Time-like Baryon Form Factors and Dispersion Relations

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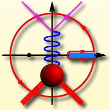
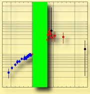
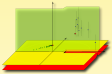
Agenda



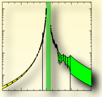
Baryon form factors and dispersion relations



Space and time data on G_E^p/G_M^p

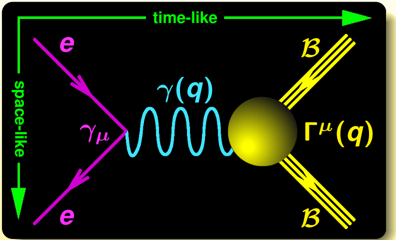


Space and time-like G_E^p/G_M^p via DR's



Asymptotic G_M^p from a DR sum rule

Baryon form factors and cross sections

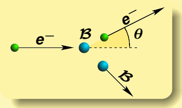


Baryon current operator (Dirac & Pauli)

$$\Gamma^\mu(q) = \gamma^\mu F_1(q^2) + \frac{i}{2M_{\mathcal{B}}}\sigma^{\mu\nu} q_\nu F_2(q^2)$$

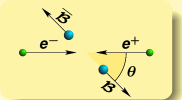
Electric and Magnetic Form Factors

$$G_E(q^2) = F_1(q^2) + \tau F_2(q^2) \qquad \tau = \frac{q^2}{4M_{\mathcal{B}}^2}$$
$$G_M(q^2) = F_1(q^2) + F_2(q^2)$$



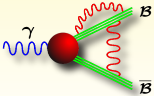
Elastic scattering (Rosenbluth)

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 E_e' \cos^2\frac{\theta}{2}}{4E_e^3 \sin^4\frac{\theta}{2}}\left[G_E^2 - \tau\left(1+2(1-\tau)\tan^2\frac{\theta}{2}\right)G_M^2\right]\frac{1}{1-\tau}$$



Annihilation

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 \beta C}{4q^2}\left[(1+\cos^2\theta)|G_M|^2 + \frac{1}{\tau}\sin^2\theta |G_E|^2\right] \qquad \beta = \sqrt{1-\frac{1}{\tau}}$$



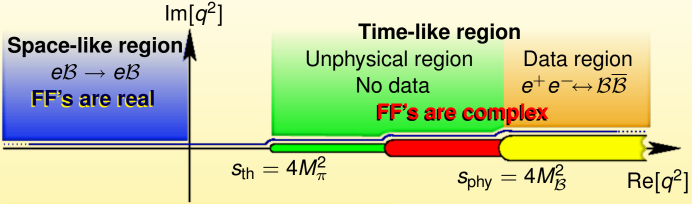
Coulomb correction

$$C = \frac{\beta/\alpha\pi}{1-\exp(-\beta/\alpha\pi)}$$

- $\mathcal{B}\overline{\mathcal{B}}$ Coulomb interaction as FSI
- Only S-wave

Analyticity of baryon form factors

q^2-complex plane



Crossing: tot. helicity $= \begin{cases} 1 \Rightarrow G_E \\ 0 \Rightarrow G_M \end{cases}$

$$G_E(4M_\mathcal{B}^2) = G_M(4M_\mathcal{B}^2)$$

QCD counting rule constrains the asymptotic behaviour

Matveev, Muradyan, Tevkheldize, Brodsky, Farrar

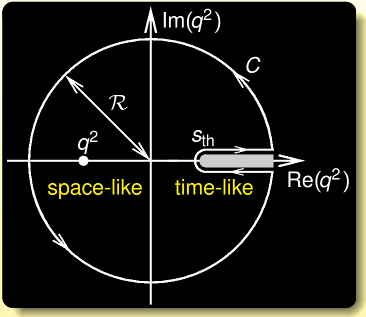
Counting rule: $q^2 \to -\infty$
$i = 1$ **Dirac,** $i = 2$ **Pauli FF**

$$F_i(q^2) \propto (-q^2)^{-(i+1)} \Rightarrow G_{E,M} \propto (-q^2)^{-2}$$

Analyticity: $q^2 \to \pm\infty$
(Phragmèn Lindelöf)

