The Strong Coupling, Charm and Bottom Quark Masses from e^+e^- Experiments

Johann H. Kühn

Karlsruhe Institute of Technology

Universität Karlsruhe (TH)
Forschungsuniversität • gegründet 1825

SFB TR9 Computational Theoretical Particle Physics

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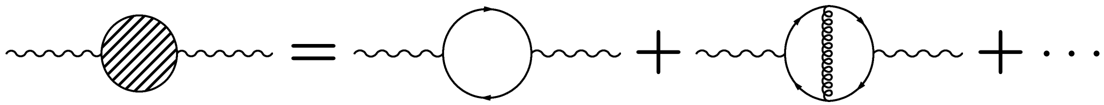
I. α_s from e^+e^- at Low Energies

1. Perturbative Predictions for R(s)

remember: $R(s) = 12\pi \, \mathrm{Im}\Pi(q^2 = s + i\epsilon)$

$$(-q^2 g_{\mu\nu} + q_\mu q_\nu)\Pi(q^2) \equiv i \int dx \, e^{i\,q\,x} \langle T \, j_\mu(x) j_\nu(0) \rangle$$



$\Pi(q^2)$ sums all intermediate states

status of theory:

- massless limit

$$R = 3 \sum_i Q_i^2 \left(1 + \frac{\alpha_s}{\pi} + \# \left(\frac{\alpha_s}{\pi}\right)^2 + \# \left(\frac{\alpha_s}{\pi}\right)^3 + \# \left(\frac{\alpha_s}{\pi}\right)^4 + \cdots \right)$$

parton model	QED Källen+ Sabry 1955	Chetyrkin+... Dine+... Celmaster 1979	Gorishny, Kataev, Larin; Surguladze, Samuel 1991	Baikov, Chetyrkin, JK 2008

- full quark-mass dependence up to $\mathcal{O}(\alpha_s^2)$

$$R_q = 3Q_q^2\left[\frac{v(3-v^2)}{2} + \frac{\alpha_s}{\pi} f_1\left(\frac{m_q^2}{s}\right) + \left(\frac{\alpha_s}{\pi}\right)^2 f_2\left(\frac{m_q^2}{s}\right) + \cdots\right]$$

Källen, Sabry 1955	Chetyrkin, JK, Steinhauser 1993

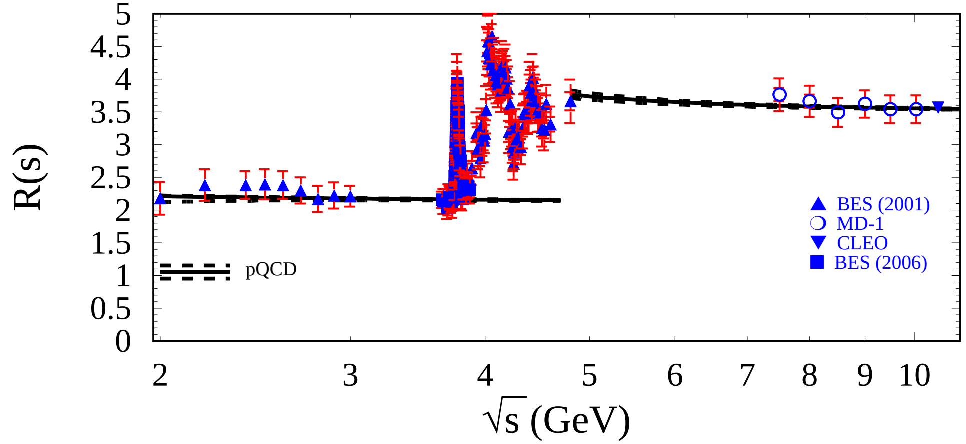
- expansion in m_q^2/s:

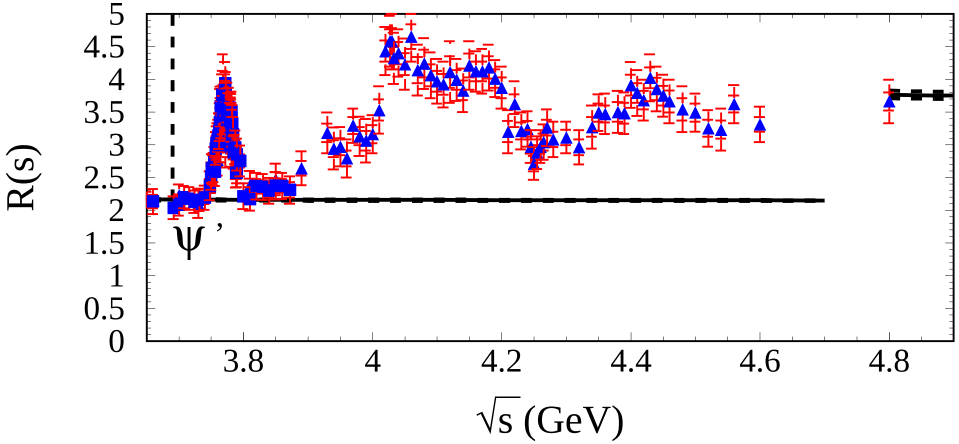
$\left(\frac{\alpha_s}{\pi}\right)^3$	$\frac{m_q^2}{s}$	Chetyrkin,JK	1990
$\left(\frac{\alpha_s}{\pi}\right)^3$	$\left(\frac{m_q^2}{s}\right)^2$	Chetyrkin,JK,Harlander	2000
$\left(\frac{\alpha_s}{\pi}\right)^4$	$\frac{m_q^2}{s}$	Baikov,Chetyrkin,JK	2003

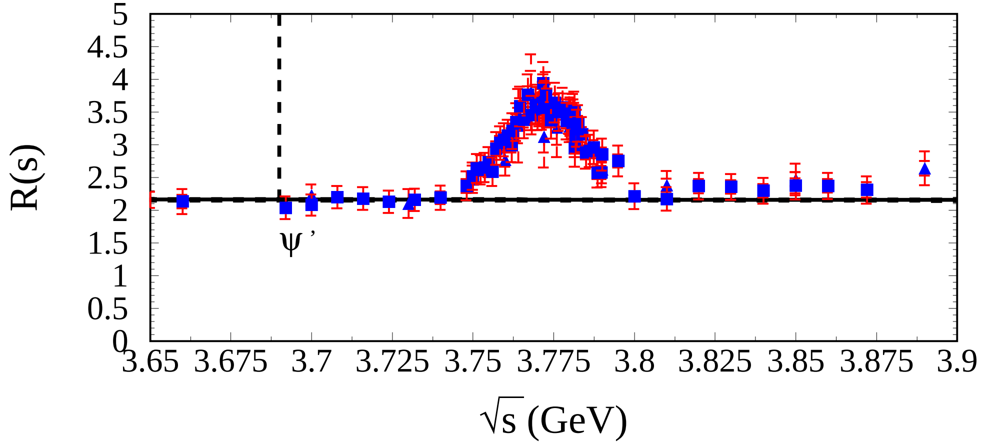
$\Rightarrow$ sufficient precision away from resonance region

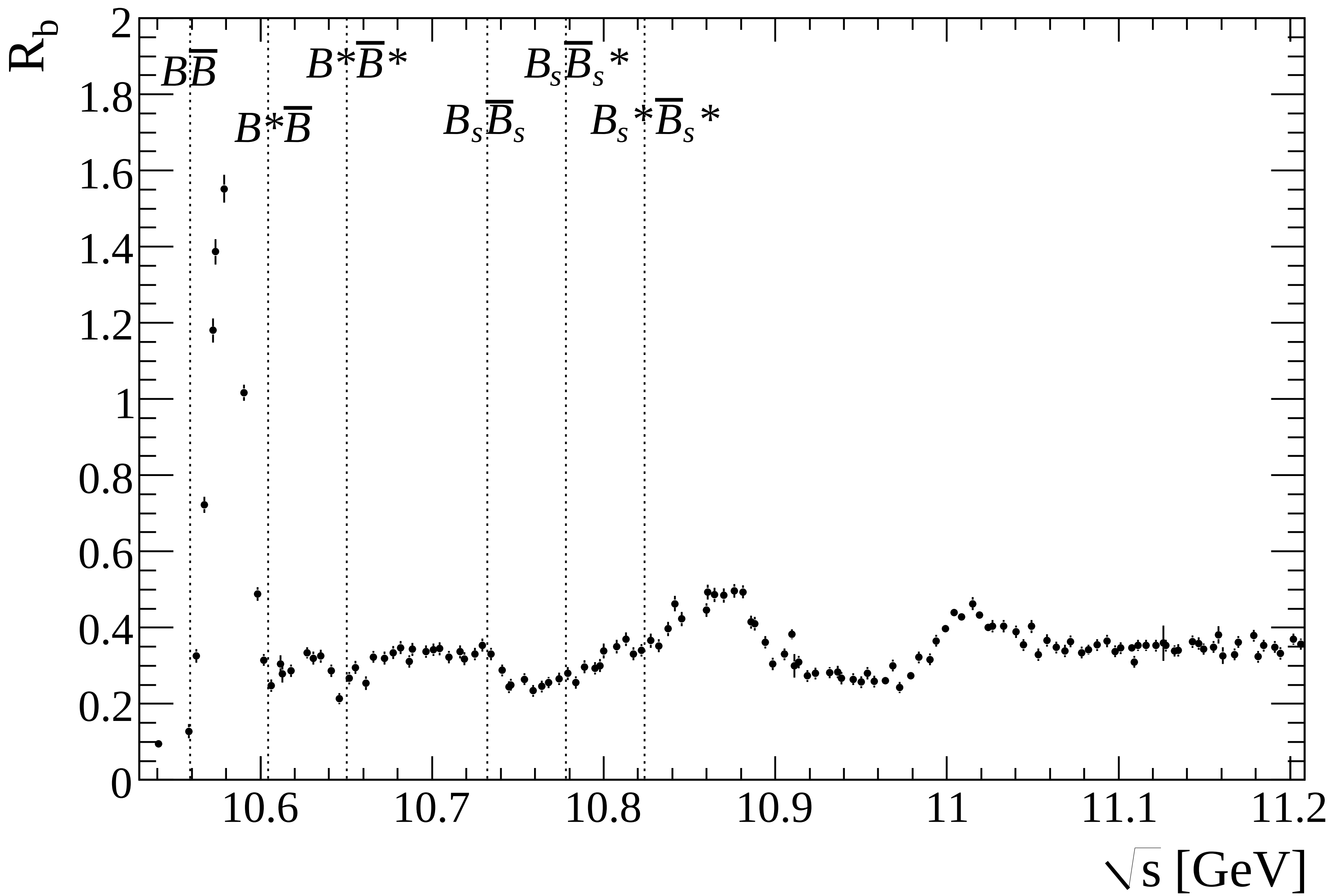
all formulas encoded in program rhad
by Harlander and Steinhauser (hep-ph/0212294)

