Two-Photon-Exchange Contribution to Proton Form Factors in Both Space-Like and Time-Like Regions

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Outline

1. Introduction

2. Two-photon-exchange contribution to elastic ep scattering

3. Two-photon-annihilation contribution to $e^+e^- \to p\overline{p}$ in hadronic model

4. Summary

Introduction: form factors of proton

One of the main problem in hadronic physics is to extract the elemental non-perturbative physical quantities, such as:

quantum number of hadrons, decay constant,
form factor,
parton distribution, distribution amplitude,
GPD, GDA, etc.

The electromagnetic(EM) form factors of proton are two of them. By the symmetry, the non-perturbative EM current matrix element of proton can be decomposed as

$$< P(p')|J_{\mu}^{EM}(0)|P(p)> \equiv \bar{u}(p')[\underline{F_1(Q^2)}\gamma_{\mu} + \underline{F_2(Q^2)}\frac{i\sigma_{\mu\nu}}{2M}q^{\nu}]u(p)$$

$$Q^2 = -q^2 = -(p'-p)^2$$

Introduction: measurement in space-like region

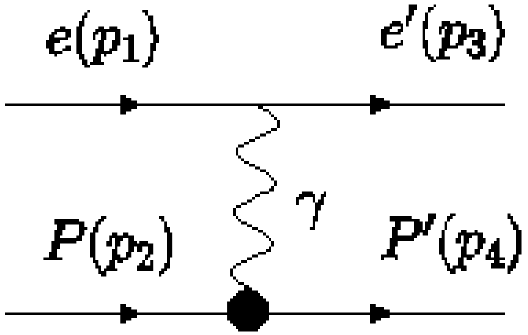
Up to now, two methods are used to extract the EM form factors of proton in the space-like region.

- Rosenbluth method:
  extract the EM form factors from the cross section of un-polarized elastic ep scattering.

- polarization method:
  extract the ratio of EM form factor from the polarization observables in elastic ep scattering.

Introduction: Rosenbluth method

For the un-polarized elastic ep scattering, taking one photon exchange approximation,



the reduced cross section is written as:

$$d\sigma \propto G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2)$$

Introduction: Rosenbluth method

$$d\sigma \propto G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2)$$

$$\tau \equiv \frac{Q^2}{4M_N^2}, \varepsilon = [1 + 2(\tau+1)\tan^2(\theta_e/2)]^{-1}$$

$$G_E = F_1 - \tau F_2, \quad G_M = F_1 + F_2$$

θ_e the scattering angle of electron in the rest frame.

By the measurements of cross sections at fixed Q^2 and different ε, the EM ffs can be extracted, and also the ratios of ffs are obtained.

Introduction: polarization method

For the polarized ep scattering

$$\vec{e} + p \to e + \vec{p}$$

there are polarization observables

P_T proton polarization perpendicular to proton momentum in the scattering plane;

P_L proton polarization parallel to proton momentum in the scattering plane.

Introduction: polarization method

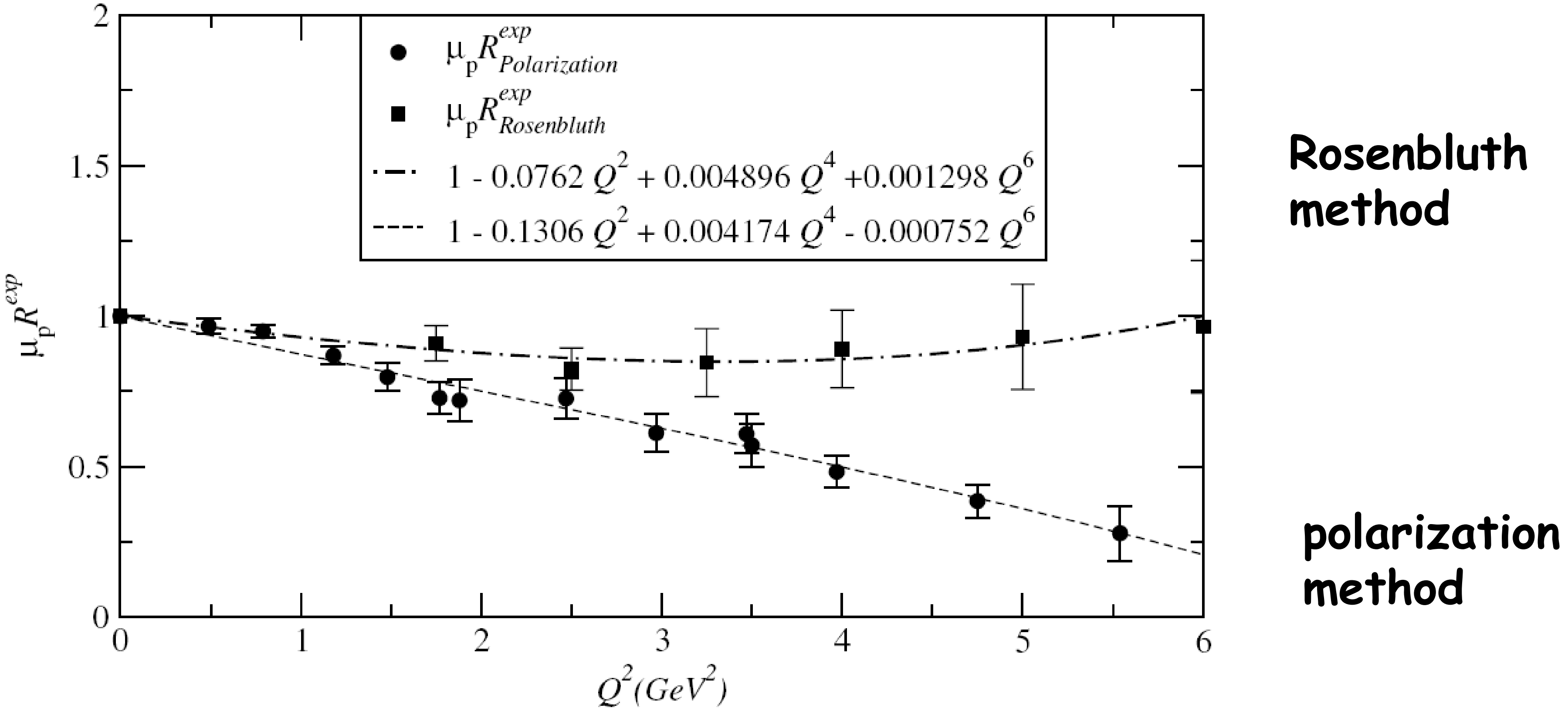
by the one photon exchange approximation, there is relation

$$R \equiv \frac{G_E}{G_M} = -\frac{P_T}{P_L}\frac{E+E'}{2M_N}\tan\frac{\theta_e}{2}$$

E,E' the energies of initial and finial electrons

The measurements of P_T, P_L at fixed Q^2 and ε, can give us the ratio of the EM ffs.

Introduction: results from experiments

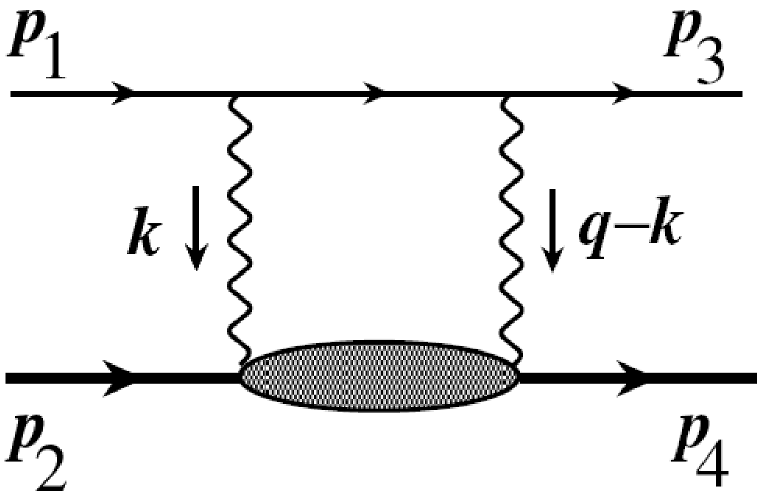


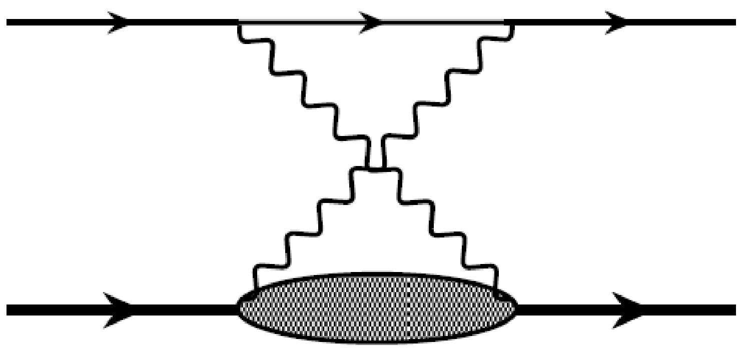
experimental values of R by Rosenbluth method and polarization method references in PRL91,142304(2003)

Un-consistent!	Which is right?

Introduction: reason-two photon exchange

Such un-consistence is explained by the two-photon-exchange effects (box diagrams of EM radiative corrections) in literatures.



