PHIPSI 09

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"Isospin Breaking corrections to di-pion tau lepton decays"

G. López Castro (Cinvestav, México)

Collabs: * A. Flores, F. Flores, G. Toledo;
* M. Davier, A. Höcker, B. Malaescu, X. Mo,
P. Wang, C. Yuan, Z. Zhang

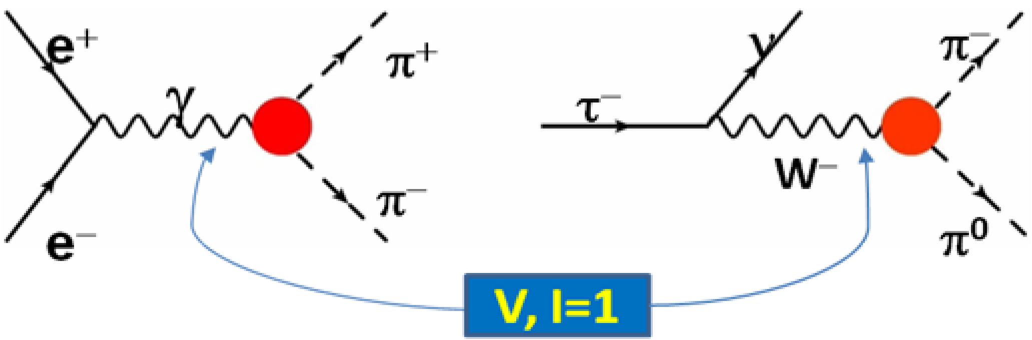
Motivations:

- 2π spectral function: $\Rightarrow$ 73% of $a_\mu^{had,LO}$, 83% of its error
- Very precise measurements (below 1% level) of the 2π spectral function and BR in τ decays.
- However, "at present, not reliable to predict $(g\text{-}2)_\mu$ because isospin breaking (IB) is not well understood" (e^+e^- vs τ discrepancy). $\Rightarrow$ Missing IB corrections?, errors in data?, both?

Some (not very) recent results on IB corrections relevant to $(g\text{-}2)_\mu$

CVC hypothesis (Isospin limit)

Replacing e+e- by τ data



$$F_0(s) = F_-(s)$$

$$\upsilon_i(s) = \frac{\beta_i^3(s)}{12}\left|F_i(s)\right|^2$$

$$\sigma(e^+e^- \to [\pi^+\pi^-]_{I=1}) = \frac{4\pi\alpha^2}{s}\upsilon(\tau^- \to \pi^-\pi^0\nu)$$

$$\upsilon(\tau^- \to \pi^-\pi^0\nu) = \frac{1}{6\left|V_{ud}\right|^2}\frac{B_{\pi^-\pi^0}}{B_e}\frac{m_\tau^8}{\left(m_\tau^2 - s\right)^2\left(m_\tau^2 + 2s\right)}\frac{1}{N_{\pi^-\pi^0}}\frac{dN_{\pi^-\pi^0}}{ds}$$

τ Branching ratios

2π mass spectrum

SOURCES OF ISOSPIN BREAKING

At fundamental level

$$\frac{1}{2}(m_u - m_d)\bar{q}\tau_3 q + \cdots, \qquad \frac{e}{2}(Q_u - Q_d)\bar{q}\gamma_\mu \tau_3 q + \cdots$$

At hadronic level: correct τ data by removing IB effects

in the isospin limit =1

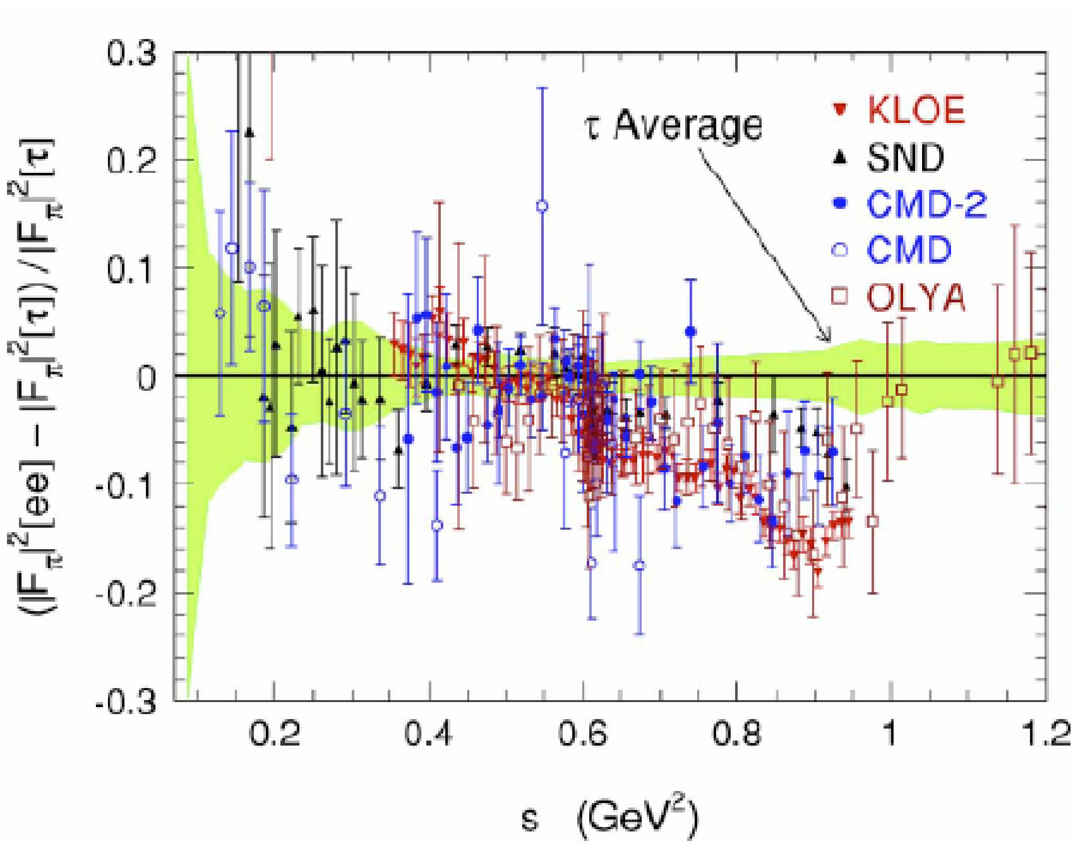
$$\upsilon(\tau^- \to \pi^- \pi^0 \nu(\gamma)) \cdot \frac{1}{S_{EW}} \frac{FSR(s)}{G_{EM}(s)} \left(\frac{\beta_0(s)}{\beta_-(s)} \right)^3 \left| \frac{F_0(s)}{F_-(s)} \right|^2$$