2. α_s from R(s), Theory vs Experiment

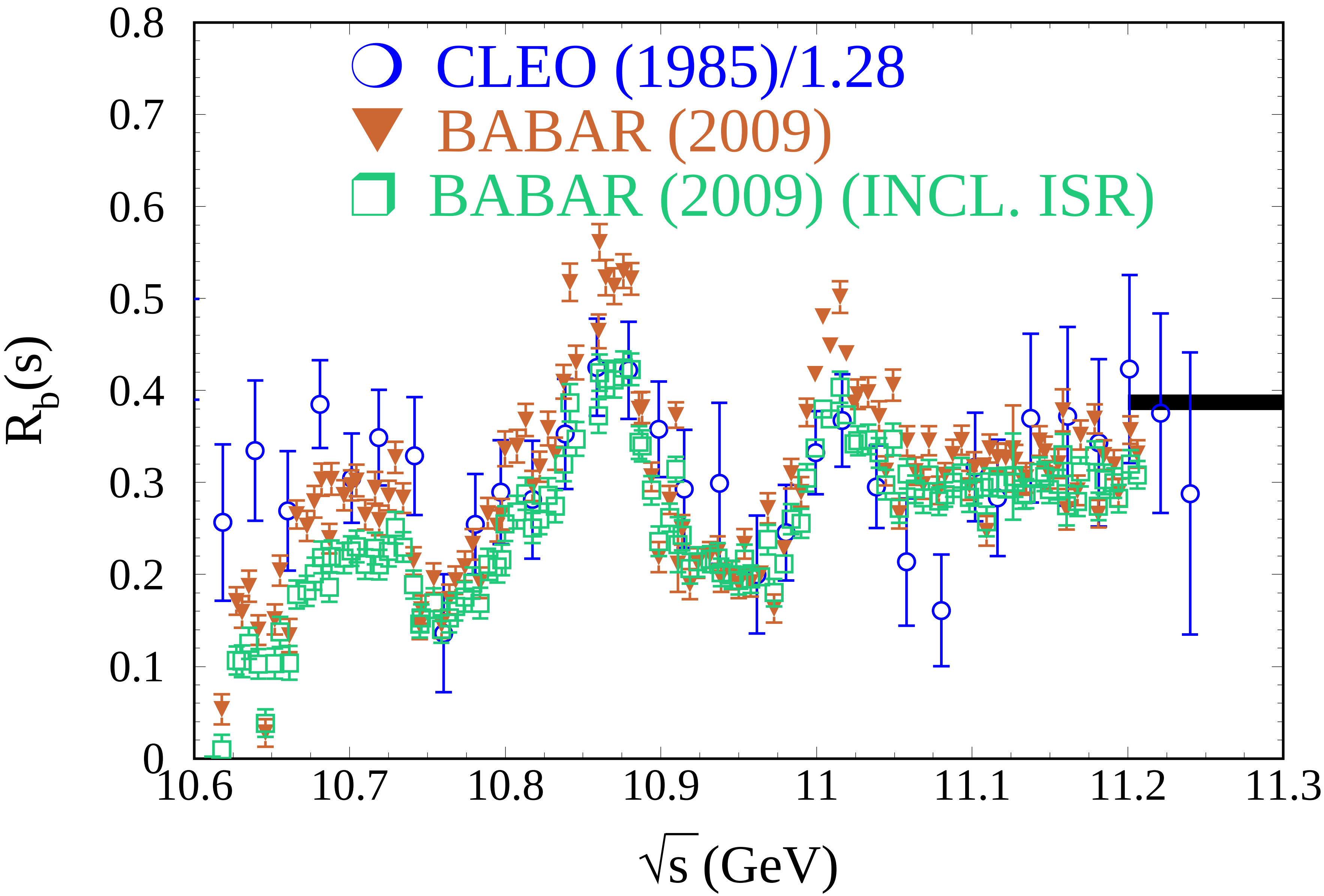








Theory: $R_b(11.2\,\mathrm{GeV}) = 0.365\,!$


CLEO (1985)/1.28
BABAR (2009)
BABAR (2009) (INCL. ISR)
$R_b(s)$
$\sqrt{s}$ (GeV)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
10.6
10.7
10.8
10.9
11
11.1
11.2
11.3

Kinematic phase space threshold suppression $\sim v$

largely compensated by Coulomb enhancement:

Sommerfeld "rescattering" $\sim 4\pi/v$

- analysis from 2001 (Steinhauser, JK)

$$\alpha_s^{(3)}(3\,\text{GeV}) = 0.369\,^{+0.047}_{-0.046}\,^{+0.123}_{-0.130}$$

from BES(2000) below 3.73 GeV

$$\alpha_s^{(4)}(4.8\,\text{GeV}) = 0.183\,^{+0.059}_{-0.064}\,^{+0.053}_{-0.057}$$

from BES(2000) at 4.8 GeV

$$\alpha_s^{(4)}(8.9\,\text{GeV}) = 0.193\,^{+0.017}_{-0.017}\,^{+0.127}_{-0.107}$$

from MD1(1996) below $7-10$ GeV

$$\alpha_s^{(4)}(10.52\,\text{GeV}) = 0.186\,^{+0.008}_{-0.008}\,^{+0.061}_{-0.057}$$

from CLEO(1998) 10.52 GeV

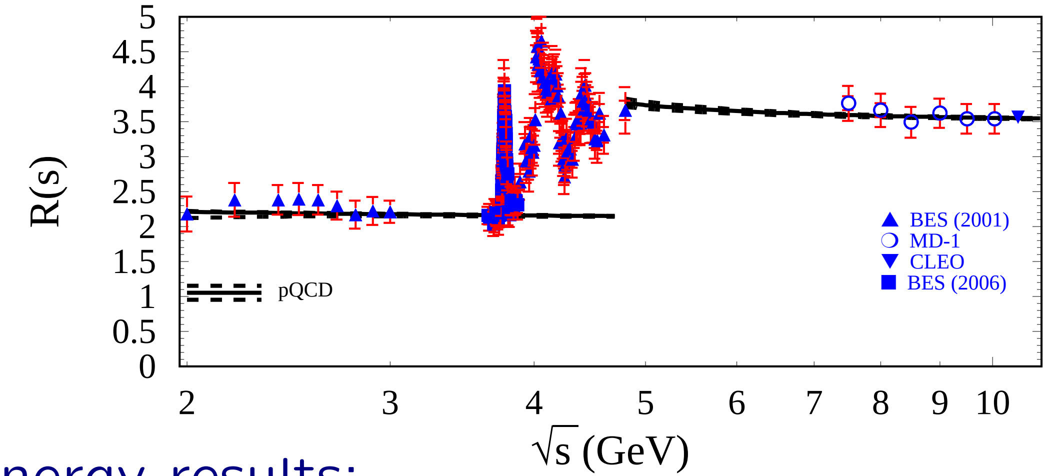
$\Rightarrow$ combine: $\alpha_s^{(4)}(5\,\text{GeV}) = 0.235\,^{+0.047}_{-0.047}$

$$\Rightarrow \qquad \alpha_s^{(5)}(M_Z) = 0.124^{+0.011}_{-0.014}$$

- analysis from 2007 (JK, Steinhauser, Teubner)

$$\alpha_s^{(4)}(9\,\mathrm{GeV}) = 0.160 \pm 0.024 \pm 0.024$$

(based on CLEO 2007
6.964 – 10.538 GeV)



combine all low energy results:

$$\Rightarrow \alpha_s^{(4)}(9\,\mathrm{GeV}) = 0.182^{+0.022}_{-0.025} \Rightarrow \alpha_s^{(5)}(M_Z) = 0.119^{+0.009}_{-0.011}$$

to be compared with

$$\alpha_s^{(5)}(M_Z) = 0.1190 \pm 0.0026 \text{ (exp)} \quad \text{from Z-decays}$$

aim: $\delta R/R < 0.5\%$!

New Result (BES II)

arXiv:0903.0900[hep-ex]

E_{cm} (GeV)	R	$\alpha_s^{(3)}(s)$	$\alpha_s^{(4)}(25\ \text{GeV}^2)$	$\alpha_s^{(5)}(M_Z^2)$
2.60	2.18	$0.266^{+0.030+0.125}_{-0.030-0.126}$		
3.07	2.13	$0.192^{+0.029+0.103}_{-0.029-0.101}$	$0.209^{+0.044}_{-0.050}$	$0.117^{+0.012}_{-0.017}$
3.65	2.14	$0.207^{+0.015+0.104}_{-0.015-0.104}$		

$\frac{\delta R}{R}$: sys $\sim 3.5\%$; stat $\sim 0.5 - 1\%$!