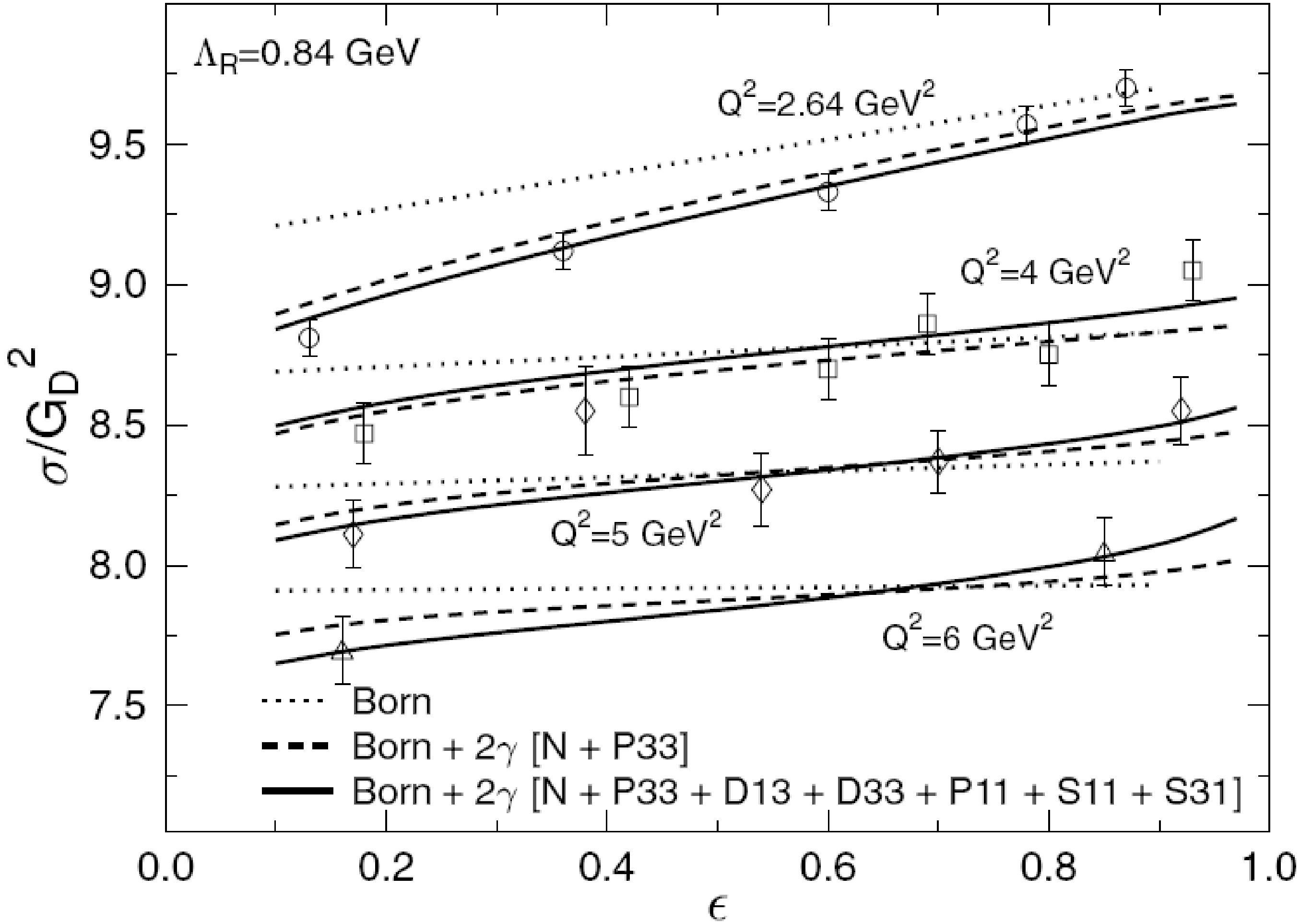


TPE in ep scattering

In literatures, four methods are mainly used to calculate the TPE correction, and also some model-independent analysis or fitting are discussed.

model-dependent dynamical calculation	hadronic model	J. A. Tjon etc; E. Tomasi-Gustafsson etc
	GPD	M. Vanderhaeghen etc
	dispersion relation	Alexander Kobushkin etc
	pQCD factorization	M. Vanderhaeghen etc
model-independent analysis or fitting		M. Vanderhaeghen etc E. Tomasi-Gustafsson etc S.N.Yang etc ………..

TPE in ep scattering: hadronic model



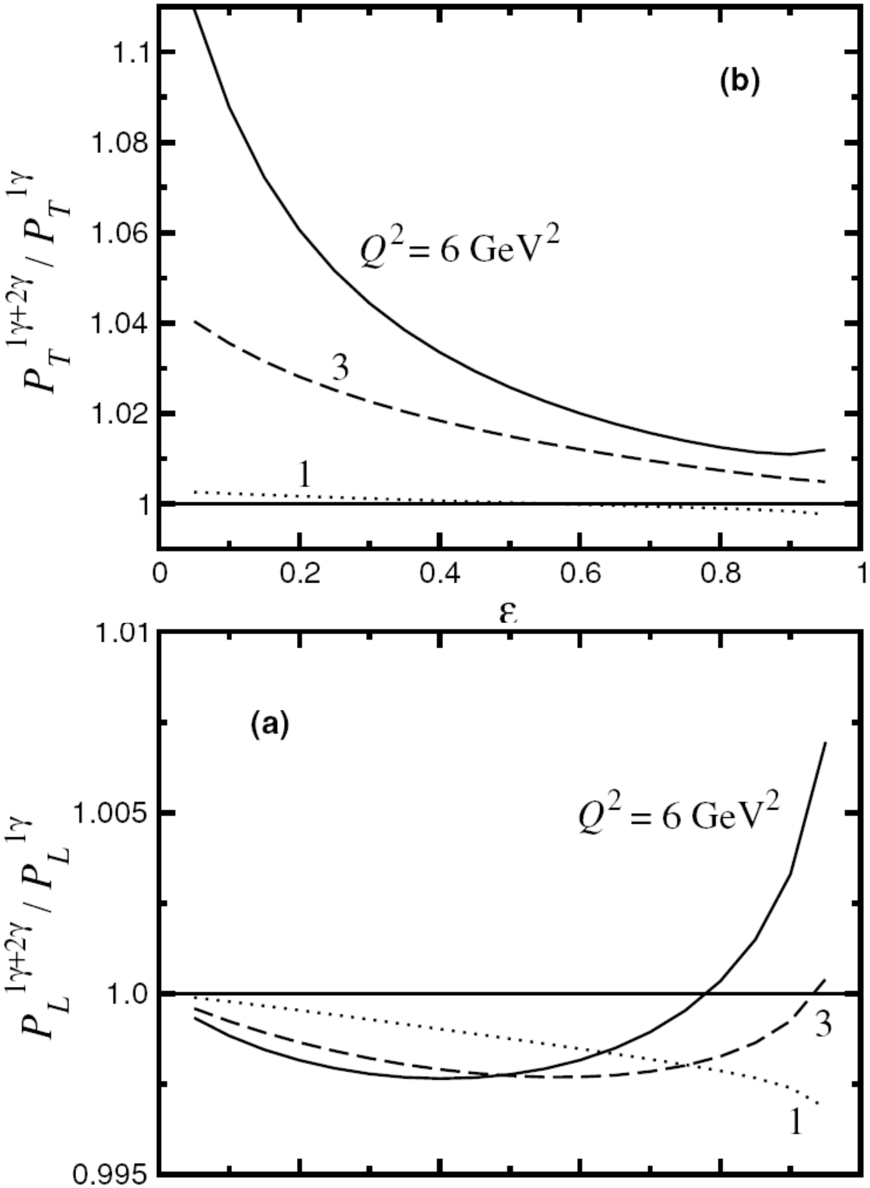
P. G. Blunden,
S. Kondratyuk,
W. Melnitchouk,
J A. Tjon,
PRL91,142304(2003),
PRL95,172503(2005),
PRC72,034612(2005),
PRC75,038201(2007).

consistent with Ex

The Born cross section is calculated with the form factors from the polarization transfer experiment.

the reduced cross section ⇨ ratio R

TPE in ep scattering: hadronic model



P. G. Blunden, W. Melnitchouk,J. A. Tjon, PRC72,034612(2005).

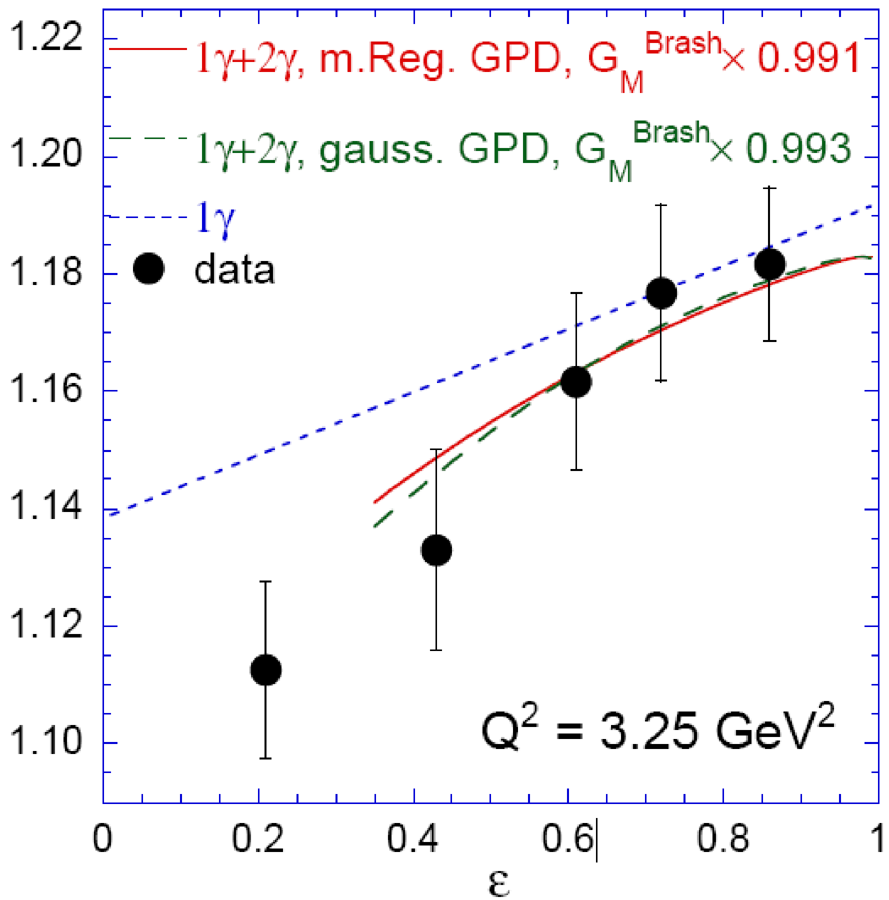
Only the N intermediate state is included.

