Lattice QCD Results on Exotics

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Disclaimer

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Outline

- **Methodologies**
  - The conventional method (the GEVP)
  - The not so conventional methods
    - Lüscher formalism
    - Other formalisms: HEFT, HALQCD, OP.
- **The scattering of charmed mesons**
  - the XYZ's
  - Prelovsek et al
  - CLQCD
  - HALQCD
- **Summary and outlooks**
  - Where we stand and
  - what to expect next

1. Methods

The first step: GEVP

in a typical lattice spectrum calculation

- A set of interpolating operators with the "right" quantum numbers: $\{\mathcal{O}_\alpha : \alpha = 1, 2, \cdots, N_{op}\}$
- Compute the correlation matrix:

$$\mathcal{C}_{\alpha\beta}(t, 0) = \langle \mathcal{O}_\alpha(t) \mathcal{O}^\dagger_\beta(0) \rangle \ , \tag{1}$$

- Solve the so-called Generalized Eigen-Value Problem (GEVP) for the eigenvalues λ_α's,

$$\mathcal{C}(t, 0) \cdot v_\alpha = \lambda_\alpha(t, t_0) \mathcal{C}(t_0, 0) \cdot v_\alpha \ , \tag{2}$$

  for some appropriately chosen t_0
- From the eigenvalues $\lambda_\alpha(t, t_0)$, extract the corresponding eigenvalues of the Hamiltonian: E_α via

$$\lambda_\alpha(t, t_0) \sim e^{-E_\alpha(t - t_0)} \ . \tag{3}$$

- Pass the E_α's to the second step

Complications

1. E_α's are NOT hadron mass values!
   - E_α is the eigenvalue of the QCD Hamiltonian
     - not even in real world, but in a latticized finite box!
   - Most hadrons are resonances
2. Many types of operators enter (operator mixing)!
   - single hadron operators
   - multi-hadron operators (esp. beyond the threshold)...

The second step

Relate the E_α's to the spectral quantity

0. E_α's are "approximate" hadron masses
   - only if the hadron is stable
   - or the hadronic resonance is "narrow" enough
   - but, what does "narrow" mean really?
1. Using a version of the Lüscher formalism
   - single channel version has matured over the years
   - multi-channel applications just appeared
   - more channels? rather complicated!
2. Other approaches
   - the Hamiltonian Effective Field Theory (HEFT) approach
   - the HAL QCD approach
   - the Optical Potential (OP) approach

Lüscher's approach

in theory (e.g. M. Lüscher, NPB354, 531, 1991)

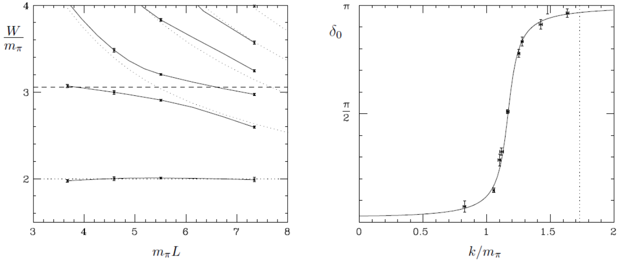
- original: single-channel spinless two-particle elastic scattering in COM frame,

$$E_\alpha(L) \Leftrightarrow \delta(E_\alpha) \ . \qquad (4)$$

$$\begin{cases} \tan\delta(\bar{k}) = \dfrac{\pi^{3/2} q}{Z_{00}(1, q^2)} \ , \\ 2\sqrt{\bar{k}^2 + m^2} = E(L) \ , \quad q = kL/(2\pi) \ . \end{cases} \qquad (5)$$

- extensions over the years
  - to particles with spin
  - to multi-channels
  - different BC's,
  - different frames,
  - ...

Lüscher's approach
in practice

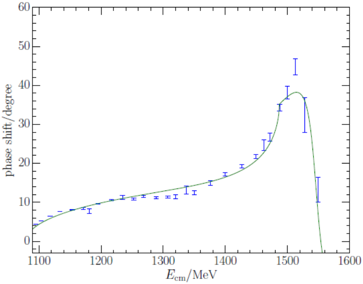


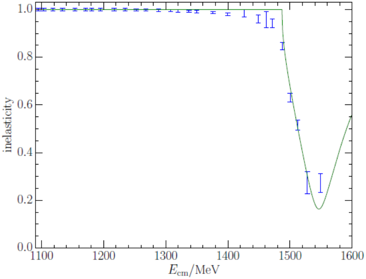
- scalar $\lambda\phi^4$ theory F. Zimmermann et al, hep-lat/9211029; NPB425, 413, 1994
- pion-pion scattering
  - quenched 1992; Gupta et al, PRD48, 388, 1993
  - unquenched since 2005 or so
- has matured in recent years