Radiative corrections inclusive γ

$\pi^\pm$-π^0 mass diff.

Leading IB: $\rho^\pm$-ρ^0 mass & width diff, ρ-ω mixing

Test of CVC in 2π production in e^+e^- and τ data



OLD analysis (DEHZ03)

IB effects from:
Alemany, Davier & Hocker (1998);

Radcorr from :
Cirigliano, Ecker & Neufeld (2002)

ee vs τ discrepancy

Important differences above the ρ mass region ⇒ two non-overlaping predictions for $a_\mu^{had, LO}$

Some recent progress on IB include:

- New evaluation of LD radcorr in $\tau \to \pi\pi\nu$: $G_{EM}(s)$
- New calculation of the $\rho^{\pm}-\rho^{0}$ width difference: $|F_0(s)/F_{-}(s)|^2$

PRD 74, (2006);
NPB(PS) 169, (2007);
PRD 76, (2007);
arXiv;0906.5443 [hep-ph]

SHORT-DISTANCE CORRECIONS

Marciano & Sirlin (78-88);
Braaten & Li, (1990);
J. Erler, Rev. Mex. Fis (2004)

Resummed $O[\alpha^n \ln^n(m_Z)]$
$O[\alpha\alpha_s{}^n \ln^n(mZ))]$ terms

$$S_{EW} = 1.0235 \pm 0.0003$$

$O(\alpha\alpha_s)$

LONG-DISTANCE CORRECTIONS

Main difference in radiative $\tau \to \pi\pi\nu\gamma$ decays

Cirigliano, Ecker & Neufeld , JHEP, (2002);
Flores, Flores, GLC, Toledo , PRD, (2006)

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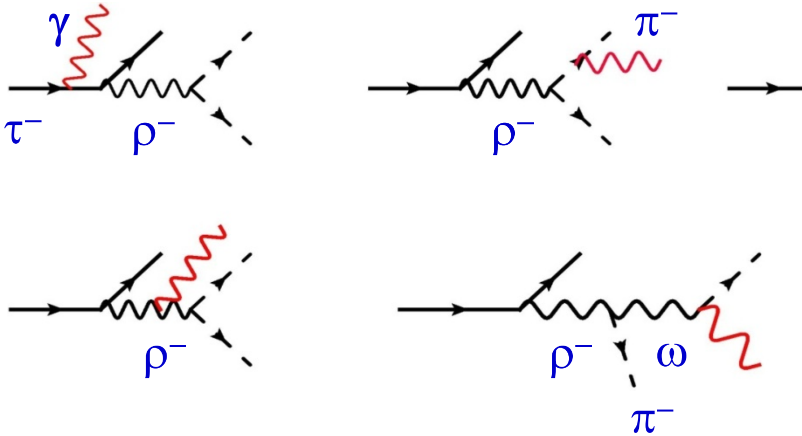

