

# Monte Carlo generator photon jets used for luminosity at $e^+e^-$ colliders<sup>\*</sup>

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**Abstract** A Monte-Carlo Generator Photon Jets (MCGPJ) to simulate Bhabha scattering as well as production of two charged muons and two photons events is discussed. The theoretical precision of the cross sections with radiative corrections (RC) is estimated to be smaller than 0.2%. The Next Leading Order (NLO) radiative corrections proportional to  $\alpha$  are treated exactly, whereas the all logarithmically enhanced contributions, related to photon jets emitted in the collinear region, are taken into account in frame of the Structure Function approach. Numerous tests of the MCGPJ as well as a detailed comparison with other MC generators are presented.

**Key words** luminosity,  $e^+e^-$ , hadrons, detectors

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## 1 Introduction

Nowadays various experiments all over the world are carried out with the aim to look for the phenomena which can not be explained by the Standard Model. All attempts to find any hints of New Physics at highest reachable energies or in precision experiments at low energies do not show its existence. But there is no doubt that after some time these attempts will be rewarded. The precision data on hadronic cross sections are required for many applications and, in particular, for the evaluation of the anomalous magnetic moment of muon,  $a_\mu = (g - 2)_\mu/2$ . One of the main ingredients in the theoretical prediction for  $a_\mu$  is the hadronic contribution related via a dispersion integral to cross sections of  $e^+e^-$  annihilation into hadrons. In the case of  $a_\mu^{\text{had}}$ , the VEPP-2000 energy range gives the major contribution both to the hadronic vacuum polarization contribution itself and to its uncertainty [1]. At high energies  $a_\mu^{\text{had}}$  can be calculated within the QCD framework whereas at low energies the experimental data are required. A numerical computation of the  $a_\mu^{\text{had}}$  value yields the result  $\sim 60$  ppm.

The goal of the new FNAL experiment is to measure  $a_\mu$  with an accuracy  $\sim 0.14$  ppm. To reduce a current systematic error of the hadronic contribution to the value  $a_\mu^{\text{had}}$  at least to the same level, the theoretical precision of the cross sections with radiative corrections (RC) should be better than 0.2% as it follows from a simple estimation:  $60 \text{ ppm} \times 0.2\% \sim 0.12 \text{ ppm}$ . This brief review shows why data on the cross sections of  $e^+e^-$  annihilation into hadrons with small systematic error are extremely important.

Inspection of the last generation experiments with CMD-2 and SND shows that there are four sources which give independent and main contributions to the total systematic error: luminosity, accelerator, detector and theory. To achieve systematics for hadronic cross sections less than 0.2% and to believe it, we should do the same at least with luminosity. Fortunately there are three purely QED processes which can be used to measure luminosity. In this case we will be able to arrange a cross check to better understand and estimate the systematics. These processes are:  $e^+e^- \rightarrow e^+e^-$ ,  $\mu^+\mu^-$ ,  $\gamma\gamma$ . We assume that VEPP-2000 will deliver a design luminosity  $10^{32} \text{ cm}^{-2}\text{s}^{-1}$  and therefore the statistical error will

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be negligibly small with respect to systematics.

## 2 Monte Carlo generator photon jets for the process $e^+e^- \rightarrow e^+e^-$

The Bhabha  $e^+e^- \rightarrow e^+e^-$  process is a main channel generally used for luminosity determination. The cross section is big enough, events have a simple topology in a detector: back-to-back tracks in drift chamber (DC) and two clusters in a barrel calorimeter associated with tracks and having equal energies. But, on the other hand, the cross section strongly depends on polar angle and contains ISR, FSR and their interference - the matrix element has very complicated form. Feynman graphs contain VP effects, cross section has asymmetric behavior against polar angle - problems with acceptance determination.