- perfect agreement with pQCD
- perfect agreement with
  - α_s from Z decay rate: $\alpha_s^{(5)}(M_Z^2) = 0.1185 \pm 0.0026$
  - τ-decays (Baikov, Chetyrkin, JK) $\alpha_s^{(3)}(m_\tau^2) = 0.332 \pm 0.005_{exp} \pm 0.015_{th} \Rightarrow \alpha_s^{(5)}(M_Z^2) = 0.120 \pm 0.019$
- relative importance of α_s^4-terms for BES e.g. at 2.606 GeV: $0.266 \pm 0.030 \pm 0.120 \quad \Rightarrow 0.286 \pm \ldots$

low energies ($\sim$ 2 GeV) of special interest

validity of pQCD? $\Rightarrow$ s-dependence!

II. Charm and Bottom Masses

in collaboration with

K. Chetyrkin, Y. Kiyo, A. Maier, P. Maierhöfer, P. Marquard, A. Smirnov,

M. Steinhauser, C. Sturm and the HPQCD Collaboration

NPB 619 (2001) 588
EPJ C48 (2006) 107
NPB 778 (2008) 05413
PLB 669 (2008) 88
NPB 823 (2009) 269
arXiv: 0907.2110
arXiv: 0907.2117

1. WHY precise masses?

B-decays:

$$\Gamma(B \to X_u l\bar{\nu}) \sim G_F^2\, m_b^5\, |V_{ub}|^2$$

$$\Gamma(B \to X_c l\bar{\nu}) \sim G_F^2\, m_b^5\, f(m_c^2/m_b^2)\, |V_{cb}|^2$$

$$B \to X_s \gamma$$

Υ-spectroscopy:

$$m\,(\Upsilon(1s)) = 2M_b - \left(\tfrac{4}{3}\alpha_s\right)^2 \frac{M_b}{4} + \ldots$$

Higgs decay (ILC)

$$\Gamma(H \to b\bar{b}) = \frac{G_F\, M_H}{4\sqrt{2}\pi}\, m_b^2(M_H)\, \tilde{R}$$

$$\tilde{R} = 1 + 5.6667 a_s + 29.147 {a_s}^2 + 41.758 {a_s}^3 - 825.7 {a_s}^4 \qquad \left(a_s \equiv \frac{\alpha_s}{\pi}\right)$$

${a_s}^4$-term = 5-loop calculation [Baikov, Chetyrkin, JK]

Yukawa Unification

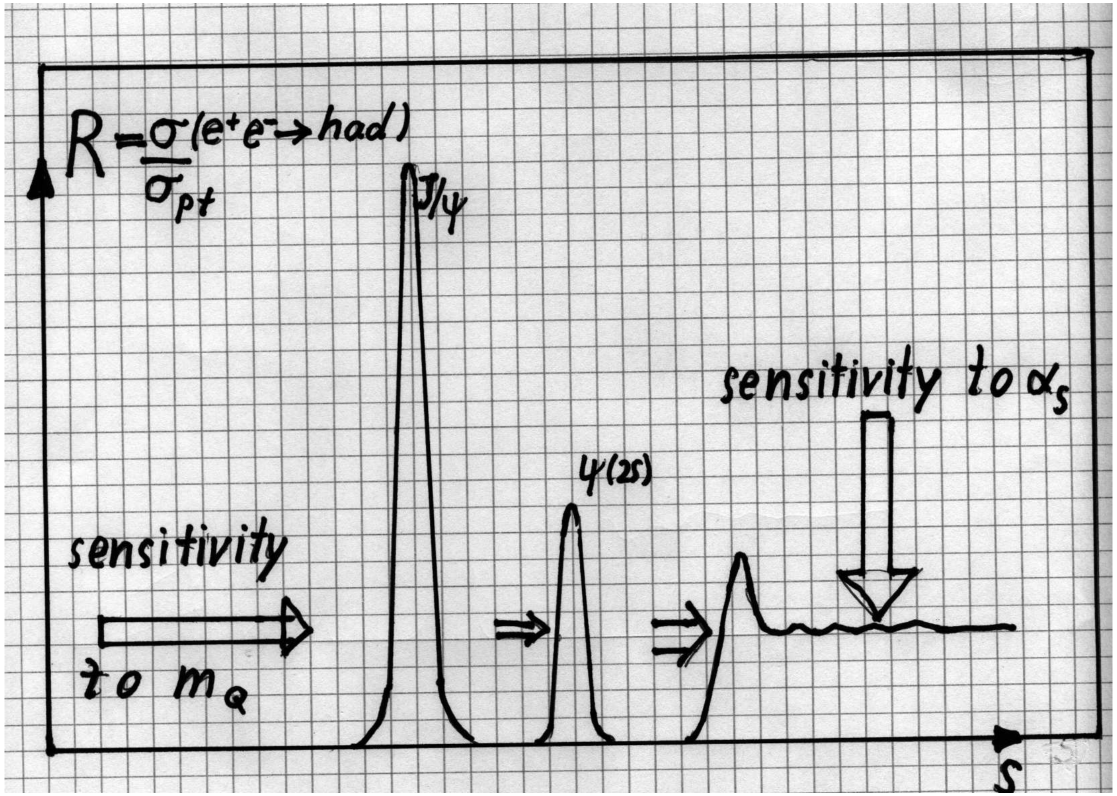
$\lambda_\tau \sim \lambda_b$ or $\lambda_\tau \sim \lambda_b \sim \lambda_t$ at GUT scale

top-bottom $\to$ $m_t / m_b \sim$ ratio of vacuum expectation values

request $\frac{\delta m_b}{m_b} \sim \frac{\delta m_t}{m_t} \Rightarrow \delta m_t \approx 1\,\text{GeV} \Rightarrow \delta m_b \approx 25\,\text{MeV}$

2. m_Q from SVZ Sum Rules, Moments and Tadpoles

Main Idea (SVZ)



Some definitions:

$$\left(-q^2 g_{\mu\nu} + q_\mu\, q_\nu\right) \Pi(q^2) \;\equiv\; i \int \mathrm{d}x\, e^{iqx} \langle T j_\mu(x) j_\nu(0) \rangle$$

with the electromagnetic current j_μ.

$$R(s) \;=\; 12\pi\, \mathrm{Im} \left[\Pi(q^2 = s + i\epsilon)\right]$$

Taylor expansion: $\Pi_Q(q^2) \;=\; Q_Q^2 \, \frac{3}{16\pi^2} \sum_{n\geq 0} \bar{C}_n\, z^n$

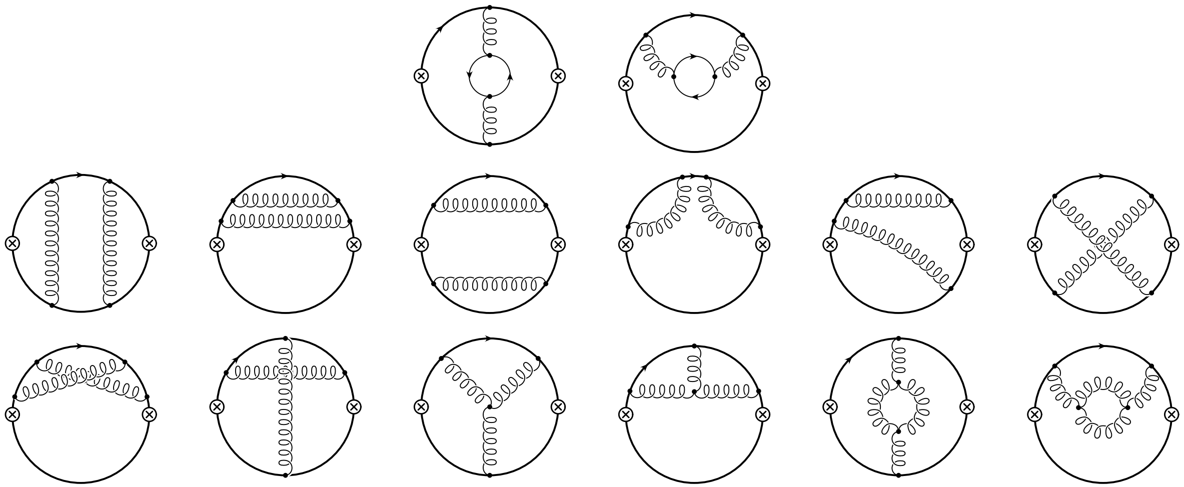
with $z = q^2/(4m_Q^2)$ and $m_Q = m_Q(\mu)$ the $\overline{\mathrm{MS}}$ mass.

$$\bar{C}_n = \bar{C}_n^{(0)} + \frac{\alpha_s}{\pi}\bar{C}_n^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 \bar{C}_n^{(2)} + \left(\frac{\alpha_s}{\pi}\right)^3 \bar{C}_n^{(3)} + \ldots$$

3. Perturbative Results

Analysis in NNLO

Coefficients $\bar{C}_n$ from three-loop one-scale tadpole amplitudes with "arbitrary" power of propagators;



- FORM program MATAD

  Coefficients $\bar{C}_n$ up to $n = 8$

  (also for axial, scalar and pseudoscalar correlators)

  (Chetyrkin, JK, Steinhauser, 1996)

- up to $n = 30$ for vector correlator

  (Boughezal, Czakon, Schutzmeier 2007)