+ others m-d terms

+ virtual corrections

$$\frac{d\Gamma^0}{ds} \Rightarrow \frac{d\Gamma^0}{ds} \times G_{EM}(s)$$

$$s = (p_{\pi^+} + p_{\pi^0})^2$$

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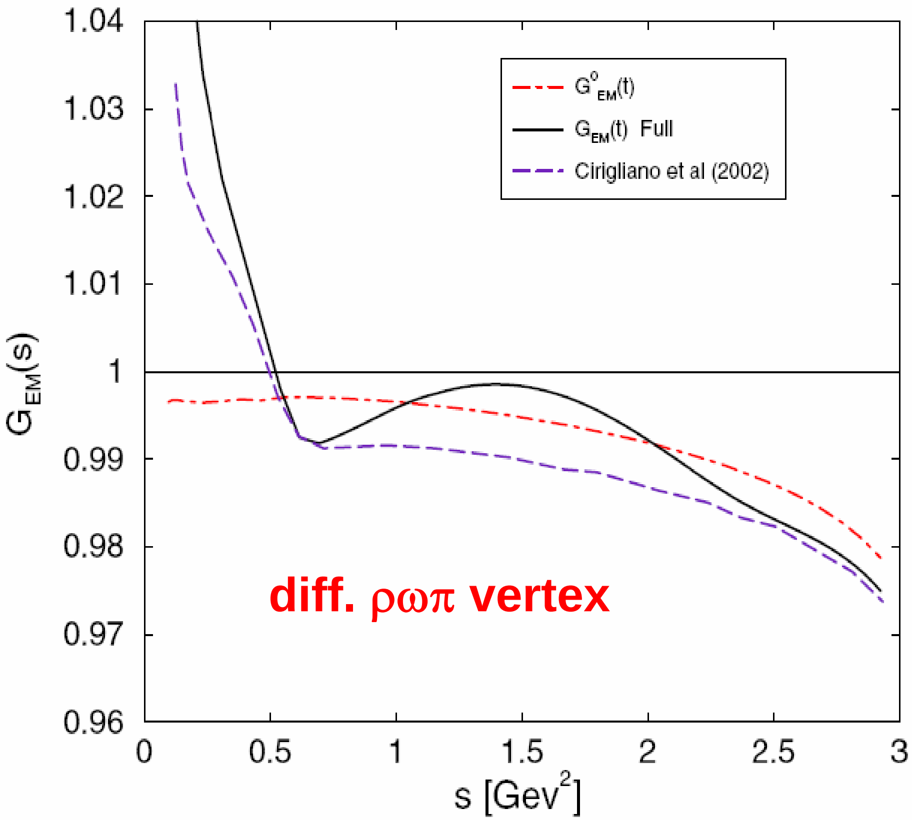
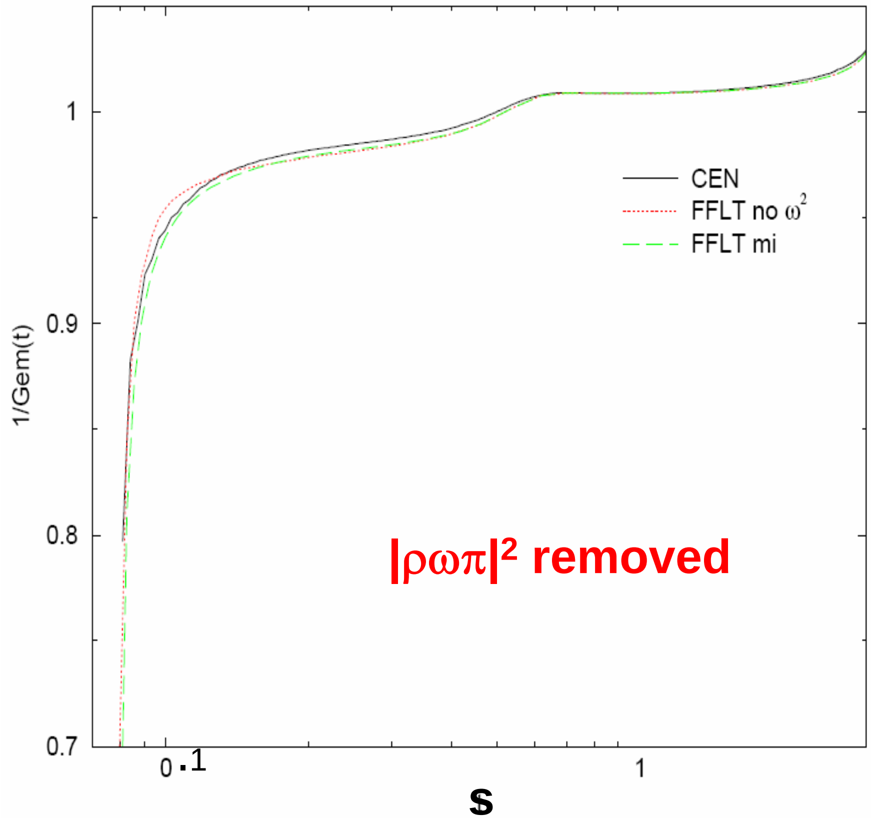




Effect on $\Delta a_\mu^{had,\,LO}(\tau)$:

-1.9×10^{-10} FFLT

-1.0×10^{-10} CEN

Appropriate when $\tau \to \nu\pi^-\omega(\to\pi^0\gamma)$ events are removed, as done by most experiments

arXiv:0906.5443 [hep-ph]

ISOSPIN BREAKING IN FORM FACTORS

$$F_0(s) = f_{\rho^0}(s)\left[1 + \delta_{\rho\omega} \cdot \frac{s}{m_\omega^2 - s - i m_\omega \Gamma_\omega}\right]$$

$$F_-(s) = f_{\rho^-}(s)$$

Leading IB effect:
Δm_ρ, $\Delta\Gamma_\rho$, $\delta_{\rho\omega}$

M. Davier et al, arXiv:0906.5443

- IB in $\rho^\pm - \rho^0$ mass (KLOE mass diff+ em mass shift for ρ^0):

$m(\rho^\pm) - m(\rho^0) = (1.0 \pm 0.9)$ MeV

- rho-omega mixing (from fit to e^+e^- data):

$$\delta_{\rho\omega} = \begin{cases} (2.00 \pm 0.06)\times 10^{-3}\, e^{i(11.6 \pm 1.8)^o}, & \text{for GS} \\ (1.87 \pm 0.06)\times 10^{-3}\, e^{i(13.2 \pm 1.7)^o}, & \text{for KS} \end{cases}$$

GS=Gounaris-Sakurai (1968) ; KS= Kühn-Santamaría (1990)

IB IN ρ WIDTH DIFFERENCE

$\Delta\Gamma_\rho = \Gamma[\rho^\pm \to \pi^\pm\pi^0 (\gamma)] - \Gamma[\rho^0 \to \pi^+\pi^-(\gamma)]$

$- 0.08$ **MeV** $(\pi\gamma, \eta\gamma, l^+l^-, ..)$

Flores, GLC, Toledo PRD 76,(2007)

- Photon inclusive rates calculated with virtual + real photons

$$\Delta\Gamma[\pi\pi(\gamma)] = \frac{g^2_{\rho\pi\pi}\sqrt{s}}{48\pi}\left[\beta_-^3(s)(1+\delta_-) - \beta_0^3(s)(1+\delta_0)\right]$$

