Q \ll m_b$
- good for charm
- bottom needs inter-/ extra-polation to m_b

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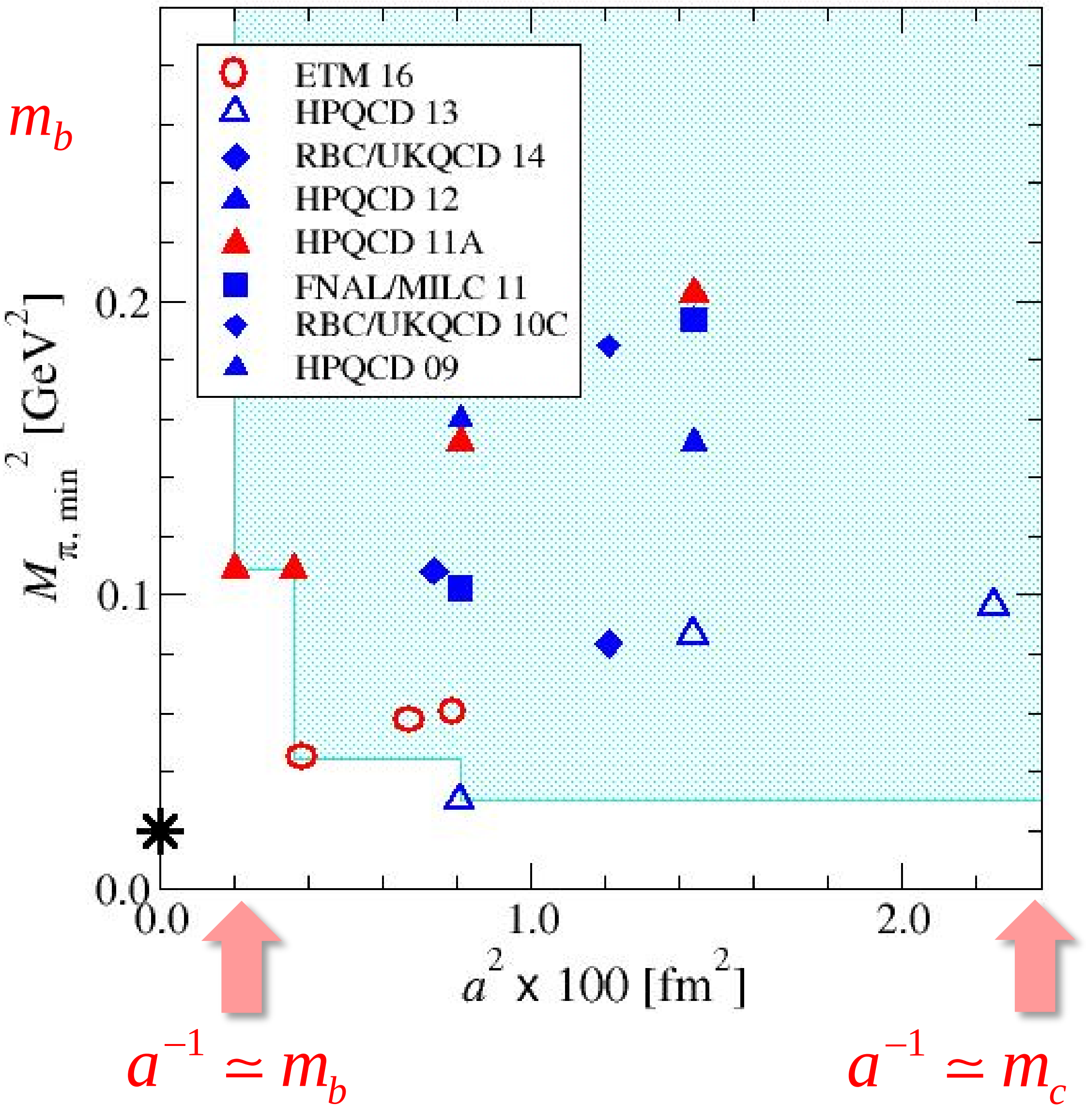

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- directly at m_c, m_b

  $\Rightarrow$ recent FFs, B_P's

- need matching to QCD, often perturbative

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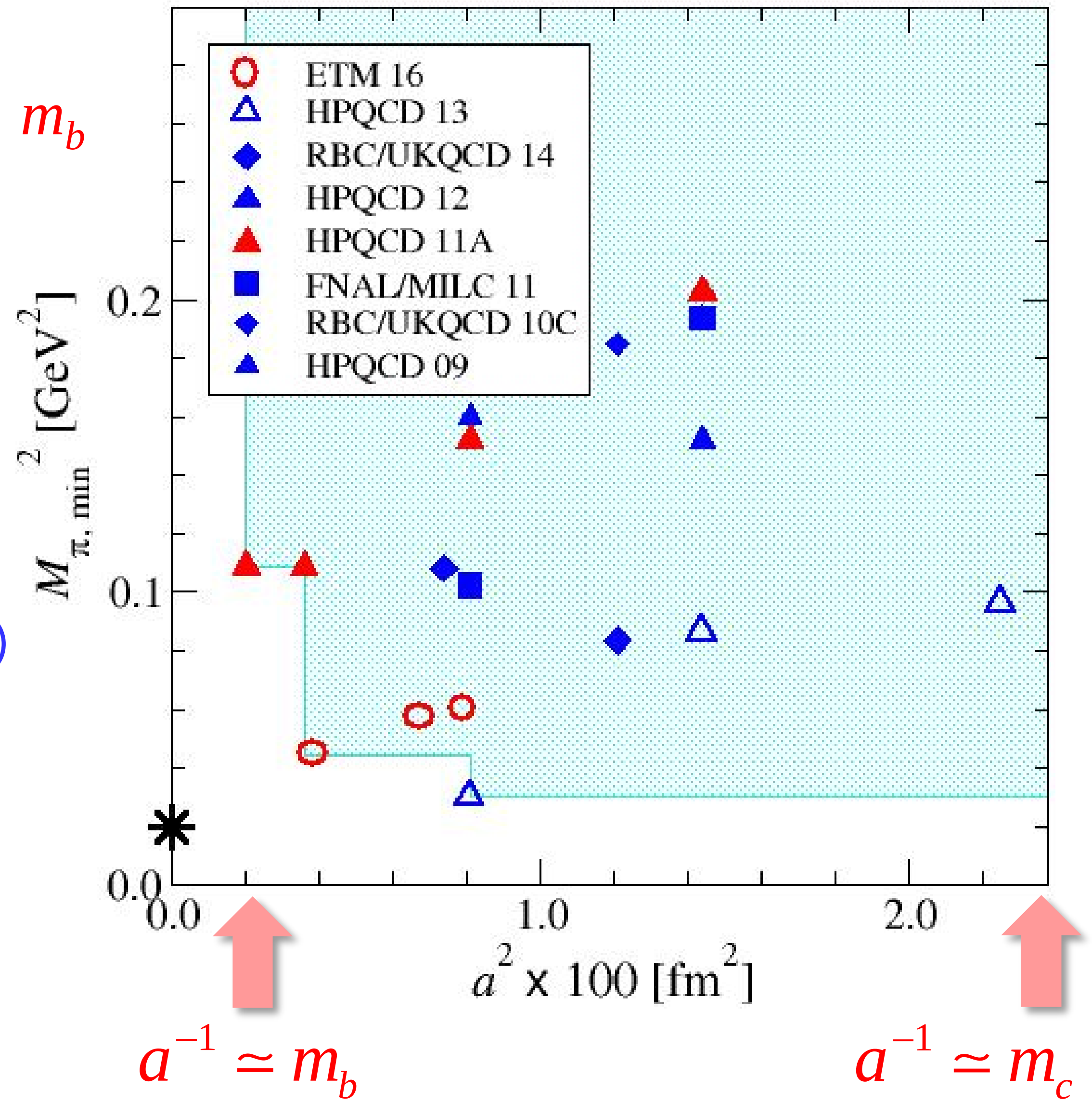

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  $\Rightarrow$ @ m_b in 5-10 yrs(?)

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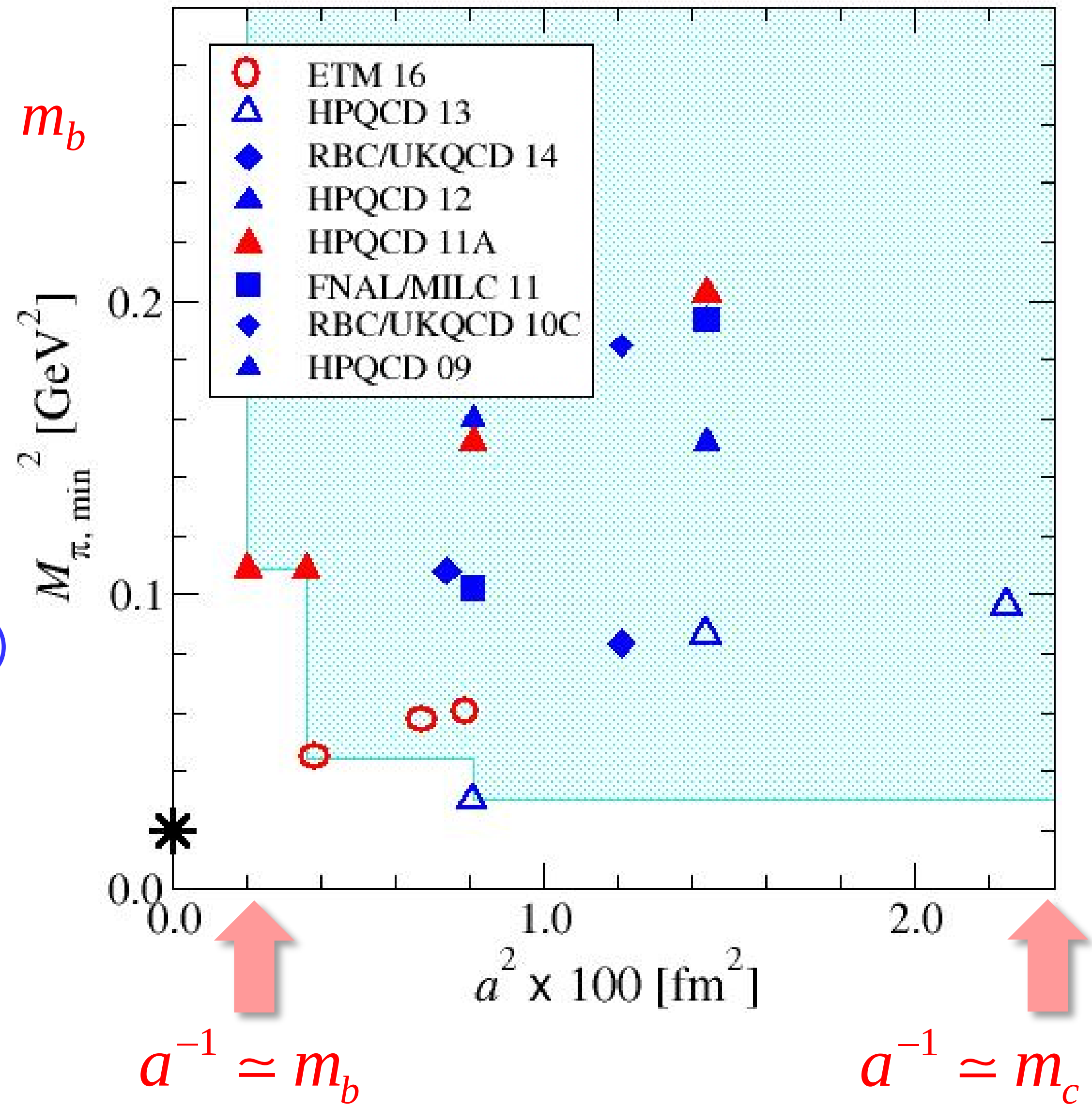

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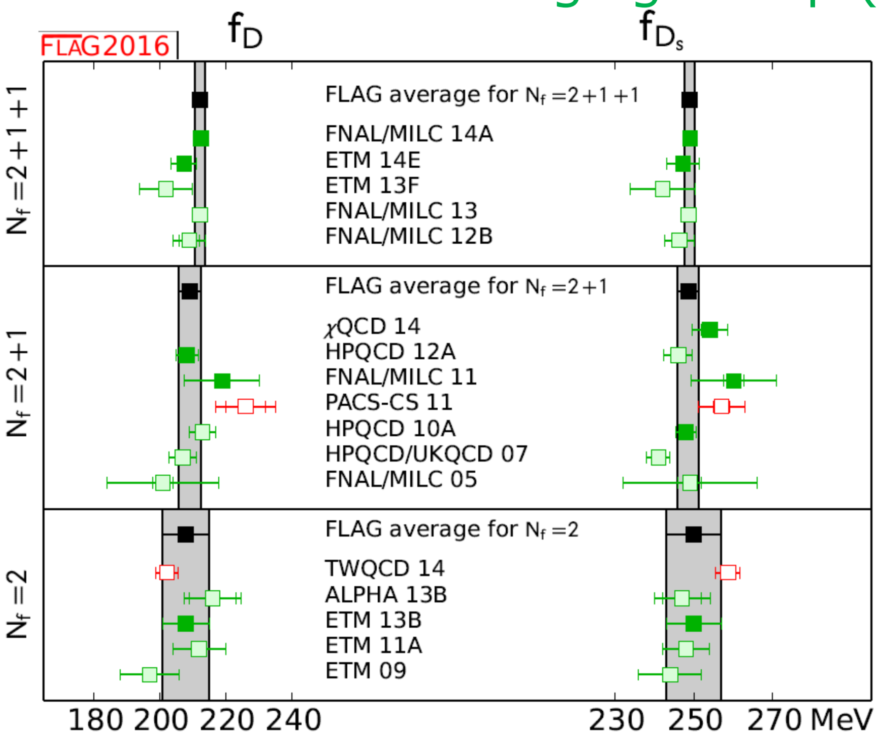
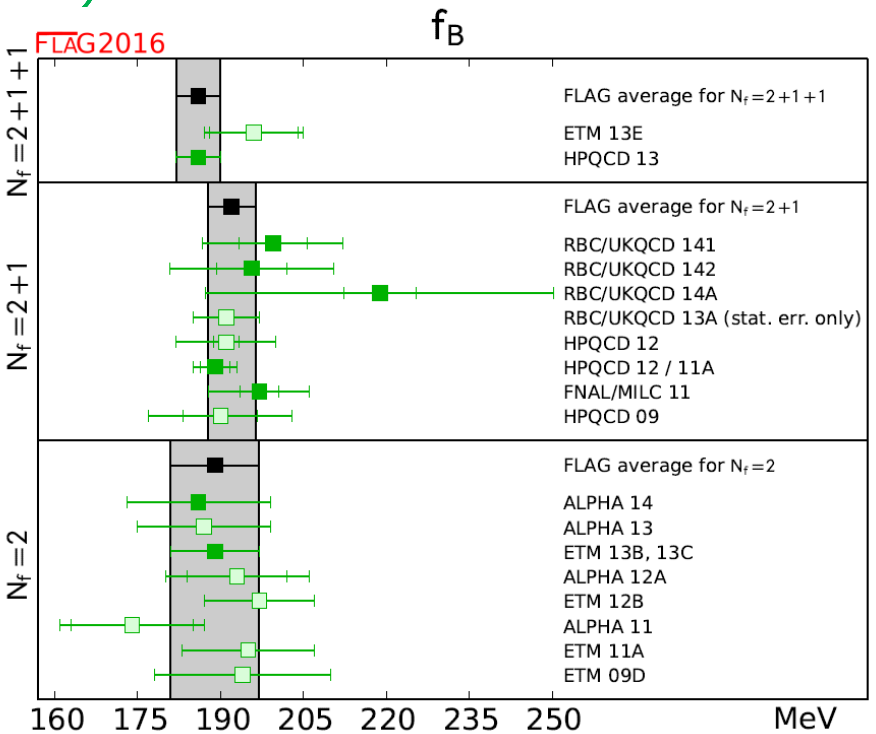
decay constants

Flavor Lattice Averaging Group (FLAG) 1607.00299





+ RBC/UKQCD 1701.02644 New!

