Nucleon form factors: the space-time connection

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Abstract Analyticity of nucleon form factors allows to derive sum rules which, using space-like and time-like data as input, can give unique information about behaviors in energy regions not experimentally accessible. Taking advantage from new time-like data on proton-antiproton differential cross section and hence the possibility to separate electric and magnetic form factors also in the time-like region, we verify the consistency of the asymptotic behavior predicted by the perturbative QCD for the proton magnetic form factor.

Key words Dispersion relations, nucleon form factors, pQCD

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1 Introduction

Nucleon electromagnetic form factors (FFs) play a key role in understanding the hadronic dynamics. They describe the coupling between a photon and a nucleon pair, and represent the only quantities, connected to the nucleon quark structure, that can be measured.

Physical FFs are defined as limit values for real q^2 (q is 4-momentum transferred by the photon) of Lorentz scalar functions that are analytic in the q^2 complex plane with a cut along the real axis (time-like region), from the theoretical threshold $q^2_{\rm theo}\equiv(2M_\pi)^2$ up to infinity. In the space-like region ($q^2<0$), where they are real functions, FF values can be extracted from the differential cross section of the elastic scattering $\mathrm{eN}\to\mathrm{eN}$ (N stands for nucleon). In the time-like region, starting from the theoretical threshold, FFs become complex functions, their moduli can be extracted above the physical threshold $q^2_{\rm phys}\equiv(2M_{\rm N})^2$, studying the angular distribution of the $\mathrm{e^+e^-}\to\mathrm{N\overline{N}}$ cross section.

The FF asymptotic behavior predicted by the perturbative QCD is a power law [1] that can be obtained either in terms of dimensional considerations, or as a consequence of a minimal gluon exchange among the constituent quarks, needed to share the photon transfer momentum.

More in detail, disregarding logarithmic corrections of the strong coupling constant, the QCD power laws, in the space-like limit: $q^2\to-\infty$, for each nucleon FFs are

$$F_1\sim(-q^2)^{-2},\quad F_2(q^2)\sim(-q^2)^{-3}$$

$$\left.\begin{array}{l} G_{\rm E}(q^2)=F_1(q^2)+\dfrac{q^2}{4M_{\rm N}^2}F_2(q^2)\\ G_{\rm M}(q^2)=F_1(q^2)+F_2(q^2)\end{array}\right\}\sim(-q^2)^{-2},\quad(1)$$

where F_1 and F_2 are the Dirac and Pauli FFs, which account for the non-spin flip and the spin flip part (further suppression factor $1/q^2$) of the nucleon electromagnetic current. $G_{\rm E}$ and $G_{\rm M}$ are, instead, the so-called electric and magnetic Sachs FFs, in the Breit frame (for small space-like transfer momenta) they represent the Fourier transforms of the charge and magnetization distributions in the nucleon.

2 Dispersion relations

Dispersion relations (DRs) allow to connect values of an analytic function in different regions of its domain. Taking advantage from analyticity and vanishing asymptotic behavior of FFs (see Sec. 1), we may define the integral relation

$$G(q^2)=\frac{1}{\pi}\int_{q^2_{\rm theo}}^{\infty}\frac{{\rm Im}\,G(s)}{s-q^2}{\rm d}s,\qquad(2)$$

with $q^2\le q^2_{\rm theo}$, for a generic FF $G(q^2)$. This DR states that real values of $G(q^2)$ can be obtained at

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any $q^2 < q^2_{\rm theo}$ integrating the imaginary part over the upper edge of the time-like cut $(q^2_{\rm theo}, \infty)$. However to use Eq. (2), in case of nucleons, we have to face two serious issues:

- the imaginary part of FFs is not measurable, one should relay in phenomenological and non-rigorous techniques to extract it from cross section data;
- even though the imaginary part was obtained from the data, its values could cover only a portion of the integration interval, starting from the physical threshold $q^2_{\rm phys}$. The so-called "unphysical region", $(q^2_{\rm theo}, q^2_{\rm phys})$, where we expect the main contributions from intermediate light-meson resonances, is not experimentally accessible.

To get around these problems we use an idea proposed for the first time in 1974 in Ref. [2].