$$G_{E,M}(-\infty) = G_{E,M}(+\infty)$$

Dispersion relations



- The form factors are **analytic** on the q^2-plane with a **multiple cut** $(s_{th} = 4M_\pi^2, \infty)$
- **Dispersion relation for the imaginary part** $(q^2 < 0)$

$$G(q^2) = \lim_{\mathcal{R}\to\infty} \frac{1}{2\pi i} \oint_C \frac{G(z)dz}{z - q^2} = \frac{1}{\pi}\int_{s_{th}}^{\infty} \frac{\mathrm{Im}G(s)ds}{s - q^2}$$

- **Dispersion relation for the logarithm** $(q^2 < 0)$

B.V. Geshkenbein, Yad. Fiz. 9 (1969) 1232.

$$\ln G(q^2) = \frac{\sqrt{s_{th} - q^2}}{\pi} \int_{s_{th}}^{\infty} \frac{\ln|G(s)|ds}{(s - q^2)\sqrt{s - s_{th}}}$$

Experimental inputs

- Space-like data on the real values of FF's from: $e^- \mathcal{B} \to e^- \mathcal{B}$ and $e^{-\uparrow} \mathcal{B} \to e^- \mathcal{B}^{\uparrow}$, with polarization
- Time-like data on moduli of FF's from: $e^+e^- \to \mathcal{B}\overline{\mathcal{B}}$
- Time-like data on G_E-G_M relative phase from: $e^+e^- \to \mathcal{B}^{\uparrow}\overline{\mathcal{B}}$ (pol.)

Theoretical ingredients

- Analyticity $\Rightarrow$ convergence relations
- Normalization and threshold values
- Asymptotic behavior

$\Downarrow$

super-convergence relations

Dispersive approach: advantages and drawbacks

Advantages

- DR's are based on unitarity and analyticity $\Rightarrow$ **model-independent approach**
- DR's relate data from different processes in different energy regions

$$\left[\begin{array}{c}\textbf{space-like}\\ \textbf{form factor}\\ e\mathcal{B}\longrightarrow e\mathcal{B}\end{array}\right] = \int \left[\begin{array}{c}\textbf{Im(form factor) or ln|form factor|}\\ \textbf{over the time-like cut } (s_{th},\infty)\\ e^+e^-\longrightarrow \mathcal{B}\overline{\mathcal{B}} + \textbf{theory}\end{array}\right]$$

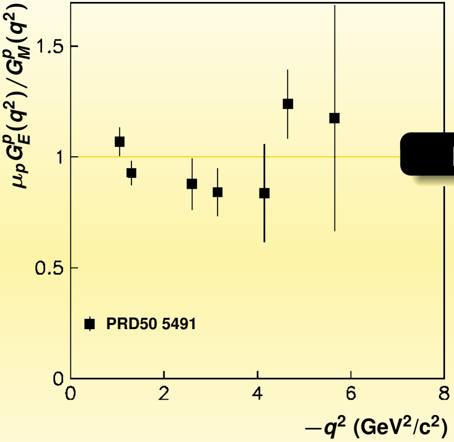
- Normalizations and theoretical constraints can be directly implemented
- Form factors can be computed in the whole q^2-complex plane

Drawbacks

- Very long-range integration
  - **Remedy #1**
    pQCD power laws
  - **Remedy #2**
    Subtracted DR's
- **No data in the unphysical region, crucial in dispersive analyses**

The data

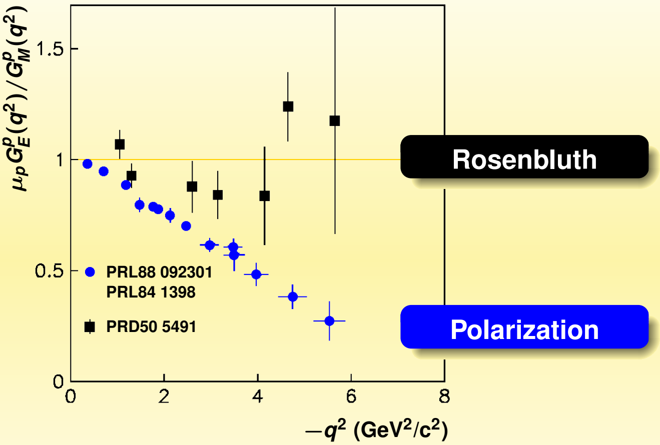
Space-like G_E^p / G_M^p measurements



Rosenbluth

Radiative corrections of polarization technique ≪ Radiative corrections in Rosenbluth method