The differential cross section with RC, transformed to ‘‘canonical form’’, was obtained in [2] and its precision was about 1%, since only  $\mathcal{O}(\alpha)$  NLO corrections were taken into account. Naturally a question arises what should be done to achieve the theoretical precision close to the pro mille level. Nowadays, the prescription is known: all enhanced contributions, coming from collinear the regions (HO), must be combined with NLO corrections. This matching can be realized using the Structure Function (SF) [3] approach. It involves a convolution of the *boosted* Born cross section with the electron (positron) SF, which describes the leading effects due to emission of photons inside narrow cones along electrons (positrons). These enhanced contributions are proportional to the large logarithm  $(\alpha/\pi)^n \ln^n(s/m_e^2)$ ,  $n = 1, 2, \dots$  and in the smoothed representation of the SF a certain part of these corrections is exponentiated and evaluated in all powers of  $n$ . The non-leading contributions proportional to  $(\alpha/\pi)$  are implemented to the MC generator exactly by means of a so-called K-factor [3]. Some part of the second order corrections  $(\alpha/\pi)^2 \ln(s/m_e^2) \sim 0.01\%$  is fortunately small and can be omitted, keeping in mind the precision  $\sim 0.1\%$ . Based on numerical calculations it was established that matching exact  $\mathcal{O}(\alpha)$  corrections with the cross section describing photon jets radiation are sufficient to achieve the theoretical accuracy of cross sections with RC of  $\sim 0.2\%$ .

The boosted Born cross section of the process  $e^-(z_1 p_-) + e^+(z_2 p_+) \rightarrow e^-(p_1) + e^+(p_2)$ , corrected for vacuum polarization factors in the  $s$  and  $t$  channels, when initial particles lost some energy by radiation of photon jets has the form [4]:

$$\frac{d\tilde{\sigma}_0^{e^+e^- \rightarrow e^+e^-}(z_1, z_2)}{d\Omega_1} = \frac{4z_1 z_2 \alpha^2}{a^2 \tilde{s}} \left( \frac{\tilde{s}^2 + \tilde{u}^2}{2\tilde{t}^2 |1 - \Pi(\tilde{t})|^2} + \frac{\tilde{t}^2 + \tilde{u}^2}{2\tilde{s}^2 |1 - \Pi(\tilde{s})|^2} + \Re e \frac{\tilde{u}^2}{\tilde{s}\tilde{t}} \frac{1}{(1 - \Pi(\tilde{s}))(1 - \Pi(\tilde{t}))} \right), \quad (1)$$

where  $z_1$  and  $z_2$  are the energy fractions of  $e^+$ ,  $e^-$  after radiation of photon jets ( $z_{1,2} = \varepsilon_{1,2}/\varepsilon_{\text{beam}}$ ),  $\Pi(\tilde{s})$  and  $\Pi(\tilde{t})$  are the photon self-energy functions in the  $s$  and  $t$  channels, respectively. The Mandelstam variables are defined as usual:  $s = 2p_- p_+$ ,  $t = -2p_- p_1$ ,  $u = -2p_- p_2$ ,  $\tilde{s} = s z_1 z_2$ ,  $\tilde{t} = -s z_1 Y_1 (1 - c_1)/2$ ,  $\tilde{u} = -s z_2 Y_1 (1 + c_1)/2$ ,  $s_1 = 2p_1 p_2$ ,  $t_1 = -2p_+ p_1$ ,  $u_1 = -2p_+ p_2$ ,  $c_1 = \cos \theta_1$ , where  $\theta_1$  is a polar angle of the final electron with respect to the electron beam direction,  $Y_1$  and  $Y_2$  are the relative energies of final  $e^-$  and  $e^+$ . Using momentum conservation low the kinematics of final particles can be reconstructed.  $z_1 + z_2 = Y_1 + Y_2$ -energy conservation;  $z_1 - z_2 = Y_1 \cos \theta_1 + Y_2 \cos \theta_2$ -momentum conservation along the  $Z$ -axis;  $Y_1 \sin \theta_1 = Y_2 \sin \theta_2$  - momentum conservation in the plane perpendicular to the  $Z$ -axis. From these equations one can find that

$$Y_1 = \frac{2z_1 z_2}{a}, \quad Y_2 = \frac{(z_1^2 + z_2^2) - (z_1^2 - z_2^2)c_1}{a},$$

$$c_2 = \frac{(z_1^2 - z_2^2) - (z_1^2 + z_2^2)c_1}{(z_1^2 + z_2^2) - (z_1^2 - z_2^2)c_1},$$