☞ complicated for multi-channels

Other approaches: the HEFT approach

see e.g. J.M.M. Hall et al, PRD87;094510,2013; arXiv:1303.4157

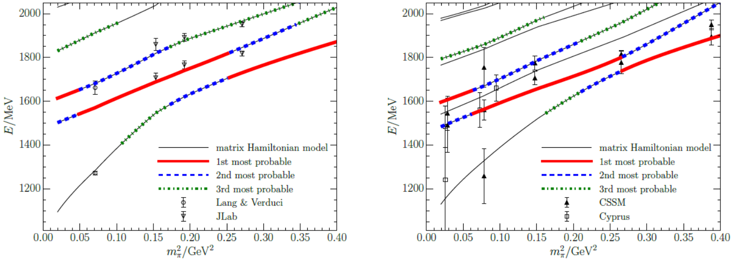




- Needs to construct the appropriate hamiltonian
- model parameters are determined by fitting low-energy data
- ☞ Example: $N^*(1535)$ $(J^P = (1/2)^-)$ study, Z.-W. Liu et al, PRL116, 082004, 2016; 1512.00140
- effective Hamiltonian in a finite volume

Other approaches: the HEFT approach

comparison of the levels



- The finite volume levels from Z.-W. Liu et al, PRL116, 082004, 2016; 1512.00140
- nicely interpolates all existing lattice data

Other approaches: the HALQCD method

see e.t. N. Ishii et al, PRL99, 022001,2007; PLB712,437,2012.

- HAL QCD: Hadrons to Atomic nuclei from Lattice QCD
- starts from the so-called NBS wavefunction (a four-point function).

☞ Ex: N-Ω interaction study:

$$F_{N\Omega}(\mathbf{x}-\mathbf{y}, t-t_0) = \langle 0|N_\alpha(\mathbf{x}, t)\Omega_{\beta,l}(\mathbf{y}, t)\bar{J}_{N\Omega}(t_0)|0\rangle \quad (6)$$

- the potential is obtained via the time-dependent HALQCD approach,

$$V_C(r) \simeq \frac{1}{2\mu}\nabla^2 R(r,t)/R(r,t) - \frac{\partial}{\partial t}\ln R(r,t)\ , \quad (7)$$

with $R(r,t) = F_{N\Omega}(t,0)/e^{-(m_N+m_\Omega)t}$.

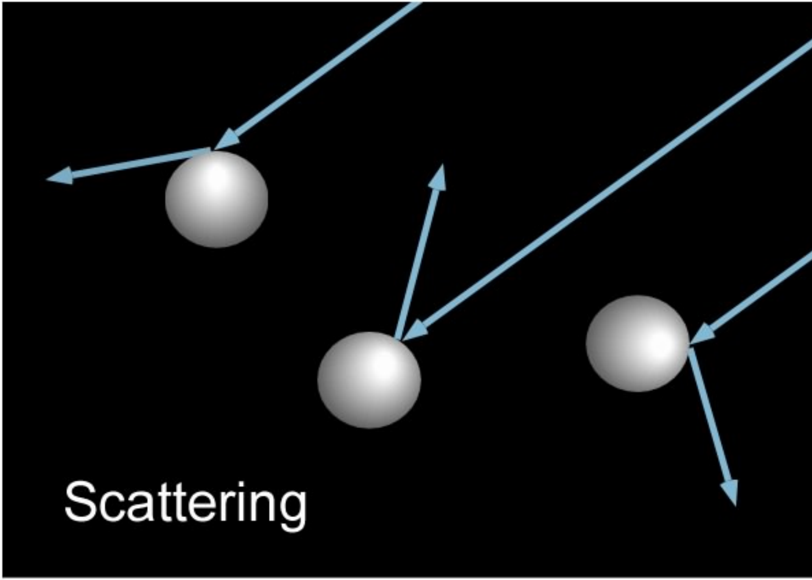
☞ no need for GEVP

Other approaches: the optical potential

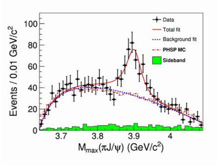
see e.g. D. Agadjanov et al, arXiv: 1603.07205

- measure the optical potential directly
  - analytically continue $W(E)$ to $W(E+i\varepsilon)$
  - taking $L \to \infty$, then $\varepsilon \to 0$
  - done by smoothing
- can handle multi-channels, or more than 2 particles
- relatively new, needs further study
- In particular, what is the relation with HALQCD approach?