+ ETM 1603.04306 New!

decay constants

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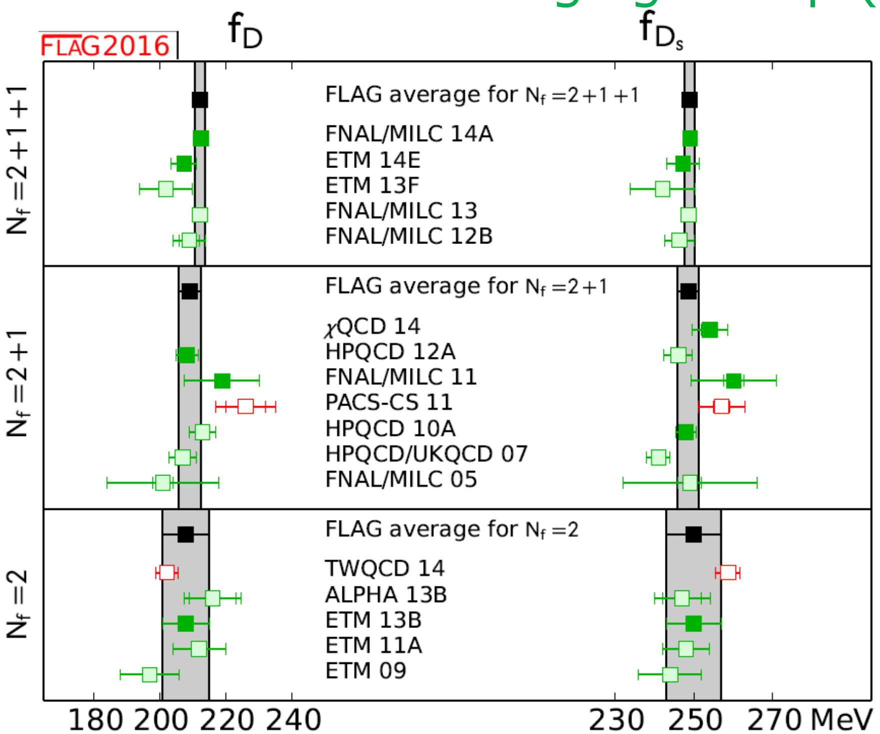
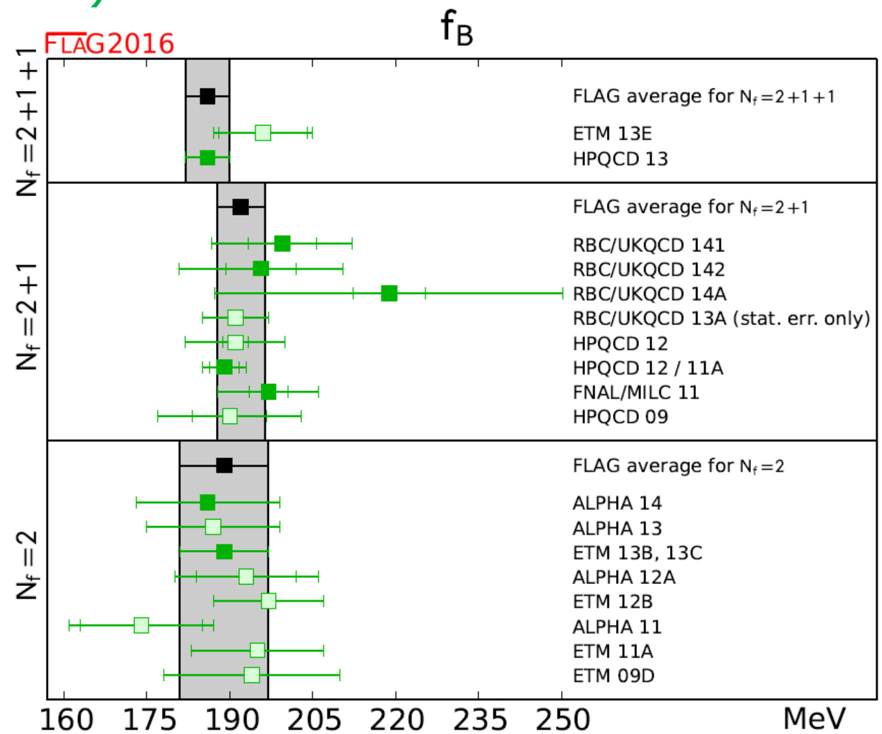




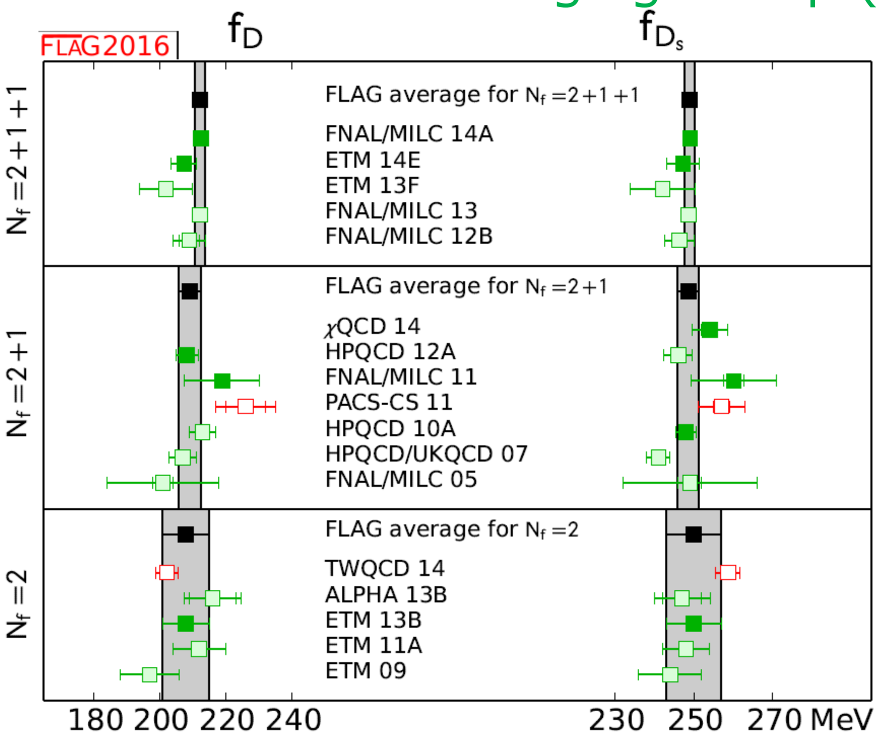
+ RBC/UKQCD 1701.02644 New!

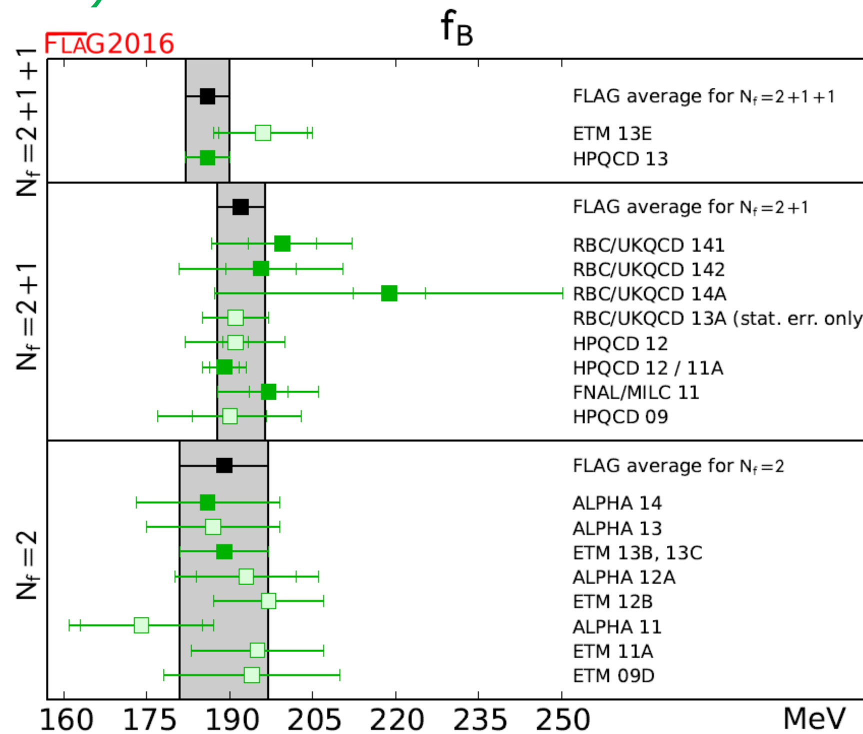
+ ETM 1603.04306 New!

- $f_{D(s)}$: fully controlled, $\Delta f_{D(s)} \sim$ 0.6% $\Leftrightarrow$ $\Delta\Gamma/2 \sim$ 2% (HFAG '16)

decay constants

Flavor Lattice Averaging Group (FLAG) 1607.00299





\+ RBC/UKQCD 1701.02644 New! + ETM 1603.04306 New!

- $f_{D(s)}$: fully controlled, $\Delta f_{D(s)} \sim 0.6\%$ $\Leftrightarrow$ $\Delta\Gamma/2 \sim 2\%$ (HFAG '16)
- $f_{B(s)}$: $\Delta f_{B(s)} \lesssim 2\%$ $\Leftrightarrow$ $\Delta\mathrm{BR}/2 \sim 3\%$ (Belle II, 50 ab^{-1})

decay constants

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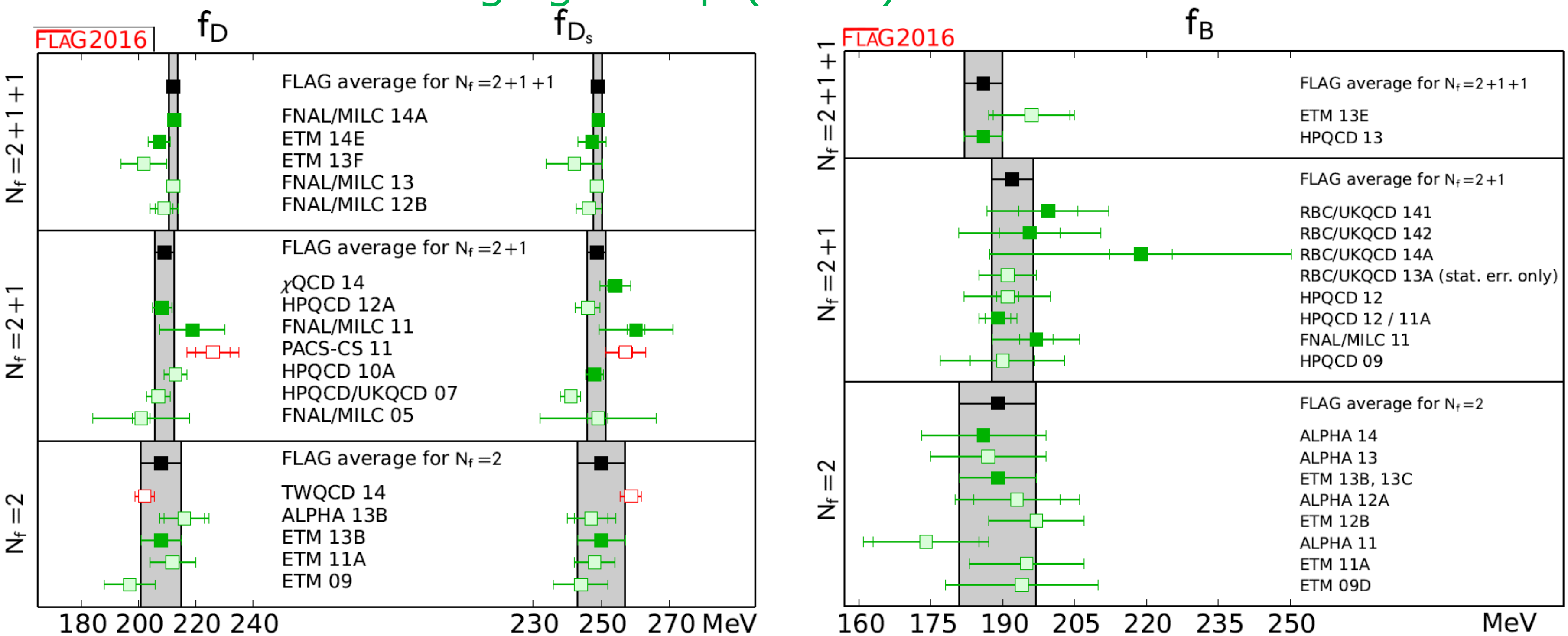

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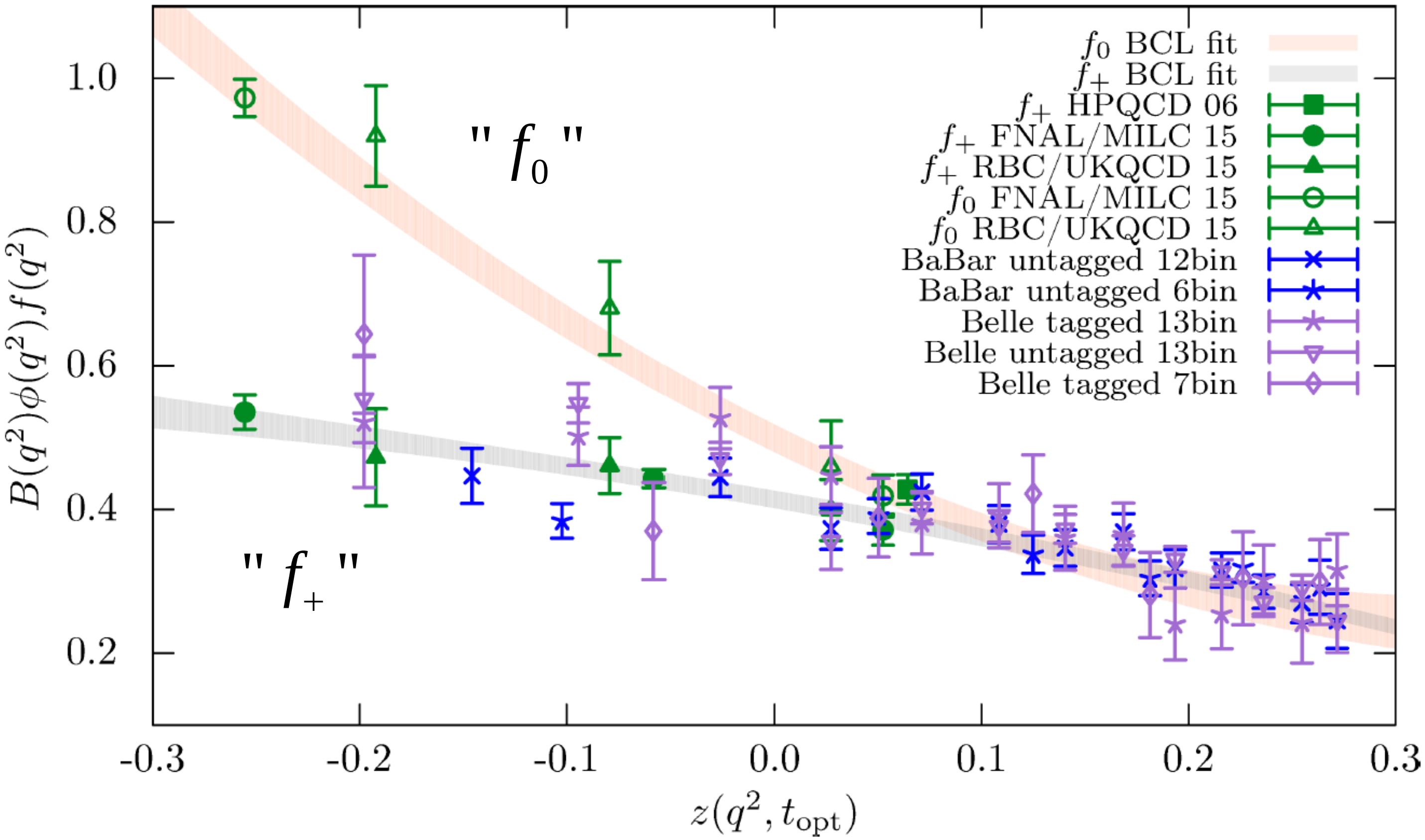
- competitive to expr't / isospin corr.s becoming relevant

semileptonic decays

$B\to\pi l\nu$: new analysis by FLAG '16



LQCD ~ expr't

modern studies by 1-2 groups

$B\to\pi$
RBC/UKQCD '15
FNAL/MILC '15

$B\to D$
FNAL/MILC '15
HPQCD '15

$B\to D^*$
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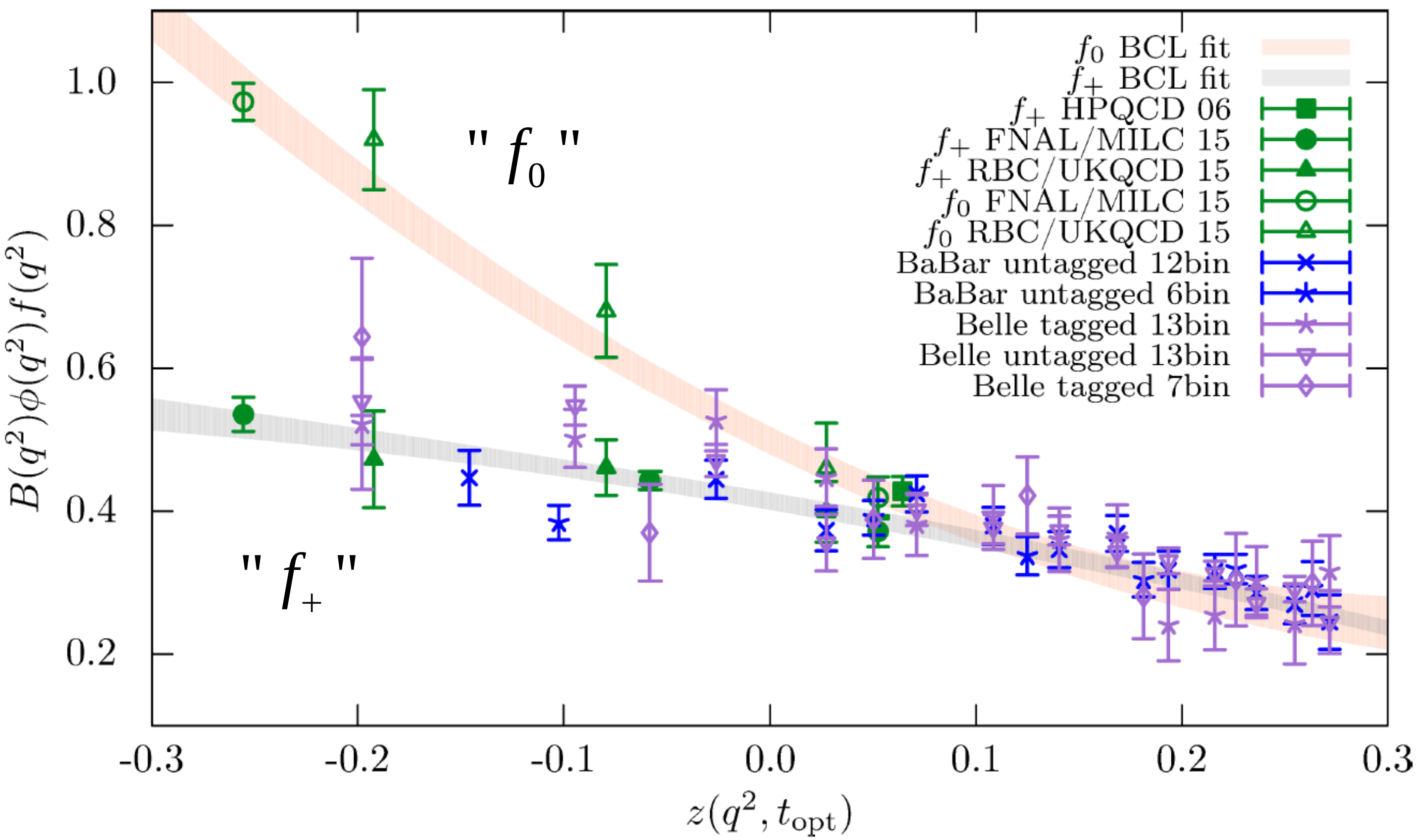

