Toward quantum gravity

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What we understand so far

$10^{10}m$: General relativity

$1m$: Newton mechanics

$10^{-10}m$:Quantum theory

$$l_p = \sqrt{\frac{\hbar G}{c^3}}$$

$$\sim 10^{-35}m$$

: Both quantum effects and gravitational effects are important -> Quantum gravity

Having QG is difficult

However there are several serious problems in constructing quantum theory of Gravity

(1) Non renormalizability of GR: Need infinitely many counter terms.

Predictability of the theory is lost.

(2) Black hole information loss problem: Black holes evaporate. The final states are always thermal , no matter what the initial state is. It indicates that unitarity does not hold in QG apparently.

Problem sets

(1) How do we formulate quantum theory of gravity?

(2) What kind of observables can we think of in QG?

(3) How the formulation resolves the apparent break down of renormalizability and unitarity?

Various approaches

- String theory
- Holography or Gauge gravity duality
- Loop quantum gravity

+ others…

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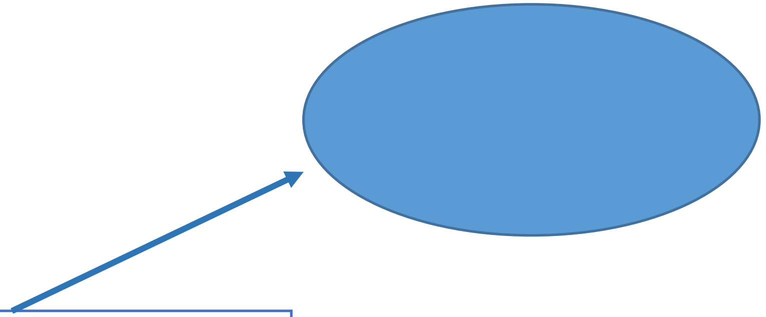
In this talk I will focus on this approach

+ others…

Holography



A quantum theory of gravity
in d+1 dimensional region

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A non gravitational QFT
living in the boundary of the region

Holographic principle states equivalence of these two theories.

Holography realizes a Quantum gravity theory in a highly non local way
Through the QFT living in the boundary of the region.

Holography

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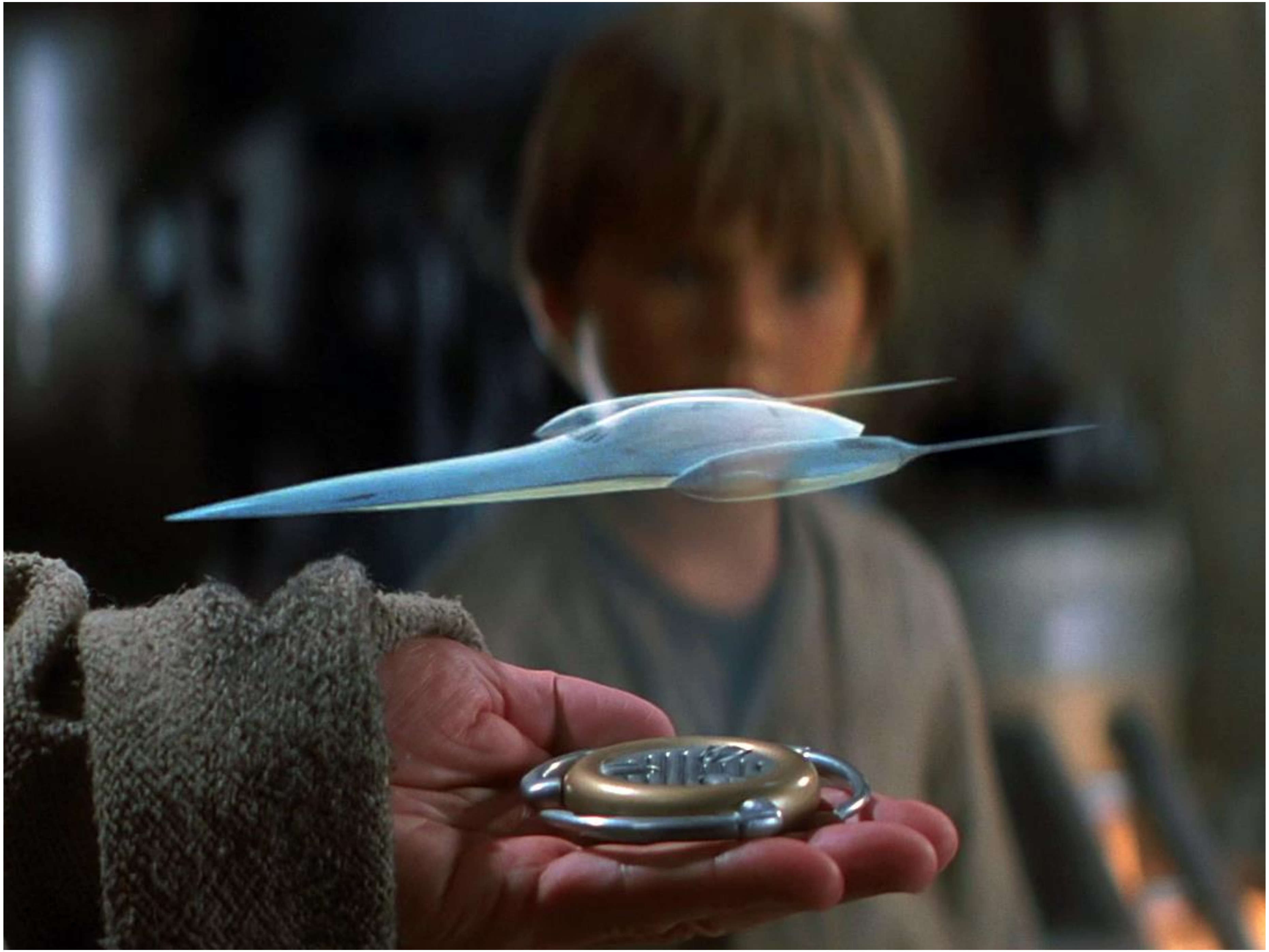
Holographic principle states equivalence of these two theories.