Space-like G_E^p / G_M^p measurements



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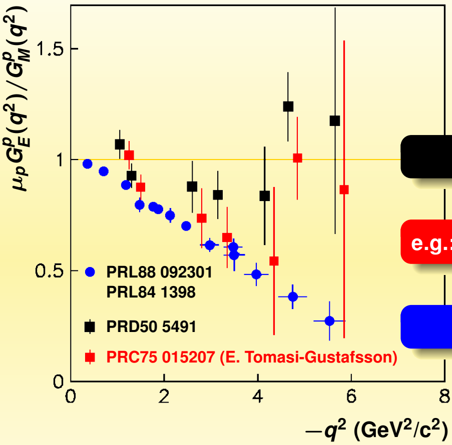
Space-like G_E^p / G_M^p measurements



Rosenbluth

e.g.: Ros. + RC, 2γ,...

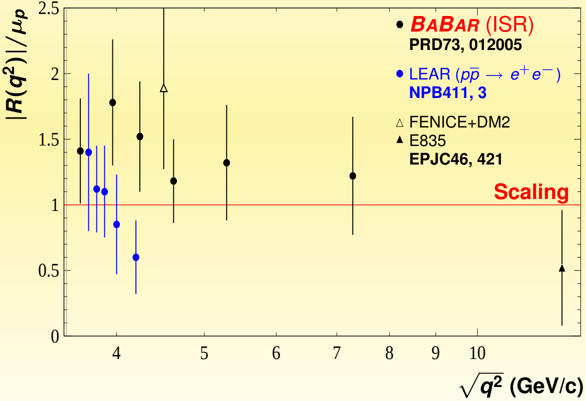
Polarization

Radiative corrections of polarization technique $\ll$ **Radiative corrections in Rosenbluth method**

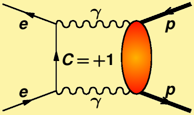
Time-like $|G_E^p/G_M^p|$ measurements

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2\beta C}{2q^2}|G_M^p|^2\left[(1+\cos^2\theta)+\frac{4M_p^2}{q^2\mu_p^2}\sin^2\theta|R|^2\right]$$

$$R(q^2) = \mu_p \frac{G_E^p(q^2)}{G_M^p(q^2)}$$



$\gamma\gamma$ exchange



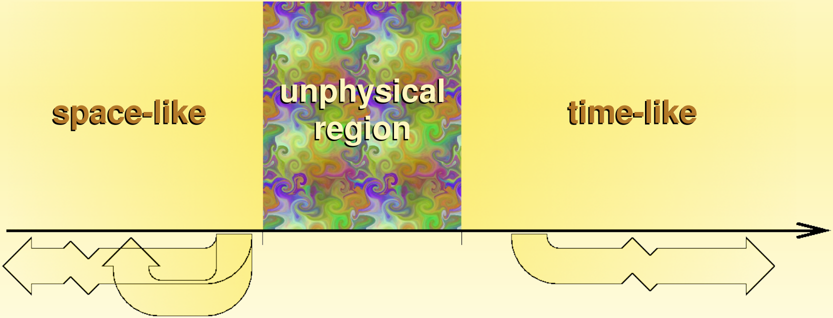
$\gamma\gamma$ exchange interferes with the Born term

Asymmetry in angular distributions

[E. Tomasi-Gustafsson and Q. H. Zhou]