where  $a = z_1 + z_2 - (z_1 - z_2)c_1$ . The emitted photons should be inside narrow cones with an opening angle  $2\theta_0$ , which should obey the follow restrictions,  $1/\gamma \ll \theta_0 \ll 1$ , where  $\gamma = \varepsilon/m_e$ . As a rule, its value is chosen as  $\sim 1/\sqrt{\gamma}$ . The cross section with one hard photon emission integrated inside these narrow cones is:

$$\frac{d\sigma_{\text{coll}}^{e^+e^- \rightarrow e^+e^- \gamma}}{d\Omega_1} = \frac{\alpha}{\pi} \int_{\Delta}^1 \frac{dx}{x} \left\{ 2 \frac{d\tilde{\sigma}_0^{e^+e^- \rightarrow e^+e^-}(1, 1)}{d\Omega_1} \left[ \left( z + \frac{x^2}{2} \right) \left( L - 1 + \ln \frac{\theta_0^2 z^2}{4} \right) + \frac{x^2}{2} \right] + \left[ \frac{d\tilde{\sigma}_0^{e^+e^- \rightarrow e^+e^-}(z, 1)}{d\Omega_1} + \frac{d\tilde{\sigma}_0^{e^+e^- \rightarrow e^+e^-}(1, z)}{d\Omega_1} \right] \left[ \left( z + \frac{x^2}{2} \right) \left( L - 1 + \ln \frac{\theta_0^2}{4} \right) + \frac{x^2}{2} \right] \right\}, \quad (2)$$

where  $L = \ln(s/m_e^2)$  is a large logarithm,  $x$ -hard photon energy in relative units,  $z = 1 - x$  is an energy fraction of incoming electron (positron), the boosted

Born cross section with reduced energies is defined in Eq. (1). The auxiliary parameter  $\Delta = \Delta\varepsilon/\varepsilon$  ( $\Delta \ll 1$ ) serves as a separator between hard and soft photons,  $\varepsilon$  is the beam energy. As it is seen, the part of this cross section has a term proportional to the large logarithm,  $(\alpha/\pi)(L-1)$ , and it is already contained in the SF [3]. So, it should be removed from this expression to avoid double counting. The remaining four terms are the non-leading corrections, named compensators, and combined with the cross

section describing hard photon emission out of narrow cones do not depend on the auxiliary parameter  $\theta_0$ . The expression for the differential cross section with one hard photon emission in the reaction  $e^-(p_-)+e^+(p_+) \rightarrow e^-(p_1)+e^+(p_2)+\gamma(k)$ , can be found in [2, 5].

Collecting all discussed above terms into one formula we get the complete expression for the *master* formula describing the process  $e^+e^- \rightarrow e^+e^- + n\gamma$ , which can be presented as follows:

$$\begin{aligned}
\frac{d\sigma^{e^+e^- \rightarrow e^+e^- + n\gamma}}{d\Omega_1} &= \int_0^1 dx_1 \int_0^1 dx_2 \int_0^1 dx_3 \int_0^1 dx_4 \frac{d\tilde{\sigma}_0(z_1, z_2)}{d\Omega_1} \times \mathcal{D}(z_1, s) \mathcal{D}(z_2, s) \mathcal{D}(z_3, \tilde{s}) \mathcal{D}(z_4, \tilde{s}) \left(1 + \frac{\alpha}{\pi} \tilde{K}_{SV}\right) \Theta + \\
&\frac{\alpha}{\pi} \int_{\Delta}^1 \frac{dx_1}{x_1} \left[ \left(1 - x_1 + \frac{x_1^2}{2}\right) \ln \frac{\theta_0^2}{4} + \frac{x_1^2}{2} \right] \frac{d\tilde{\sigma}_0(z_1, 1)}{d\Omega_1} \Theta + \frac{\alpha}{\pi} \int_{\Delta}^1 \frac{dx_2}{x_2} \left[ \left(1 - x_2 + \frac{x_2^2}{2}\right) \ln \frac{\theta_0^2}{4} + \right. \\
&\left. \frac{x_2^2}{2} \right] \frac{d\tilde{\sigma}_0(1, z_2)}{d\Omega_1} \Theta + \frac{\alpha}{\pi} \int_{\Delta}^1 \frac{dx_3}{x_3} \left[ \left(1 - x_3 + \frac{x_3^2}{2}\right) \ln \frac{\theta_0^2 z_3^2}{4} + \frac{x_3^2}{2} \right] \frac{d\tilde{\sigma}_0(1, 1)}{d\Omega_1} \Theta + \\
&\frac{\alpha}{\pi} \int_{\Delta}^1 \frac{dx_4}{x_4} \left[ \left(1 - x_4 + \frac{x_4^2}{2}\right) \ln \frac{\theta_0^2 z_4^2}{4} + \frac{x_4^2}{2} \right] \frac{d\tilde{\sigma}_0(1, 1)}{d\Omega_1} \Theta + \frac{4\alpha}{\pi} \frac{d\tilde{\sigma}_0(1, 1)}{d\Omega_1} \ln \frac{u}{t} \ln \Delta + \\
&\frac{\alpha^3}{2\pi^2 s} \int_{\substack{k^0 > \Delta\varepsilon \\ \theta_\gamma > \theta_0}} R_{\text{hard}}^{e^+e^- \rightarrow e^+e^- \gamma} \frac{d\Gamma}{d\Omega_1} \Theta, \tag{3}
\end{aligned}$$