If this is true, holography realizes a Quantum gravity theory in a highly non local way through the QFT living in the boundary of the region.



Holography (2)

- This idea is inspired from physics of black hole. (Stationary) Black holes are thermodynamic objects which have their own temperature and entropy.

- The Bekenstein Hawking formula of black hole entropy states that entropy of a black hole is proportional to its area of the event horizon,

$$S_{BH} = \frac{A}{4G}$$

S

Holography (3)

- In usual non gravitational QFT, thermal entropy is proportional to the volume of the system. On the contrary to this, black hole entropy is proportional to the area of the event horizon of the black hole, which is the boundary of the spacetime.

- This suggests that d.o.f of the black hole are localized on the horizon.

- It had been difficult to make the statement explicit , but people found explicit examples (AdS /CFT corespondence) in mid 90's based on better understanding of non perturbative structure of string theory.

AdS/CFT correspondence [Maldacena]

Quantum gravity (especially string theory) in Anti de Sitter space (AdS) AdS_{d+1}

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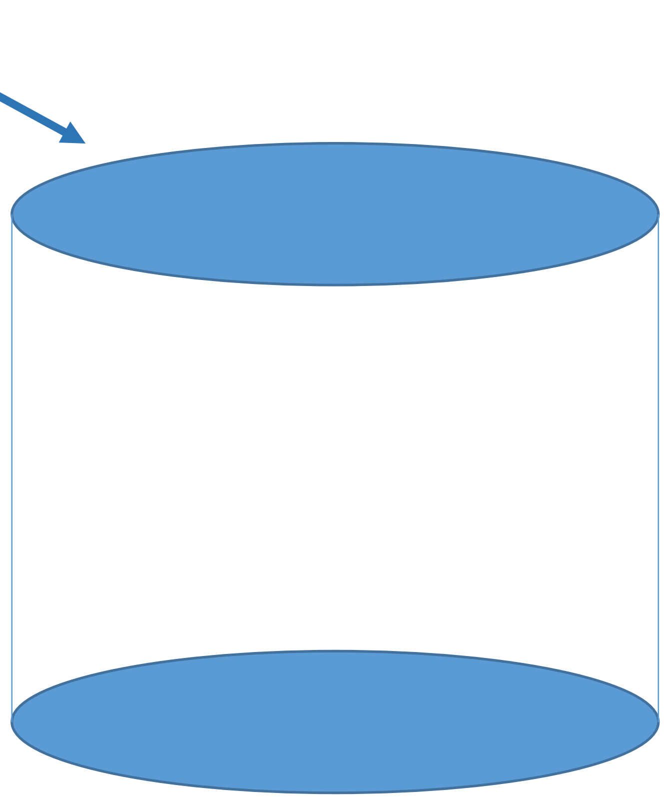
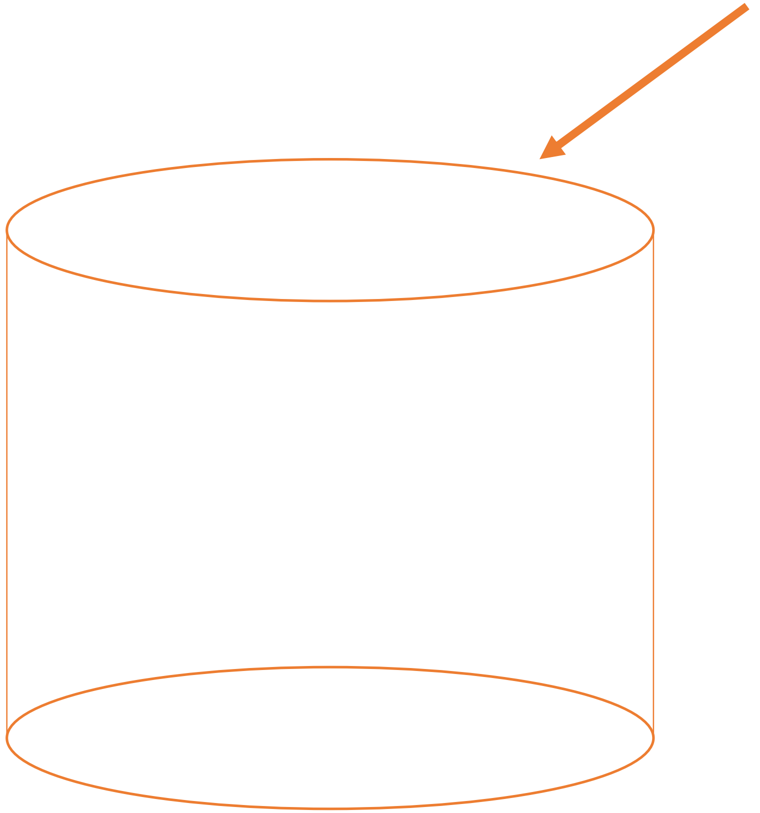
A Conformal field theory living on ∂AdS_{d+1}



AdS/CFT correspondence [Maldacena]

Quantum gravity (especially string theory) in Anti de Sitter space (AdS) AdS_{d+1} = A Conformal field theory living on ∂AdS_{d+1}



Examples

Type IIB string theory on $\mathrm{AdS}_5 \times S^5$ $\Longleftrightarrow$ N=4 SU(N) SYM in 4d