The results shows the TPE corrections to the ratio R by polarization method is positive and as large as +4% for $Q^2 = 3$ and as large as +10% for $Q^2 = 6$ at small ε . $\varepsilon = 0.1$

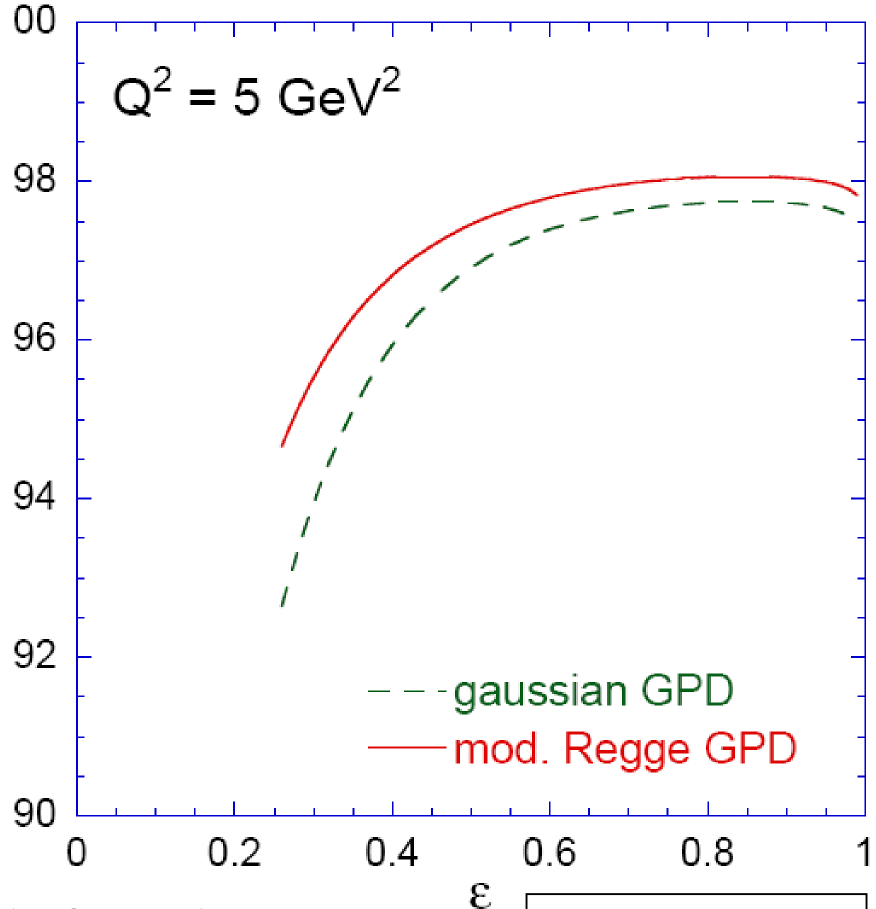
$$\frac{P_T^{1\gamma+2\gamma} / P_T^{1\gamma}}{P_L^{1\gamma+2\gamma} / P_L^{1\gamma}}$$

TPE in ep scattering: GDP

Cross section for ep elastic scattering



2-γ corrections to polarization ratio - proton



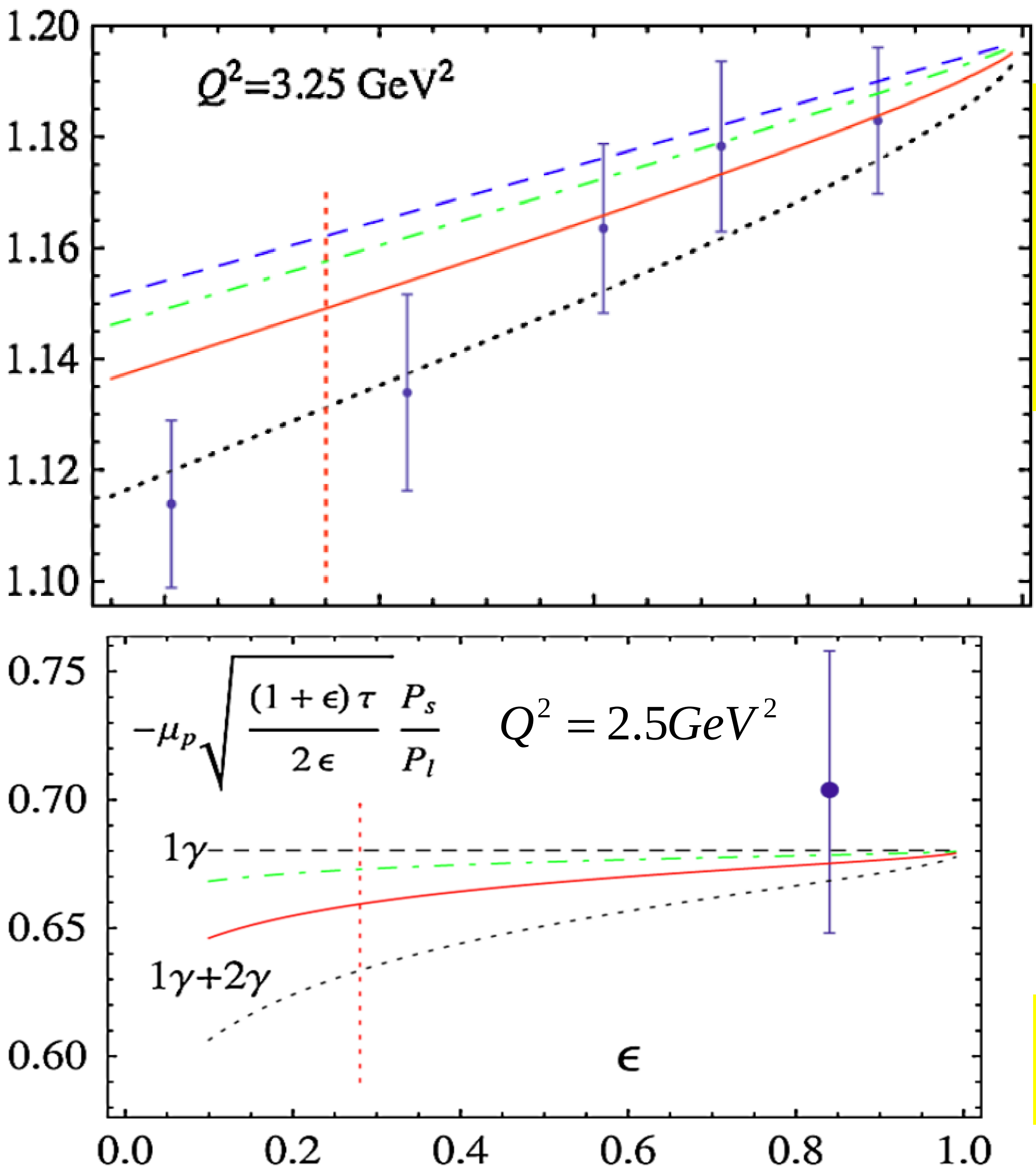
$\sigma_R \ / \ (\mu_p G_{dipole})^2$ consistent with Ex

$\frac{P_T^{1\gamma+2\gamma} \ / \ P_T^{1\gamma}}{P_L^{1\gamma+2\gamma} \ / \ P_L^{1\gamma}}$, %

-5% $\varepsilon = 0.3$
-7%

Y.C. Chen, A.V. Afanasev etc PRL93,122301(2004),PRD72,013008 (2005).