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- expect new studies *e.g.* $B \to \pi l \nu$ JLQCD, FNAL/MILC @ Lat'17
- BSM, rare decays : *e.g.* $B \to \pi$ f_T FNAL/MILC 1507.01618
- $B \to D^*$ @ nonzero recoil ($d\Gamma/dw \propto (w^2\text{-}1)^{1/2}$) (*cf.* FNAL/MILC @ Lat'17)

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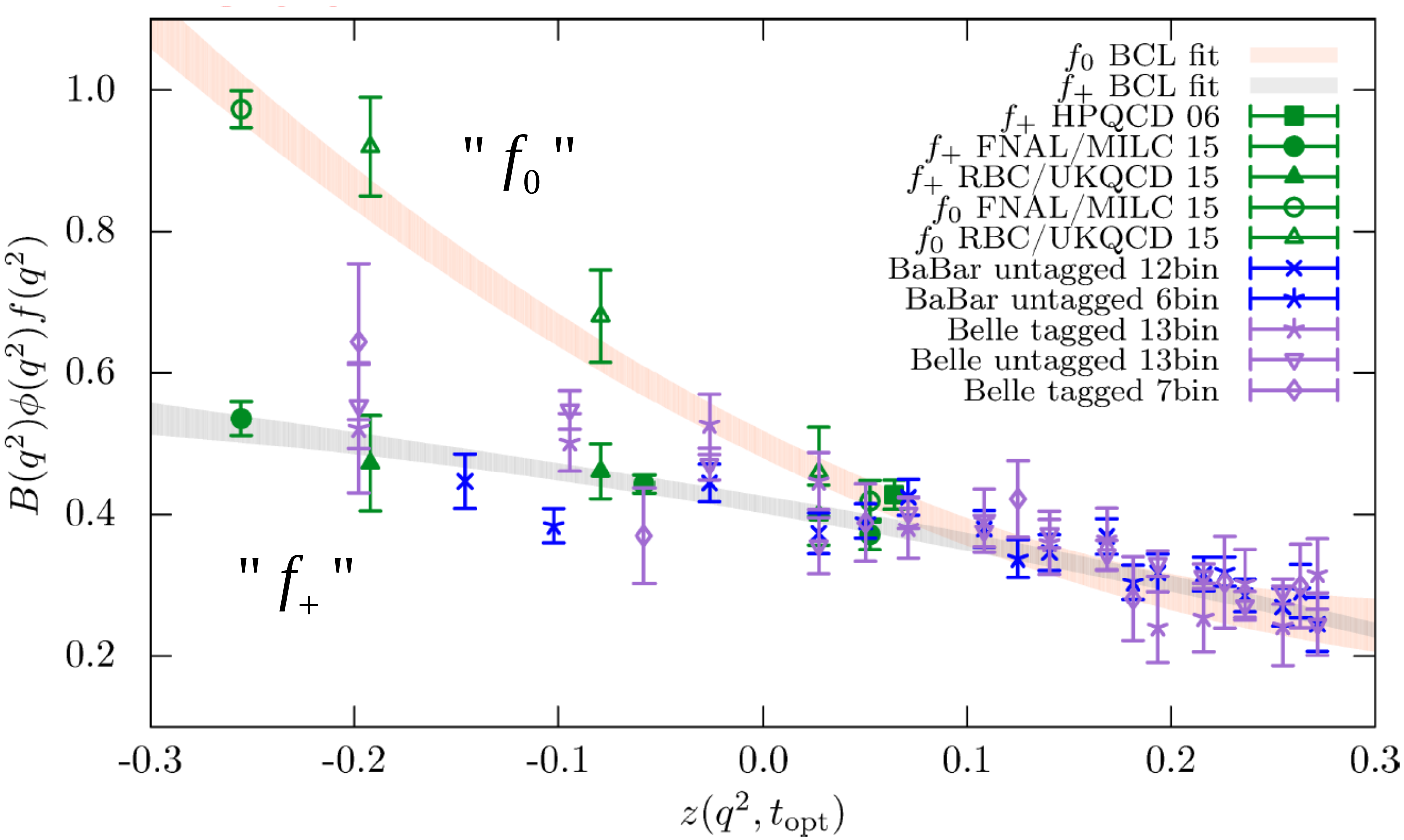

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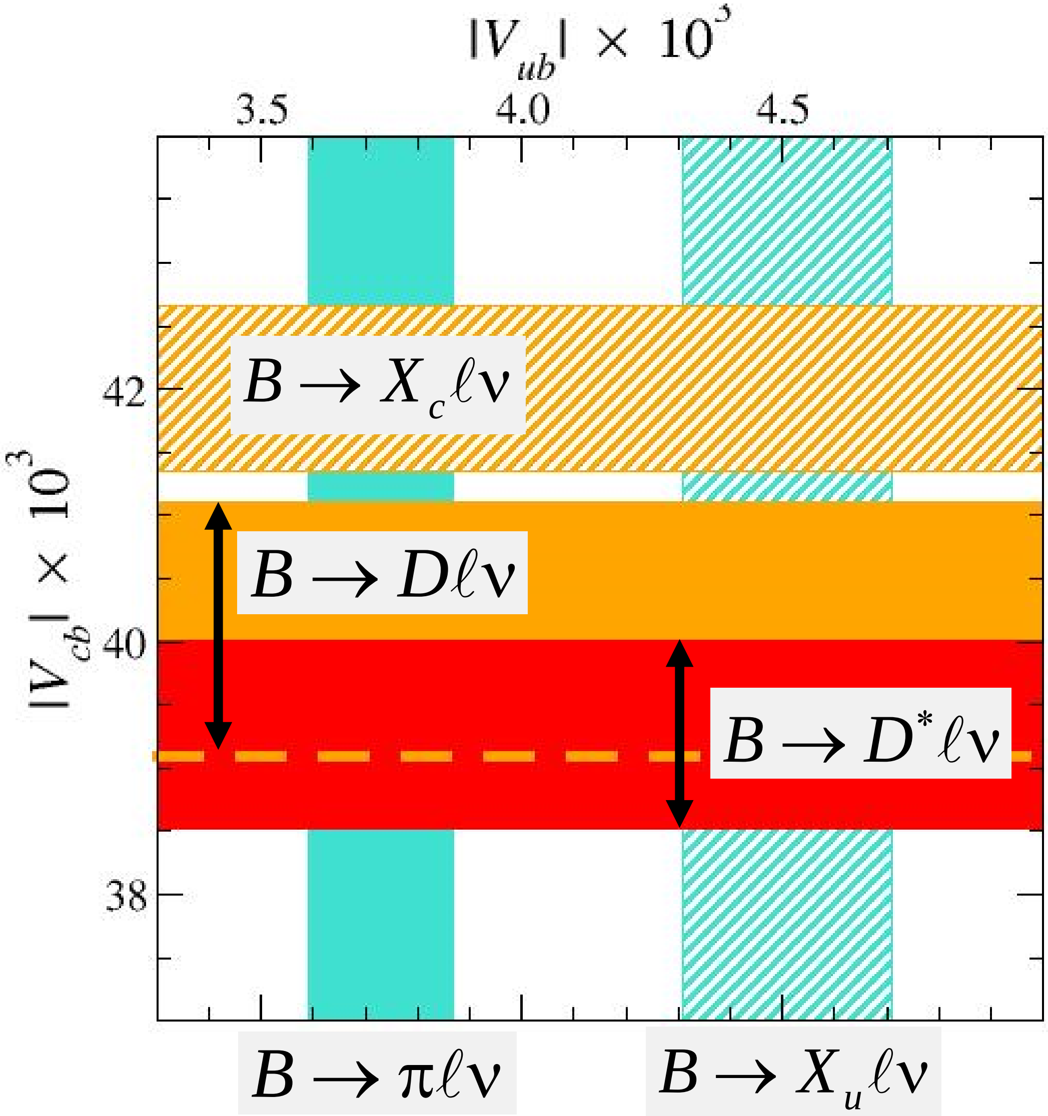
in a few years …

exclusive vs inclusive



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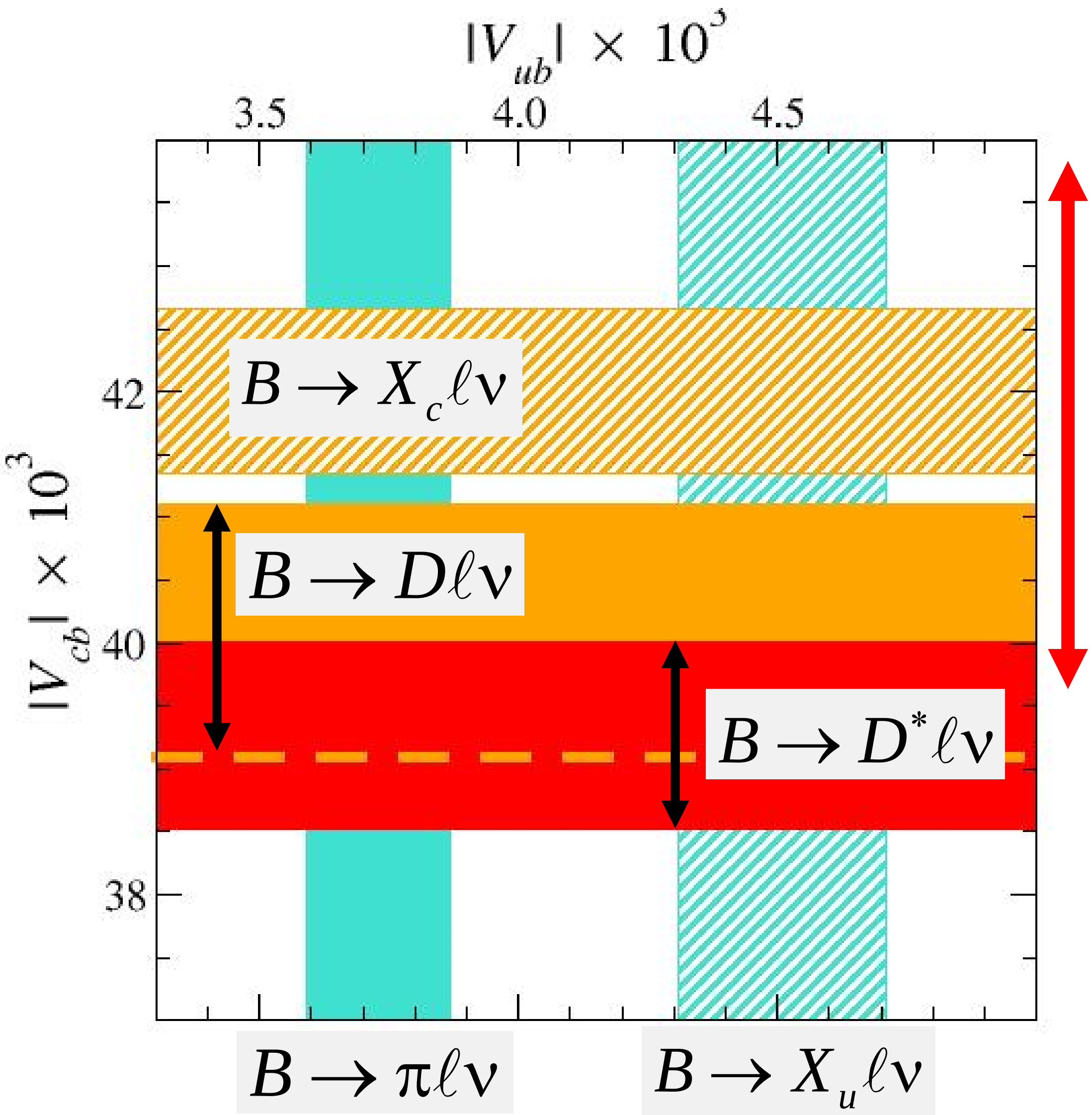

$B \to D^* \ell \nu$

Bigi-Gambino-Schacht '17

w/ new Belle data '17

- 2 param.s for w-dep.

  ⇒ systematic shift in $|V_{cb}|$

  ⇔ LQCD @ nonzero recoil

Future lattice fits	χ^2/dof	$\lvert V_{cb}\rvert$
CLN	56.4/37	0.0407 (12)
CLN+LCSR	59.3/40	0.0406 (12)
BGL	28.2/33	0.0409 (15)
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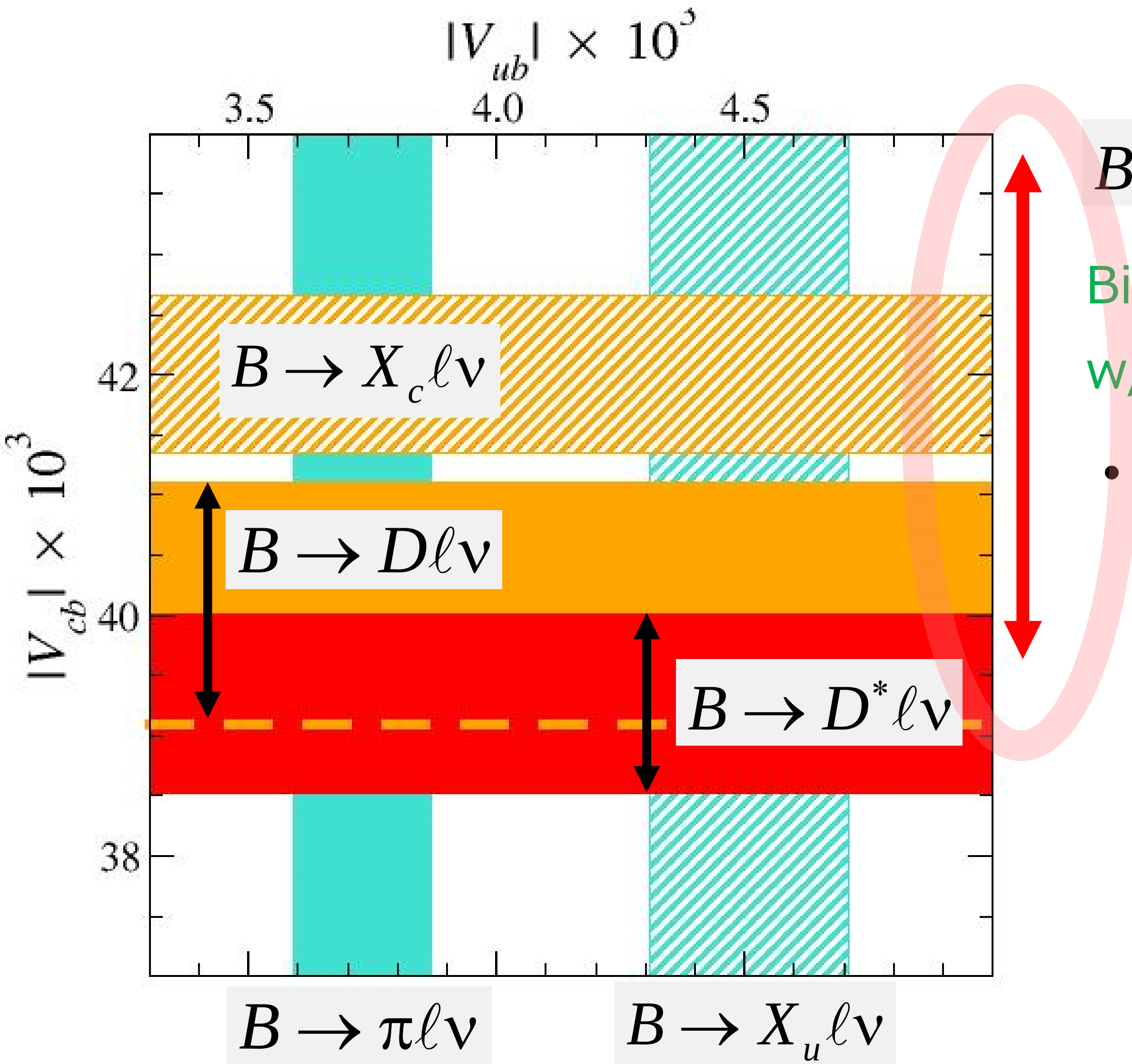