M theory on $AdS_4 \times S^7/Z_k$ $\Longleftrightarrow$ N=6 ABJM theory in 3d

These holographic relations are derived from two descriptions (open string picture and closed string picture)of extremal (BPS) black hole and by taking the near horizon limit. There are other holographic relations which are not string theory based, for example,

Vasiliev type higher spin gravity on AdS_4 Singlet sector of O(N) model

How to use holography

- One can consider two ways to use holography,   gravity <-> CFT

(1) Using the bulk gravity to understand the boundary CFT.

(2)Using the boundary CFT to understand the bulk (Quantum) gravity

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Using gravity to understand CFT

- Take the duality between type IIb string and N=4 SU(N) SYM for example.

- The t' Hooft limit of the SYM side corresponds to weakly coupled, semi classical limit (super gravity)of string theory.

This allows us to compute various quantities of strongly coupled large

N CFT (hard), from weakly coupled semi classical gravity (easy) through the holographic relation. This relation has been used to compute baryon masses and various transport coefficients, for example.

How to use holography

- One can consider two ways to use holography,   gravity <-> CFT

(1) Using gravity to understand CFT.

(2)Using CFT to understand quantum gravity

Using CFT to understand (Quantum) gravity

- Through the holographic relation, the boundary CFT can be regarded as a non perturbative definition of bulk quantum gravity.