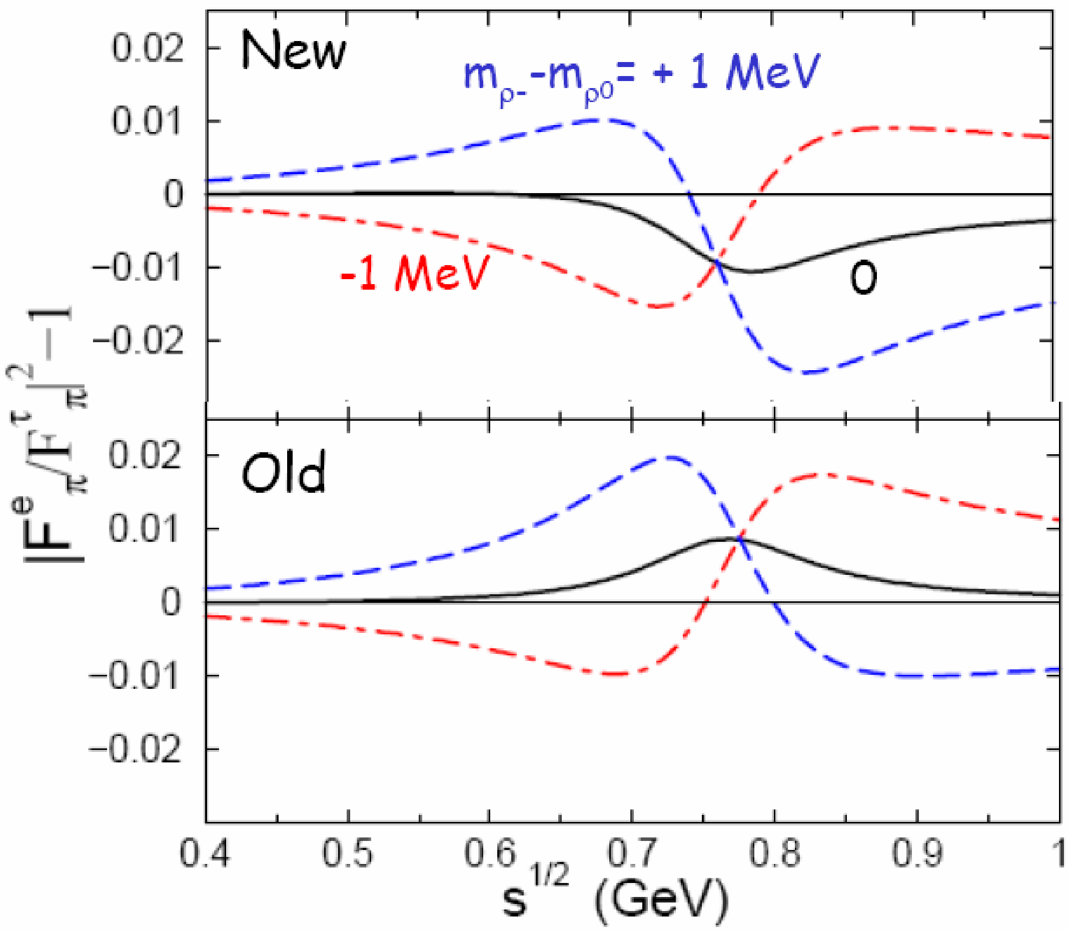
$$= (-0.76 \pm 0.08)\ MeV, \quad at\ \sqrt{s} = m_\rho$$

- Comparison with previous results which only included effects of hard photons but neglect rad. corrections [**P.Singer, PR (1963)**]

$$\Delta\Gamma[\pi\pi(\gamma)] = (0.49 \pm 0.58)\ MeV$$

Alemany, Davier, Hocker (1998)

Effects of $\rho^{\pm}$–ρ^{0} mass & width difference in the ratio of I=1 components of pion form factors