- up to $n = 30$ for vector, axial, scalar and pseudoscalar correlators

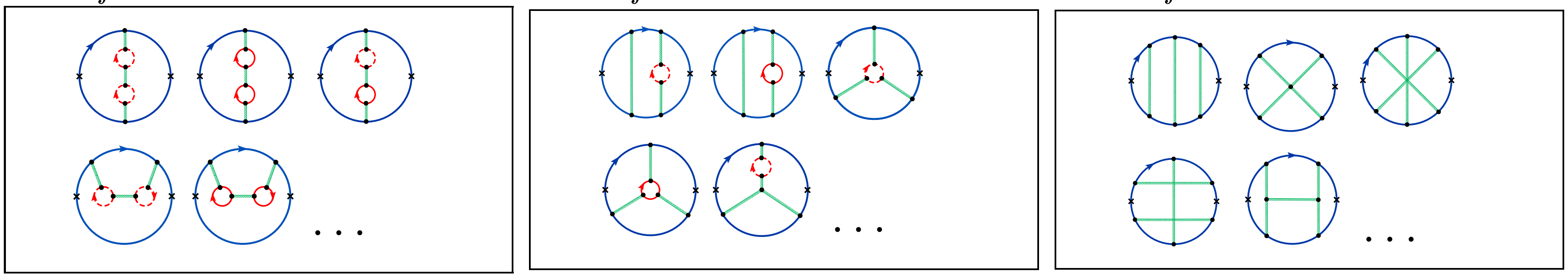
  (A. Maier, P. Maierhöfer, P. Marquard, 2007)

Analysis in N^{3}LO

Algebraic reduction to 13 master integrals (Laporta algorithm);

numerical and analytical evaluation of master integrals

n_f^2-contributions $\quad$ n_f^1-contributions $\quad$ n_f^0-contributions



 : heavy quarks,  : light quarks,

n_f: number of active quarks

$\Rightarrow$ About 700 Feynman-diagrams

$\bar{C}_0$ and $\bar{C}_1$ in order α_s^3 (four loops!) (2006)

➪ Reduction to master integrals

(Chetyrkin, JK, Sturm; Boughezal, Czakon, Schutzmeier)

➪ Evaluation of master integrals numerically or analytically in terms of transcendentals.

All master integrals known analytically and double checked.

(Schröder + Vuorinen, Chetyrkin et al., Schröder + Steinhauser, Laporta, Broadhurst, Kniehl et al.)

Similar approach: four-loop ρ parameter

(K. G. Chetyrkin, M. Faisst, JK, P. Maierhöfer, C. Sturm)

New developments

- ➪ $\bar{C}_2$, $\bar{C}_3$
  (Maier, Maierhöfer, Marquard, A. Smirnov, 2008)

- ➪ $\bar{C}_4 - \bar{C}_{10}$: extension to higher moments by Padé method, using analytic information from low energy ($q^2 = 0$), threshold ($q^2 = 4m^2$), high energy ($q^2 = -\infty$) (Kiyo, Maier, Maierhöfer, Marquard, 2009)

Relation to measurements

$$\mathcal{M}_n^{\text{th}} \equiv \frac{12\pi^2}{n!}\left(\frac{\mathrm{d}}{\mathrm{d}q^2}\right)^n \Pi_c(q^2)\Bigg|_{q^2=0} = \frac{9}{4}Q_c^2\left(\frac{1}{4m_c^2}\right)^n \bar{C}_n$$

Perturbation theory: $\bar{C}_n$ is function of α_s and $\ln\frac{m_c^2}{\mu^2}$

dispersion relation:

$$\Pi_c(q^2) = \frac{q^2}{12\pi^2}\int \mathrm{d}s\,\frac{R_c(s)}{s(s-q^2)} + \text{subtraction}$$

$$\Rightarrow \mathcal{M}_n^{\text{exp}} = \int \frac{\mathrm{d}s}{s^{n+1}} R_c(s)$$

constraint: $\mathcal{M}_n^{\text{exp}} = \mathcal{M}_n^{\text{th}}$

$\Rightarrow$ m_c

4. Experimental Input and Results

Ingredients (charm)

experiment:

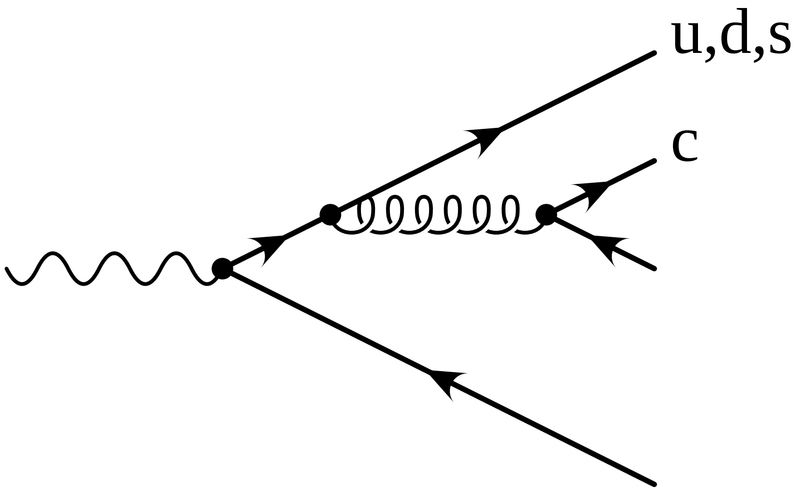
- $\Gamma_e(J/\psi, \psi')$ from BES & CLEO & BABAR
- $\psi(3770)$ and $R(s)$ from BES
- $\alpha_s = 0.1187 \pm 0.0020$

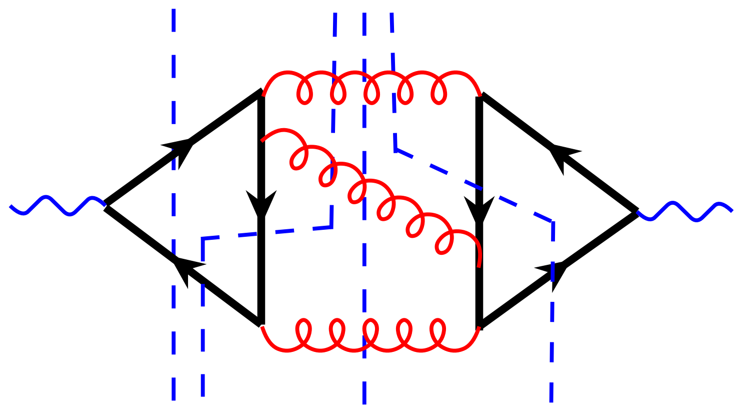
theory:

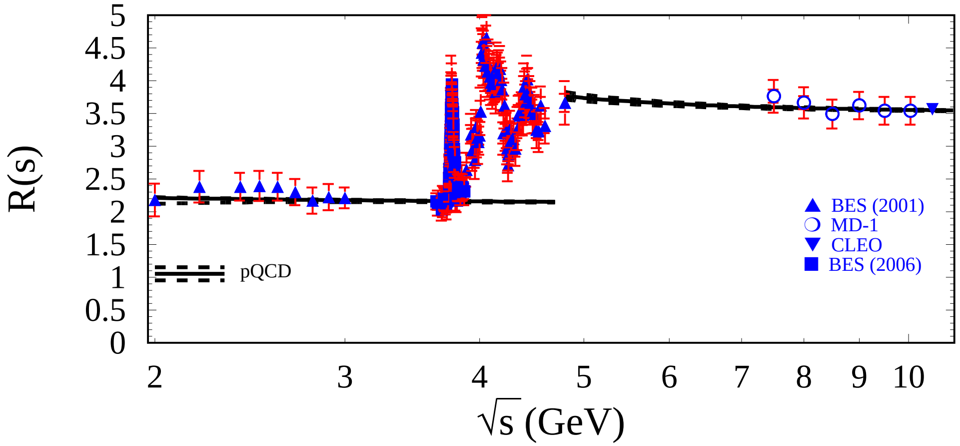
- N^{3}LO for $n = 1, 2, 3, 4$
- include condensates

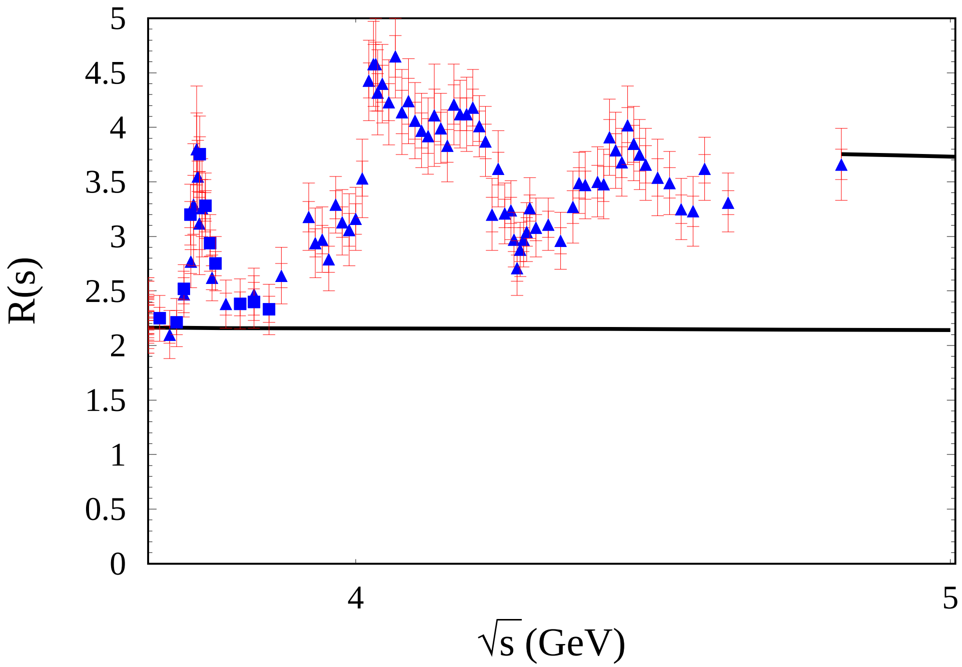
$$\delta\mathcal{M}_n^{\text{np}} = \frac{12\pi^2 Q_c^2}{(4m_c^2)^{(n+2)}} \left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle a_n \left(1 + \frac{\alpha_s}{\pi} \bar{b}_n\right)$$