Dispersive analysis of the ratio

$$R = \mu_p G_E^p / G_M^p$$



The dispersive approach for $R(q^2)$

We start from the imaginary part of the ratio $\boldsymbol{R(q^2)}$,
written in the most general and model-independent way as

$$I(q^2) \equiv \mathrm{Im}[R(q^2)] = \text{series of orthogonal polynomials}$$

Theoretical constraints can be applied directly on this function $I(q^2)$

Dispersion Relations →

The function $R(q^2)$ is reconstructed in time and space-like regions

Additional theoretical conditions and the experimental constraints can be imposed on the obtained analytic expression of $\boldsymbol{R(q^2)}$

Parameterization and constraints

ImR is parameterized by two series of orthogonal polynomials $T_i(x)$

$$\mathrm{Im}R(q^2) \equiv I(q^2) = \begin{cases} \sum_i C_i T_i(x) & x = \frac{2q^2 - s_{\mathrm{phy}} - s_{\mathrm{th}}}{s_{\mathrm{phy}} - s_{\mathrm{th}}} \quad s_{\mathrm{th}} \leq q^2 \leq s_{\mathrm{phy}} \\ \sum_j D_j T_j(x') & x' = \frac{2s_{\mathrm{phy}}}{q^2} - 1 \quad q^2 > s_{\mathrm{phy}} \end{cases}$$

Theoretical conditions on Im$R(q^2)$

- $R(4M_\pi^2)$ is real $\Longrightarrow I(4M_\pi^2) = 0$
- $R(4M_N^2)$ is real $\Longrightarrow I(4M_N^2) = 0$
- $R(\infty)$ is real $\Longrightarrow I(\infty) = 0$

Theoretical conditions on $R(q^2)$

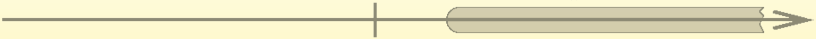
- Continuity at $q^2 = 4M_\pi^2$
- $R(4M_N^2)$ is real and Re$R(4M_N^2) = \mu_p$

Experimental conditions on $R(q^2)$ and $|R(q^2)|$

- Space-like region ($q^2 < 0$) data for R from JLab and MIT-Bates
- Time-like region ($q^2 \geq 4M_N^2$) data for $|R|$ from FENICE+DM2, *BABAR* , and E835

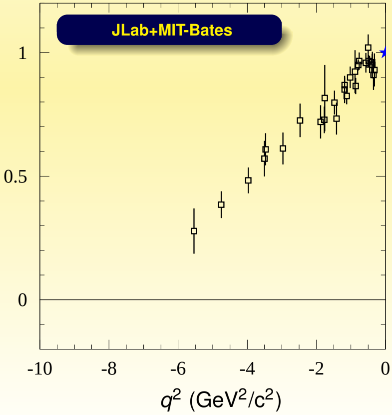
$R(q^2)$

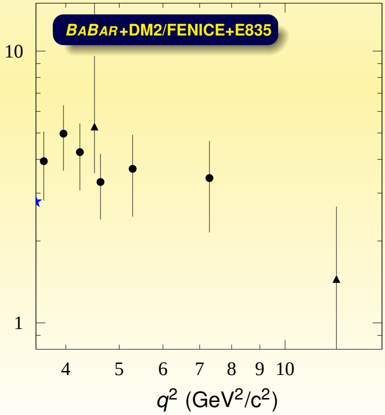
$$R(q^2) = R(0) + \frac{q^2}{\pi}\int_{4M_\pi^2}^{\infty}\frac{\mathrm{Im}R(s)}{s(s-q^2)}ds$$