- The boundary CFT realizes the bulk quantum gravity in the highly non local manner.

- The main question is how the bulk spacetime dynamic is emergent from the boundary CFT.

- Recent studies suggest that entanglement is the key to understand the question.

What is entanglement

- We say a quantum state is entangled when it is not a product of two states.

Singlet state is an example of entangled state. $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 |\uparrow\rangle_2\right)$

An essential ingredient of quantum computation.

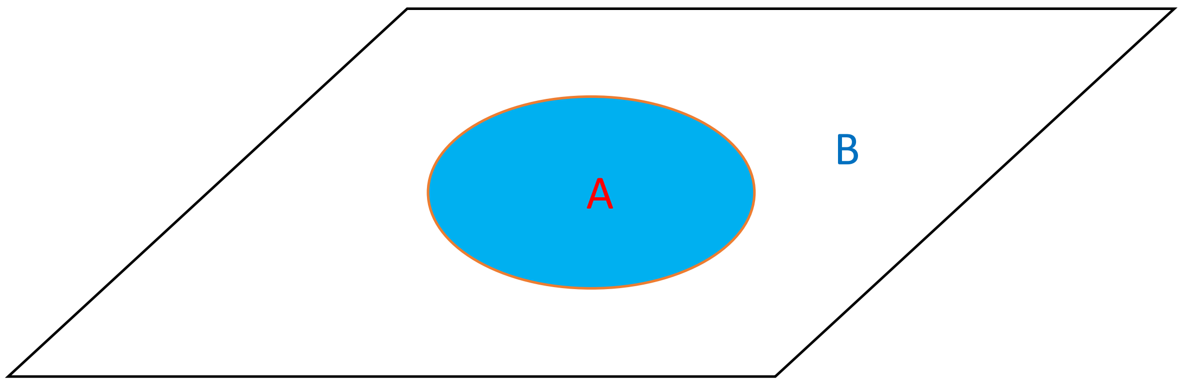
Entanglement entropy measures the amount of entanglement of the state.

$$\rho = \mathrm{tr}_1 |\psi\rangle\langle\psi| \qquad S_{EE}(\rho) = -\mathrm{tr}\rho \log \rho$$

Entanglement in quantum field theory

- There is a natural notion of entanglement in quantum field theory. Divide a time slice of the spacetime into two parts, the Hilbert space is also divided into two parts.

$$H_{tot} = H_A \times H_B$$



$$\rho_A = \mathrm{tr}_B |\psi\rangle\langle\psi| \qquad S_{EE} = -\mathrm{tr}\rho_A \log \rho_A$$

Entanglement in quantum field theory

In QFT vacuum has large amount of entanglement proportional to
to the area of entangling surface. (Rindler effect)

$$S_{EE}(\rho_{vac}) = \frac{\mathrm{A}_\Sigma}{\epsilon^{d-2}} + \cdots$$

Entanglement entropy of slightly excited states $\rho = \rho_{vac} + \delta\rho$ can be computed perturbatively

$$S_{EE}(\rho) = S_{EE}(\rho_{vac}) + S^{(1)}(\delta\rho) + S^{(2)}(\delta\rho) + \cdots$$

Entanglement first law in CFT

- The variation of entanglement entropy at first order is given by

$$S^{(1)}(\delta\rho) = \mathrm{tr}\left[K\delta\rho\right] \qquad\qquad K = -\log\rho_{vac}$$

K is called modular Hamiltonian of the vacuum which is the generator of the boost with respect to the region.