- estimate of non-perturbative terms (oscillations, based on Shifman)
- careful extrapolation of R_{uds}
- careful definition of R_c


u,d,s
c




R(s)
√s (GeV)
pQCD
BES (2001)
MD-1
CLEO
BES (2006)


R(s)
√s (GeV)

Contributions from

- narrow resonances: $R = \frac{9\,\Pi\, M_R \Gamma_e}{\alpha^2(s)}\,\delta(s - M_R^2)$ (PDG)
- threshold region ($2\,m_D$ – 4.8 GeV) (BESS)
- perturbative continuum ($E \geq$ 4.8 GeV) (Theory)

n	$\mathcal{M}_n^{\text{res}}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\text{thresh}}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\text{cont}}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\text{exp}}$ $\times 10^{(n-1)}$	$\mathcal{M}_n^{\text{np}}$ $\times 10^{(n-1)}$
1	0.1201(25)	0.0318(15)	0.0646(11)	0.2166(31)	−0.0001(2)
2	0.1176(25)	0.0178(8)	0.0144(3)	0.1497(27)	0.0000(0)
3	0.1169(26)	0.0101(5)	0.0042(1)	0.1312(27)	0.0007(14)
4	0.1177(27)	0.0058(3)	0.0014(0)	0.1249(27)	0.0027(54)

Different relative importance of resonances vs. continuum for $n = 1, 2, 3, 4$.

Results (m_c)

arXiv: 0907:2110

n	$m_c(3\,\text{GeV})$	exp	α_s	μ	np	total
1	986	9	9	2	1	13
2	976	6	14	5	0	16
3	978	5	15	7	2	17
4	1004	3	9	31	7	33

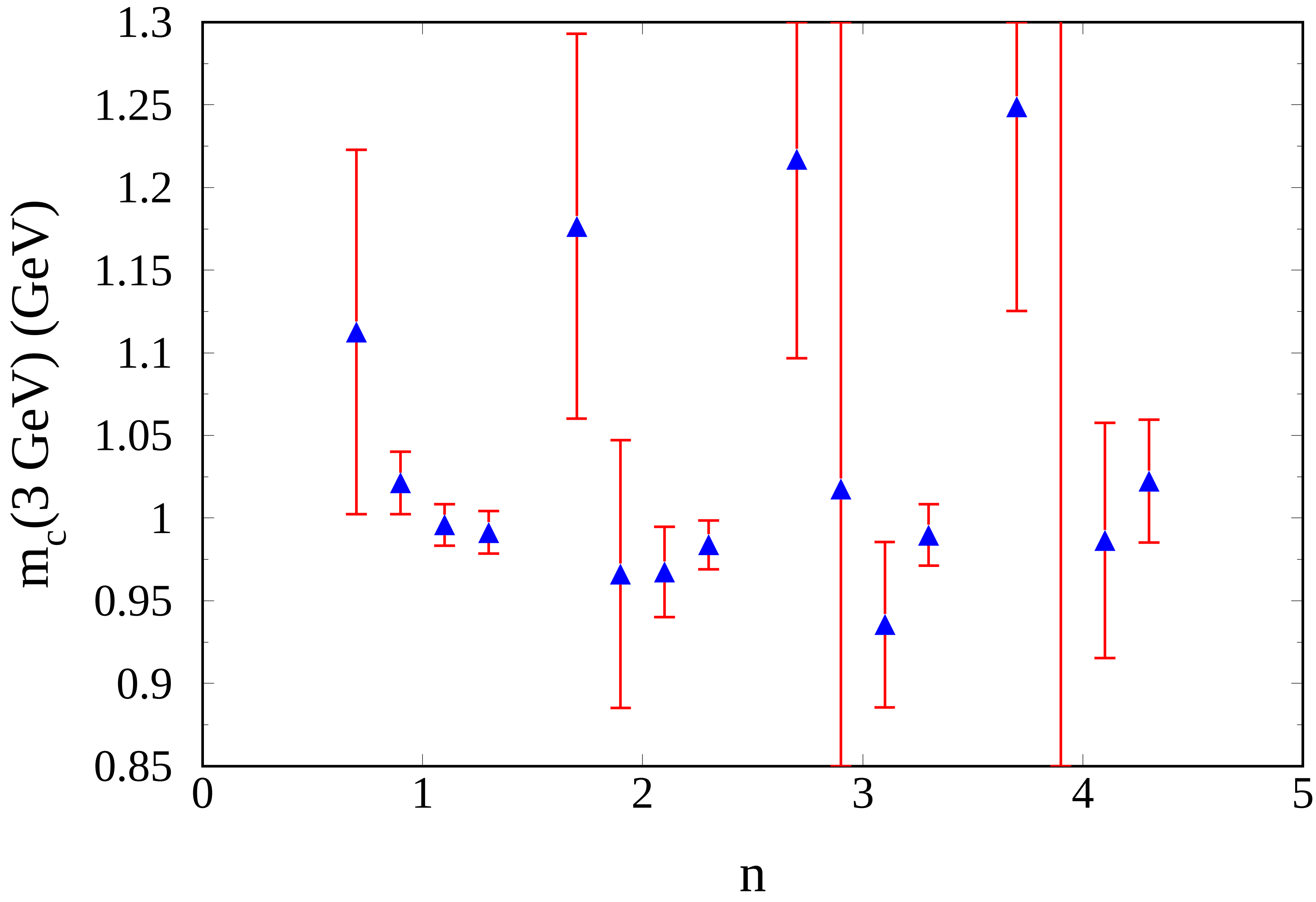
Remarkable consistency between $n = 1, 2, 3, 4$

and stability ($\mathcal{O}(\alpha_s^2)$ vs. $\mathcal{O}(\alpha_s^3)$);

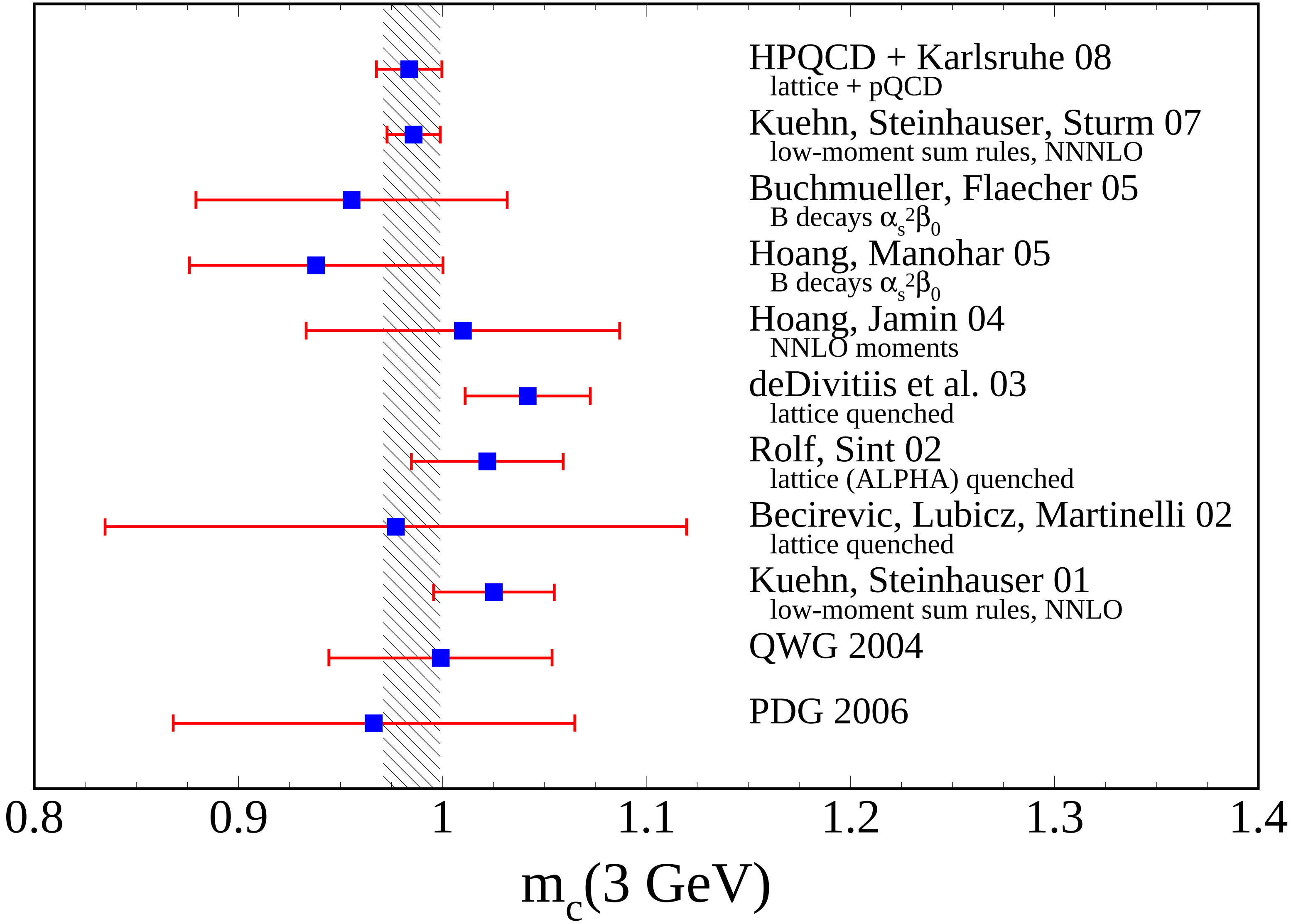
prefered scale: $\mu = 3\,\text{GeV}$,

conversion to $m_c(m_c)$:

- $m_c(3\,\text{GeV}) = 986 \pm 13\,\text{MeV}$
- $m_c(m_c) = 1279 \pm 13\,\text{MeV}$



1.3
1.25
1.2
1.15
1.1
1.05
1
0.95
0.9
0.85
0
1
2
3
4
5
m_c(3 GeV) (GeV)
n


HPQCD + Karlsruhe 08
lattice + pQCD
Kuehn, Steinhauser, Sturm 07
low-moment sum rules, NNNLO
Buchmueller, Flaecher 05
B decays $\alpha_s^2\beta_0$
Hoang, Manohar 05
B decays $\alpha_s^2\beta_0$
Hoang, Jamin 04
NNLO moments
deDivitiis et al. 03
lattice quenched
Rolf, Sint 02
lattice (ALPHA) quenched
Becirevic, Lubicz, Martinelli 02
lattice quenched
Kuehn, Steinhauser 01
low-moment sum rules, NNLO
QWG 2004
PDG 2006
0.8
0.9
1
1.1
1.2
1.3
1.4
m_c(3 GeV)

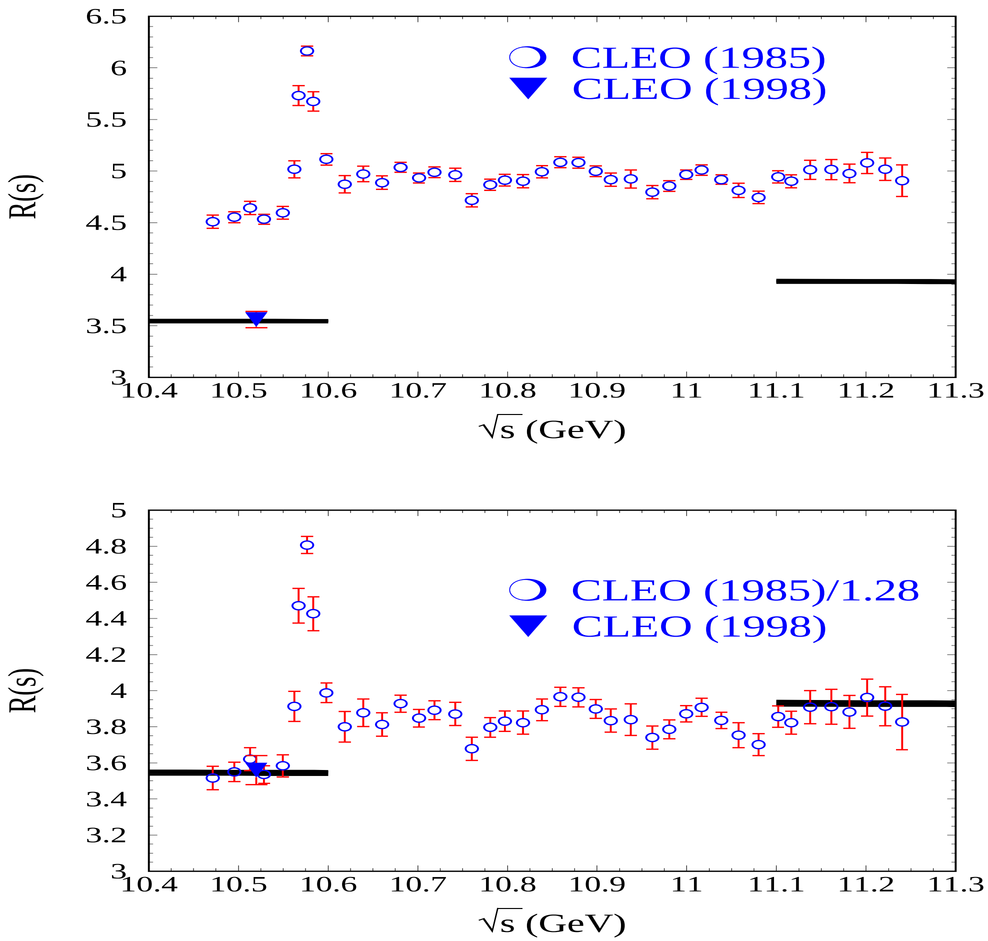
Experimental Ingredients for m_b

Contributions from

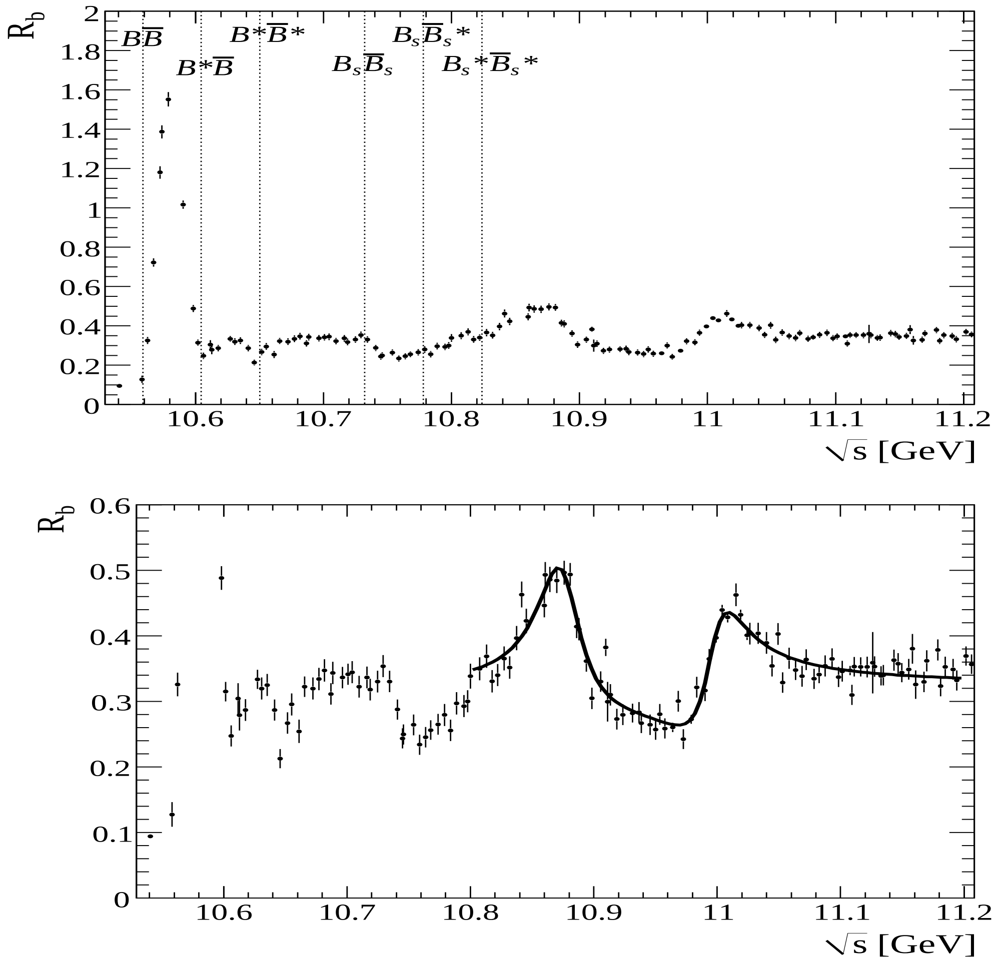
- narrow resonances ($\Upsilon(1S) - \Upsilon(4S)$) (PDG)
- threshold region (10.618 GeV – 11.2 GeV) (BABAR 2009)
- perturbative continuum ($E \geq 11.2$ GeV) (Theory)
- different relative importance of resonances vs. continuum for $n = 1, 2, 3, 4$

n	$\mathcal{M}_n^{\text{res,(1S−4S)}}$ $\times 10^{(2n+1)}$	$\mathcal{M}_n^{\text{thresh}}$ $\times 10^{(2n+1)}$	$\mathcal{M}_n^{\text{cont}}$ $\times 10^{(2n+1)}$	$\mathcal{M}_n^{\text{exp}}$ $\times 10^{(2n+1)}$
1	1.394(23)	0.287(12)	2.911(18)	4.592(31)
2	1.459(23)	0.240(10)	1.173(11)	2.872(28)
3	1.538(24)	0.200(8)	0.624(7)	2.362(26)
4	1.630(25)	0.168(7)	0.372(5)	2.170(26)