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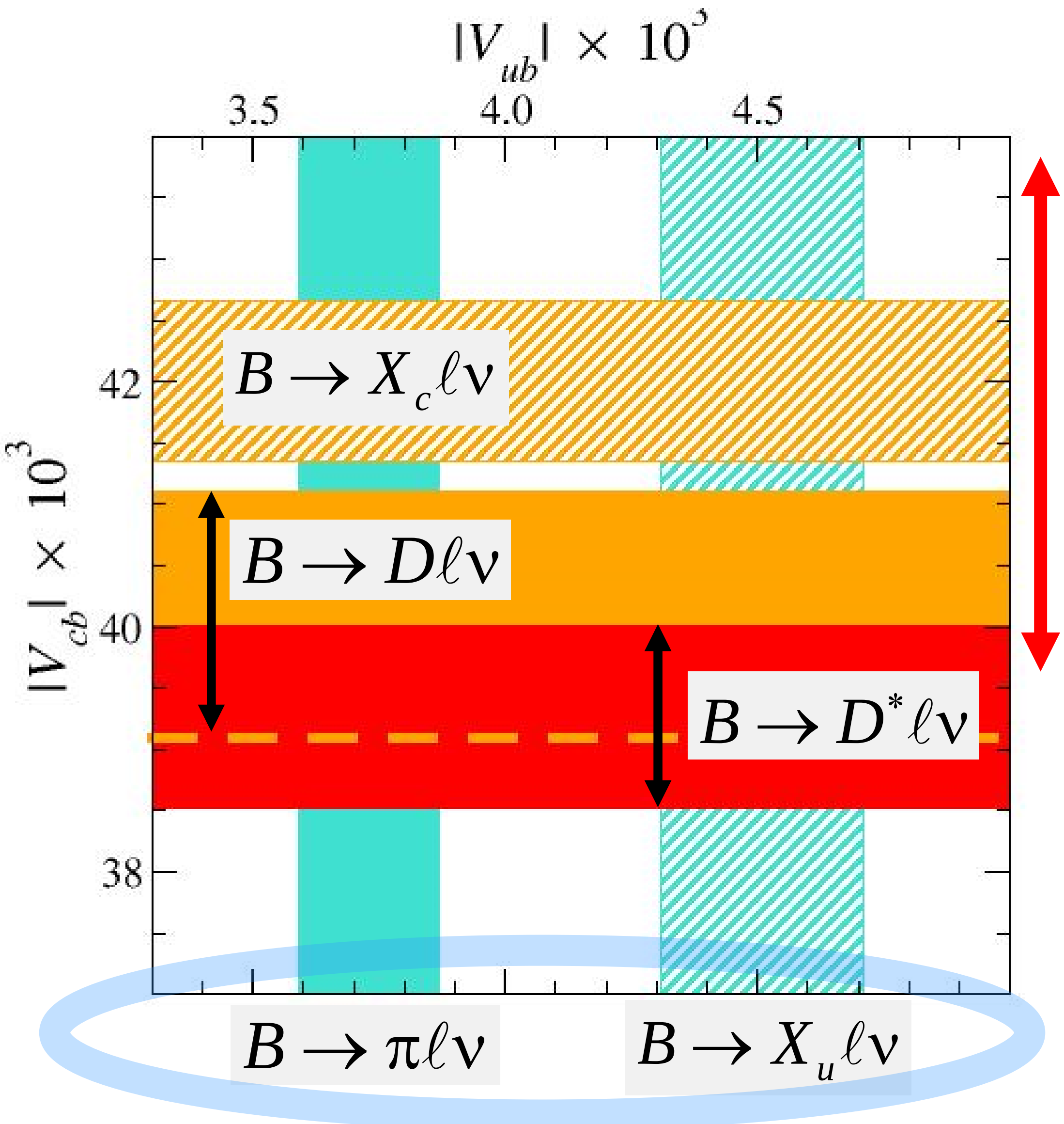

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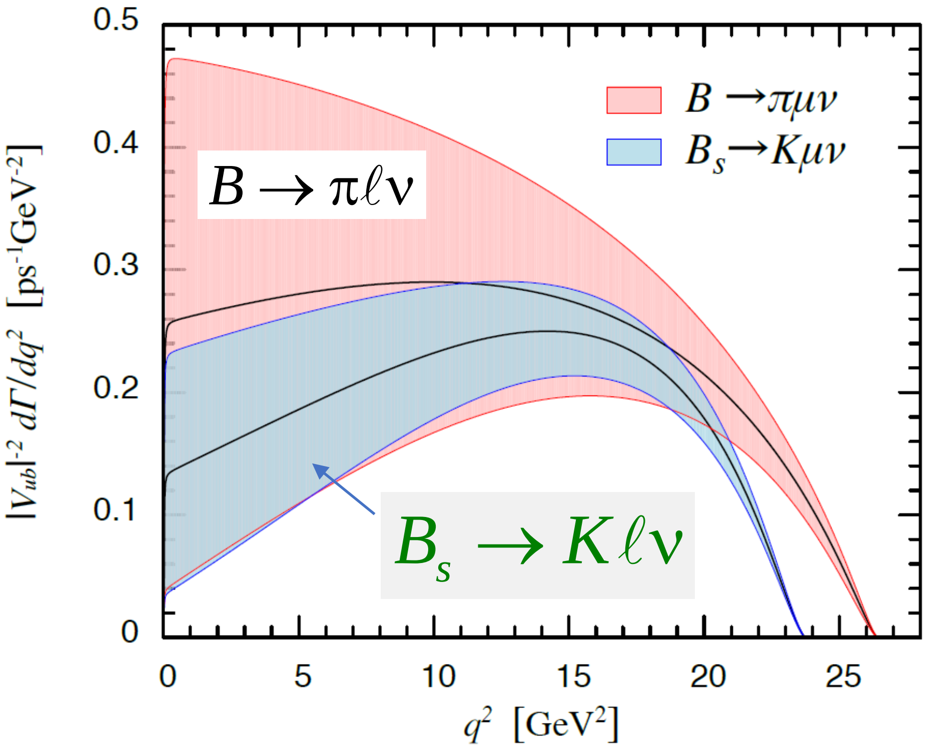
other excl. modes may help / inclusive decays on the lattice?

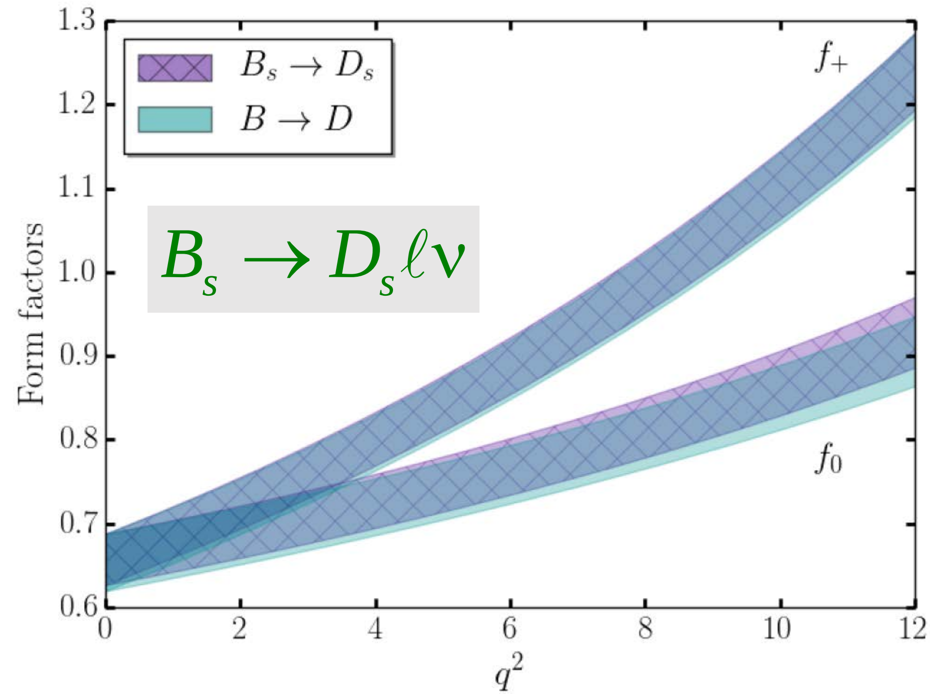
non "conventional" modes

RBC/UKQCD 1501.05373



+ ALPHA 1601.04277 New!

HPQCD 1703.09728 New!



⋯ and B_c decays Mathur@Lat'17

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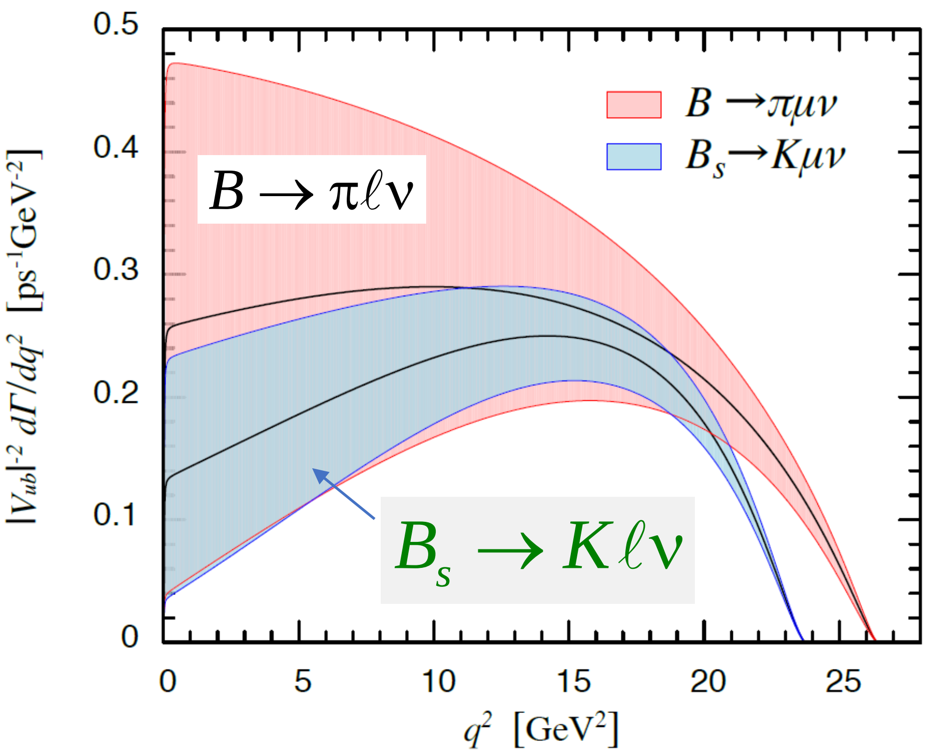

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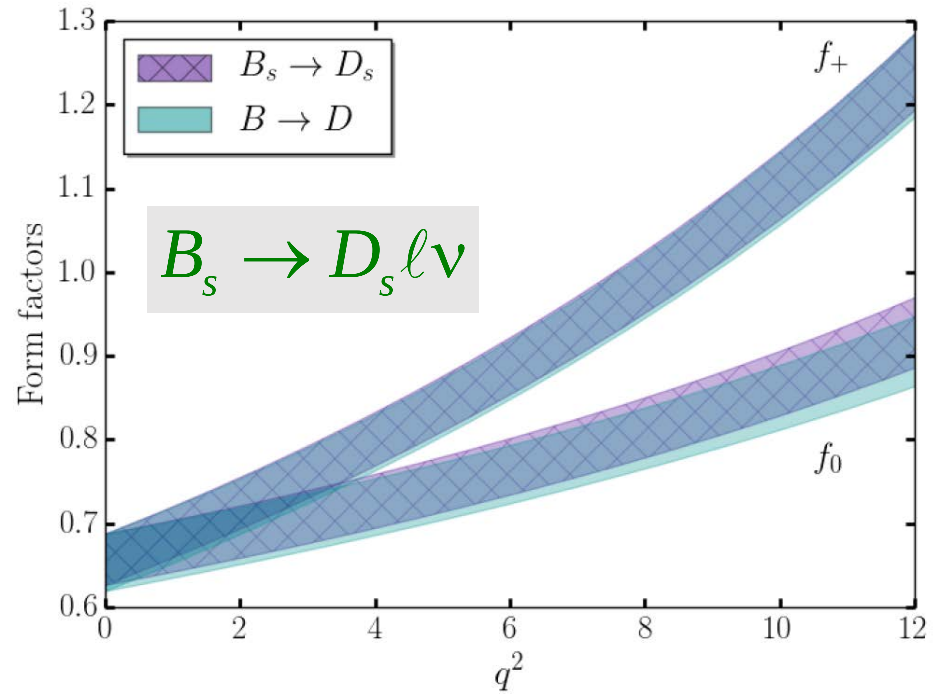

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$\cdots$ and B_c decays Mathur@Lat’17

- statistics + systematics : equally or better controlled
- key = feasibility of precision experiments: σ, BGs, ν

baryon decays

semileptonic decays provide independent determination of $|V_{qq'}|$

- Detmold et al. ‘15: $\Lambda_b \to p l \nu, \Lambda_c l \nu \Rightarrow |V_{cb}|/|V_{ub}|$
- Meinel 1611.09696 New!: $\Lambda_c \to \Lambda l \nu \Rightarrow |V_{cs}|$

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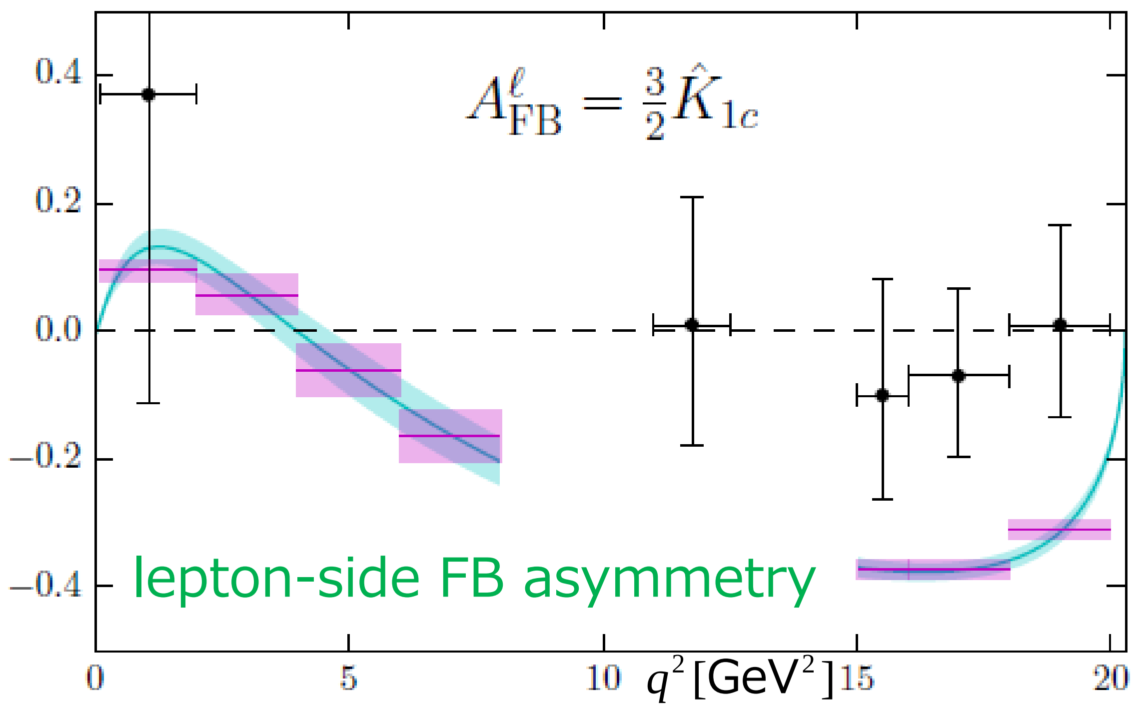
Detmold-Meinel 1602.01399 New!

$\Lambda_b \to \Lambda ll$ rare decays

- FFs $\Rightarrow$ $\mathrm{dBR}/\mathrm{d}q^2$ and angular observables



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- 3.3σ tension in A_{FB}^l?