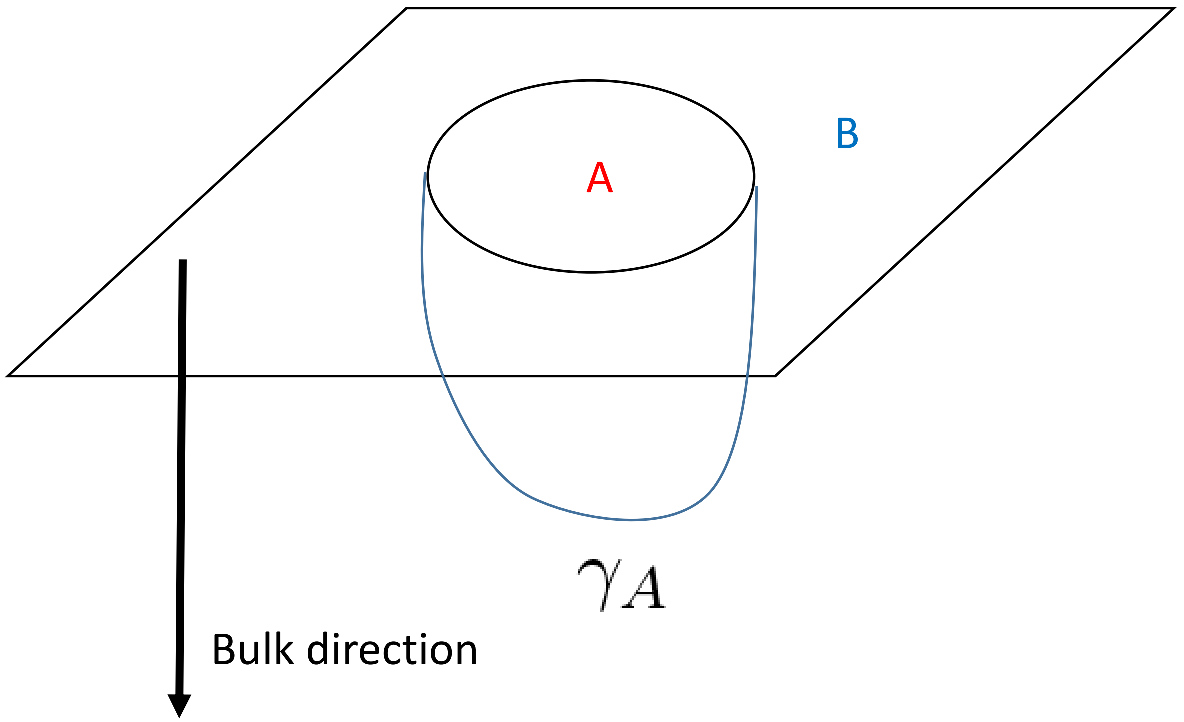
In CFT, when the region is spherical , the modular Hamiltonian has the simple form

$$K = \int_0^R dr\frac{R^2 - r^2}{2R}T_{00}(r)$$

Entanglement entropy in holography

There is a simple holographic dictionary that enable us to calculate CFT entanglement entropy from the area of bulk minimal surface γ_A

[Ryu Takayanagi]



$$S_{EE} = \frac{\text{Area}(\gamma_A)}{4G}$$

Entanglement first law and linearized Einstein eq

It can be shown that entanglement first law relation in the CFT implies the linearized Einstein equation is satisfied in the Bulk.

$$S^{(1)}(\delta\rho) = \mathrm{tr}\left[K\delta\rho\right]$$ In the CFT $\Rightarrow$ $$\Box h_{\mu\nu} = 0$$ In the bulk

This suggests that entanglement structure of states in the boundary CFT determines the dynamics of bulk gravity.

Entanglement at second order and Non linear dynamics

[Sarosi, T.U]

One can also calculate the second order change of entanglement Entropy. The contribution of each operator O to is given by an integral of 4 point block.

$$S^{(2)} = C^2_{VVO} \int \frac{ds}{\cosh^2 \frac{s}{2}} \mathcal{F}_O(s)$$

One can rewrite it into the bulk language. Namely this is equivalent to the integral of bulk stress tensor (canonical energy)

$$S^{(2)} = \int d\Sigma^{\mu} \xi^{\nu} T_{\mu\nu}(\phi)$$

where is the bulk field dual to O, and ξ^{μ} is the bulk boost generator. Combine this with the first law result we get linearized Einstein equations with matter stress tensor,

$$\Box h_{\mu\nu} = T_{\mu\nu}$$

Entanglement at second order and Non linear dynamics

- Take O to be the CFT stress tensor, we get graviton contribution to The canonical energy, and this is nothing but first non linear part of Einstein equations.

One can also systematically derive the bulk quantum corrections From the CFT analysis.

Conclusions

- Toward quantum gravity from holography
- AdS/CFT correspondence provides concrete realization of holography
- In the AdS/CFT framework entanglement plays a key role for reconstructing bulk dynamics.

Thank you !