TPE in ep scattering: pQCD



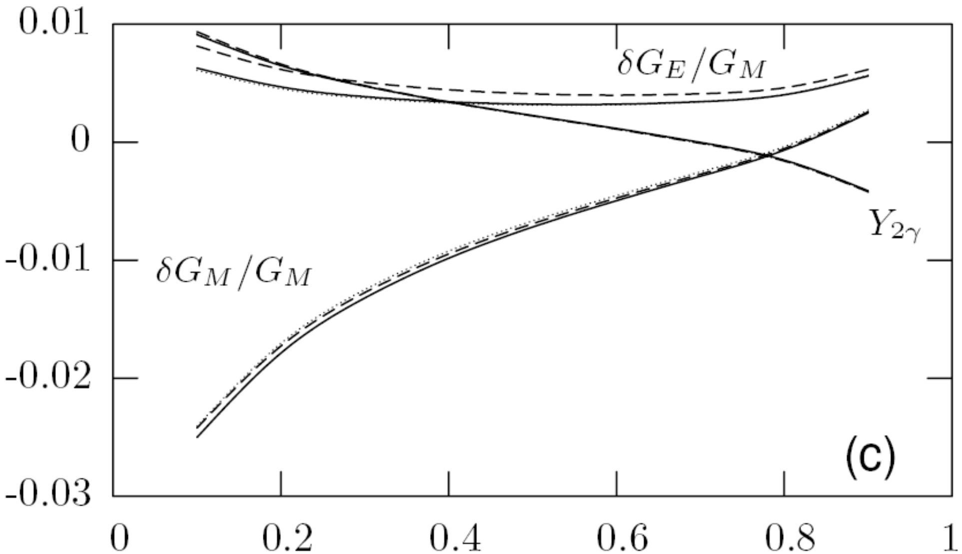
Dash-Blue: 1γ
Dotted black: $1\gamma+2\gamma$ (COZ)
Solid red: $1\gamma+2\gamma$ (BLW)
Dash-dotted green:
$1\gamma+2\gamma$ (QCDSF)

consistent with Ex

-7%(COZ)
-3%(BLW)
$\varepsilon = 0.3$

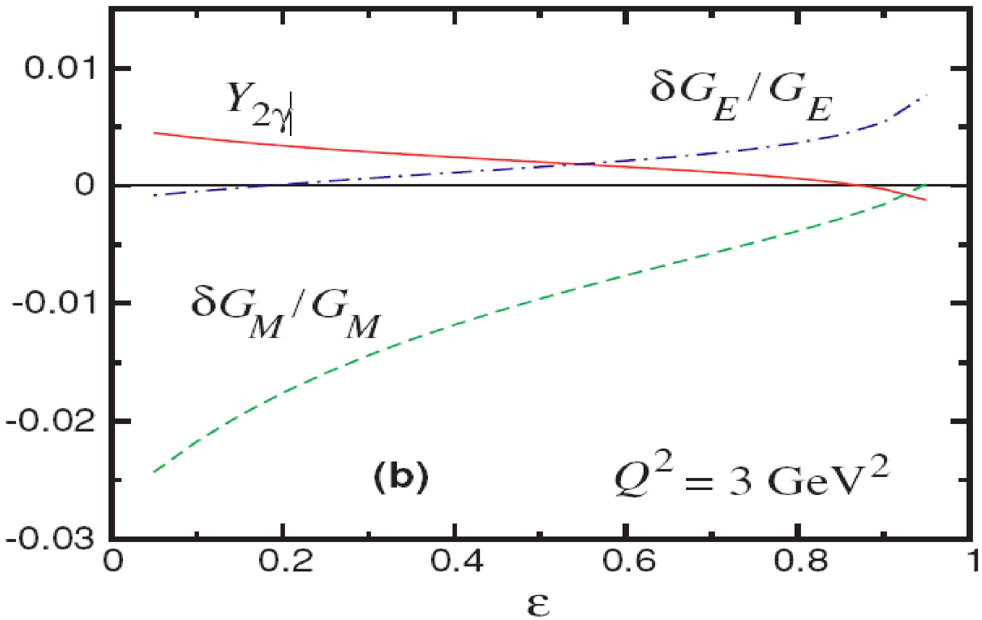
Nikolai Kivel, Marc Vanderhaeghen
PRL103,092004(2009).

TPE in ep scattering: dispersion method vs. HM



Dmitry Borisyuk, Alexander Kobushkin, PRC74,065203(2006).

consistent with each other



P. G. Blunden, W. Melnitchouk,J. A. Tjon, PRC72,034612(2005).

Only the N intermediate state is included.

TPE in ep scattering: summary of the results

1: the corrections to un-polarized cross sections:
consistent.

2:the corrections to ratio R in polarization method:
un-consistent at high Q^2 and small ε .

- hadronic model: positive and as large as +4% for $Q^2=3$ and as large as +10% for $Q^2=6$ at $\varepsilon=0.1$.(where only N is included)

- GDPs and pQCD: negative about -5% for $Q^2=2.5,5$ at $\varepsilon=0.3$.

Which is right?

JLab/Hall C exp. E-04-019

TPE in ep: re-discussion of hadronic model

Two problem exists in the calculation of hadronic model when including the Delta(1232) intermediated state :

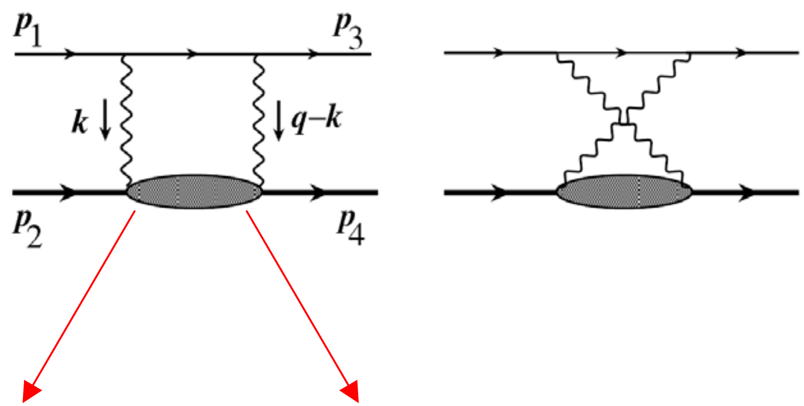
- vertex relation
- Coulomb term

P. G. Blunden,
S. Kondratyuk,
W. Melnitchouk,
J A. Tjon,
PRL95,172503(2005),
PRC75,038201(2007).

TPE in ep: re-discussion of hadronic model

vertex relation

P. G. Blunden, S. Kondratyuk,W. Melnitchouk,J A. Tjon, <u>PRL95,172503(2005)</u>, <u>PRC75,038201(2007)</u>.



$$\Gamma_{\gamma N\to\Delta} = \gamma_0[\Gamma_{\gamma\Delta\to N}]^{+}\gamma_0$$

We modify it as

$$\Gamma_{\gamma N\to\Delta}(p,q) = -\gamma_0[\Gamma_{\gamma\Delta\to N}(p,-q)]^{+}\gamma_0$$

p is the momentum of proton, q is the momentum of incoming photon.

TPE in ep: re-discussion of hadronic model

The explicit expression

$$\Gamma^{\mu\alpha}_{\gamma\Delta\to N} = \frac{-F_\Delta(q_1^2)}{M_N^2}[g_1(g^\alpha_\mu \hat{k}\hat{q}_1 - k_\mu\gamma^\alpha\hat{q}_1 - \gamma_\mu\gamma^\alpha k\cdot q_1 + \gamma_\mu\hat{k}q_1^\alpha)$$
$$+g_2(k_\mu q_1^\alpha - k\cdot q_1 g^\alpha_\mu) + g_3/M_N(q_1^2(k_\mu\gamma^\alpha - g^\alpha_\mu\hat{k})$$
$$+q_{1\mu}(q_1^\alpha\hat{k} - \gamma^\alpha k\cdot q_1))]\gamma_5 T_3,$$

$$\Gamma^{\mu\alpha}_{\gamma\to\bar{N}\Delta} = \frac{-F_\Delta(q_2^2)}{M_N^2}(k)T_3^+\gamma_5[g_1(g^\beta_\nu\hat{q}_2\hat{k} - k_\nu\hat{q}_2\gamma^\beta - \gamma^\beta\gamma_\nu k\cdot q_2 + \hat{k}\gamma_\nu q_2^\beta)$$
$$+g_2(k_\nu q_2^\beta - k\cdot q_2 g^\beta_\nu) \boxed{-} g_3/M_N(q_2^2(k_\nu\gamma^\beta - g^\beta_\nu\hat{k})$$
$$+q_{2\nu}(q_2^\beta\hat{k} - \gamma^\beta k\cdot q_2))].$$