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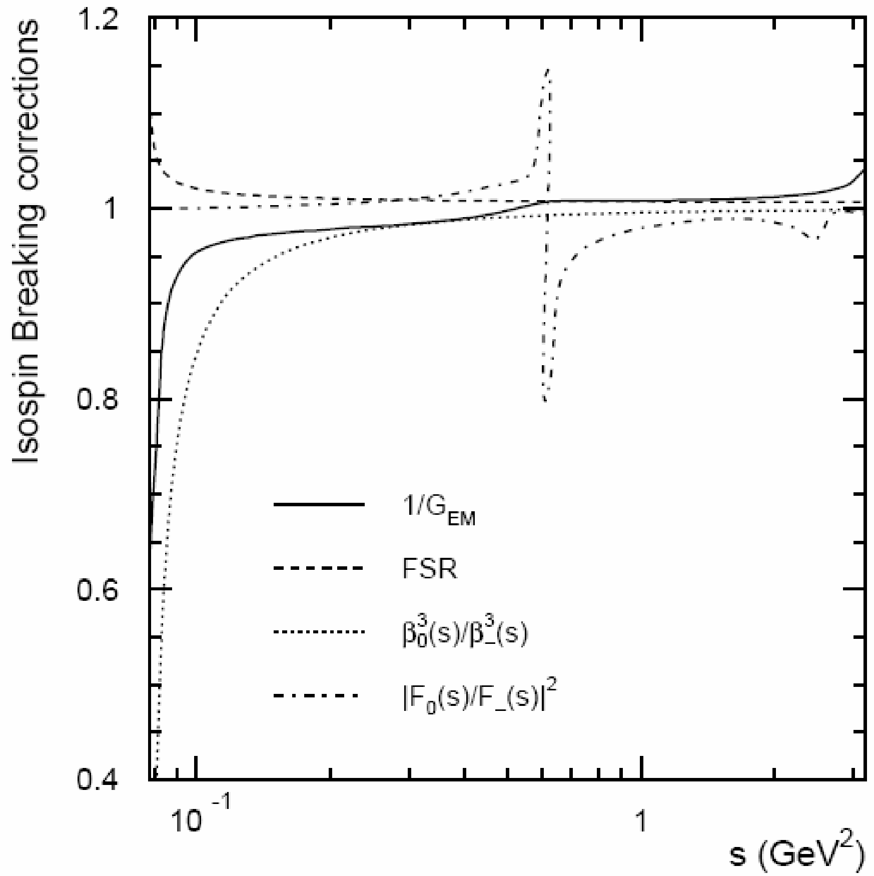
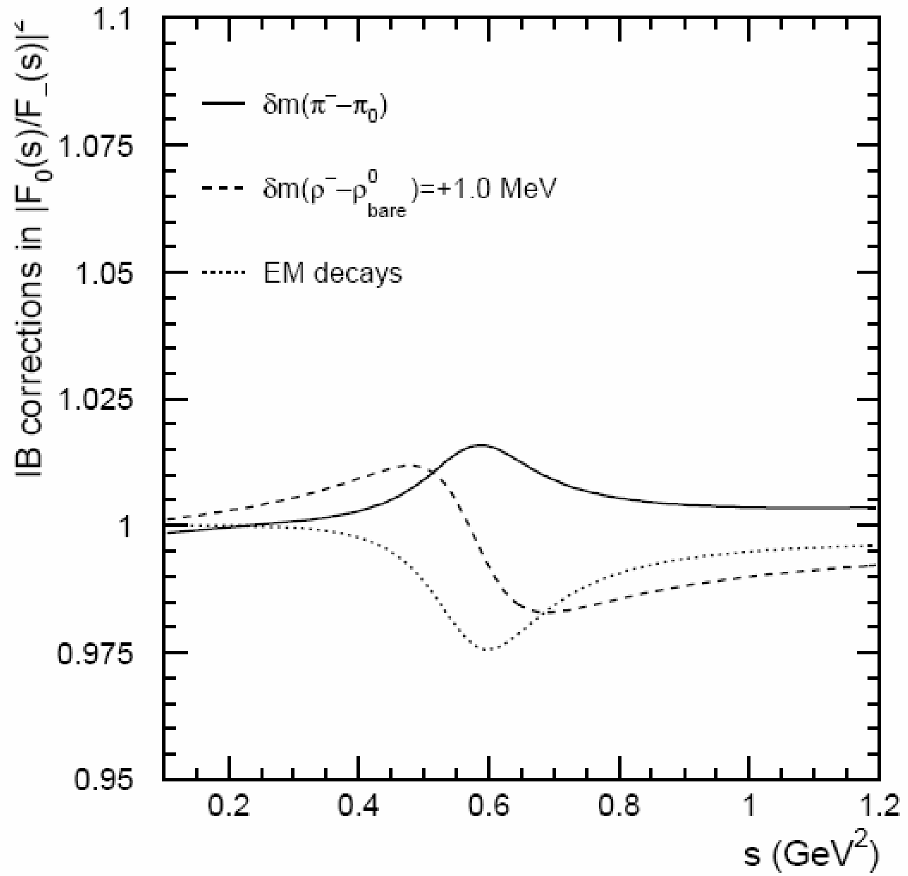