where  $x_{1,2,3,4}$  are the relative energies of photon jets emitted along the initial and final particles;  $z_{1,2,3,4} = 1 - x_{1,2,3,4}$  are the energy fractions of electrons and positrons after radiation of photon jets;  $\Theta(\text{cuts})$  is a step function equal to 1 (0) if the kinematic variables obey (or not) selection criteria; the expression for  $\tilde{K}_{SV}(\tilde{\theta}_1)$  can be found in [2, 4]. The cutoff energy  $\Delta\varepsilon$  was chosen at ten electron masses to optimize the efficiency of event simulation ( $\Delta = \Delta\varepsilon/\varepsilon \sim 1\%$ ). The selection criteria adopted here were similar to those used in CMD-2 data analysis [6].

The cross section dependence on the auxiliary parameter  $\Delta\varepsilon$  is shown in Fig. 1 after integration over the remaining kinematic variables for the c.m. energy of 900 MeV. It is seen that variations are inside the claimed precision while  $\Delta\varepsilon$  changes by a factor of  $10^4$ . The cross section stability with an auxiliary parameter  $\theta_0$  is presented in Fig. 2. It is a remarkable result since averaged deviation does not exceed  $\pm 0.1\%$  level.

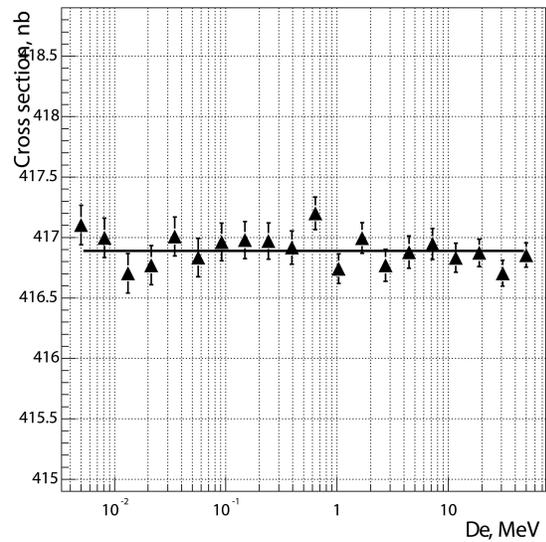


Fig. 1. Cross section dependence on the auxiliary parameter  $\Delta\varepsilon$ .

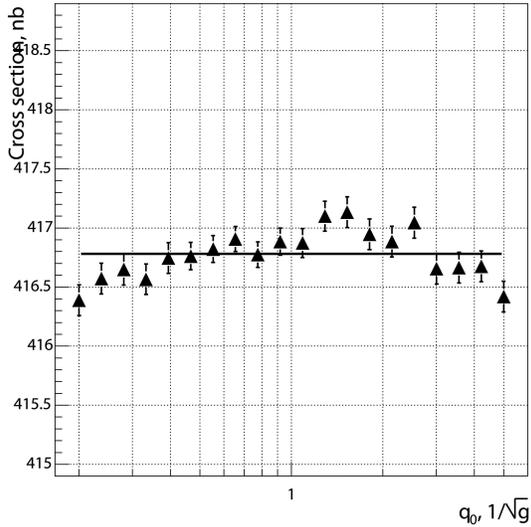


Fig. 2. Cross section dependence on the auxiliary parameter  $\theta_0$  after integration over the remaining kinematic variables.