$\Leftrightarrow$ P_5' anomaly in $B \to K^*ll$

(LHCb '13,⋯)

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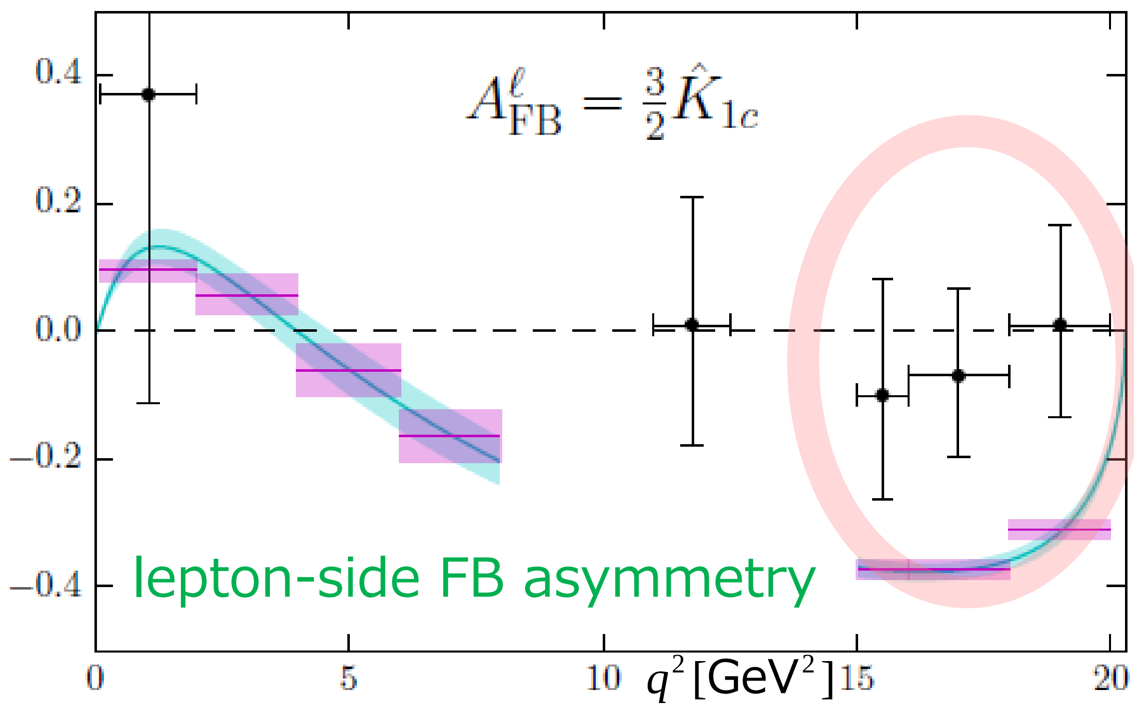

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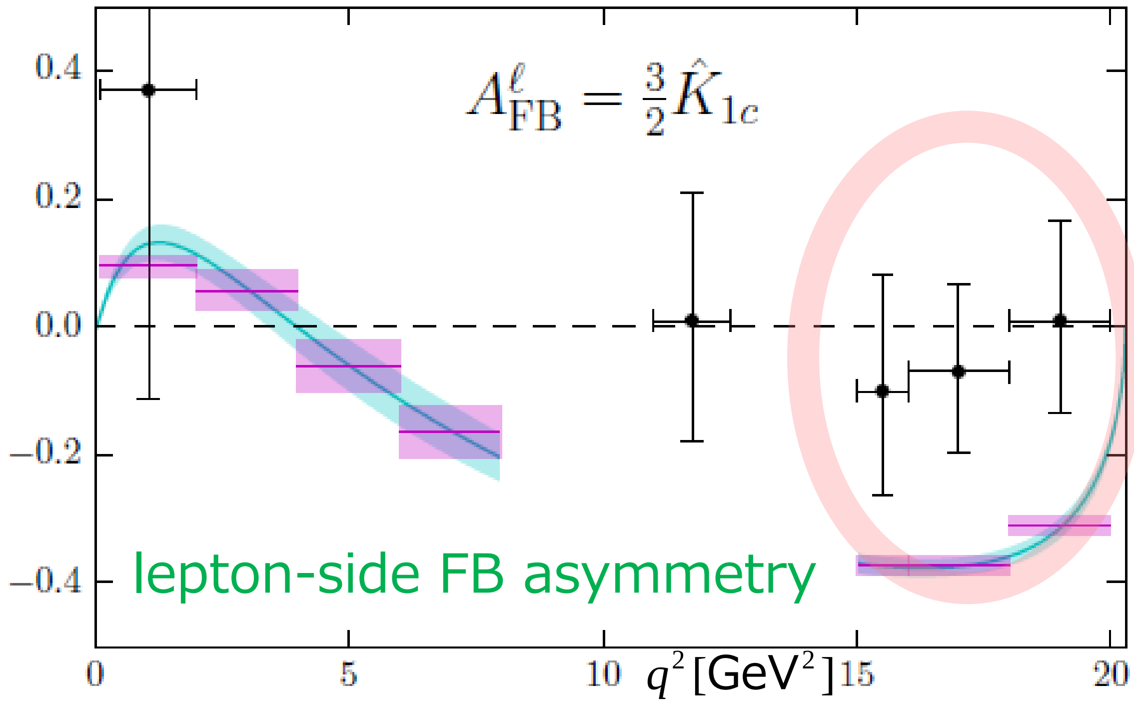

heavy M_π, finite V?

$\Rightarrow$ systematics to be checked, possible in relatively short term

inclusive decays

D, B strong decays : framework under development (long term)

inclusive decays

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Hashimoto 1703.01881 New!

$$|\mathcal{M}|^2 = |V_{qb}|^2 G_F^2 M_B l^{\mu\nu} W_{\mu\nu} \Leftrightarrow \quad T_{\mu\nu} = i\int d^4x\, e^{-iqx} \langle B | T[J_\mu^\dagger(x) J_\nu(0)] | B \rangle$$

hadronic tensor $\quad W = -\pi^{-1}\,\mathrm{im}[T] \quad$ forward scattering ME

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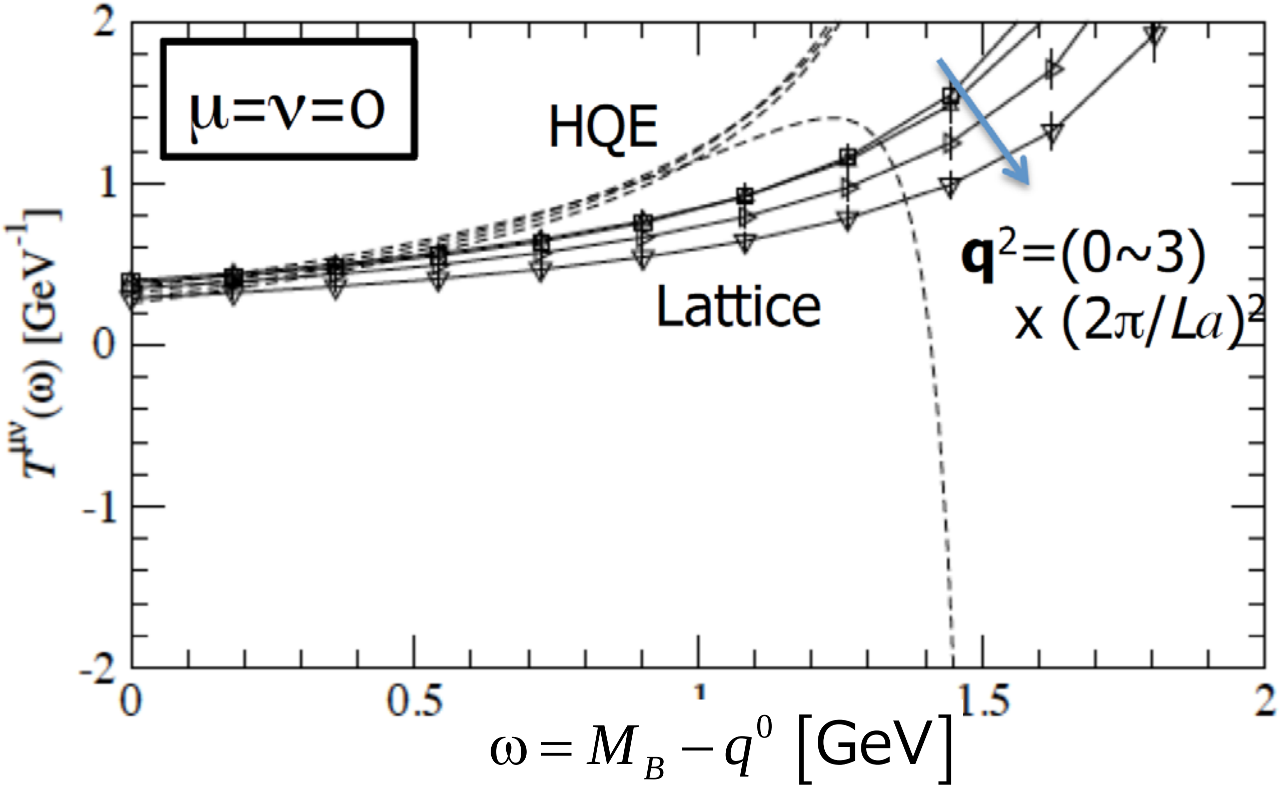


exploratory

- $B_s \rightarrow X_c l\nu$ @ $q^2=0$
- marginal agreement w/ $O(1/M, \alpha_s^0)$ HQE
- $q^2 \neq 0$, $B \rightarrow X_c$, X_u

inclusive decays

LQCD @ unphysical kinematics

$$v\cdot q > (v\cdot q)_{\max}$$

contour integral using expr'tal data $\left(v\cdot q > \sqrt{q^2}\right)$ and pQCD $\left(v\cdot q \le \sqrt{q^2}\right)$

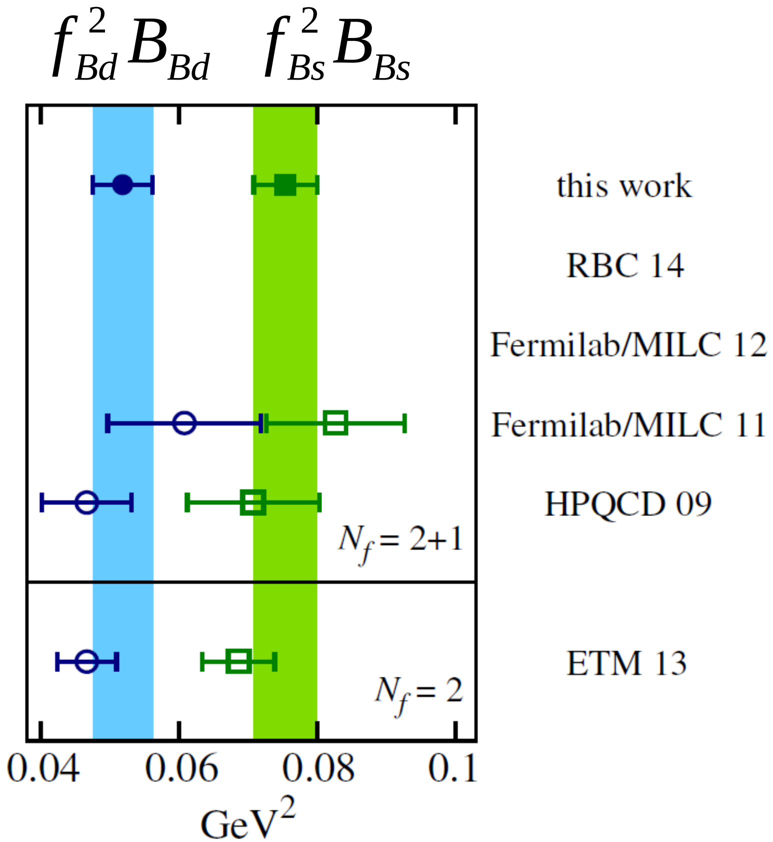
$$T(v\cdot q) = \frac{1}{\pi}\int_{-\infty}^{(v\cdot q)_{\max}} d(v\cdot q')\,\frac{\mathrm{im}\left[T(v\cdot q)\right]}{v\cdot q' - v\cdot q}$$

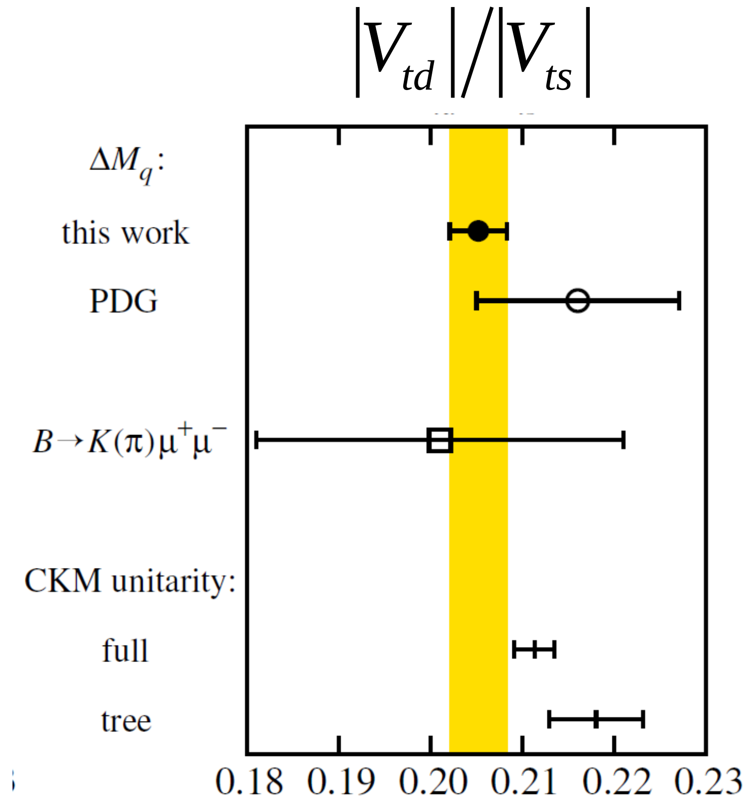
a key = contour integral under BGs and experimental cuts

see also Hansen, Meyer, Robaina 1704.08993 New!

B meson mixing

FNAL/MILC 1602.03560 New!





- x2-3 improvement: more realistic (stat., a^{-1}, M_π), renorm.

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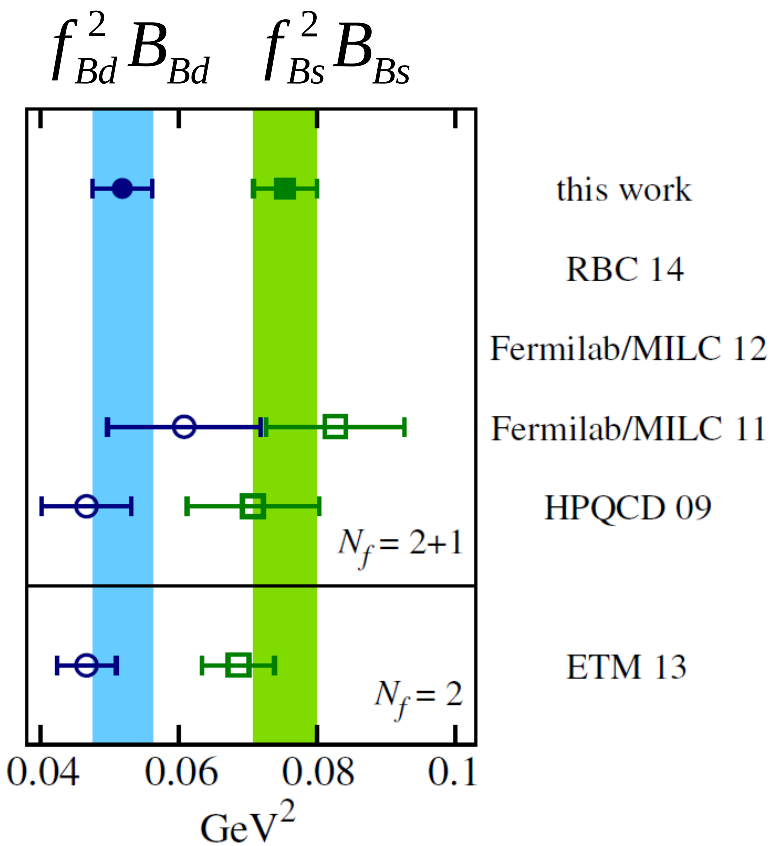