$R(q^2)$ space-like

$|R(q^2)|$ time-like





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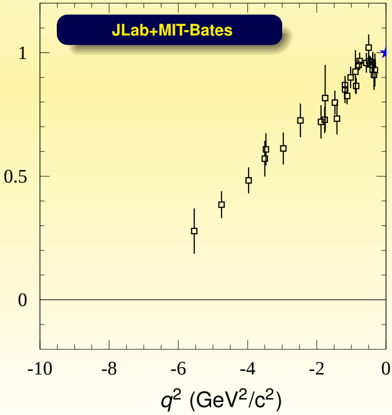
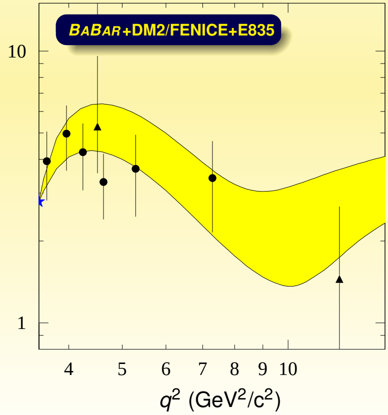
$$R(q^2) = R(0) + \frac{q^2}{\pi}\int_{4M_\pi^2}^{\infty}\frac{\mathrm{Im}R(s)}{s(s-q^2)}ds$$

$R(q^2)$ space-like



$|R(q^2)|$ time-like

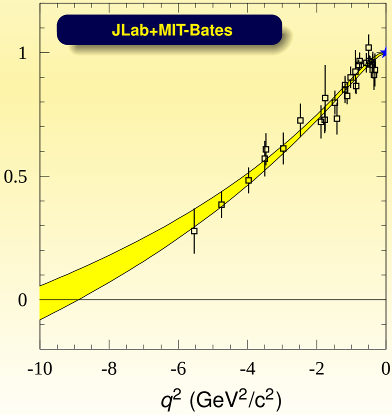


$R(q^2)$

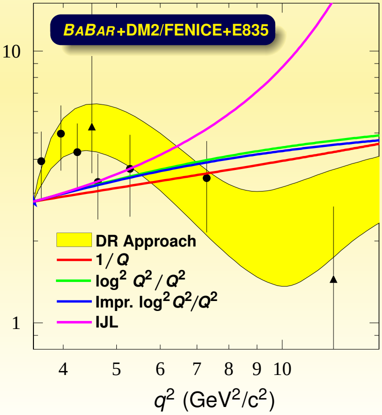
EPJA32 421

$$R(q^2) = R(0) + \frac{q^2}{\pi}\int_{4M_\pi^2}^{\infty} \frac{\mathrm{Im}R(s)}{s(s-q^2)}\,ds$$

$R(q^2)$ space-like



$|R(q^2)|$ time-like



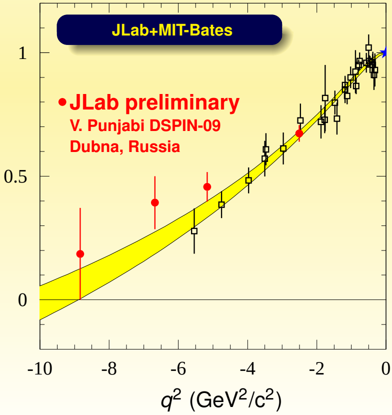
$R(q^2)$

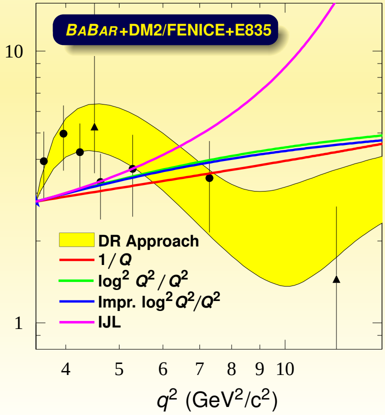
$$R(q^2) = R(0) + \frac{q^2}{\pi}\int_{4M_\pi^2}^{\infty} \frac{\mathrm{Im}R(s)}{s(s-q^2)}ds$$

