PWA on Zc states

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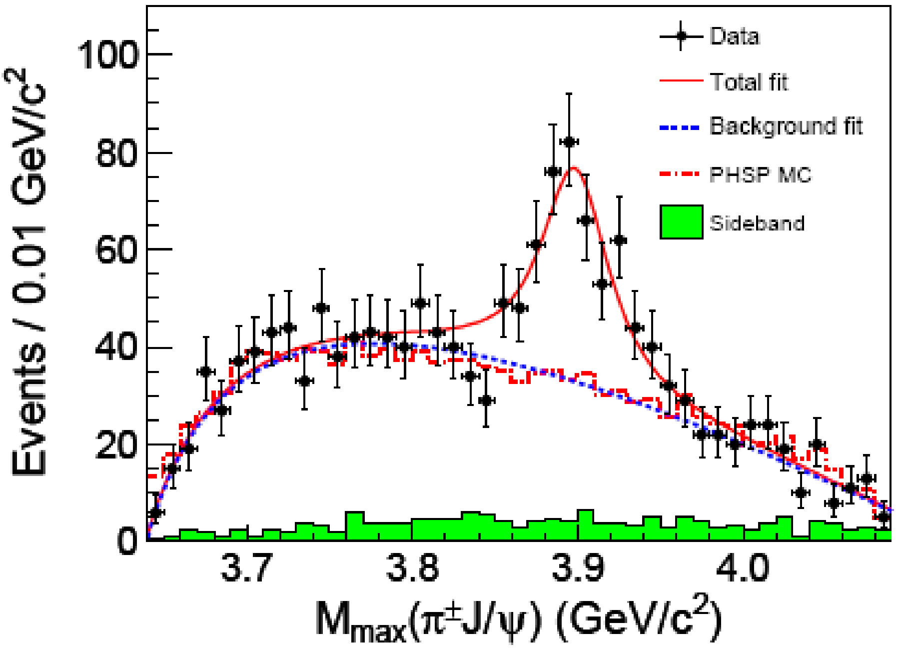
(For BESIII collaboration)

Outline:

- Introduction
- Data sets
- Amplitude construction
- Partial wave analysis results
- Systematic uncertainties
- Summary

Introduction

- Observation of Zc at BESIII [**Phys.Rev.Lett. 110 (2013) 252001]**



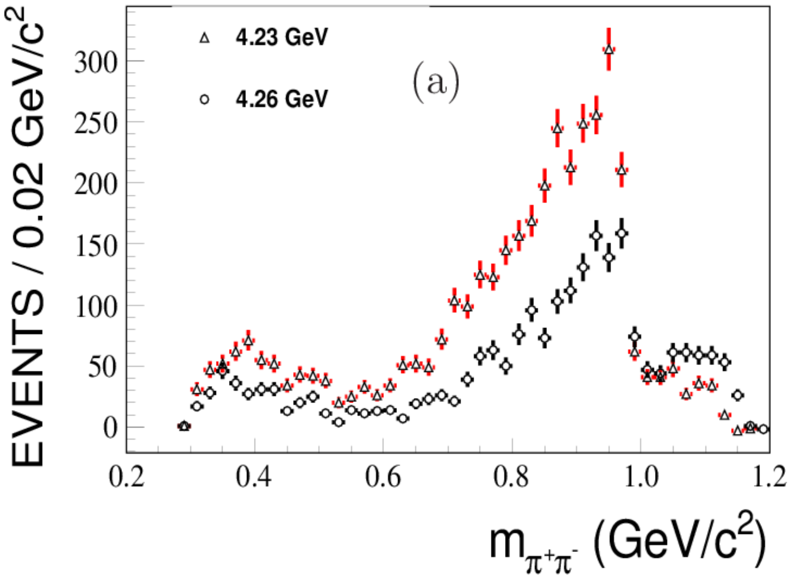
From SPIRE HEP Database (21st, Apr):