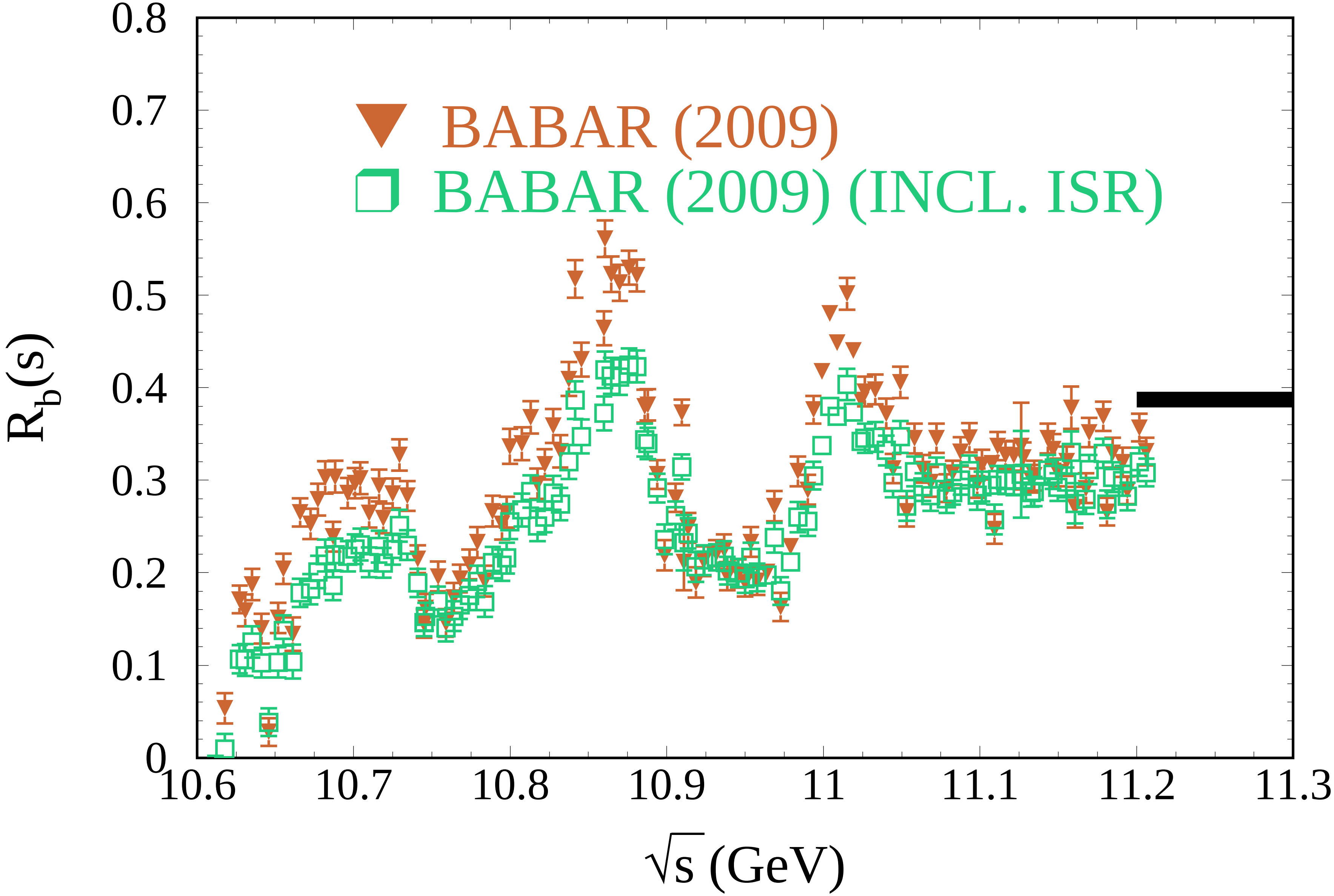
Improvements Based on Recent Babar Results

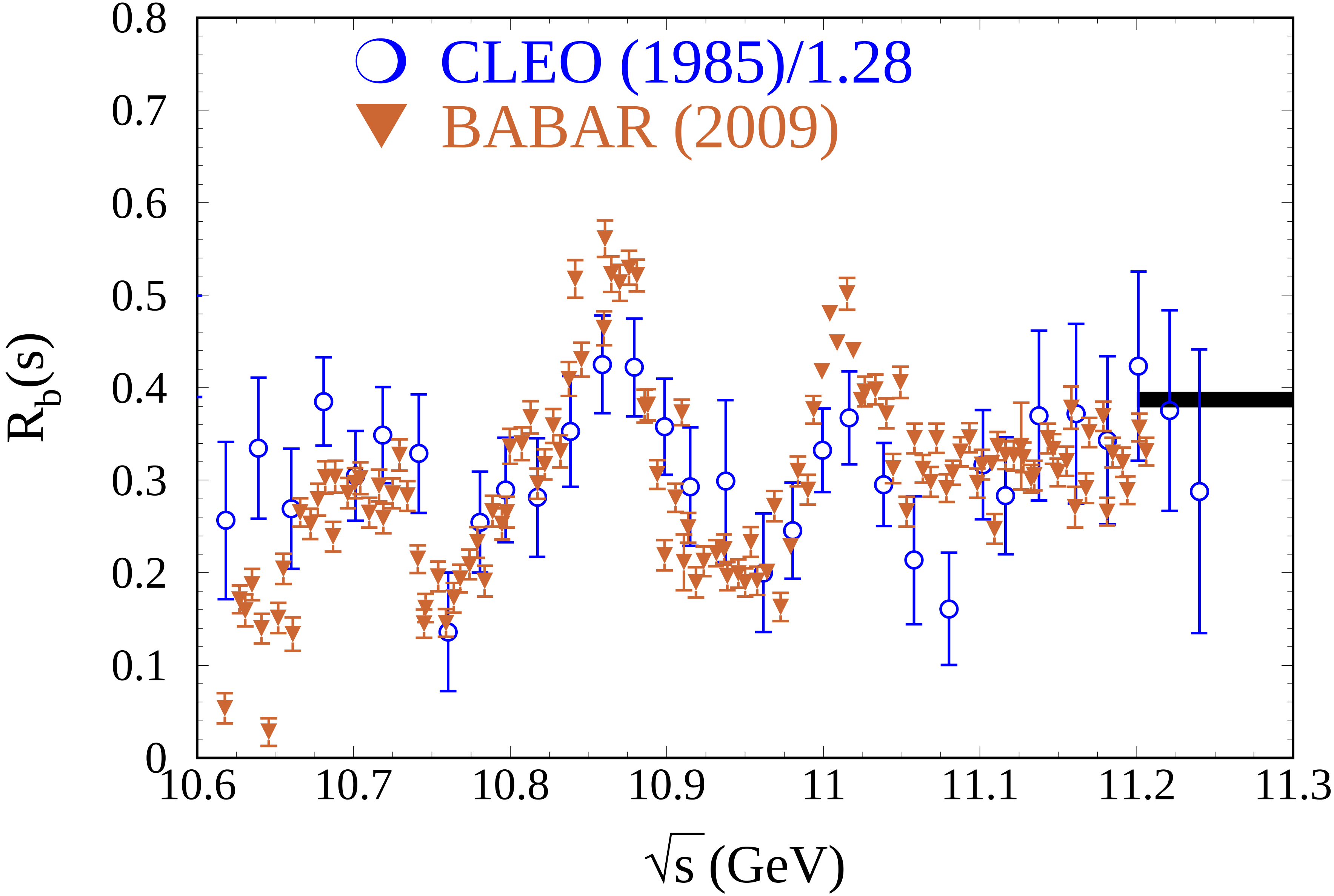


uncertainties after “renormalization” estimated to be 10%
⇒ dominant contribution to error



2% systematic experimental error;
Deconvolute ISR and apply radiative corrections

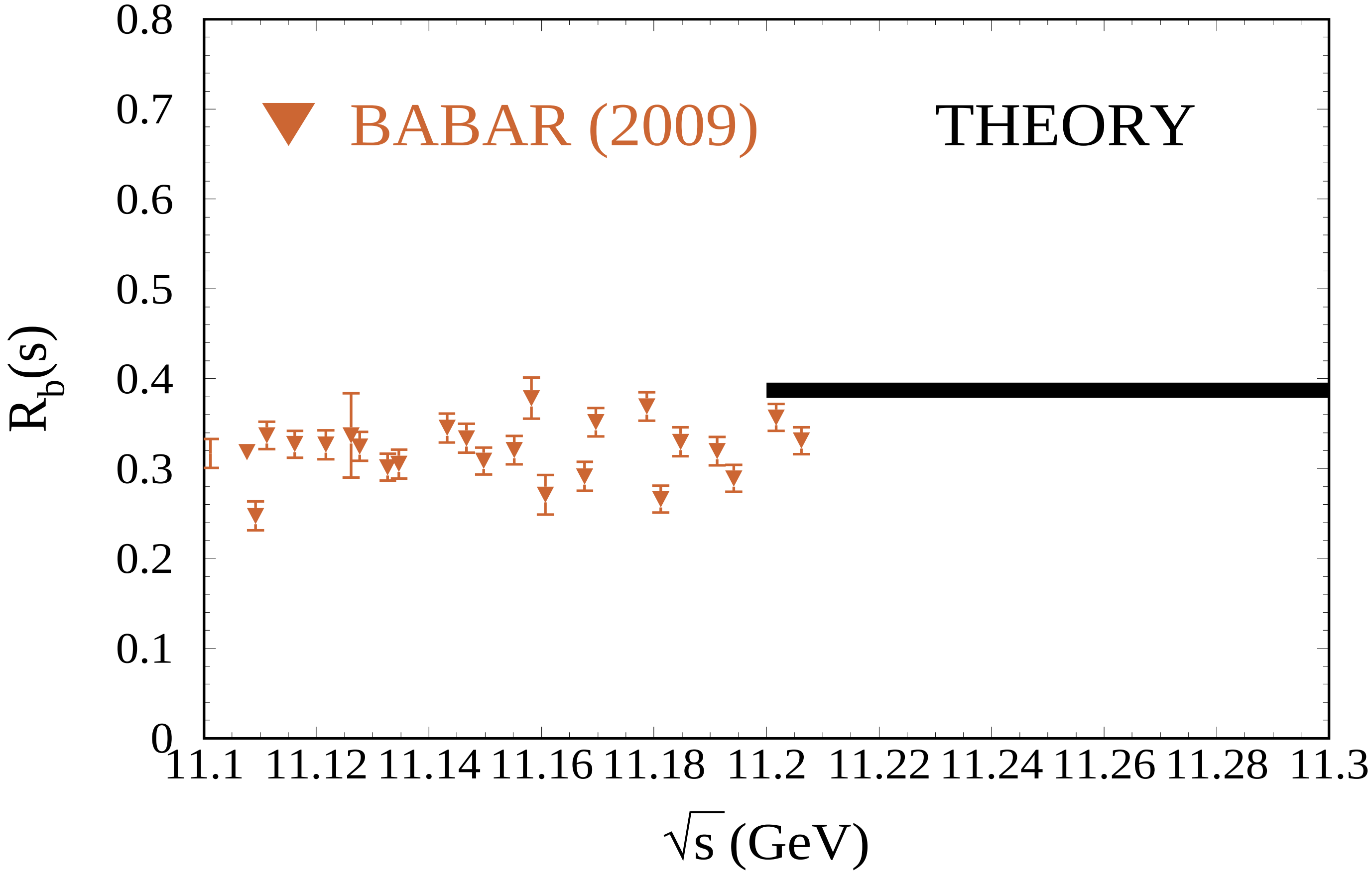

BABAR (2009)
BABAR (2009) (INCL. ISR)
$R_b(s)$
$\sqrt{s}$ (GeV)
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
10.6
10.7
10.8
10.9
11
11.1
11.2
11.3