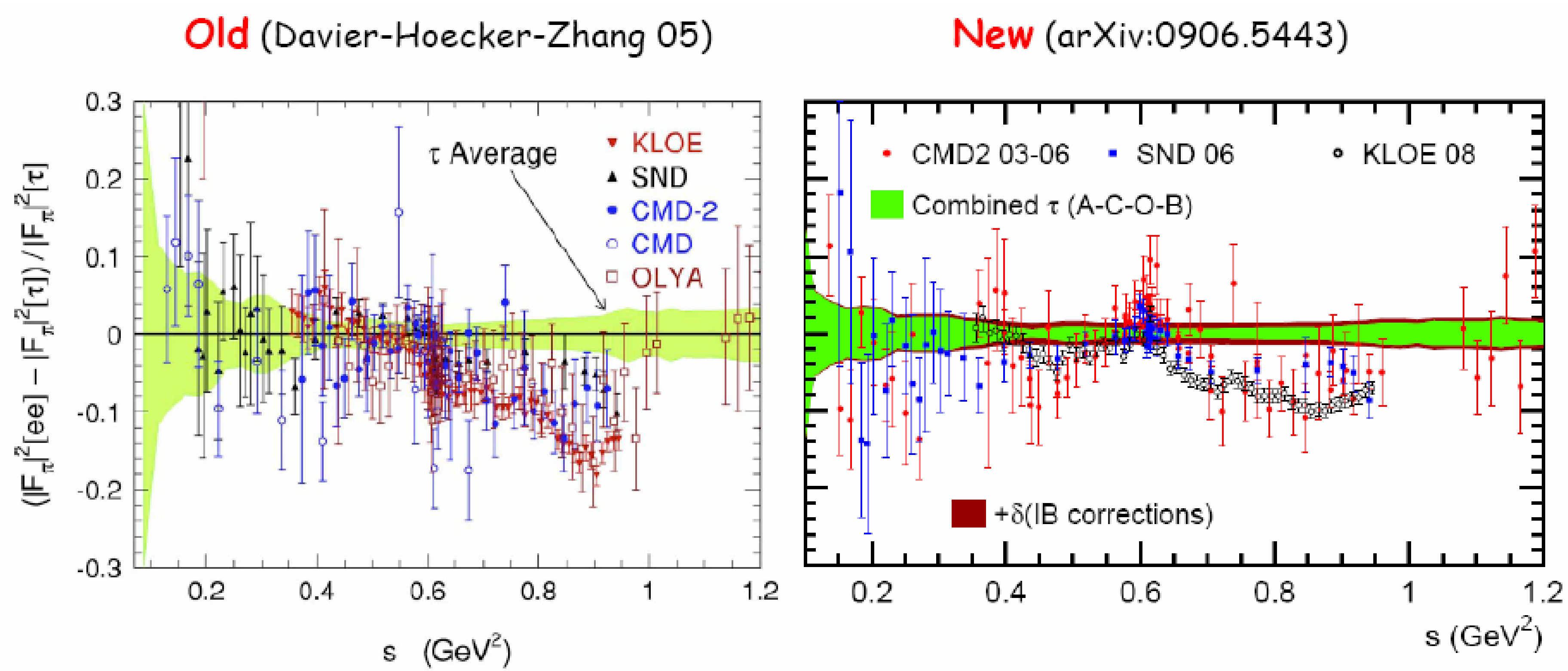


s-dependent IB corrections

IB in the ratio of I=1 form factors

Comparison of ee vs. IB corrected τ data

Z. Zhang, EPS 2009



Relative normalization and shape in better agreement than before (**Davier-Eidelman-Hocker-Zhang 2003**).
Small deviations remain above m_ρ^2 (large deviations for KLOE data)

IB Corrections in $\Delta a_\mu^{had,\ LO}(\tau)$

Davier et al arXiv:0906.5443 [hep-ph]

Source	$\Delta a_\mu^{had,LO}[\pi\pi,\tau]$ (10^{-10}) GS model	KS model	OLD DEHZ(03)
S_{EW}	-12.21 ± 0.15		-12.1 ± 0.3
G_{EM}	-1.92 ± 0.90		-1.0
FSR	$+4.67 \pm 0.47$		------
ρ–ω interference	$+2.80 \pm 0.19$	$+2.80 \pm 0.15$	$+3.5 \pm 0.6$
$m_{\pi^\pm} - m_{\pi^0}$ effect on σ	-7.88		-7.0
$m_{\pi^\pm} - m_{\pi^0}$ effect on Γ_ρ	$+4.09$	$+4.02$	$+4.2$
$m_{\rho^\pm} - m_{\rho^0_{bare}}$	$0.20^{+0.27}_{-0.19}$	$0.11^{+0.19}_{-0.11}$	0.0 ± 2.0
$\pi\pi\gamma$, electrom. decays	-5.91 ± 0.59	-6.39 ± 0.64	-1.4 ± 1.2
Total	-16.07 ± 1.22	-16.70 ± 1.23	-13.8 ± 2.4
	-16.07 ± 1.85		

Adding FSR to DEHZ (03), net change becomes - 6.94 × 10^{-10}

$\Rightarrow$ Closer predictions for $a_\mu^{had,LO}$, from e+e- and τ data

BR prediction based on IB corrected e+e– data

$$\Delta B^{CVC}(\tau \to \pi\pi\nu) = \frac{3B_e|V_{ud}|^2}{2\pi\alpha^2 m_\tau^2} \int_{s_{\min}}^{m_\tau^2} ds\, \sigma^0_{\pi^+\pi^-}(s) \left(1 - \frac{s}{m_\tau^2}\right)^2 \left(1 + \frac{2s}{m_\tau^2}\right)$$

$$\times \left[\frac{S_{EW} G_{EM}(s)}{FSR} \frac{\beta_-^3}{\beta_0^3} \left| \frac{F_-(s)}{F_0(s)} \right|^2 - 1 \right]$$

$$= (+0.69 \pm 0.22)\%$$

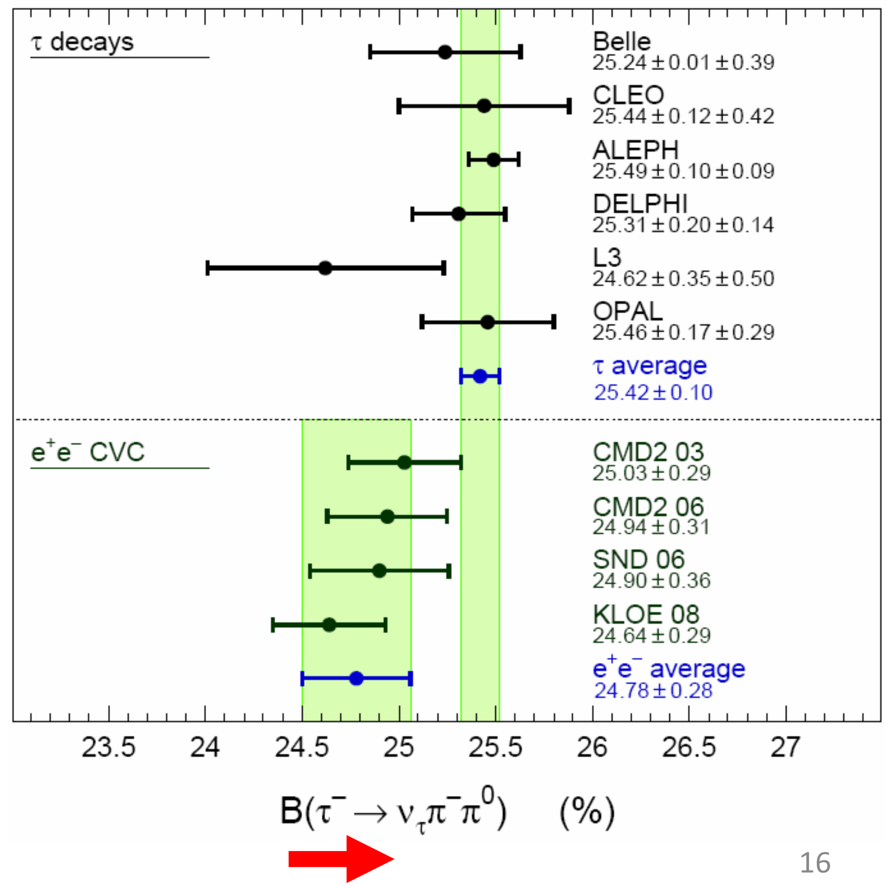
Discrepancy reduced from

$(0.92 \pm 0.21)\%$ (**DEHZ 2003**)