3 Sum rule

The idea [2] consists in using the DR of eq. (2) for the function:

$$\phi(z) = A_L(z,s)\frac{\ln G(z)}{\sqrt{q^2_{\rm theo} - z}}, \tag{3}$$

where $A_{\rm L}(z,s)$, with s real and positive, is an analytic function used to suppress the FF contribution in the time-like region $(0,s)$, and it can be chosen requiring

$$\int_0^s A_L(z,s)^2 {\rm d}z \ll 1\,.$$

Following the suggestion given in Ref. [2] we use the definition

$$A_L(z,s) = \sum_{l=0}^{L}\frac{2l+1}{(L+1)^2}P_l\left(1-2\frac{\sqrt{s}-\sqrt{z}}{\sqrt{s}+\sqrt{z}}\right), \tag{4}$$

where P_l is the Legendre polynomial of degree l, while the upper limit L represents an "attenuation-power indicator". Following the definition of Eq. (4), $A_L(z,s)$ is analytic in z with a cut along the whole negative real axis. The imaginary part of $\phi(x)$ is then (x is real)

$$\operatorname{Im}\phi(x) = \begin{cases} \dfrac{\operatorname{Im} A_L(x,s)\ln G(x)}{\sqrt{q^2_{\rm theo}-x}} & x \le 0 \\[2ex] 0 & 0 < x < q^2_{\rm theo} \\[2ex] \dfrac{A_L(x,s)\ln|G(x)|}{\sqrt{x-q^2_{\rm theo}}} & x \ge q^2_{\rm theo}\,. \end{cases} \tag{5}$$

We consider the proton magnetic FF normalized to its magnetic moment $\mu_{\rm p}$, i.e.: $G(q^2) = G^{\rm p}_{\rm M}(q^2)/\mu_{\rm p}$, having no poles (from analyticity), nor zeros (our assumption) in the q^2 complex plane, the $\phi(z)$ function (see Eq. (3)) is still analytic and the DR of Eq. (2) now reads

$$\phi(q^2) = \frac{1}{\pi}\int_{-\infty}^0 \frac{\operatorname{Im}\phi(t)}{t-q^2}{\rm d}t + \frac{1}{\pi}\int_{q^2_{\rm theo}}^{\infty}\frac{\operatorname{Im}\phi(s)}{s-q^2}{\rm d}s\,,$$

for $0 < q^2 < q^2_{\rm theo}$. In particular at $q^2 = 0$, having the normalizations $G(0) = 1$ and $\phi(0) = 0$, the previous relation becomes the identity

$$\int_{-\infty}^0 \frac{\operatorname{Im}\phi(t)}{t}{\rm d}t = -\int_{q^2_{\rm theo}}^{\infty}\frac{\operatorname{Im}\phi(s)}{s}{\rm d}s\,, \tag{6}$$

in terms of A_L and FF, i.e. using the definition of Eq. (5), we have

$$\int_{-\infty}^0 \frac{\operatorname{Im}A_L(t,s)\ln G(t)}{t\sqrt{q^2_{\rm theo}-t}}{\rm d}t = -\int_{q^2_{\rm theo}}^{\infty}\frac{A_L(s',s)\ln|G(s')|}{s'\sqrt{s'-q^2_{\rm theo}}}{\rm d}s' \simeq -\int_{s}^{\infty}\frac{A_L(s',s)\ln|G(s')|}{s'\sqrt{s'-q^2_{\rm theo}}}{\rm d}s'\,. \tag{7}$$

The approximate identity of Eq. (7) holds thanks to the attenuation in the region $(0,s)$ provided by the function $A_L(z,s)$.

Finally the sum rule we want to use is obtained from Eq. (7) in the special case with $s = q^2_{\rm phys}$

$$\int_{-\infty}^0 \frac{\operatorname{Im}A_L(t,q^2_{\rm phys})\ln G(t)}{t\sqrt{q^2_{\rm theo}-t}}{\rm d}t \simeq -\int_{q^2_{\rm phys}}^{\infty}\frac{A_L(s',q^2_{\rm phys})\ln|G(s')|}{s'\sqrt{s'-q^2_{\rm theo}}}{\rm d}s'\,. \tag{8}$$

This identity involves only measurable quantities, i.e.: real values of the proton magnetic FF in the space-like region (left-hand side), and modulus, only from the physical threshold, in the time-like region (right-hand side).

4 Check for the asymptotic behavior

We verify the compatibility of space and time-like data on $G^{\rm p}_{\rm M}(q^2)$ with the QCD power law behavior given in Eq. (1), using the "space-time connection" provided by the sum rule of Eq. (8).