The tune comparison with the BHWIDE code [7] is presented in Fig.3 for the VEPPP-2000 energy range. It is seen that relative difference of the cross sections is inside a band with a width smaller than 0.1%. The difference against acollinearity angle  $\Delta\theta$  is plotted in Fig. 4. One can see that the scale and sign of the difference depend on the particular choice of  $\Delta\theta$ . The reason of the difference about  $\sim 0.5\%$  for  $\Delta\theta \sim 0.05$  rad due to the fact that all photons (except one in our approach) are emitted strongly along the motion of electrons (positrons) whereas in the BHWIDE code they have some angular distribution. The MCGPJ code correctly reproduces different spectra only when the angle  $\Delta\theta$  is several times bigger than  $\theta_0 \sim 0.03$ . The difference of  $\sim 0.3\%$  for the large acollinearity angles  $|\Delta\theta| \sim 1$  rad is due to the fact that BHWIDE code produces one hard photon only.

It is important to reliably estimate the total theoretical precision of the cross section with RC. In order to quantify a theoretical uncertainty, independent comparison has been performed with the generator based on Ref. [2], where  $\mathcal{O}(\alpha)$  corrections are treated exactly. It was found that the relative difference of cross sections is more than 1% for small acollinearity angles  $\Delta\theta < 0.1$  rad (Fig. 5) and it is less than  $\sim 0.2\%$  for acollinearity angles  $\sim 0.25$  rad. From that immediately follows: radiation of two and more photons (jets) in collinear regions contributes by the amount  $\sim 0.2\%$  only. Therefore, we can conclude that the theoretical precision of the Bhabha cross section with RC is certainly better than  $\sim 0.2\%$  for soft selection criteria.

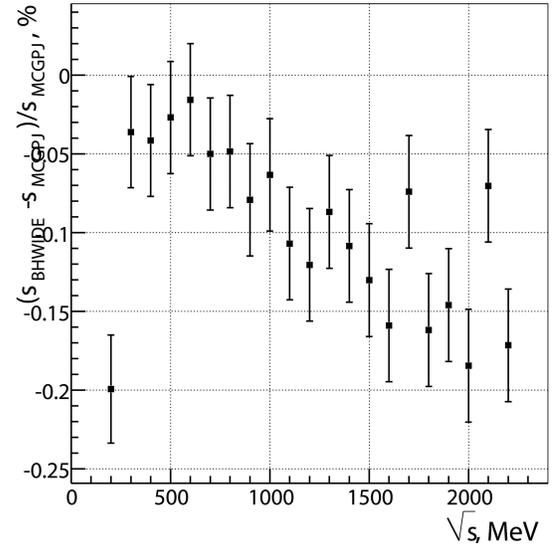


Fig. 3. Relative difference of the cross sections calculated with MCGPJ and BHWIDE as a function of the c.m.energy.

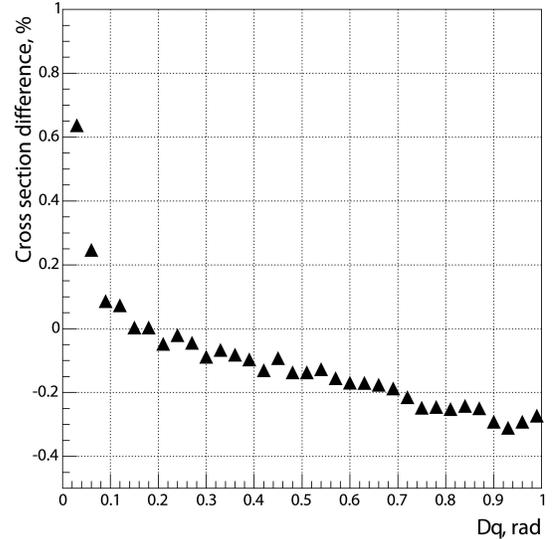


Fig. 4. Relative difference between the cross sections calculated with MCGPJ and BHWIDE versus the acollinearity angle  $|\Delta\theta|$ .