$\boxed{+}$

P. G. Blunden, S. Kondratyuk,W. Melnitchouk,J A. Tjon, PRL95,172503(2005), PRC75,038201(2007).

TPE in ep: re-discussion of hadronic model

Coulomb term

P. G. Blunden, S. Kondratyuk,W. Melnitchouk,J A. Tjon, PRL95,172503(2005), PRC75,038201(2007).

$$g_3 = g_c = 0, -2$$

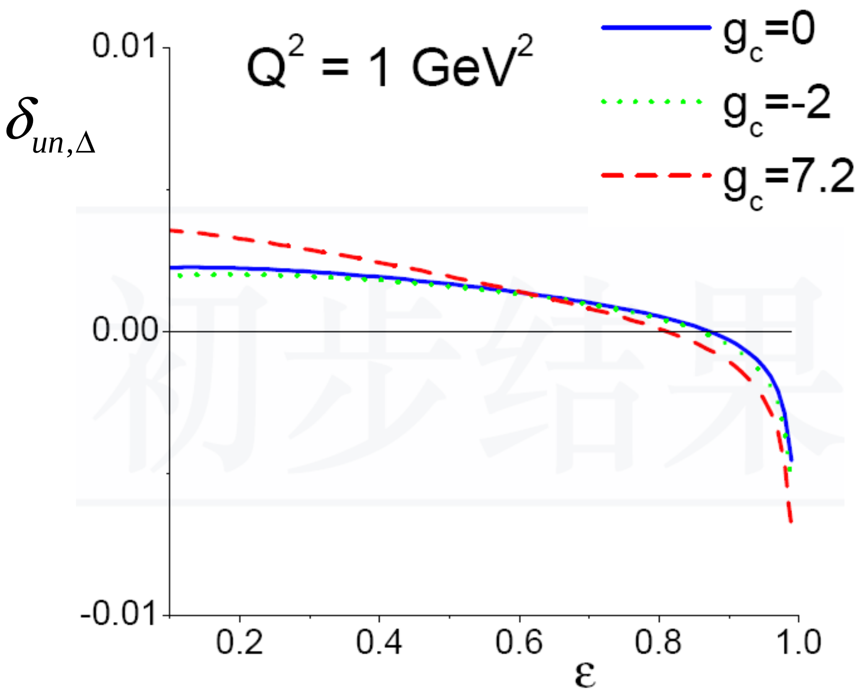
We take

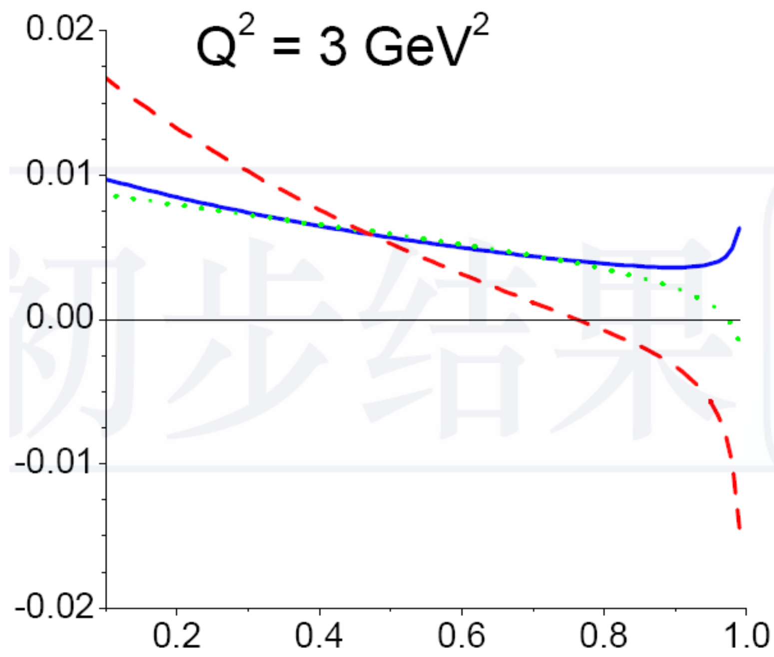
$$g_3 = g_c = 7.2$$

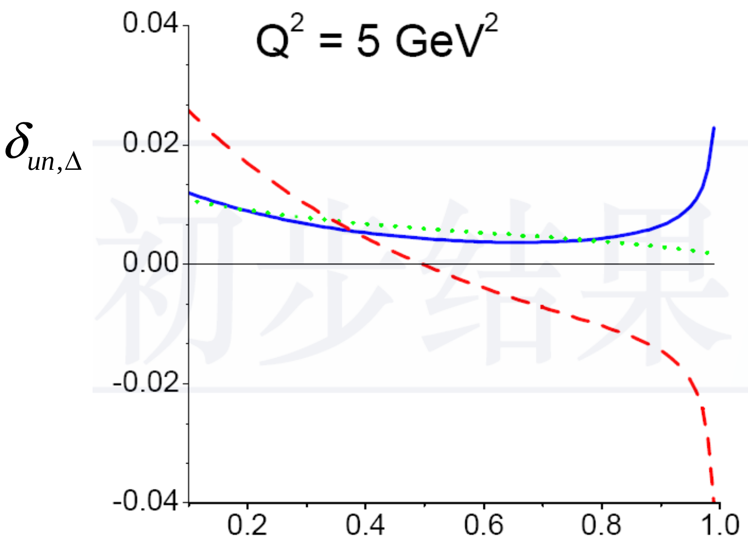
Also the form factors F_Δ are modified.

By these modification, the new results:

TPE in ep: results for un-polarized case



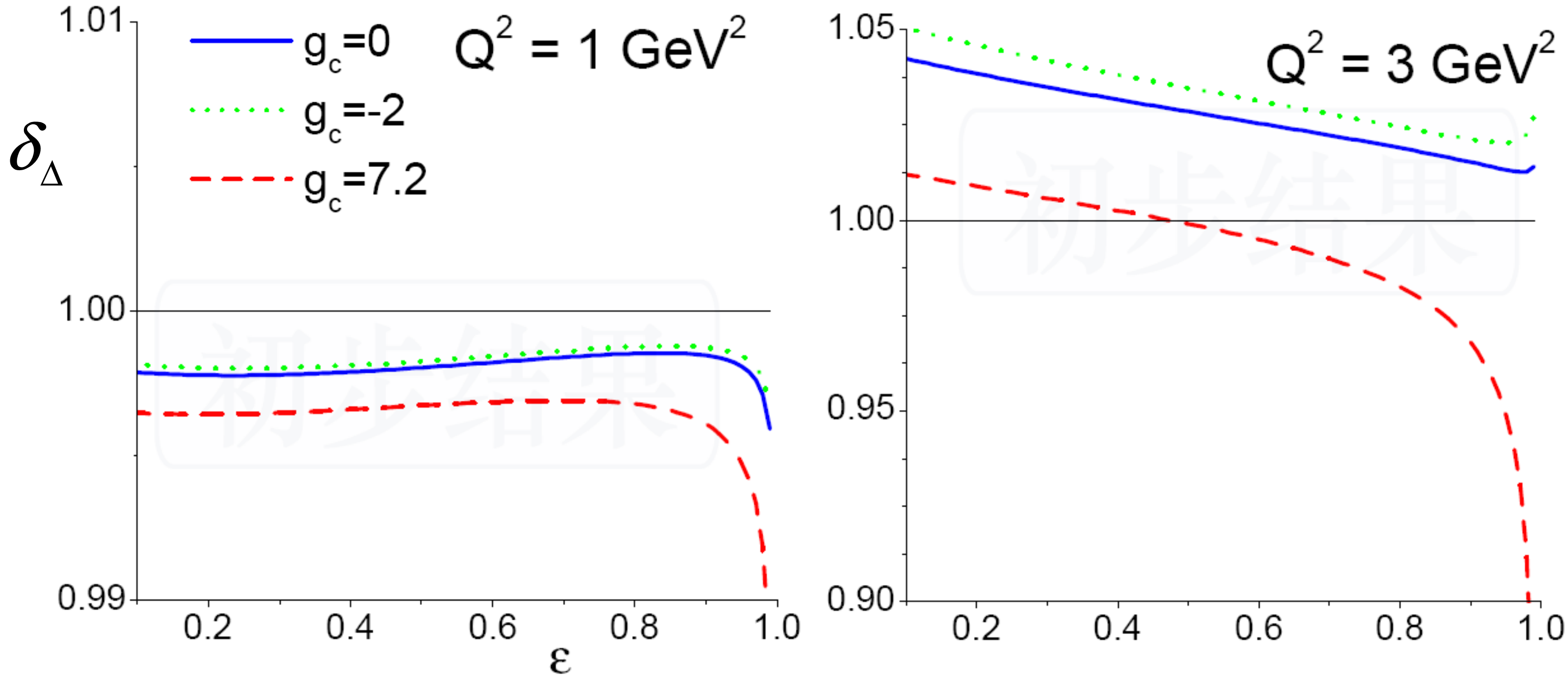




$$\delta_{un,\Delta} \equiv \frac{\sigma_{un}^{2\gamma(\Delta)}}{\sigma_{un}^{1\gamma}}$$