$\Downarrow$

$(0.60 \pm 0.10_\tau \pm 0.28_{ee})\%$

arXiv:0906.5443 [hep-ph]



Comments, conclusions

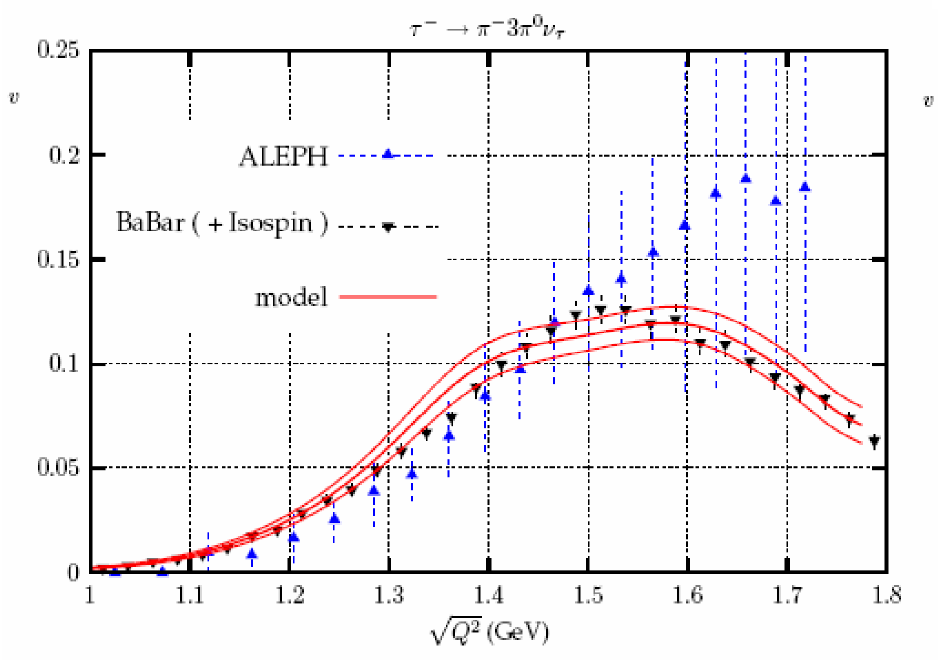
- Recent progress in calculation of IB corrections :
  - Long-distance radiative corrections to $\tau \to \pi\pi\nu$
  - Width difference of $\rho^{\pm}-\rho^{0}$ mesons
- Reduced discrepancies (better agreement with CVC) between:
  - Weak & em pion form factors,
  - Predictions for $(g\text{-}2)_{\mu}$ from e^+e^- and τ (**M. Davier, next talk**)
  - Prediction for $B_{\tau}(\pi\pi)$ from e^+e^- data and direct measurements

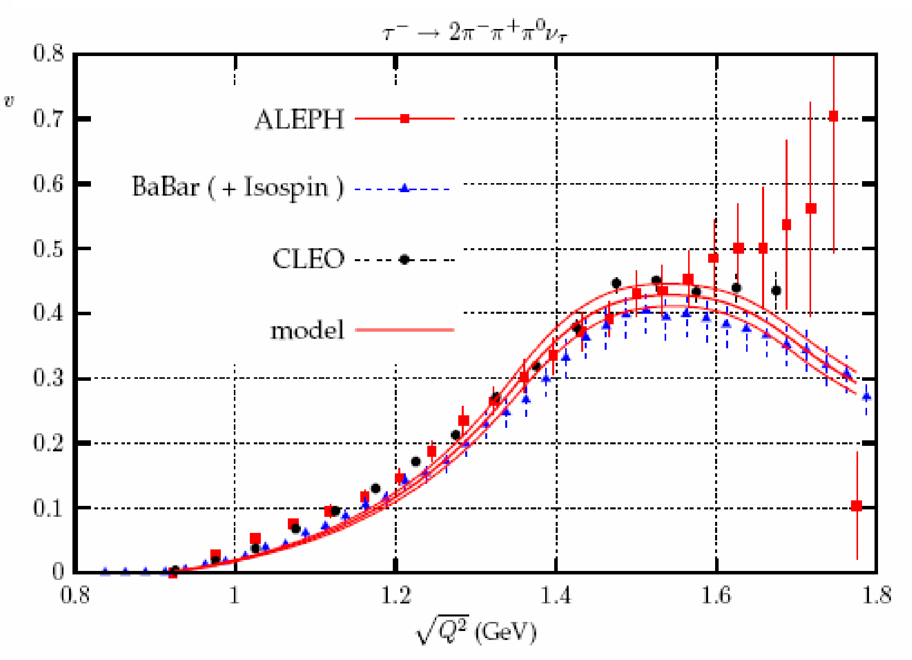
Most important changes induced by $\rho^{\pm}-\rho^{0}$ width difference. IB in pion FF's for $\sqrt{s} > m_{\rho}$ still remains.