In Fig. 6 a two-dimensional plot is presented. The points on the plot correspond to electron's and positron's energies. A different population of events is observed far aside from the area where the most part of the events are concentrated. About  $\sim 1\%$  events have correlated low energies and they are distributed predominantly along a corridor which extends from the right upper corner to the left bottom one of this plot. The appearance of these events due to simultaneous radiation of two jets with close energies along either initial or final particles. Events of this type never appear, if a MC generator with one radiated photon is used.

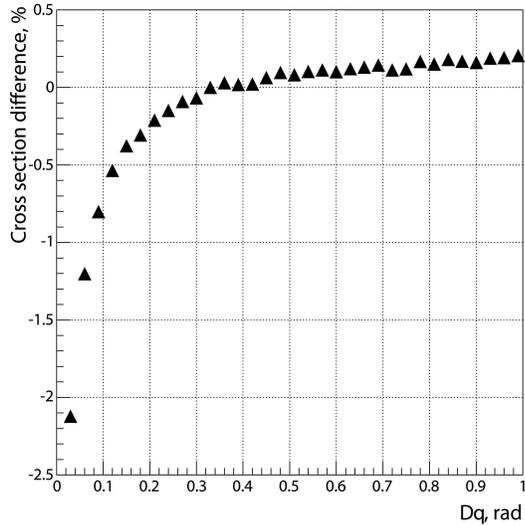


Fig. 5. Relative difference between the cross sections calculated with the MCGPJ code and the generator based on Ref. [2] versus the acollinearity angle  $|\Delta\theta|$ .

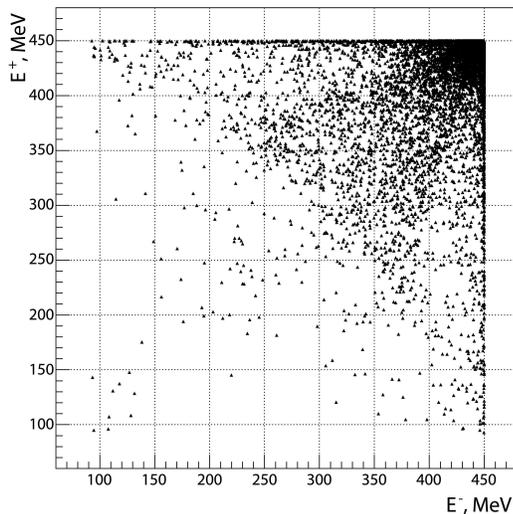


Fig. 6. Two-dimensional plot of simulated events (MCGPJ). The points in this plot correspond to the electron and positron energies. The influence of the condition  $\Delta\theta < 0.25$  rad can be seen as an arc-like smooth border.

### 3 MC generator for the process $e^+e^- \rightarrow \mu^+\mu^-$

The channel  $e^+e^- \rightarrow \mu^+\mu^-$  is also a purely QED process and fine for luminosity determination. The magnitude of cross section is ten times smaller than that of Bhabha, nevertheless, it is not a problem for colliders with high luminosity. It is important that the cross section practically does not depend on the polar angle of a muon track and it has symmetrical

form with respect to transformation:  $\sigma_{\mu\mu}(\cos(\theta)) = \sigma_{\mu\mu}(\cos(\pi-\theta))$ . The Feynman diagram contains only s-channel – direct way to extract vacuum polarization effects which are required for many applications. The cross section contains RC with photon jets radiation along motion of initial particles. The final state radiation (FSR) is considered with the next to leading order (NLO) corrections only. It is enough to provide the theoretical accuracy better than a 0.1% [8]. This process can provide an alternative method to better understand and correctly estimate the systematic error for luminosity, but it will meet problems with muons ID at high energies. In addition to that, a careful study of the double ratio  $(\sigma_{\mu\mu}^{\text{th}}/\sigma_{ee}^{\text{th}})/(\sigma_{\mu\mu}^{\text{exp}}/\sigma_{ee}^{\text{exp}})$  can serve as a power tool to check experimentally the accuracy of theoretical calculations. It was done with CMD-2 data [9] and the fit result for this ratio was found to be:  $-1.7\% \pm 1.4\%_{\text{st.}} \pm 0.7\%_{\text{syst.}}$ . At CMD-3 we plan to move down systematics and statistics at least by a factor of 3.