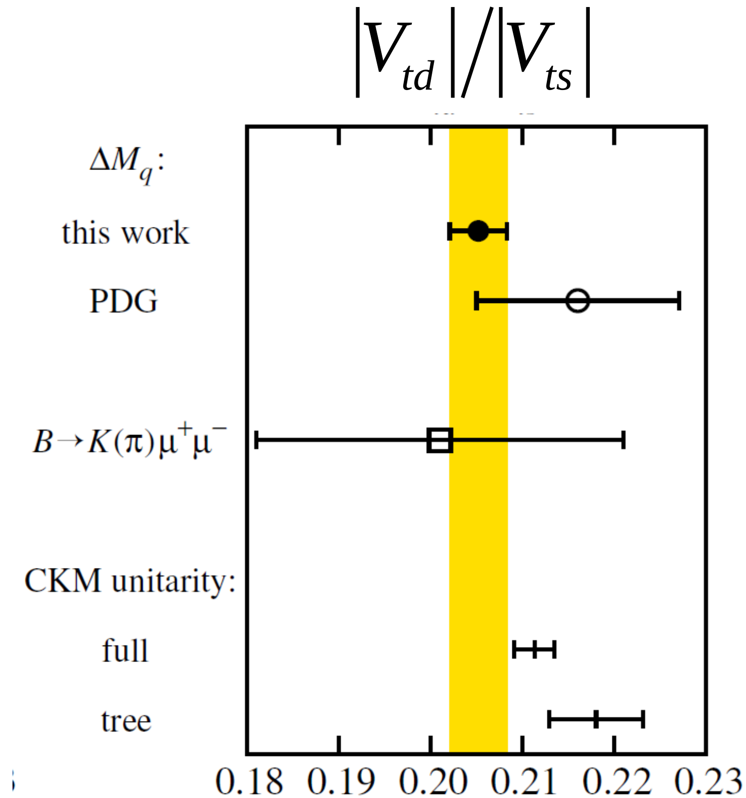

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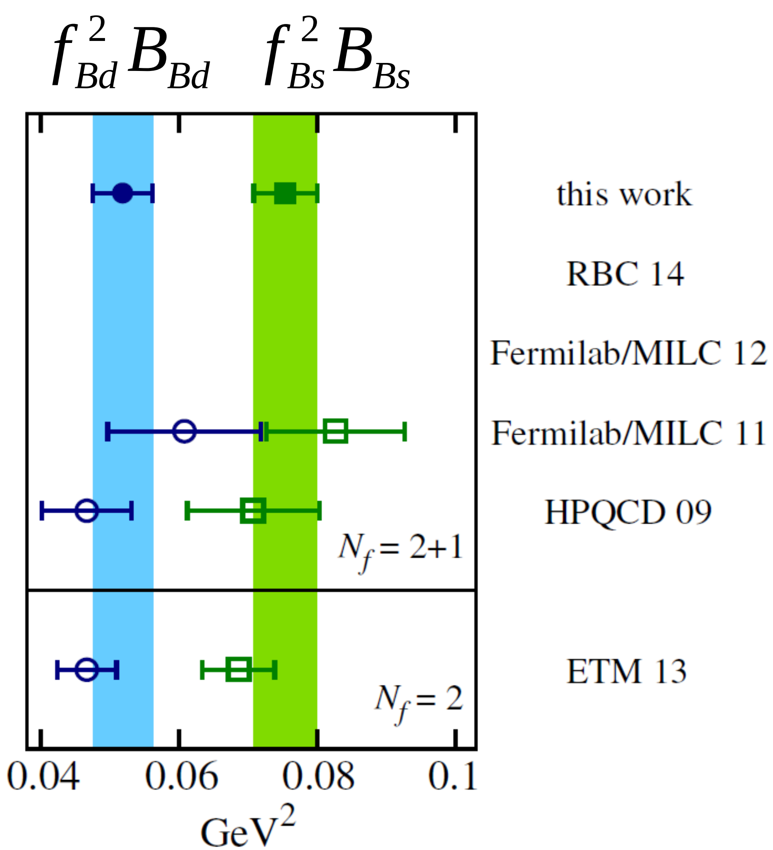
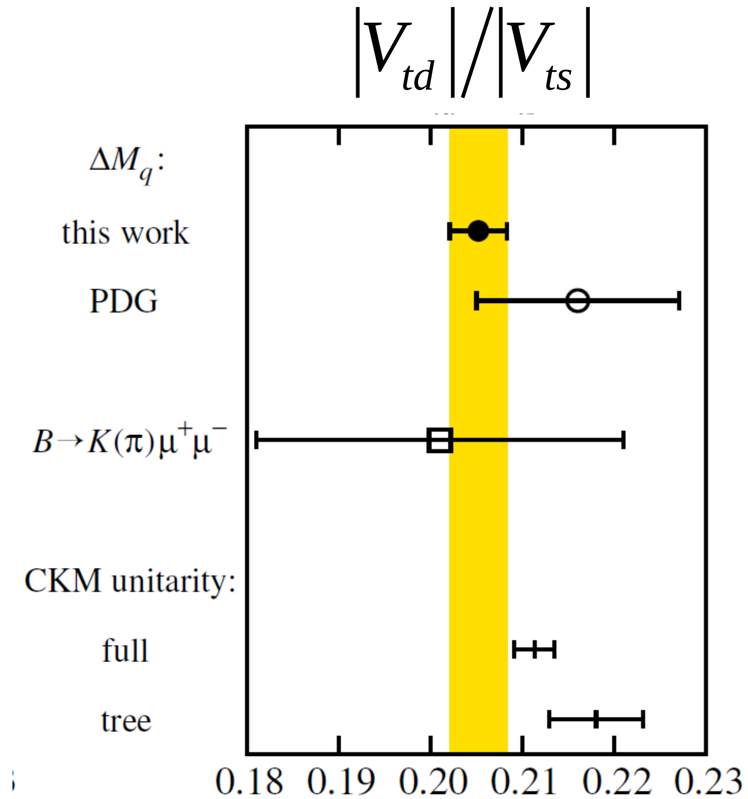
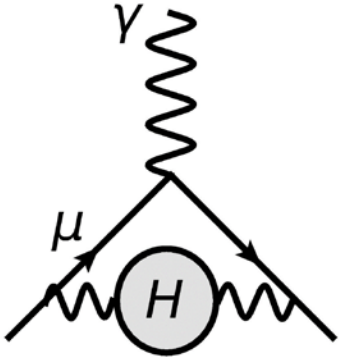




- x2-3 improvement: more realistic (stat., a^{-1}, M_π), renorm.
- 2σ tension in ΔM_d, $\Delta M_d / \Delta M_s$ ⇒ independent calculations?
- BSM and D-mixing MEs available FNAL/MILC 1706.04622 New!

muon g-2 : HVP



- $R(e^+e^- \to hadrons) \Rightarrow \Delta a_\mu^{\mathrm{HVP}} \sim 0.6\%$
- purely theoretical estimate?

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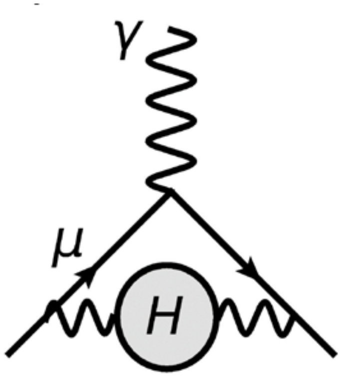

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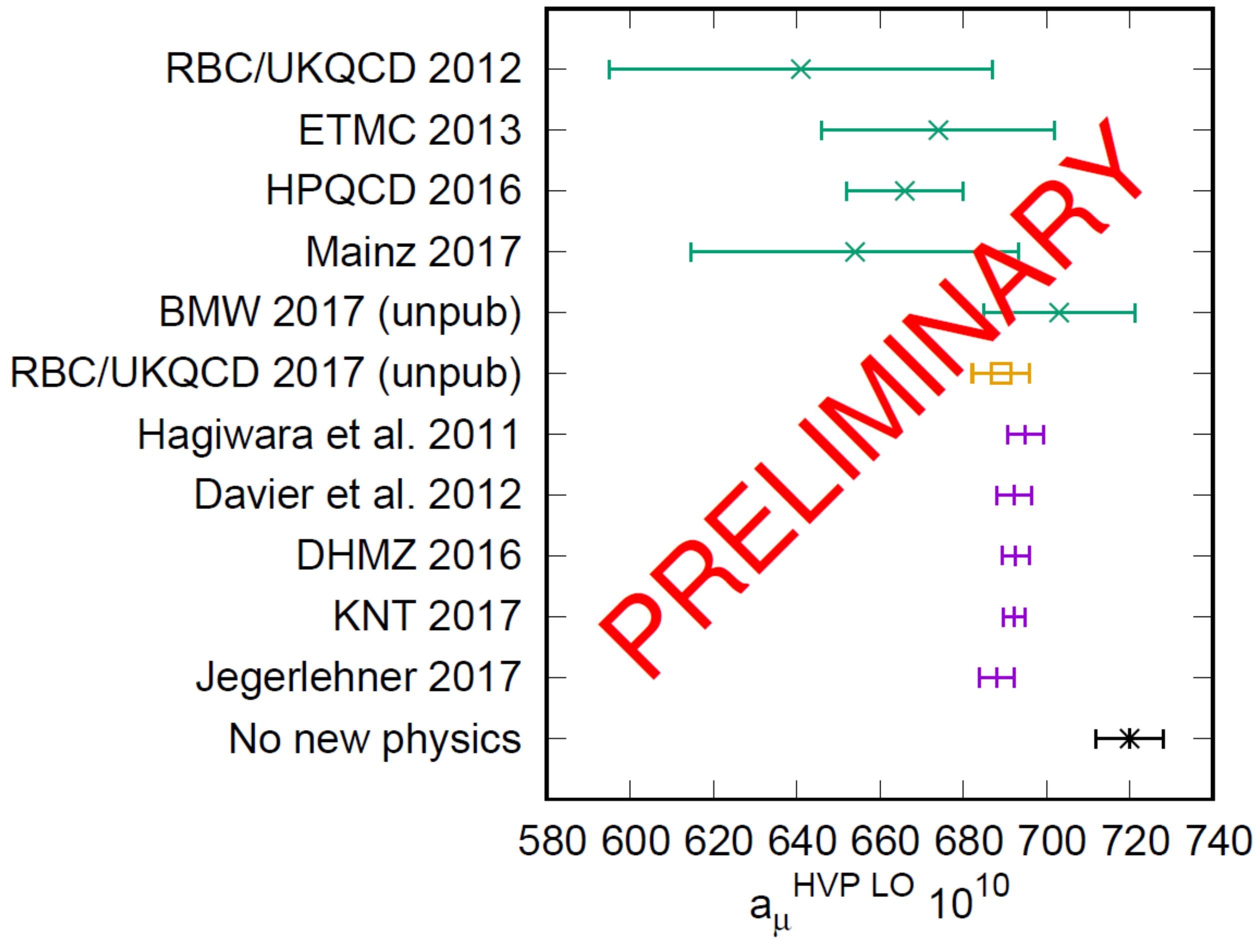
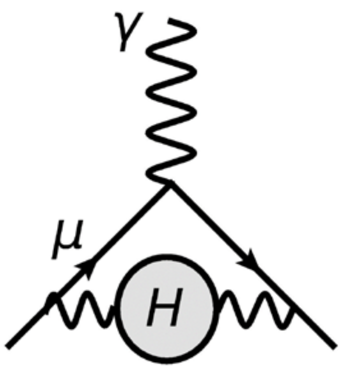
Lattice '17: plenary Lehner



new results to control

- stat. error
- $a \neq 0$, $V \neq \infty$, m_q
- isospin
- charm

muon *g*-2 : HVP



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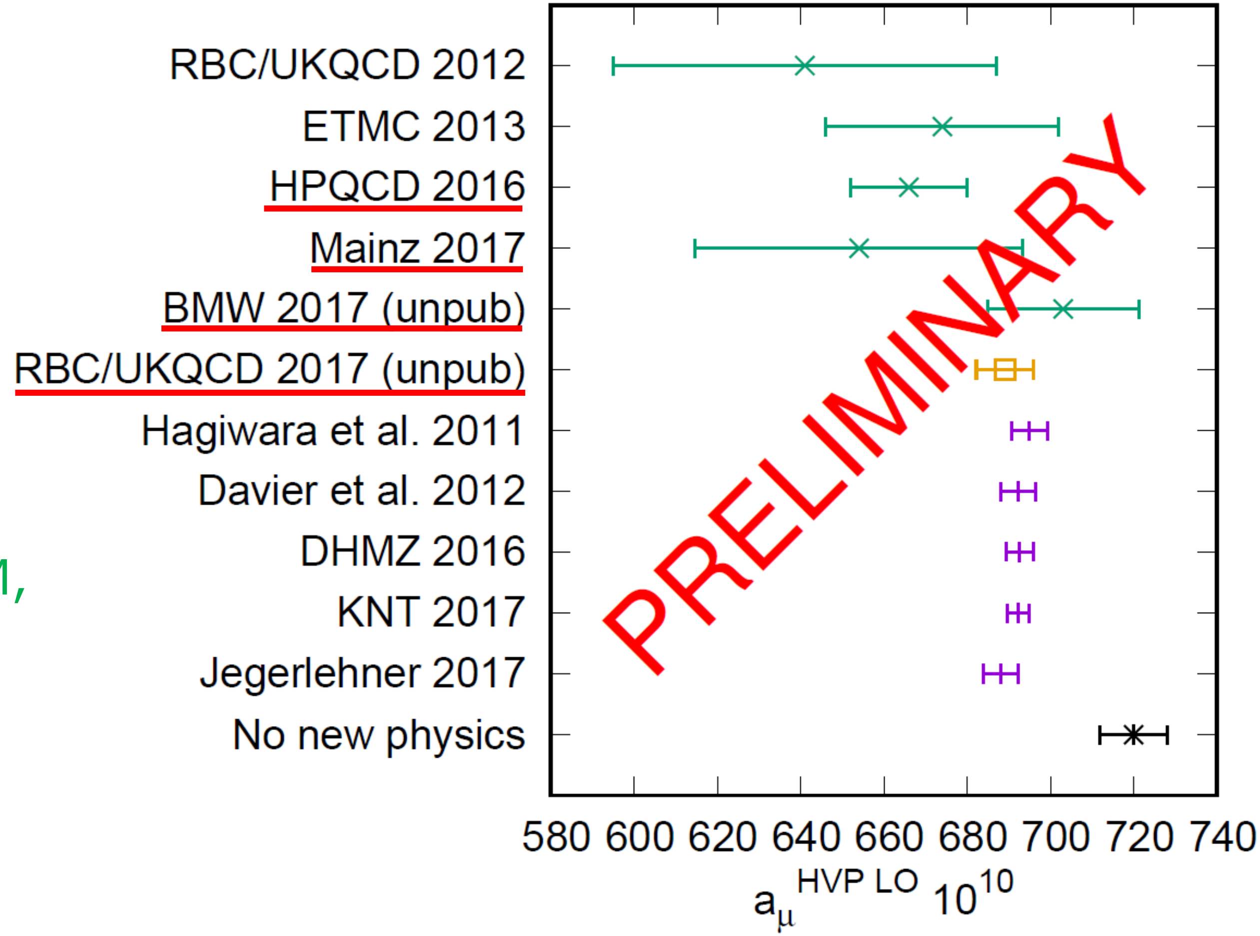
(also papers/talks by ETM, PACS, Golterman et al.)



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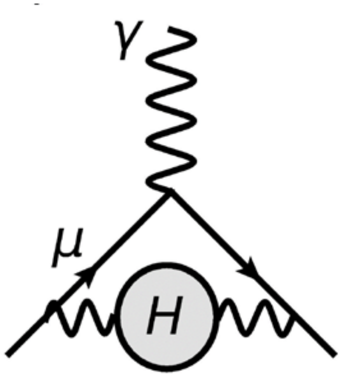

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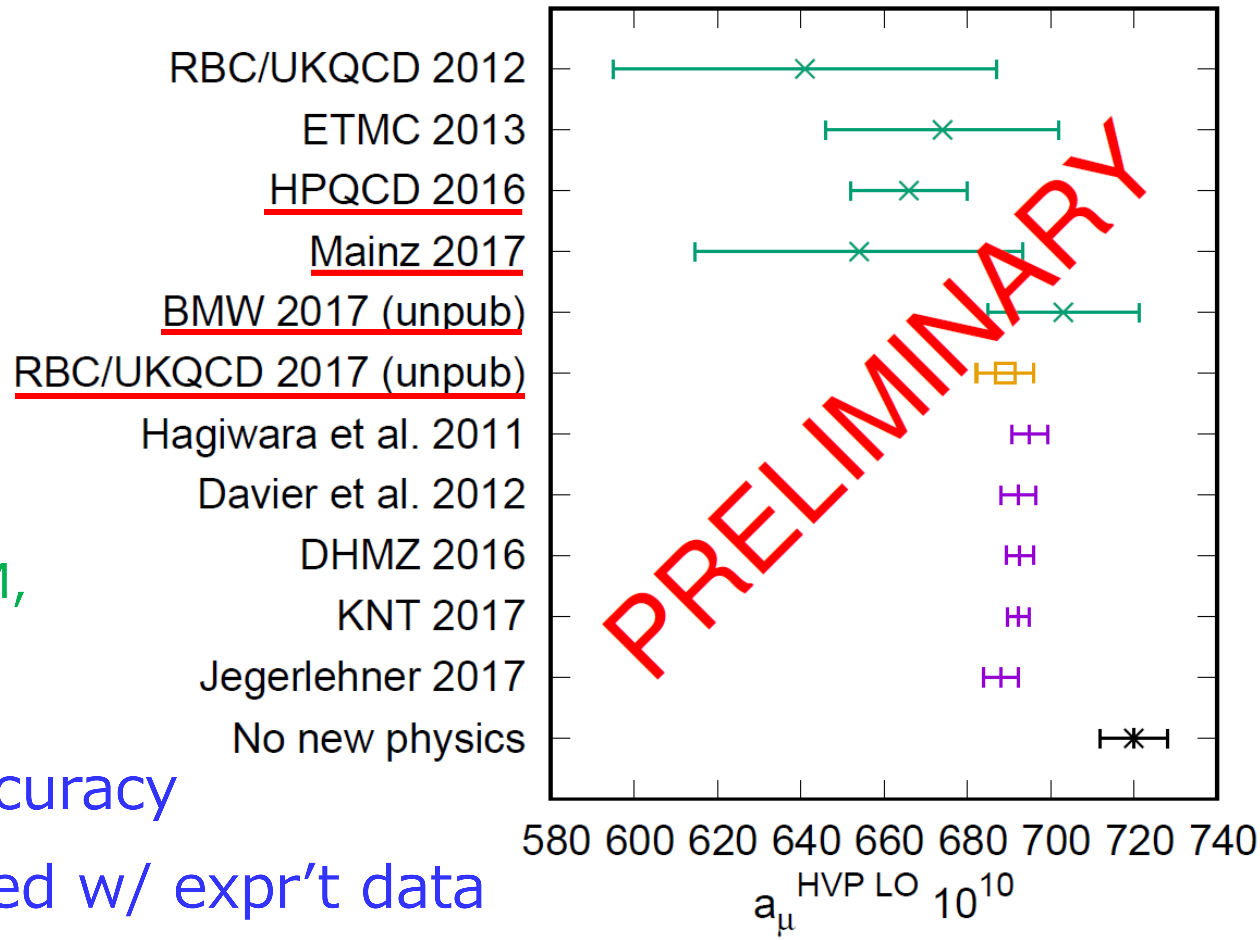
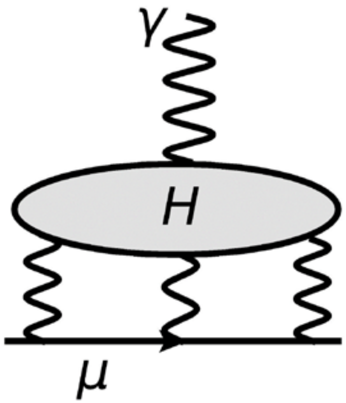


achieving 1 - 2% accuracy

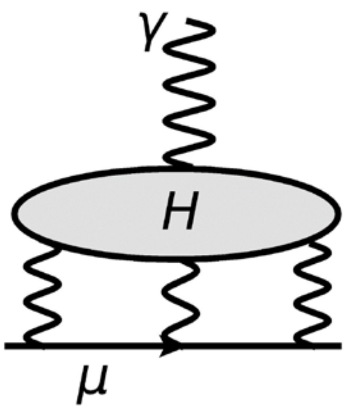
combined w/ expr't data

muon g-2 : LbL



- 4 point function
- model estimate $a_\mu^{\mathrm{LbL}}=10.5(2.6)\mathrm{x}10^{-10}$
- dominant contributions in QCD in progress

muon g-2 : LbL



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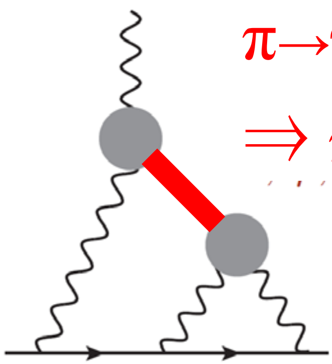
Gerardin et al. 1607.08174 New!