Backup slides

IB in 4π channel (Czyz, Kühn, Wapienik 2008)

IB corrections applied: S_{EW} and phase space



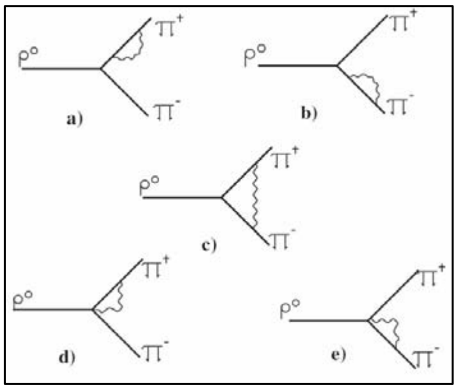


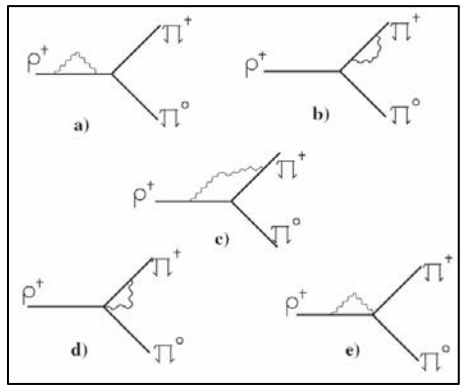
No IB observed within present accuracy

$\Delta\Gamma_\rho = \Gamma[\rho^\pm \to \pi^\pm\pi^0 (\gamma)] - \Gamma[\rho^0 \to \pi^+\pi^-(\gamma)]$

– 0.08 MeV ($\pi\gamma$, $\eta\gamma$, l^+l^-, ..)

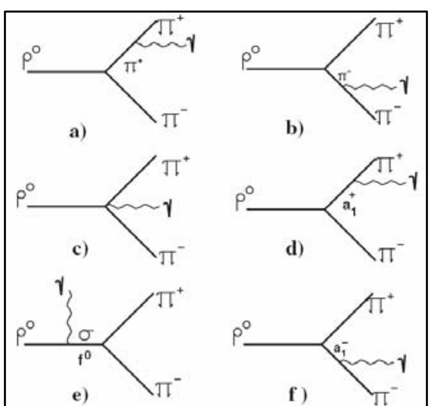
Flores, GLC, Toledo
PRD 76,(2007)

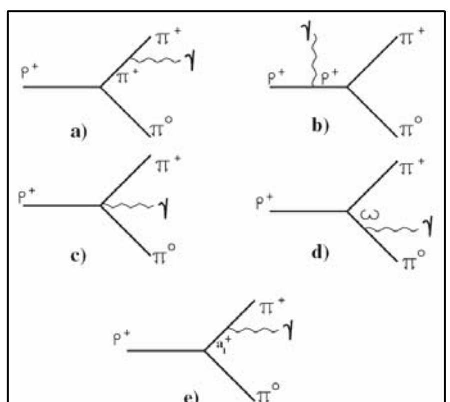




Γ

$$\Gamma_{\rho^+} - \Gamma_{\rho^0} = \begin{cases} (-3.6 \pm 2.4)\ MeV, & KLOE03 \\ (-0.76 \pm 0.07)\ MeV, & This\ calc. \end{cases}$$





For Eγ > 50 MeV

$$\frac{\Gamma^{\exp}\left(\rho^0 \to \pi^+\pi^-\gamma\right)}{\Gamma^{theo}\left(\rho^0 \to \pi^+\pi^-\gamma\right)} = 0.86 \pm 0.14$$

Comparison of radiative branching ratios:

$$\Delta_\rho \equiv \Gamma(\rho\to\pi\pi\gamma)/\Gamma(\rho\to\pi\pi)$$

S63 = P. Singer, PR 130 (1963)
Ours = F. Flores-Baez, GLC, G. Toledo, PRD76, (2007)

$E_\gamma^{min}(MeV)$	$\Delta_{\rho^0}(S63)$	$\Delta_{\rho^0}(Ours)$	$\Delta_{\rho^+}(S63)$	$\Delta_{\rho^+}(Ours)$
15	23.0	23.0	8.3	8.2
30	16.0	16.0	5.1	5.8
45	12.0	12.0	3.7	4.5
60	9.7	9.9	3.0	3.7
105	5.3	5.8	1.6	2.3

BR $\times 10^{-3}$

Small differences due to Interference w/ $\rho^+\to\pi^+\omega$
$\quad\quad\lfloor\ \pi^0\gamma$

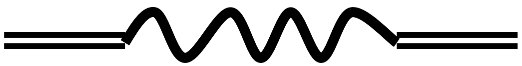
IB in $\rho^{\pm}-\rho^{0}$ masses:

Physical masses (zero of real part of the pole propagator) measured by KLOE [**Phys . Lett. B561, 55 (2003)**] give:

$$m_{\rho^+}^{phys} - m_{\rho^0}^{phys} = (-0.4 \pm 0.9)\ MeV$$

IB in ρ meson masses:

$$m_{\rho^+}^{phys} = m_0 + \delta m_{\rho^+}^{loops}$$



$$m_{\rho^0}^{phys} = m_0 + \delta m_{\rho^0}^{loops} + \delta m_{\rho^0}^{\gamma-\rho\ mixing}$$

$$\delta m_{\rho^0}^{\gamma-\rho\ mixing} = 3\Gamma(\rho^0 \to e^+e^-)/(2\alpha) = 1.45\ MeV$$

Masses entering pion FF are $m_\rho = m_0 + \delta m_\rho^{loops}$

$$m_{\rho^+} - m_{\rho^0} = (1.0 \pm 0.9)\ MeV$$