CLEO (1985)/1.28
BABAR (2009)
$R_b(s)$
$\sqrt{s}$ (GeV)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
10.6
10.7
10.8
10.9
11
11.1
11.2
11.3

n	$\mathcal{M}_n^{\text{threshold}}$ $\times 10^{(2n+1)}$ CLEO	$\mathcal{M}_n^{\text{threshold}}$ $\times 10^{(2n+1)}$ BABAR
1	0.296(32)	0.287(12)
2	0.249(27)	0.240(10)
3	0.209(22)	0.200(8)
4	0.175(19)	0.168(7)

- consistency between BABAR and CLEO
- reduction of experimental error in this region by factor 3, total error by factor 2/3



slight problem at the interface between theory and experiment at $\sqrt{s} = 11.2\,\text{GeV}$!

(Systematic error of BABAR $2-3\%$, difference $\sim 10\%$)

An independent measurement would be desirable (BELLE ?!)

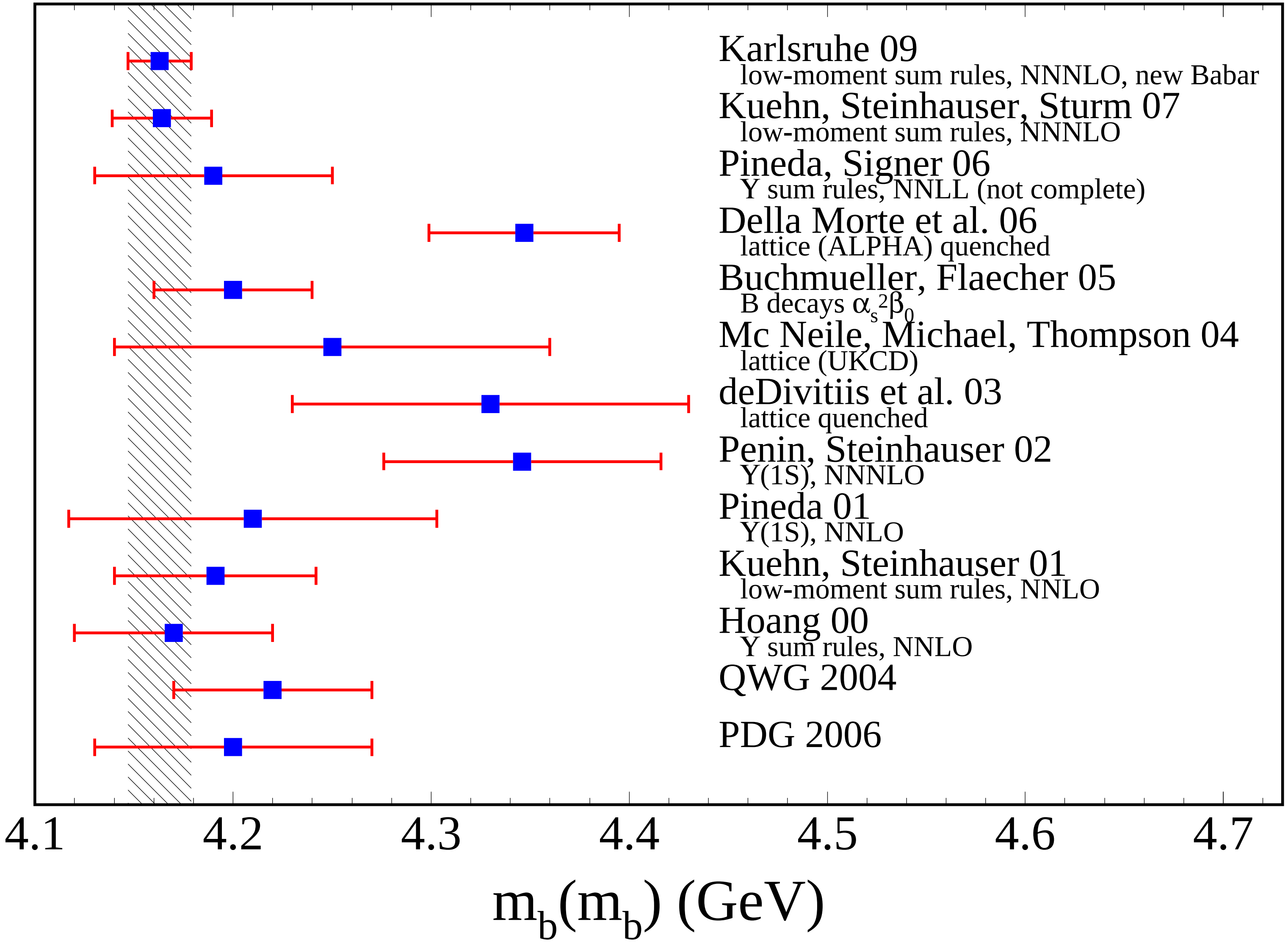
n	$m_b(10\,\text{GeV})$	exp	α_s	μ	total	$m_b(m_b)$
1	3597	14	7	2	16	4151
2	3610	10	12	3	16	4163
3	3619	8	14	6	18	4172
4	3631	6	15	20	26	4183

Consistency ($n = 1, 2, 3, 4$) and stability ($\mathcal{O}(\alpha_s^2)$ vs. $\mathcal{O}(\alpha_s^3)$);

(slight dependence on n could result from input into $\mathcal{M}^n_{\text{exp}}$)

- $m_{\text{b}}(10\,\text{GeV}) = 3610 \pm 16\,\text{MeV}$
- $m_{\text{b}}(m_{\text{b}}) = 4163 \pm 16\,\text{MeV}$

well consistent with KSS 2007


Karlsruhe 09
low-moment sum rules, NNNLO, new Babar
Kuehn, Steinhauser, Sturm 07
low-moment sum rules, NNNLO
Pineda, Signer 06
Υ sum rules, NNLL (not complete)
Della Morte et al. 06
lattice (ALPHA) quenched
Buchmueller, Flaecher 05
B decays $\alpha_s^2\beta_0$
Mc Neile, Michael, Thompson 04
lattice (UKCD)
deDivitiis et al. 03
lattice quenched
Penin, Steinhauser 02
Υ(1S), NNNLO
Pineda 01
Υ(1S), NNLO
Kuehn, Steinhauser 01
low-moment sum rules, NNLO
Hoang 00
Υ sum rules, NNLO
QWG 2004
PDG 2006
4.1
4.2
4.3
4.4
4.5
4.6
4.7
$m_b(m_b)$ (GeV)

α_s-dependence

$$m_c(3\ \text{GeV}) = \left(986 - \frac{\alpha_s - 0.1189}{0.002} \cdot 9 \pm 10\right)\ \text{MeV}$$

$$m_b(10\ \text{GeV}) = \left(3610 - \frac{\alpha_s - 0.1189}{0.002} \cdot 12 \pm 11\right)\ \text{MeV}$$

$$m_b(m_b) = \left(4163 - \frac{\alpha_s - 0.1189}{0.002} \cdot 12 \pm 11\right)\ \text{MeV}$$

$$m_b(M_Z) = \left(2835 - \frac{\alpha_s - 0.1189}{0.002} \cdot 27 \pm 8\right)\ \text{MeV}$$

$$m_b(161.8\ \text{GeV}) = \left(2703 - \frac{\alpha_s - 0.1189}{0.002} \cdot 28 \pm 8\right)\ \text{MeV}$$

III. Lattice & pQCD

(HPQCD + Karlsruhe, Phys. Rev. D78, 054513)

lattice evaluation of pseudoscalar correlator

$\Rightarrow$ replace experimental moments by lattice simulation

input: $M(\eta_c) \widehat{=} m_c\,,\quad M(\Upsilon(1S)) - M(\Upsilon(2s)) \widehat{=} \alpha_s$

pQCD for pseudoscalar correlator available:

"all" moments in $\mathcal{O}(\alpha_s^2)$

three lowest moments in $\mathcal{O}(\alpha_s^3)$.

lowest moment: dimensionless: $\sim \left(\bar{C}^{(0)} + \frac{\alpha_s}{\pi}\bar{C}^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 \bar{C}^{(2)} + \left(\frac{\alpha_s}{\pi}\right)^3 \bar{C}^{(3)} + \ldots\right)$

$\Rightarrow$ $\alpha_s(3\text{GeV}) \Rightarrow \alpha_s(M_Z) = 0.1174(12)$

higher moments: $\sim m_c^2 \times \left(1 + \ldots \frac{\alpha_s}{\pi} \ldots\right)$

$\Rightarrow$ $m_c(3\text{GeV}) = 986(10)$ MeV

to be compared with 986(13) MeV from e^+e^- !

SUMMARY

$m_c(3\,\text{GeV}) = 986(13)$ MeV — $e^+e^- +$ pQCD

$m_c(3\,\text{GeV}) = 986(10)$ MeV — lattice $+$ pQCD

$m_b(10\,\text{GeV}) = 3610(16)$ MeV
$m_b(m_b) = 4163(16)$ MeV — $e^+e^- +$ pQCD

Important role of low energy $e + e^-$ annihilation for α_s and m_Q