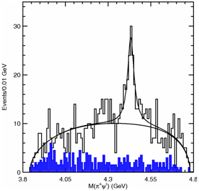
2. Charmed meson scattering and the XYZ's



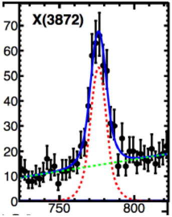
The XYZ particles
and other threshold exotics



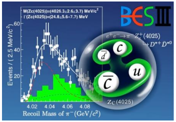
$Z_c(3900)$



$Z(4430)$



$X(3872)$



$Z_c(4025)$

- quarkonium-like states: valence quark structure $Q\bar{Q}q'\bar{q}$
- Neutral ones, $q = q'$, e.g. $X(3872)$, $Y(4260)$, etc.
- Charged ones, $q \neq q'$, $Z_c(3900)$, $Z_c(4025)$, $Z(4430)$, etc.
- Close to thresholds of mesons: $Q\bar{q}$ and $\bar{Q}q'$

Plus the newly discovered pentaquark states: P_c^+, etc.

S. Prelovsek et al study on Z_c's

S. Prelovsek et al, PRD91 014504 2015; 1405.7623

- Focus on $I^G(J^{PC}) = 1^+(1^{+-})$ sector;
- gauge ensemble: $N_f = 2$ improved Wilson fermion
- One volume ($L \sim 2$fm)
- one lattice spacing ($a \sim 0,124$fm)
- one pion mass value ($m_\pi \sim 266$MeV)

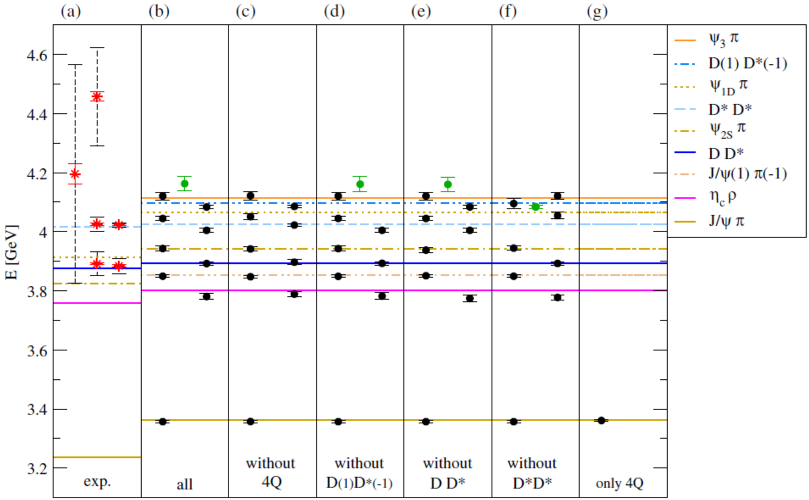
☞ Main pro: used many interpolating operators: DD^*, D^*D^*, $\rho\eta_c$, $J/\psi\pi$ and tetra-quark operators !

☞ Main strategy: Study E_α's and compare with the free case!

Implicit judgements & assumptions

- aware that E_α's are not mass values!
- however, too many channels to be taken care of
- not using Lüscher
- ☞ implicitly assuming weak interactions

S. Prelovsek et al study on Z_c's



- Study E_α's and compare with the free case!

S. Prelovsek et al, PRD91 014504 2015; 1405.7623

CLQCD's study on Z_c's

CLQCD, PRD89 094506 (2014);CLQCD, PRD92 054507 (2015)

- Focus on $I^G(J^{PC}) = 1^+(1^{+-})$ sector;
- gauge ensemble: $N_f = 2$ twisted mass fermion
- One volume ($L \sim 2.1$fm)
- one lattice spacing ($a \sim 0.067$fm)
- three pion mass values ($m_\pi \sim 300 - 485$MeV)

☞ Main strategy: Single out the most important channel near threshold (single-channel approximation)

Charmed meson near-threshold scattering

$N_f = 2$ twisted mass confs., using Lüscher

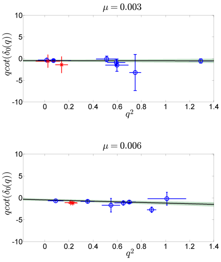
- $(D^*\bar{D}^*)^{\pm}$ ($Z_c(4025)$)

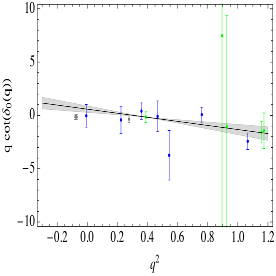
  CLQCD, PRD92 054507 (2015)