$\mathrm{Re}\,q^2$

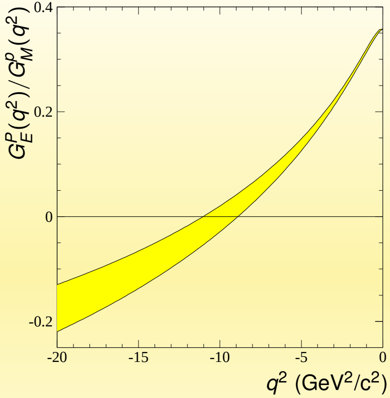
$R(q^2)$ space-like

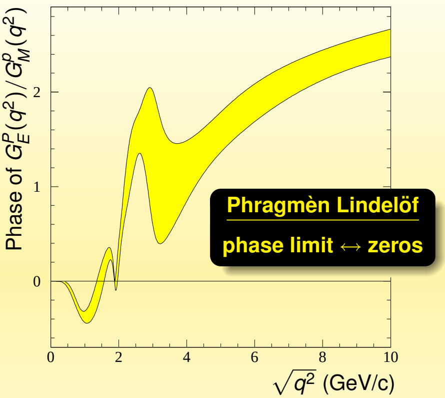
$|R(q^2)|$ time-like





$G_E^P(q^2)/G_M^p(q^2)$: zero and phase





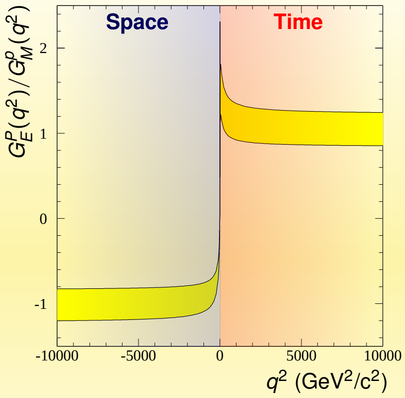
Space-like zero

$$t_0^{BABAR} = (-10 \pm 1)\ \text{GeV}^2$$

Phase from DR

$$\phi(q^2) = -\frac{\sqrt{q^2 - s_0}}{\pi}\ \text{Pr}\int_{s_0}^{\infty} \frac{\ln|R(s)|ds}{\sqrt{s - s_0}(s - q^2)}$$

Asymptotic $G_E^P(q^2)/G_M^p(q^2)$



- **Real asymptotic values for G_E^p/G_M^p**

$$\frac{G_E^p}{G_M^p} \xrightarrow[|q^2|\to\infty]{} -1.0 \pm 0.2$$

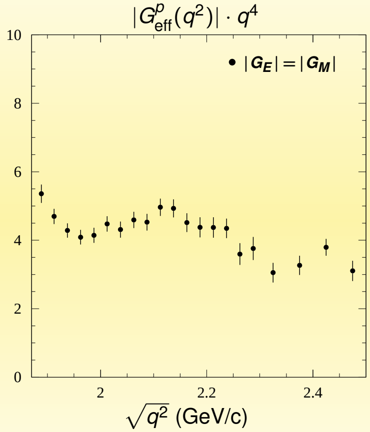
- **Asymptotic behaviour of F_2/F_1**

$$\frac{q^2}{4M_N^2}\left|\frac{F_2}{F_1}\right| \xrightarrow[|q^2|\to\infty]{} \left|\frac{G_E^p}{G_M^p} - 1\right| = 2.0 \pm 0.2$$

pQCD prediction

$$\left|\frac{G_E^p(q^2)}{G_M^p(q^2)}\right| \xrightarrow[|q^2|\to\infty]{} 1$$

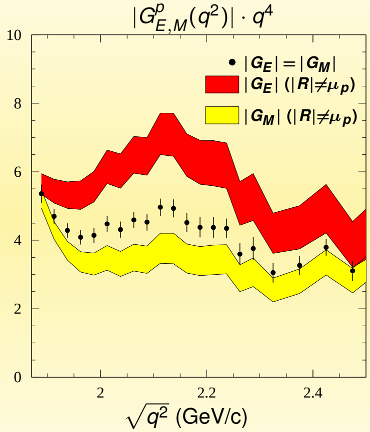
$|G_E^p(q^2)|$ and $|G_M^p(q^2)|$ from $\sigma_{p\bar{p}}$ and DR



$$|G_{\text{eff}}(q^2)|^2 = \frac{\sigma_{p\bar{p}}(q^2)}{\frac{4\pi\alpha^2\beta C}{3s}}\left(1 + \frac{1}{2\tau}\right)^{-1}$$