using the new vertex relation

TPE in ep: results for polarized case

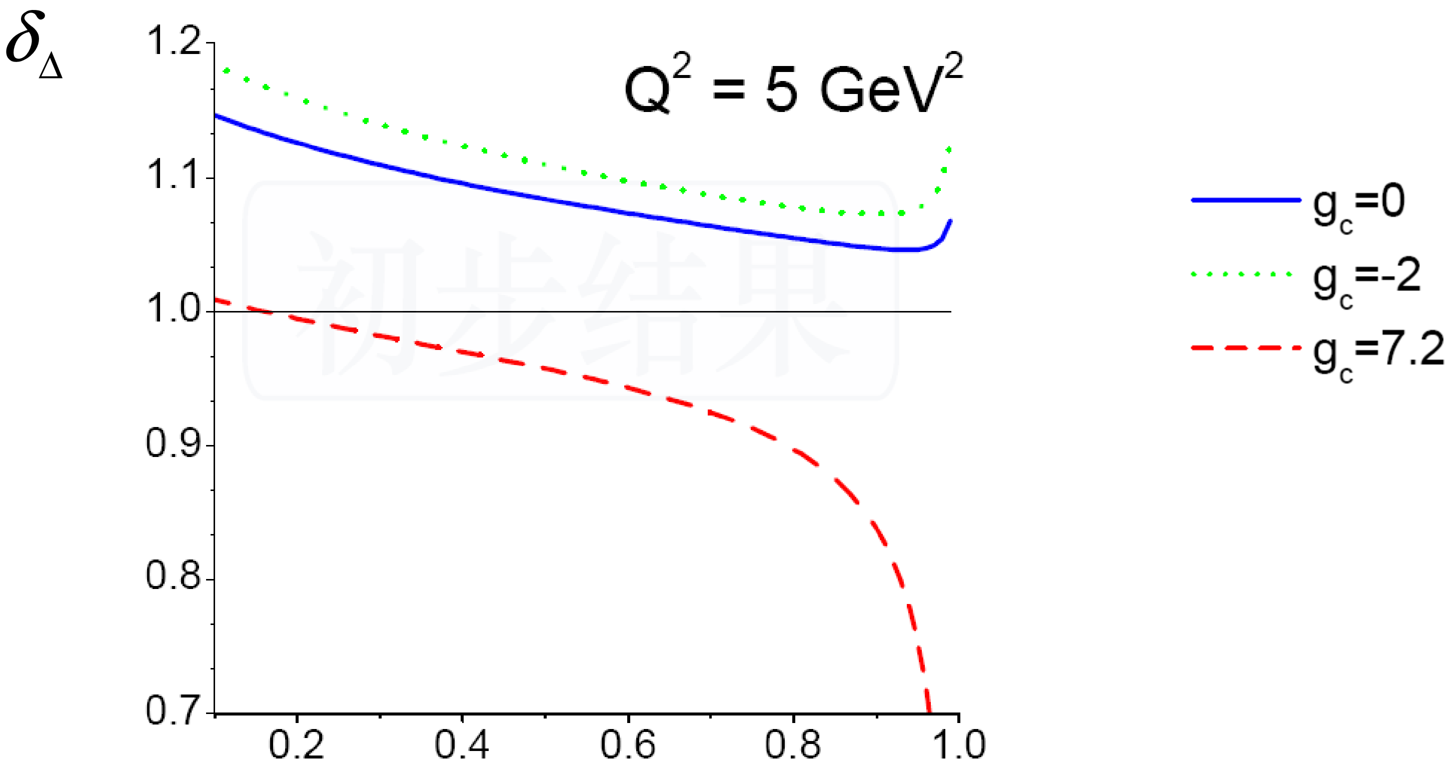


$$\delta_{\Delta} \equiv \frac{P_T^{1\gamma+2\gamma(\Delta)} \,/\, P_T^{1\gamma}}{P_L^{1\gamma+2\gamma(\Delta)} \,/\, P_L^{1\gamma}}$$

using the new vertex relation

$$R_{Phy} = R_{Exp} /(\delta_N + \delta_{\Delta})$$

TPE in ep: results for polarized case, high Q^2



For high Q^2 and large ε , it shows surprising properties: the correction to P_T is very large in this region.

Un-physical ?

TPE in ep: modify?

Problem ?

Is hadronic model not reasonable when including Delta(1232) at such Q^2 and ε ? What is the valid region?

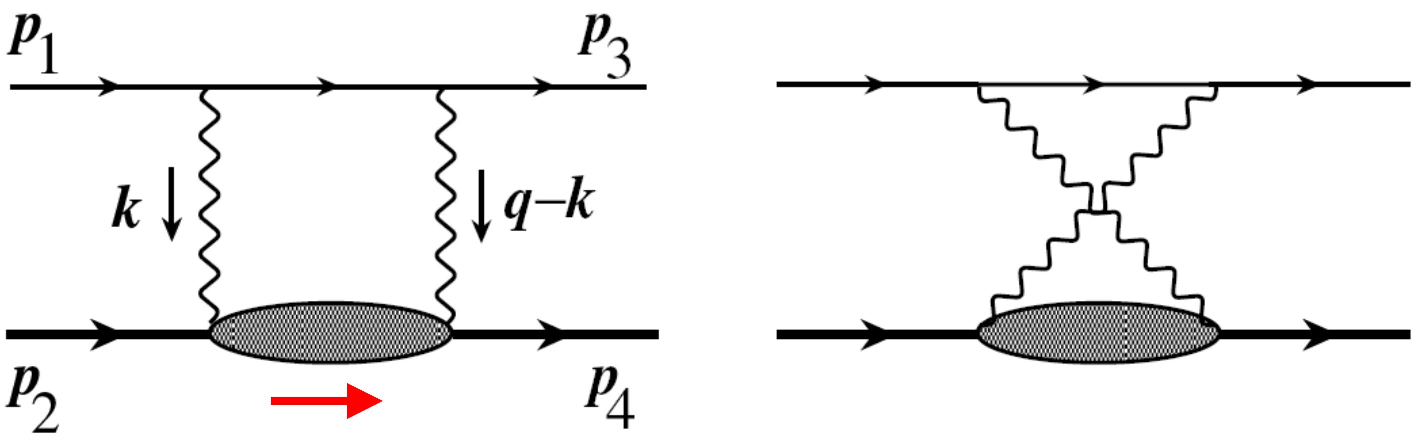
$$Q^2 = 3, 5\,?$$

$$\varepsilon = 0.6, 0.8, 0.9\,?$$

or modify?

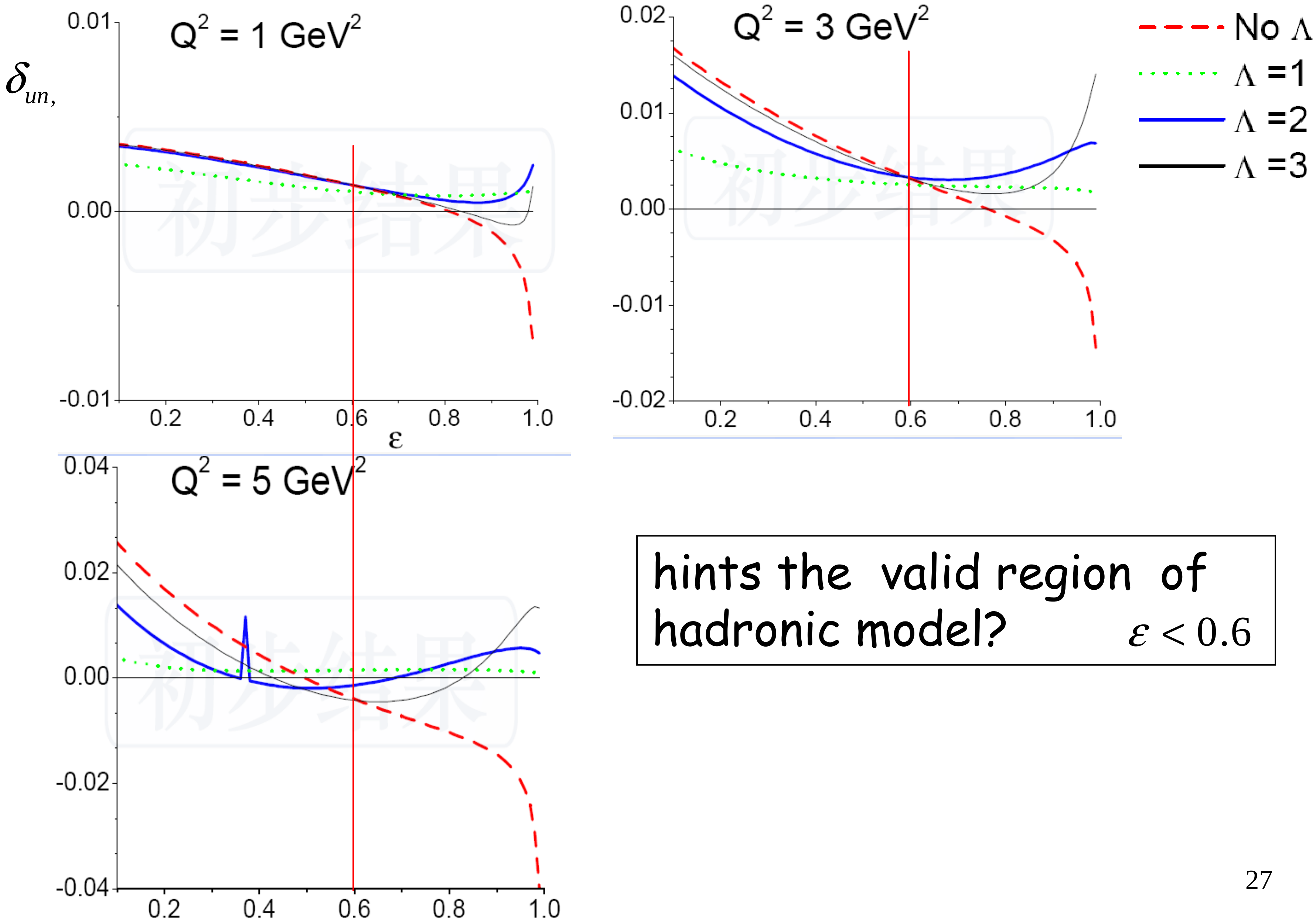
TPE in ep: modify?

to regular the behavior when including Delta(1232), we add a factor by hand in the loop:



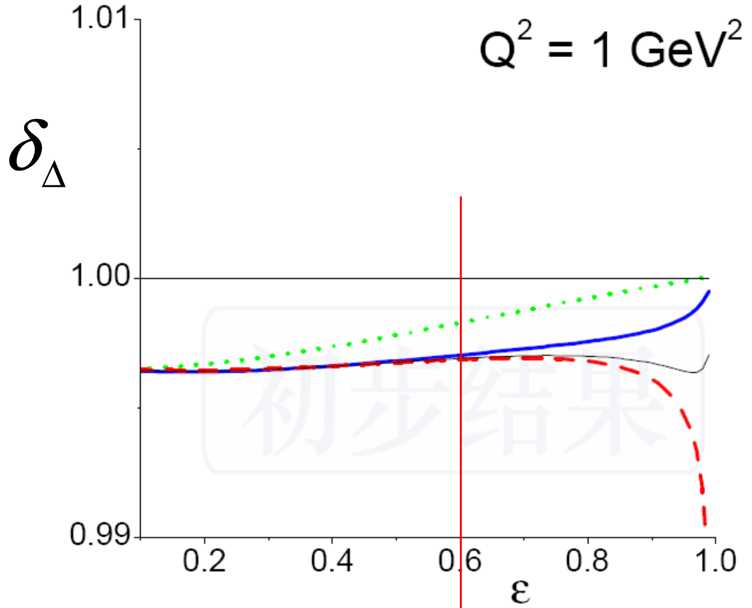
$$f(p_2+k)=\frac{\Lambda^4}{((p_2+k)^2-M_\Delta^2)^2+\Lambda^4}$$