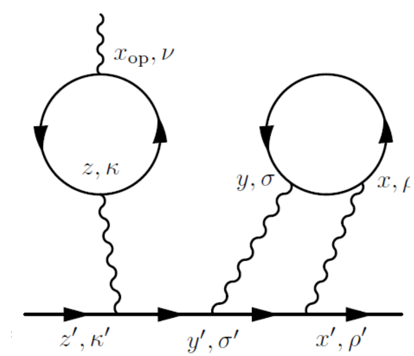


π→γ*γ* FF

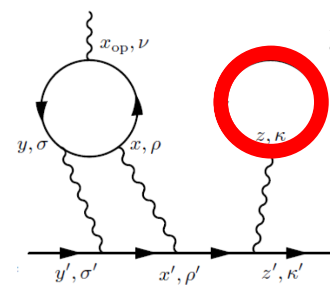
⇒ π pole

Knetcht-Nyfferer '02

RBC/UKQCD 1610.04603 New!



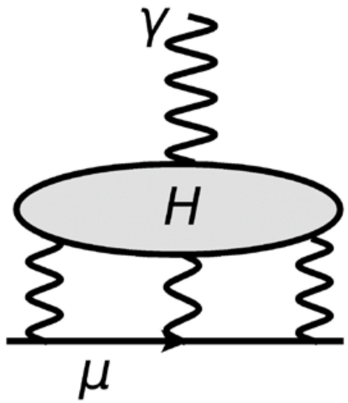
$O(m_s - m_{ud})$



u, d, s

…

muon g-2 : LbL



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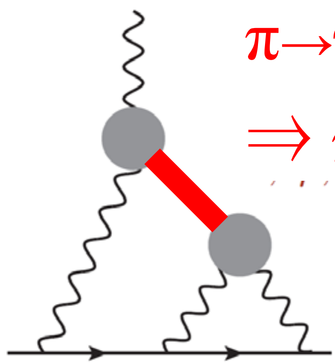

$\pi\to\gamma^*\gamma^*$ FF

$\Rightarrow$ π pole

Knetcht-Nyfferer ’02

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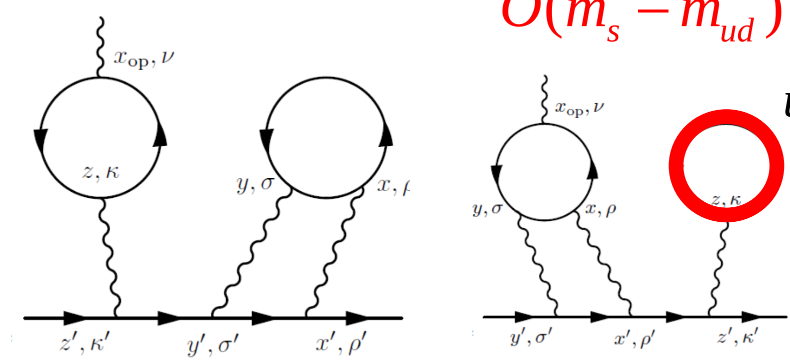


u, d, s

…

$$a_\mu^{\mathrm{LbL}} = 6.50(0.83)_{\mathrm{stat}} \times 10^{-10}$$

$$a_\mu^{\mathrm{LbL}} = 5.35(1.35)_{\mathrm{stat}} \times 10^{-10}$$

good consistency / systematically improvable

summary

- substantial progress for the search of new physics in collaboration with flavor factories (Belle II, LHCb, BESIII, ⋯)

- "gold-plated" quantities: calculated with fully controlled errors
  expecting more studies on semileptonic, rare decays, mixing

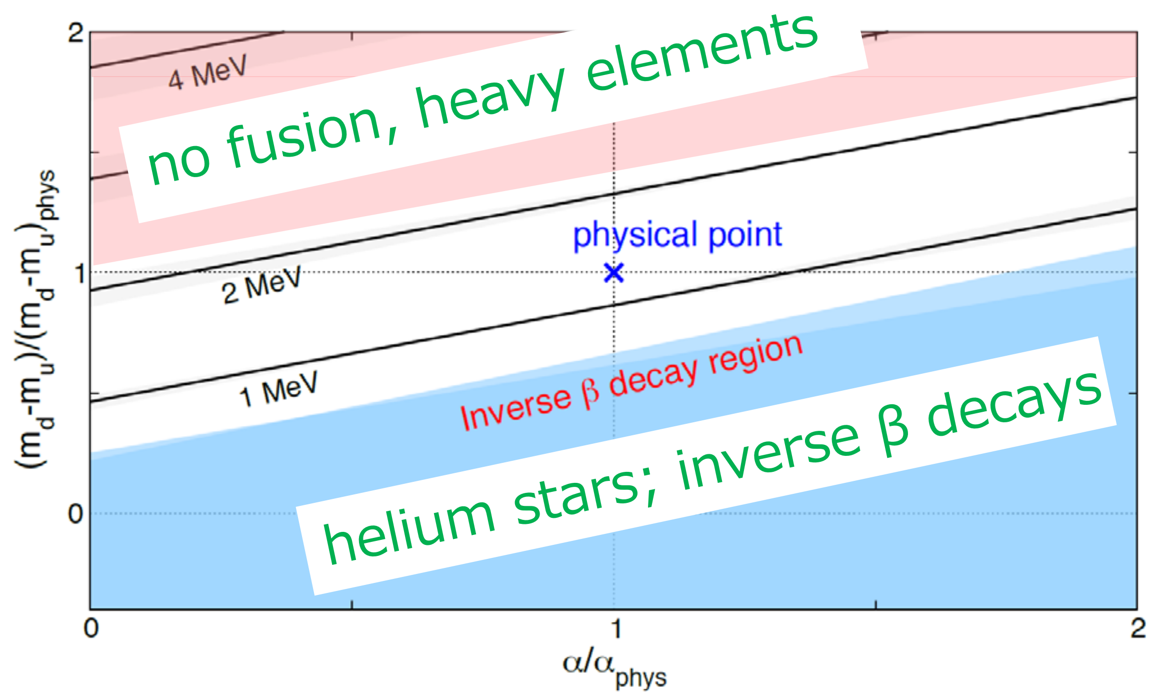
- continuous efforts for $K \to \pi\pi$
  framework under active development for D (and B)

- new ideas for inclusive decays
  more R&D both in theory and experiment sides

Backup slides

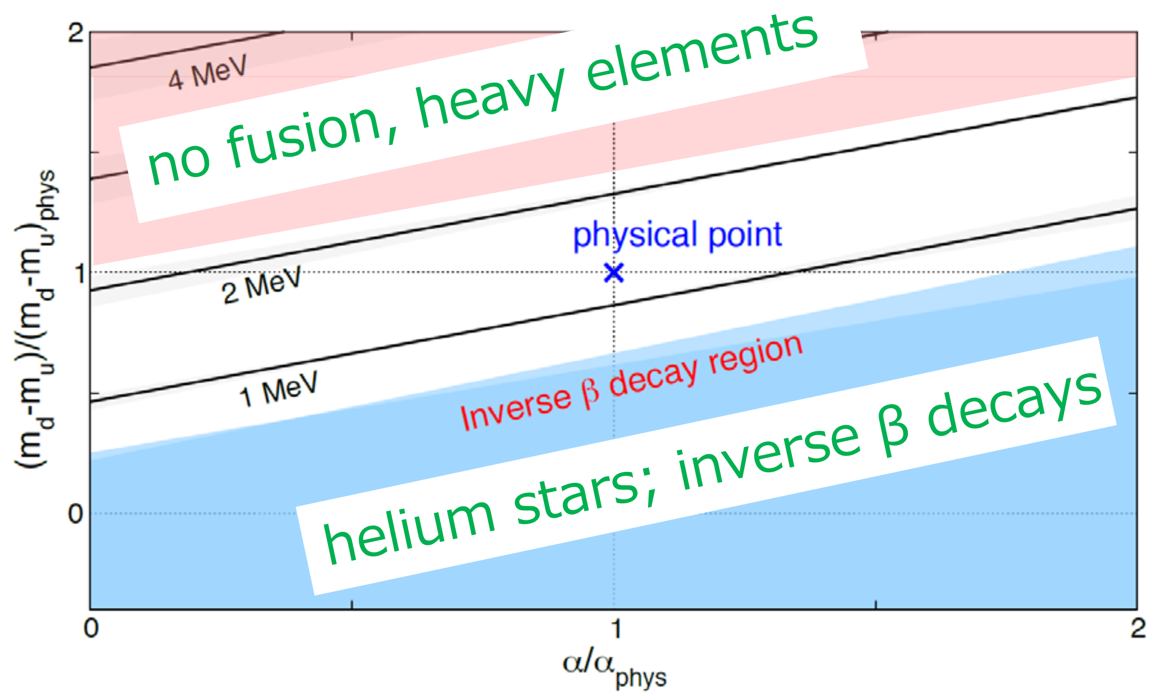
isospin splittings

- ‰ level needed



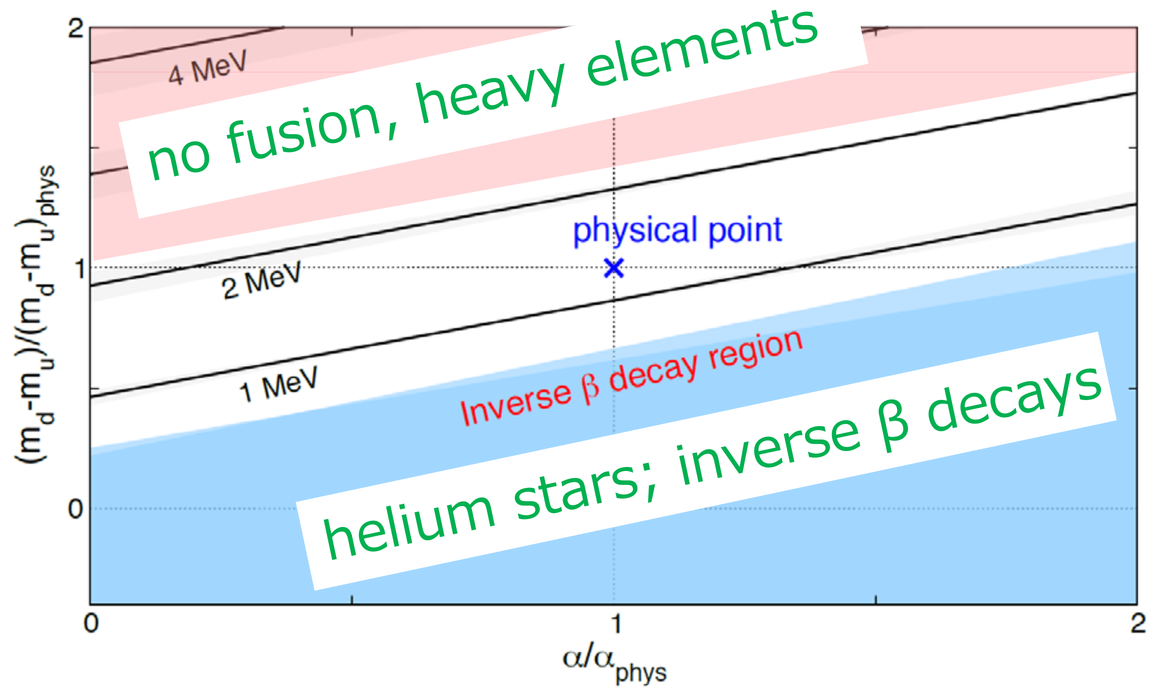
isospin splittings

- ‰ level needed
- $m_u \neq m_d$: straightforward



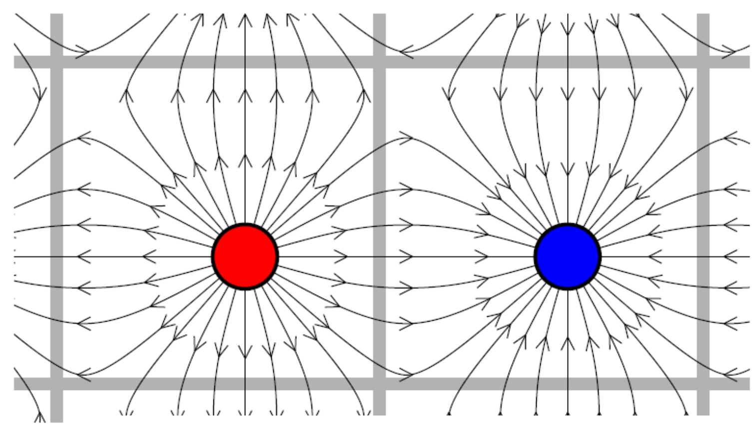
isospin splittings

- ‰ level needed
- $m_u \neq m_d$:
  straightforward



- EM corrections
  ⇔ QED on finite/periodic lattice
  - boundary condition (QED_C)
  - photon mass (QCD_m)
  - photon field ($QED_{\{TL,L,SF\}}$)