- **Usually what is extracted from the cross section $\sigma(e^+e^- \longrightarrow p\bar{p})$ is the effective time-like form factor $|G^p_{\text{eff}}|$ obtained assuming $|G_E^p| = |G_M^p|$ i.e. $|R| = \mu_p$**
- **Using our parametrization for R and the BABAR data on $\sigma(e^+e^- \longrightarrow p\bar{p})$, $|G_E^p|$ and $|G_M^p|$ may be disentangled**

$|G_E^p(q^2)|$ and $|G_M^p(q^2)|$ from $\sigma_{p\bar{p}}$ and DR

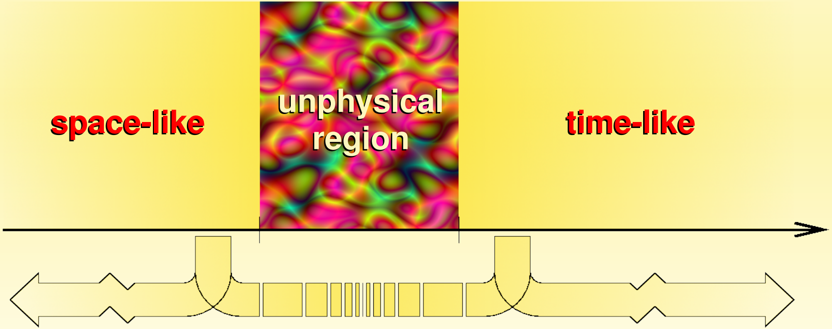


$$|G_M(q^2)|^2 = \frac{\sigma_{p\bar{p}}(q^2)}{\frac{4\pi\alpha^2\beta C}{3s}}\left(1+\frac{|R(q^2)|}{2\mu_p\tau}\right)^{-1}$$

- **Usually what is extracted from the cross section $\sigma(e^+e^-\longrightarrow p\bar{p})$ is the effective time-like form factor $|G^p_{\rm eff}|$ obtained assuming $|G_E^p| = |G_M^p|$ i.e. $|R| = \mu_p$**
- **Using our parametrization for R and the *BaBar* data on $\sigma(e^+e^-\longrightarrow p\bar{p})$, $|G_E^p|$ and $|G_M^p|$ may be disentangled**

Dispersive analysis of G_M^p

The asymptotic behavior

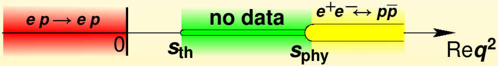


Dispersion relations and sum rules

Geshkenbeĭn, Ioffe, Shifman Yad. Fiz. 20, 128 (1974)

- **DR's connect space and time values of a form factor $G(q^2)$**

$$G(q^2) = \frac{1}{\pi}\int_{s_{th}}^{\infty} \frac{\mathrm{Im}G(s)ds}{s - q^2}$$



Drawbacks

- **The imaginary part is not experimentally accessible**
- **There are no data in the unhysical region $[s_{th}, s_{phy}]$**
- **We need to know the asymptotic behavior**

- **They applied the DR for the imaginary part to the function**

$$\phi(z) = f(z)\frac{\ln G(z)}{z\sqrt{s_{th} - z}} \quad \text{with} \quad \int_0^{s_{phy}} f^2(z)dz << 1$$

Advantages

- **The DR integral contains the modulus $|G(s)|$**
- **The unhysical region contribution is suppressed**

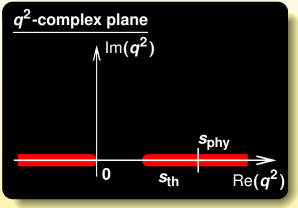
Drawback

- **Zeros of $G(z)$ are poles for $\phi(z)$**

Attenuation of the unphysical region

Strategy

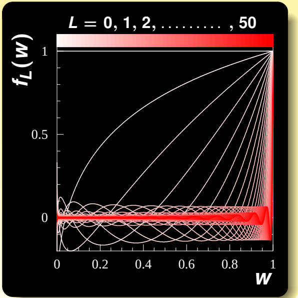
- Use the function $\phi(z) = f(z)\dfrac{\ln G(z)}{z\sqrt{s_{\rm th} - z}}$
- $f(z)$, is analytic with the cut $(-\infty, 0)$
- $f(z) = f_L(w) = \sum_{l=0}^{L} \dfrac{2l+1}{(L+1)^2} P_l(1-2w)$, $w = \dfrac{\sqrt{s_{\rm phy}} - \sqrt{z}}{\sqrt{s_{\rm phy}} + \sqrt{z}}$