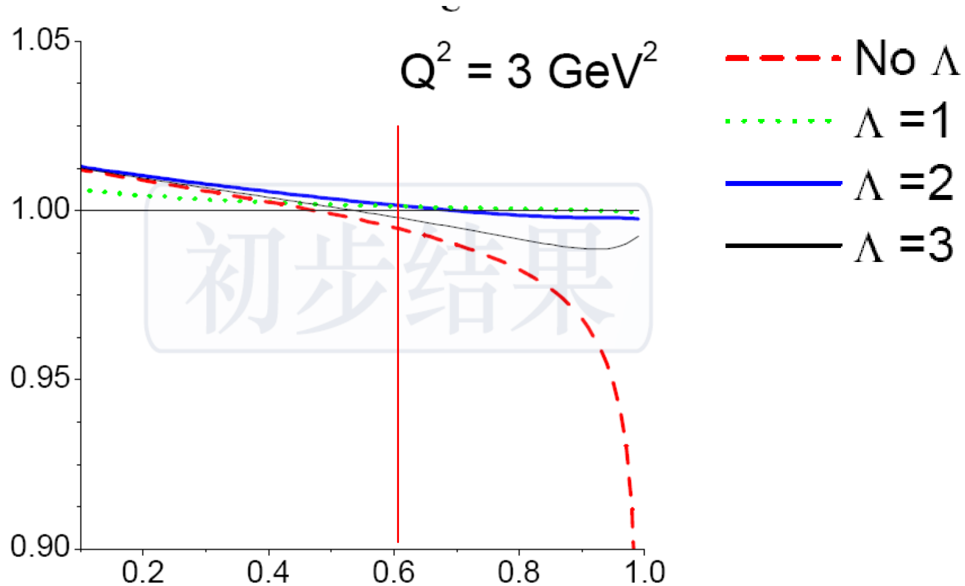
TPE in ep: results for un-polarized case

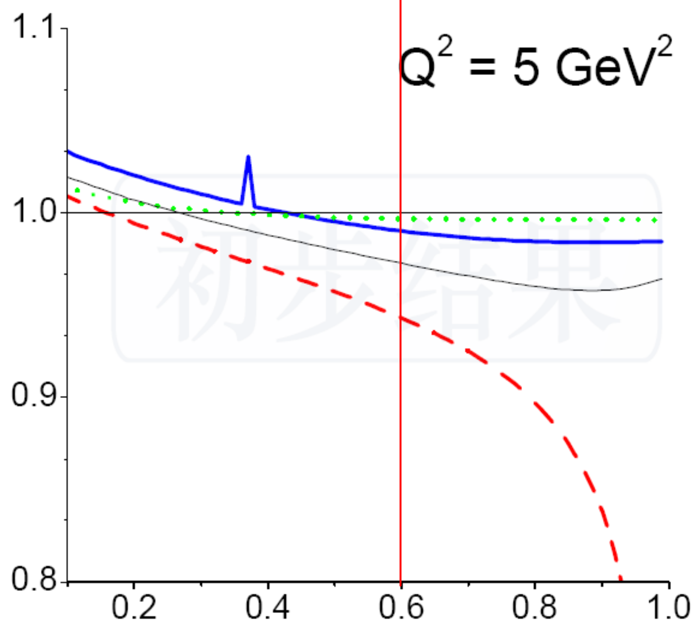


hints the valid region of
hadronic model? $\varepsilon < 0.6$

TPE in ep: results for polarized case







hints the valid region of hadronic model? $\varepsilon < 0.6$

Does not change the total correction to ratio in polarization methods:
N+D: positive

TPE in ep: summary

1. The experiment (JLab/Hall C exp. E-04-019) maybe distinguish which model is reasonable for the TPE correction in polarization methods:

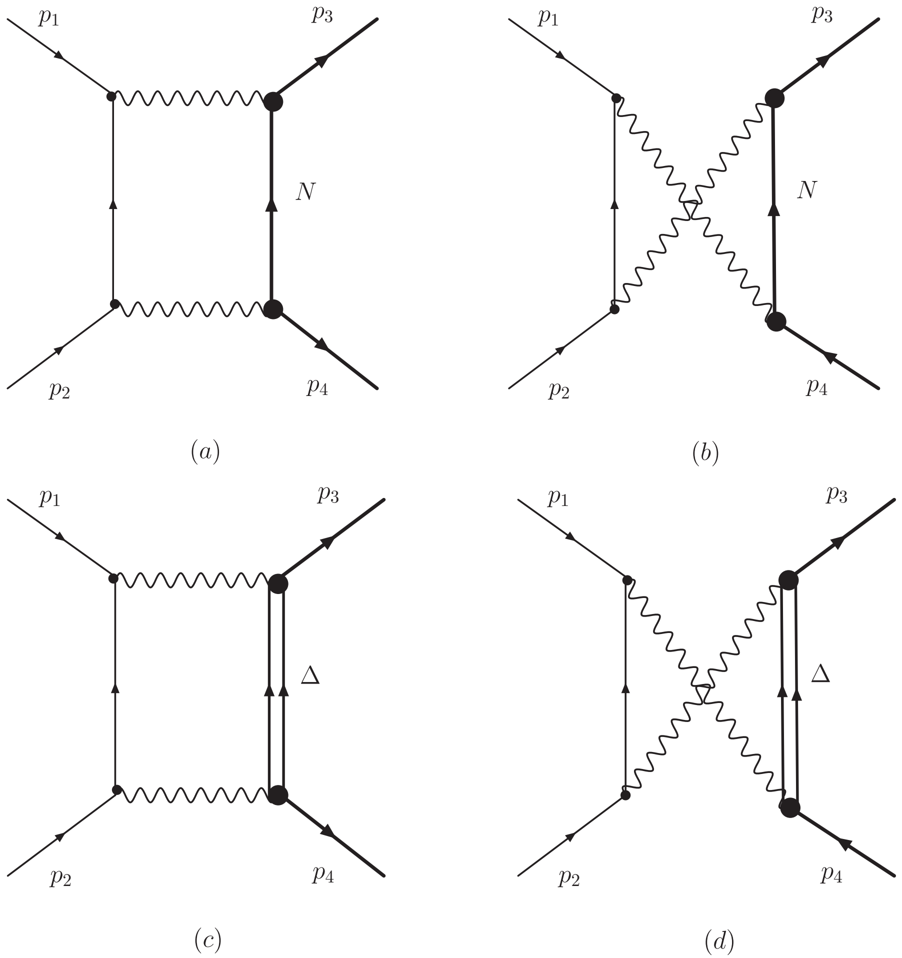
   positive or negative?

2. How to combine those methods is still a problem (the valid region of different methods, not only for the TPE, but also the for γZ exchange).

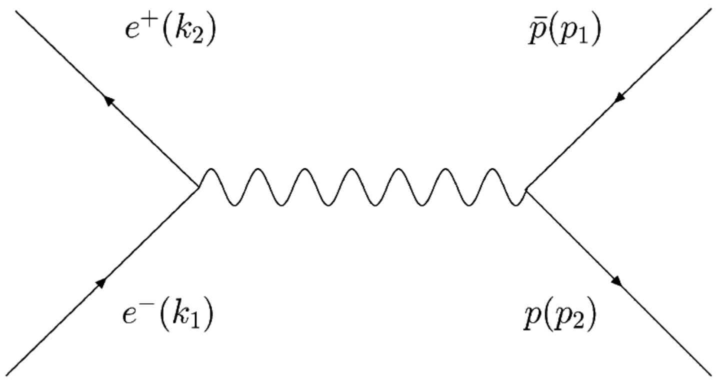
PRL99,262001(2007), PRC79,062501(2009)(RC);
PRL100,082003(2008), PRC79, 055201 (2009);
PRL102,091806(2009);
arXiv:0903.1098

TPE in time-like region

We simply apply the hadronic model to $e^+e^- \to p\bar{p}$ to give an estimate of two-photon-annihilation correction



TPE in time-like region: observables



one-photon annihilation

Un-polarized cross section

$$\left(\frac{d\sigma}{d\Omega}\right)_{CM} = \frac{\alpha^2\sqrt{1-4M_N^2/q^2}}{4q^2} \times \left(|G_M|^2(1+\cos^2\theta) + \frac{1}{\tau}|G_E|^2\sin^2\theta\right)$$

TPE in time-like region: observables

Consider the un-polarized incoming positron, longitudinally polarized incoming electron, and the polarized antiproton in the final state,the cross section