  ⇒ Patella @ Lattice’16

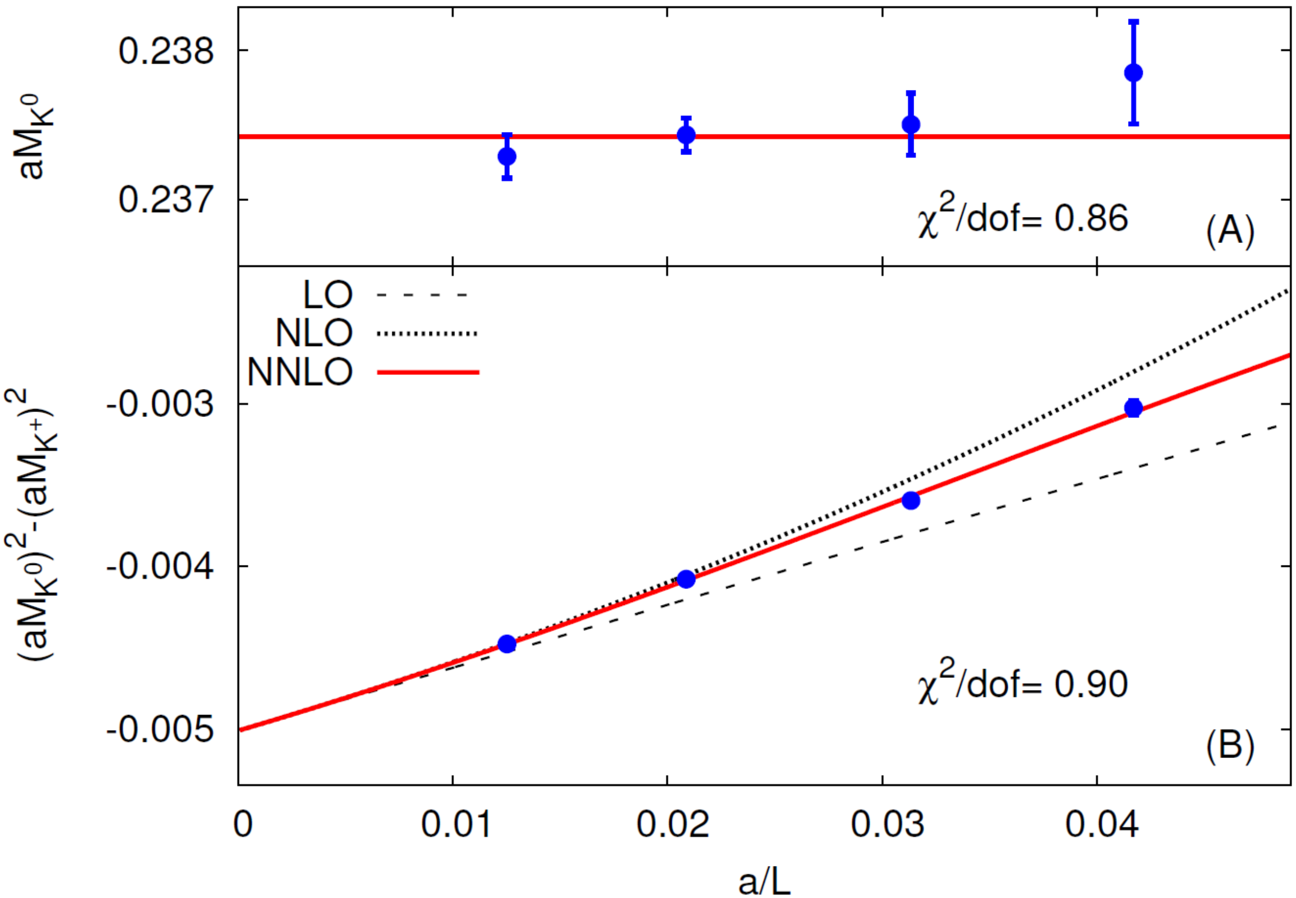


C boundary condition: Polley ‘93

EM correction

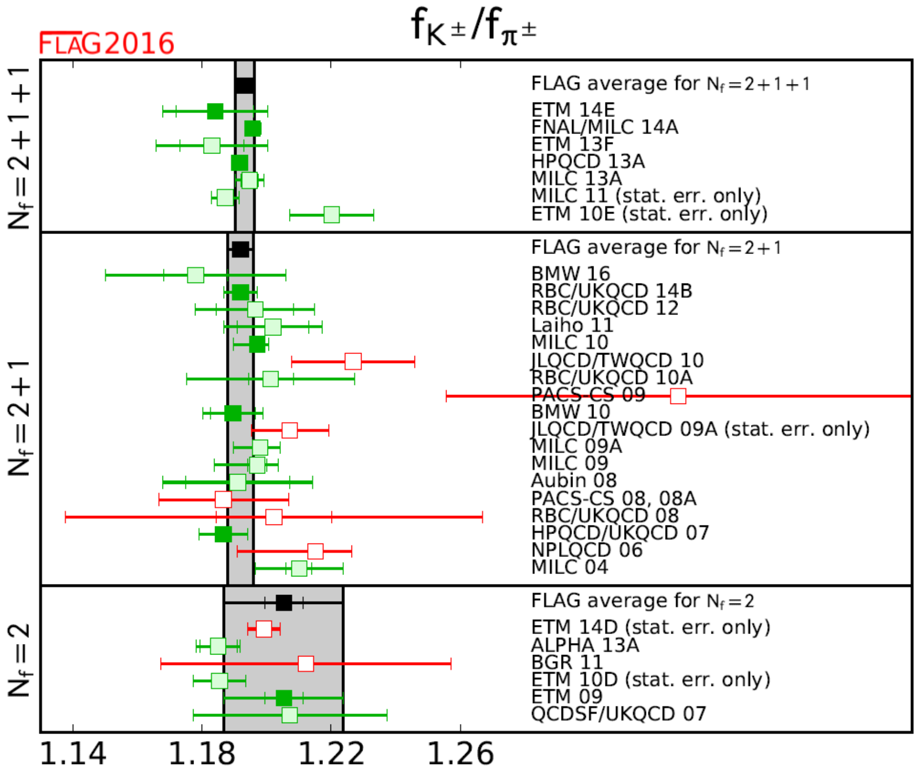
finite volume effects on M_{K0} and $M_{K0}{}^2$-$M_{K+}{}^2$

BMW, 1406.4088

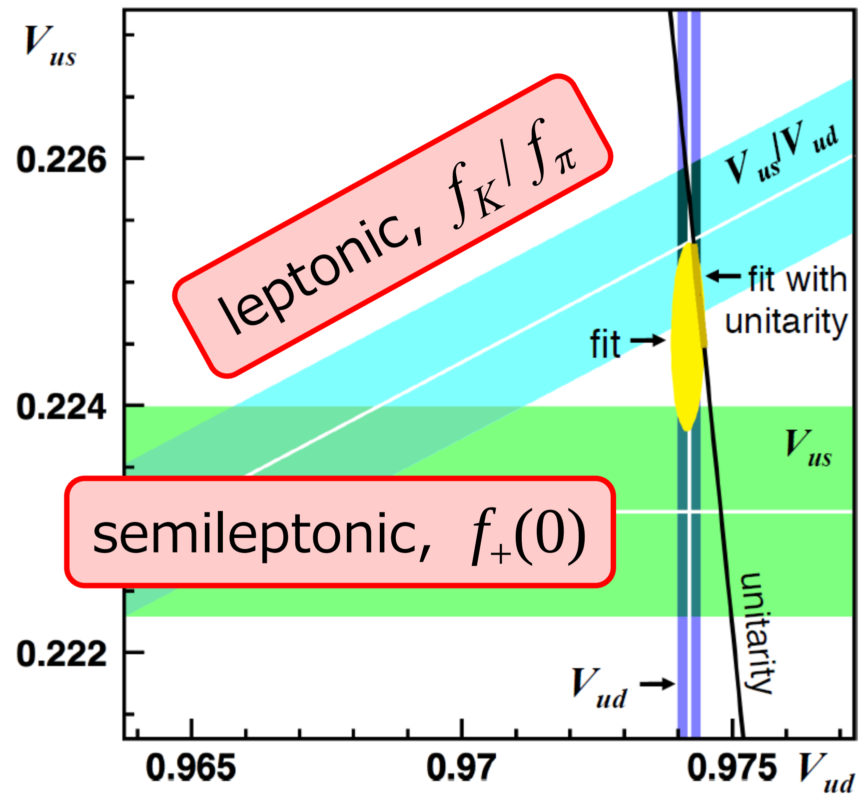


kaon (semi)leptonic decays

Flavor Lattice Averaging Group (FLAG) ‘16



Moulson @ CKM’16

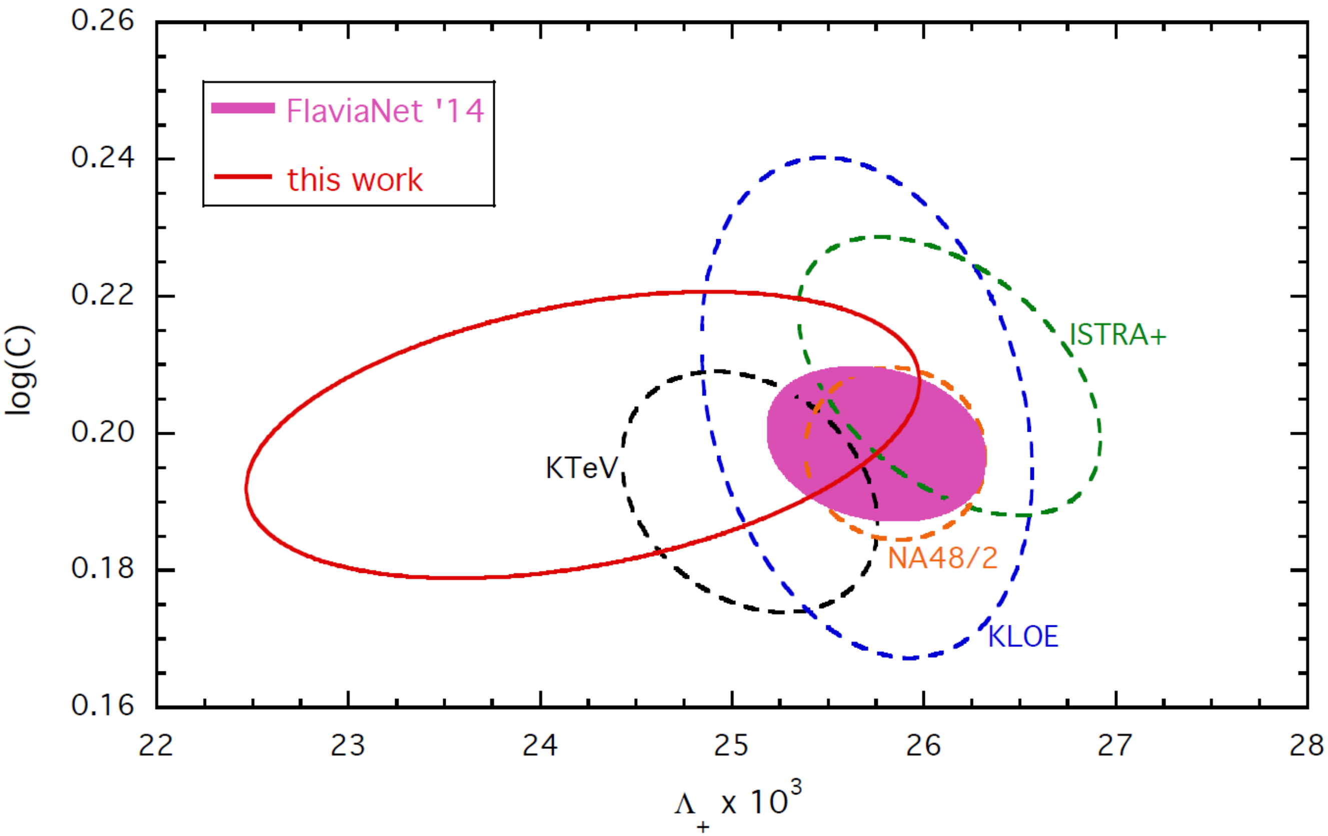


- precision frontier: ΔMEs ~0.3% ⇔ Δ(BR,τ,SU(2)) ~ 0.2-0.6%
- $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -5(5)\times10^{-4}$ ⇒ NP @ O(10) TeV
- FF shape (ETM’16, JLQCD’17) / isospin correction (next)

FF shape

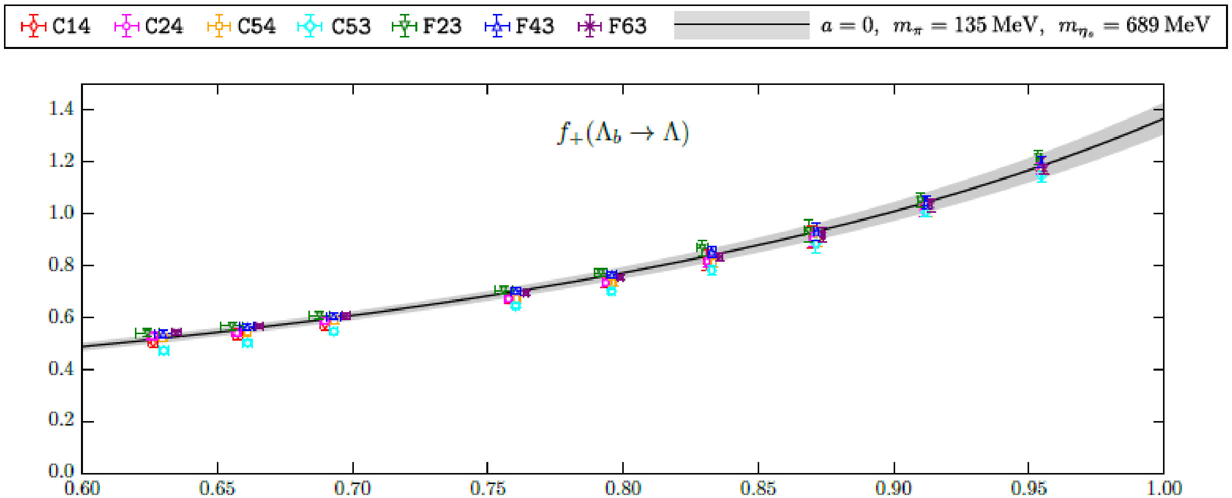
slopes of $K \to \pi l \nu$ FFs

ETM, 1602.04113



JLQCD, 1705.00884

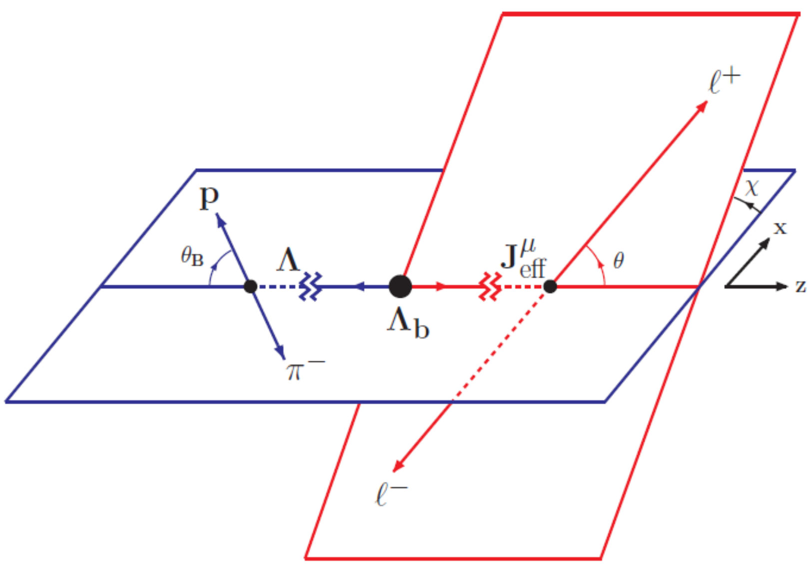
Λ_b decays

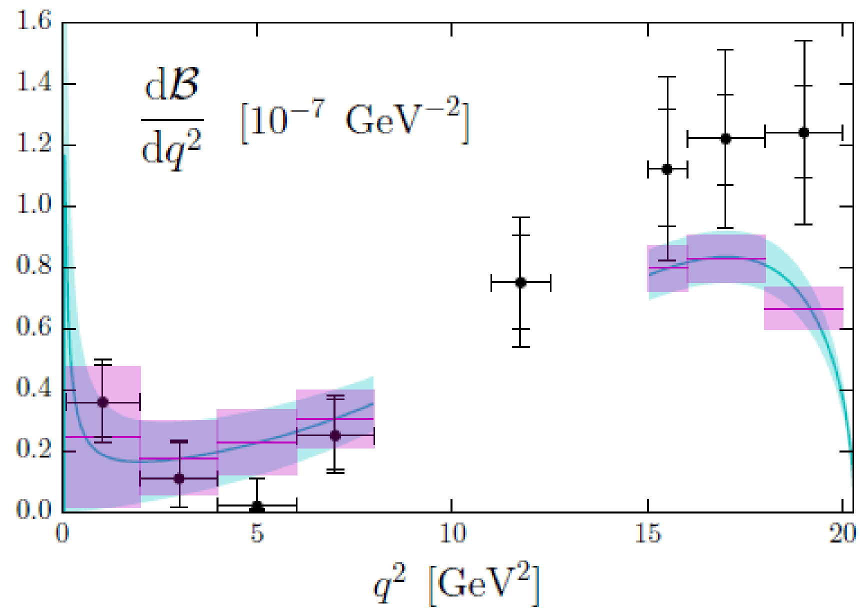


$f_{+,0,\perp}$

$g_{+,0,\perp}$

$h_{+,,\perp}$





inclusive decays

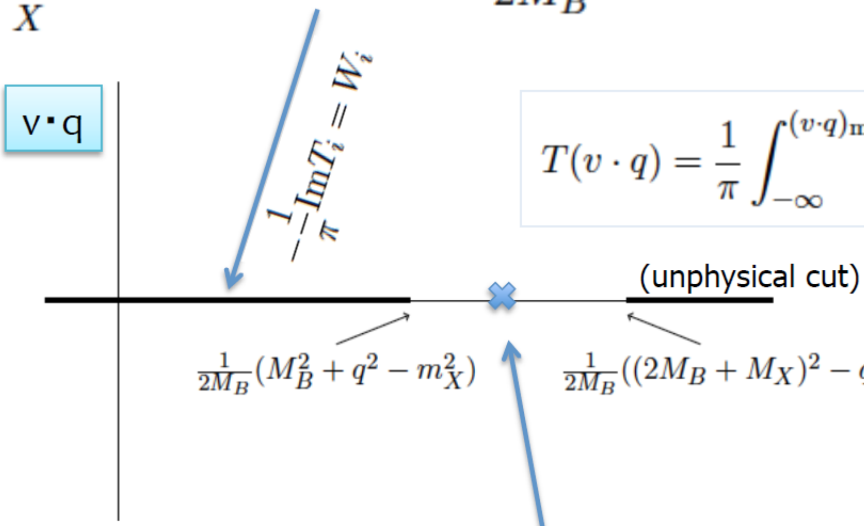
Hashimoto @ Lattice '17

Decay amplitude: $|\mathcal{M}|^2 = |V_{qQ}|^2 G_F^2 M_B l^{\mu\nu} W_{\mu\nu}$ (function of v·q and q^2)

Structure function:

$$W_{\mu\nu} = \sum_X (2\pi)^3 \delta^4(p_B - q - p_X) \frac{1}{2M_B} \langle B(p_B)|J_\mu^\dagger(0)|X\rangle\langle X|J_\nu(0)|B(p_B)\rangle$$

$$T(v\cdot q) = \frac{1}{\pi}\int_{-\infty}^{(v\cdot q)_{\max}} d(v\cdot q') \frac{\mathrm{Im}T(v\cdot q')}{v\cdot q' - v\cdot q}$$



calculable on the lattice
in the unphysical kinematical regime

expr't including $|V_{cb}|^2$

Matrix element:

$$T_{\mu\nu} = i\int d^4x\, e^{-iqx} \frac{1}{2M_B} \langle B|T\{J_\mu^\dagger(x) J_\nu(0)\} B\rangle$$