The same approach was adopted to create a MC generator to simulate muon pairs production in the reaction  $e^-(z_1p_-)+e^+(z_2p_+) \rightarrow \mu^-(p_1)+\mu^+(p_2)$ , when initial particles radiate some energy by emission of photon jets in collinear regions. This cross section was studied in detail elsewhere [5] and presented in differential form, keeping the relevant information about kinematics of final particles. It was elucidated by M.Voloshin [8] that to achieve systematics with FSR smaller than 0.1% it is sufficient to take into account emission one photon only. All enhanced second-order corrections proportional to  $\alpha^2$  gain the cross section  $\sim 0.04\%$  near threshold and decrease with energy.

A tune comparison with the KKMC [10] and BabaYaga@NLO [11] codes has been performed. The theoretical accuracy of the formulae on which KKMC and BabaYaga are based on is about  $\sim 0.1\%$ . Perfect agreement was found between different distributions at the level of precision  $\pm 0.2\%$ .

### 4 Monte-Carlo generator for the process $e^+e^- \rightarrow \gamma\gamma$

The reaction  $e^+e^- \rightarrow \gamma\gamma$  is a purely QED process too and cross section is big enough to apply for luminosity determination. The events of this process also have a simple signature in the detector: no tracs in DC and back-to-back clusters in the calorimeter with equal energies. It is utmost important that the cross section contains RC due to ISR only and Feynman graphs do not contain VP effects contrary to that of

Bhabha or diagram for muon pairs production. The cross section is even with respect to transformation of the photon polar angle:  $\theta_\gamma \rightarrow \pi - \theta_\gamma$ , is a powerful instrument to study acceptance systematics. The LXe calorimeter in CMD-3 will be able to detect a point of photon conversion with spatial resolution about 1 mm. As a result, the luminosity can be measured with accuracy of that of Bhabha, but, obviously, the systematics will be absolutely different.

A similar approach as for muons was adopted to create a MC generator (MCGPJ) to simulate two-photons production in the reaction  $e^-(z_1 p_-) + e^+(z_2 p_+) \rightarrow \gamma\gamma$ , when initial particles radiate some energy by emission of photon jets in the collinear regions. This cross section was studied in detail elsewhere [12] and presented in a differential form, keeping the relevant information about kinematics of final particles. Unfortunately, up to now there is not MC event generator with similar accuracy in order to make the tune comparison.

## 5 Summary and concluding remarks

A MC event generator for the processes  $e^+e^- \rightarrow$

$e^+e^-$ ,  $\mu^+\mu^-$ ,  $\gamma\gamma$ , with photon jets radiation in collinear regions was developed (MCGPJ). The enhanced contributions coming from the collinear regions are taken into account by means of the SF approach. As a result, a theoretical accuracy of the cross sections with RC is estimated as  $\sim 0.2\%$ . Comparison with the wellknown codes BHWIDE, KKMC and BabaYaga@NLO shows the remarkable agreement for many simulated spectra and cross sections. Relying upon the above brief review a main conclusion can be done: theoretical predictions aiming at a 0.1% precision must include contributions of both exact  $\mathcal{O}(\alpha)$  corrections and all higher-order  $\mathcal{O}(\alpha^n L^n)$  logarithmically enhanced corrections coming from collinear regions. Exploiting all discussed above features of different processes we plan to reduce the systematic error of luminosity in the forthcoming experiment with CMD-3 at the VEPP-2000 to the level of  $\sim 0.2\text{--}0.3\%$ . It is worth to mention here, that the MCGPJ generator covers many other hadronic processes and computer time required to simulate events is significantly smaller ( $10^2 - 10^3$  times) with respect to the BHWIDE, BabaYaga@NLO and especially for KKMC codes.

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