  $(D\bar{D}^*)^{\pm}$ ($Z_c(3900)$)

  CLQCD, PRD89 094506 (2014)

  - TBC utilized
  - 3 m_π values: 300,425,485MeV
  - weakly repulsive interaction found
  - no indication of a bound state





$$q \cot \delta(q^2) = \frac{1}{a_0} + \frac{1}{2} r_0 q^2 + \cdots ,$$

☞ need more ensembles (with $N_f = 2+1+1$) to inspect chiral & finite volume behavior

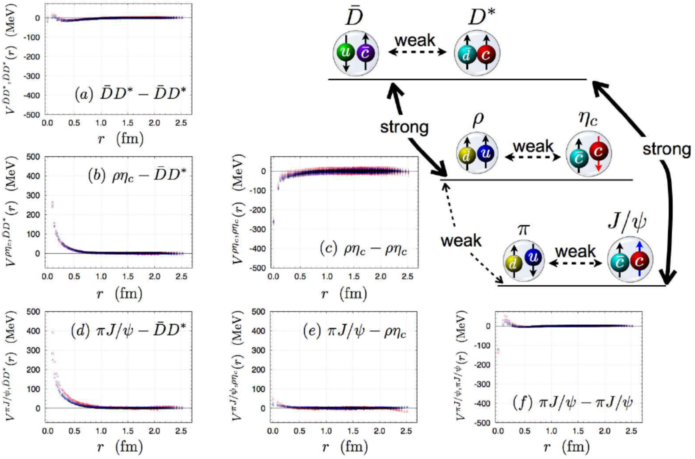
For $Z(4430)$

$N_f = 2$ twisted mass confs., using Lüscher

- $(\bar{D}_1 D^*)^{\pm}$ ($Z(4430)$) CLQCD, Phys.Rev. D93 (2016)
  - attractive interaction shows up
  - appears to be more attractive than the quenched results G. Meng et al, PRD80 034503 (2009)
  - some indications of a bound state seen
  - however, needs more volumes

The HALQCD approach

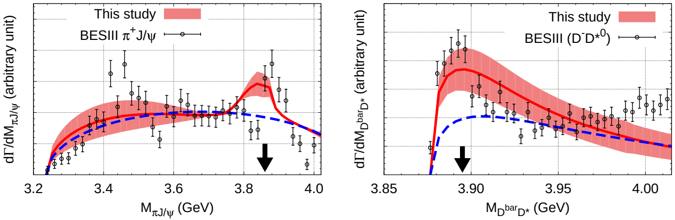
$Z_c(3900)$, 1602.03465



- $Z_c(3900)$ is NOT a conventional resonance
- appears to be a coupled channel effect arising from the $\pi J/\psi - \bar{D}D^* - \eta_c\rho$ coupling

The HALQCD approach

$Z_c(3900)$, 1602.03465

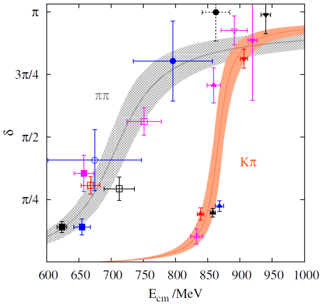


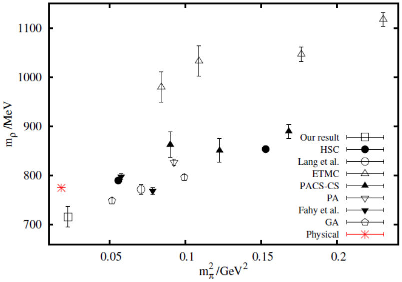
- 2+1 improved Wilson
- one volume ($L \sim 2.9$fm)
- one lattice spacing ($a \sim 0.09$fm)
- three pion masses (410 – 700MeV)

- model $Y(4260) \to \pi \bar{D} D^*, \pi\pi J/\psi$ three-body decays using experimental data from BESIII
- spectator π plus re-scattering via $V^{\alpha\beta}$

Summary and outlooks

1. conventional computations have come to the precision era
   - we can reproduce the ρ resonance nicely!





Summary and outlooks

2. The near-threshold exotics have been studied in LQCD
   - looking at E_α's directly $\Rightarrow$ negative
   - single-channel scattering $\Rightarrow$ negative
   - staggered fermion by Fermilab $\Rightarrow$ negative
   - HALQCD $\Rightarrow$ coupled-channel effects?
   - what's next?
3. More studies are needed
   - coupled-channel Lüscher to cross-check
   - more systematic studies

Thank you for your patience!