The function $f(z)$, with $f_L(0)=1$, minimizes:

$$\int_0^1 f_L^2(w)\,dw$$

and suppresses the unphysical region contribution



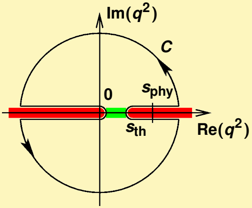
Attenuated DR and sum rule

- New DR with variable suppressed region $[0, s_{\rm phy}]$........$[G(q^2)$ has no zeros$]$

$$\oint_C \phi(z)dz = 0$$

$$\Downarrow$$

$$\underbrace{-\int_{-\infty}^{0} \frac{{\rm Im}[f(t)] \ln G(t)}{t\sqrt{s_{\rm th} - t}} dt}_{\text{Space-like}} = \underbrace{\int_{s_{\rm th}}^{\infty} \frac{f(s) \ln |G(s)|}{s\sqrt{s - s_{\rm th}}} ds}_{\text{Time-like}}$$



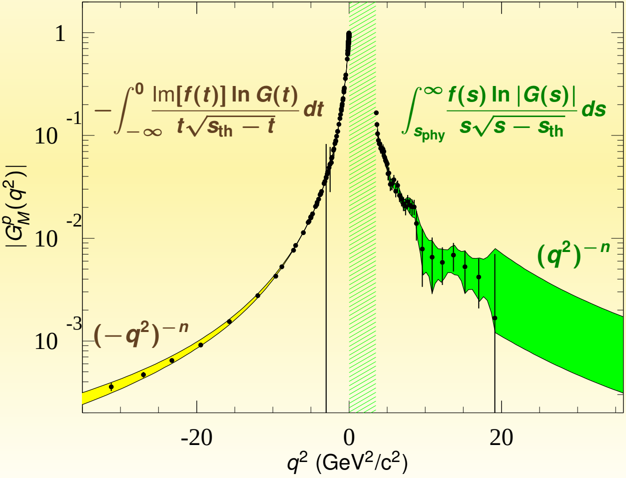
- Convergence relation to test asymptotic power behaviour of G_M^p

$$\underbrace{-\int_{-\infty}^{0} \frac{{\rm Im}[f(t)] \ln G(t)}{t\sqrt{s_{\rm th} - t}} dt}_{\text{Space-like data} + (-t)^{-n}} = \int_{s_{\rm th}}^{\infty} \frac{f(s) \ln |G(s)|}{s\sqrt{s - s_{\rm th}}} ds \approx \underbrace{\int_{s_{\rm phy}}^{\infty} \frac{f(s) \ln |G(s)|}{s\sqrt{s - s_{\rm th}}} ds}_{\text{Time-like data} + s^{-n}}$$

n is the free parameter

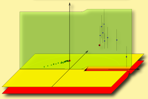
Sum rule: result for G_M^p

$$G_M^p(q^2) \underset{|q^2|\to\infty}{\propto} |q^2|^{-(2.27\pm0.36)}$$



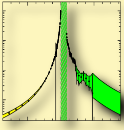
. . . in summary

G_E^p / G_M^p



- Space-like zero for G_E^p
- Asymptotic limit $G_E^p / G_M^p \underset{q^2 \to \pm\infty}{\longrightarrow} -1$
- Time-like form factors separation

$G_M^p(\pm\infty)$

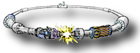


Space + time-like "fixed" data for $|G_M^p|$ + analyticity

$\Downarrow$

Confirmation of the pQCD asymptotic behavior:

$$G_M^p \propto (q^2)^{-2}$$



BESIII, VEPP2000, Belle2, Panda, SuperB (?)