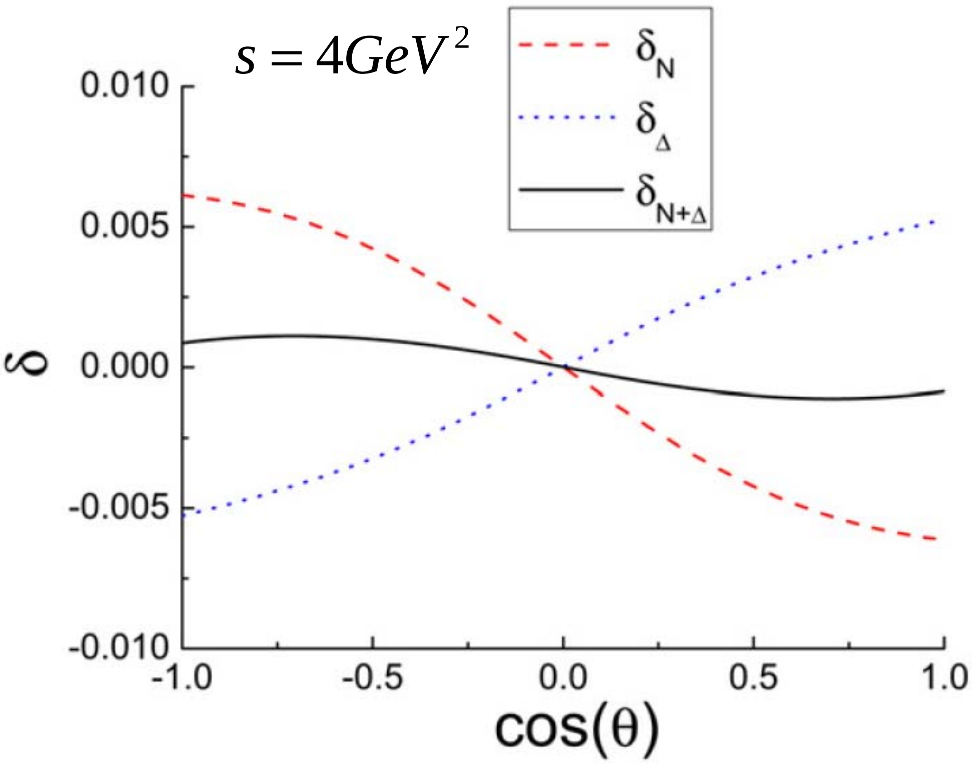
$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_{un}}{d\Omega}[1 + P_y\xi_y + \lambda_e P_x\xi_x + \lambda_e P_z\xi_z].$$

double spin polarization observables P_x and P_z

$$P_x = -\frac{2\sin\theta}{D\sqrt{\tau}}\{\mathrm{Re}[\tilde{G}_M\tilde{G}_E^*] + \mathrm{Re}[\tilde{G}_M\tilde{F}_3^*]\sqrt{\tau(\tau-1)}\cos\theta\}$$

$$P_z = \frac{2}{D}\{|\tilde{G}_M|^2\cos\theta - \mathrm{Re}[\tilde{G}_M\tilde{F}_3^*]\sqrt{\tau(\tau-1)}\sin^2\theta\}$$

TPE time-like: results for un-polarized case



properties:

1: odd function of $\cos\theta$

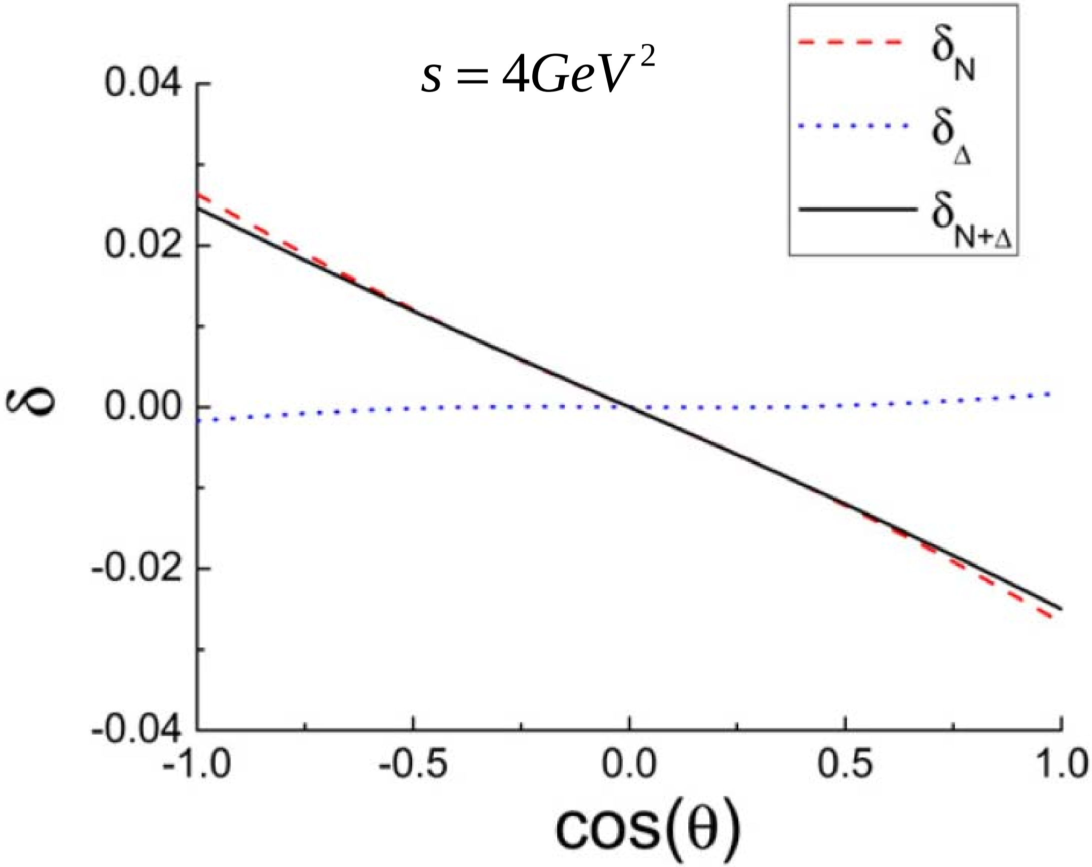
2: contributions from N and Delta(1232) intermediate state are opposite.

3: total TPE contributions are small;

$$\delta_{2\gamma} = 2\frac{Re\{\overline{\mathcal{M}_{2\gamma}\mathcal{M}_0^\dagger}\}}{|\mathcal{M}_0|^2}$$

D.Y. Chen, Y.B.Dong, H.Q.Zhou, PRC78,045208(2008), PLB675,305(2009).

TPE time-like: results for P_x



$$\delta(P_x) = \frac{P_x^{1\gamma\otimes 2\gamma}}{P_x^{1\gamma}}$$

properties:

1: odd function of $\cos\theta$

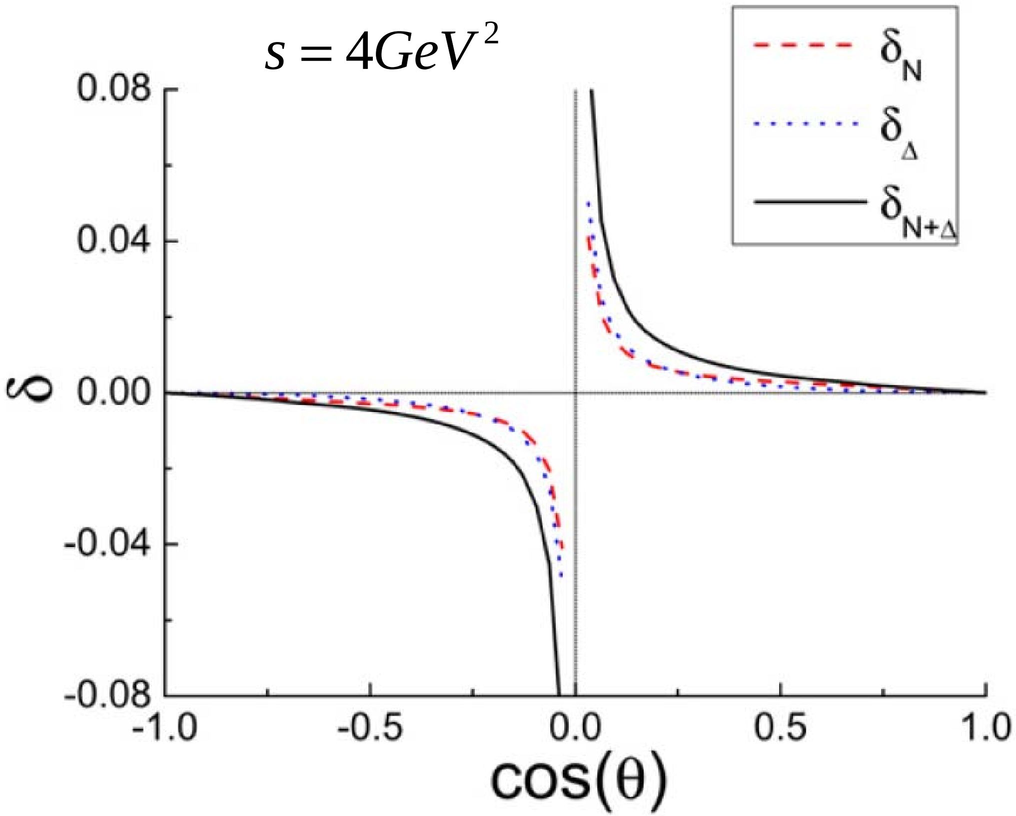
2: main contribution from N and the Delta(1232) contribution is very small.

3: relatively larger at

$$\cos\theta = \pm 1$$

4: the absolute contributions are small.

TPE time-like: results for P_z



properties:

1: odd function of $\cos\theta$

2: $P_z^{1\gamma}(\pi/2) = 0$

$P_z^{1\gamma\otimes 2\gamma}(\pi/2) \neq 0$

3: non-zero of P_z at $\pi/2$ reveal TPE and the large correction suggests it may deserve to be considered in the experiment near $\pi/2$.

$$\delta(P_z) = \frac{P_z^{1\gamma\otimes 2\gamma}}{P_z^{1\gamma}}$$

Summary

- TPE in ep scattering played important roles while its corrections to polarization observables are not clear now.

- How to combine the four methods is still a problem.

- TPE contributions in $e^+e^- \to p\bar{p}$ at small s are usually small and is relative larger to polarization observable P_z. This may be a considerable quantity to see TPE directly.

- Other methods to estimate the TPE contribution in time-like region?