More in detail, in the space-like region we define the real FF as a combination of a fit of several data sets [3] (247 data points) and a power law, i.e.

$$G_{\rm SL}(q^2) = \begin{cases} G_{\rm SL}^{\rm fit}(q^2) & q^2_{\rm min} \le q^2 \le 0 \\ \\ G_{\rm SL}^{\rm fit}(q^2_{\rm min})(q^2_{\rm min}/q^2)^n & q^2 \le q^2_{\rm min}\,, \end{cases} \tag{9}$$

where $q^2_{\rm min} \sim -30$ GeV^2 is the energy of the lower data point.

In the time-like region the situation is more troublesome, indeed all data are on the modulus of an effective FF which corresponds to $|G_{\rm M}^{\rm p}(q^2)|$ only when $|G_{\rm M}^{\rm p}(q^2)| = |G_{\rm E}^{\rm p}(q^2)|$, but that happens, by definition (see Eq. (1)), solely at $q^2 = q^2_{\rm phys}$! Hence, to extract genuine values of $|G_{\rm M}^{\rm p}(q^2)|$ we used the BaBar data on the $e^+e^- \to p\overline{p}$ total cross section and angular distribution [4] together with a dispersive technique to disentangle $|G_{\rm E}^{\rm p}(q^2)|$ and $|G_{\rm M}^{\rm p}(q^2)|$ [5].

Similarly to what we did in the space-like region, the modulus of the FF in the time-like region is defined as

$$G_{\rm TL}(q^2) = \begin{cases} G_{\rm TL}^{\rm fit}(q^2) & 0 \le q^2 \le q^2_{\rm max} \\ \\ G_{\rm TL}^{\rm fit}(q^2_{\rm max})(q^2_{\rm max}/q^2)^n & q^2 \ge q^2_{\rm max}\,, \end{cases} \tag{10}$$

where $q^2_{\rm max} \sim 20$ GeV^2 is the energy of the higher BaBar data point.

In both definitions of Eq. (9) and Eq. (10) we used the same power law as a consequence of the Phragmèn-Lindelöf theorem [6]. Such a theorem states that, not only the power n which rules the asymptotic behavior, but also the limit must be the same, i.e.

$$\lim_{q^2 \to -\infty} G_{\rm SL}(q^2) = \lim_{q^2 \to +\infty} G_{\rm TL}(q^2)\,.$$

It follows that the sum rule of Eq. (8), once all the theoretical and experimental information have been considered, becomes an equation with only one unknown the power n, and the result we obtained is

$$n = 2.27 \pm 0.36\,.$$

Figure 1 summarizes this result. The shaded bands represent: the fit functions in the data regions, and the power laws at high space and time-like energies ($q^2 < -30$ GeV^2 and $q^2 > 20$ GeV^2). The lined central area is the unphysical region, which does not contribute to the sum rule, being suppressed by the function $A_L(q^2_{\rm phys}, z)$.

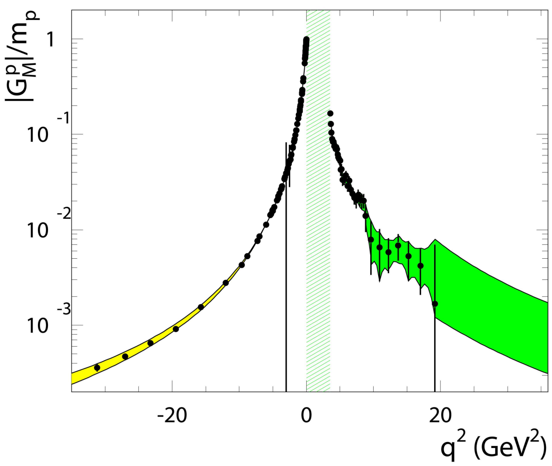

Fig. 1. Modulus of the normalized proton magnetic FF in space-like (left) and time-like (right) region. The bands represent the obtained description (see text), the points are the data [3–5]. The lined central area is the unphysical region.

In conclusion, using the sum rule of Eq. (8), based on analyticity properties of FFs, we have shown that experimental data in space and time-like region are consistent with the QCD asymptotic behavior. In particular, we found a power law for $G_{\rm M}^{\rm p}(q^2)$ which is in good agreement with the perturbative QCD expectation.

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