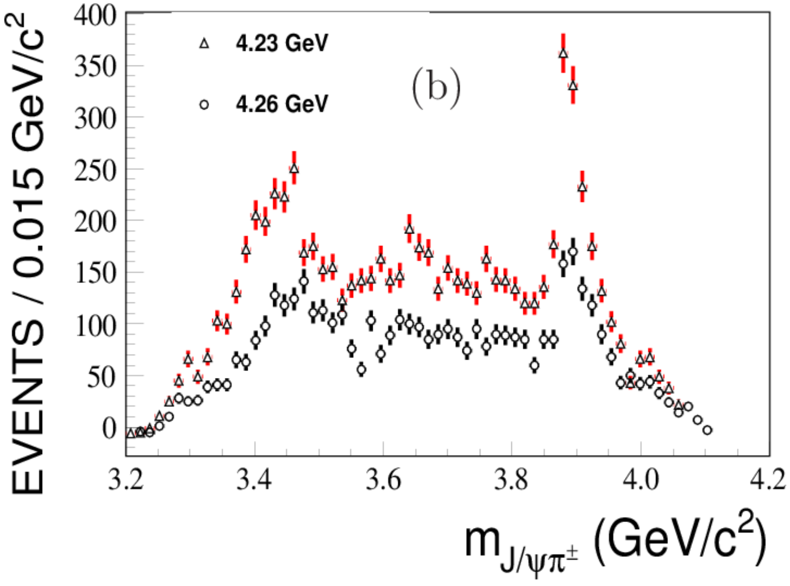
1. Tetraquarks
   - arXiv:1110.1333, 1303.6857
   - arXiv:1304.0345, 1304.1301
2. Hadronic molecules
   - arXiv:1303.6608, 1304.2882, 1304.1850
3. Four quark state (1 or 2)
   - arXiv:1304.0380
4. Meson loop
   - arXiv:1303.6355
   - arXiv:1304.4458
5. ISPE model
   - arXiv:1303.6842
6. ...

What's J^P?

- Luminosity; 525 pb^{-1} (4.26GeV)
- Observed candidates $\pi^+\pi^-J/\psi$: 1447

Data sets

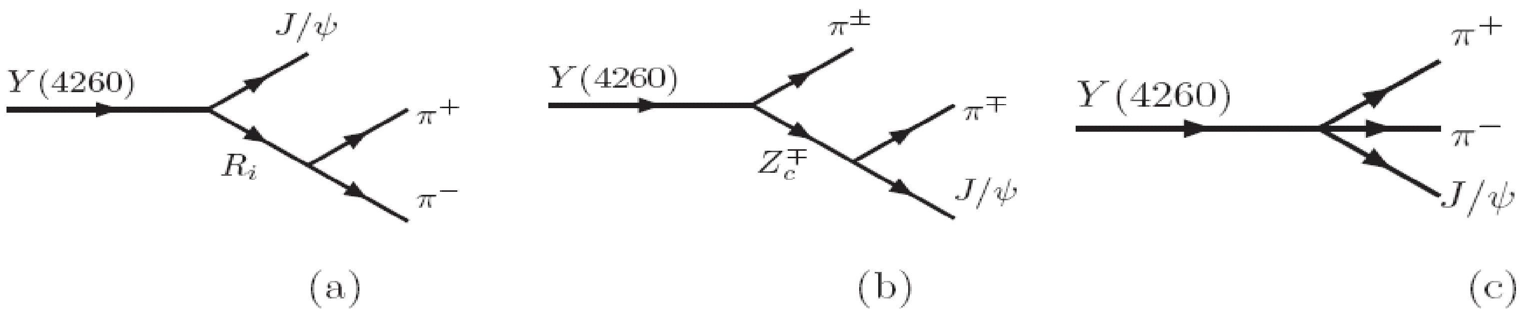




Event selection criteria: Phys.Rev.Lett. 110 (2013) 252001

√s (GeV)	*L* (pb^{-1})	Events	backgrounds
4.23	1092	4415	365
4.26	827	2447	272
sum	1919	6862	637

Amplitude construction



(a): $$A_1(\lambda_0,\lambda_2) = \sum_{\lambda_1,j} F^{Y}_{\lambda_1,\lambda_2}(r_1) D^{1*}_{\lambda_0,\lambda_1-\lambda_2}(\theta_0,\phi_0) BW_j(m_{\pi^+\pi^-}) F^{R_j}_{0,0}(r_2) D^{J_1*}_{\lambda_1,0}(\theta_1,\phi_1),$$

(b): $$A_2(\lambda_0,\lambda_2) = \sum_{\lambda_1,j} F^{Y}_{\lambda_1,0}(r_1) D^{1*}_{\lambda_0,\lambda_1}(\theta_0,\phi_0) BW_j(m_{J/\psi\pi}) F^{Z_c}_{\lambda_2,0}(r_2) D^{J_1*}_{\lambda_1,\lambda_2}(\theta_1,\phi_1),$$

$$F^{J}_{\lambda\nu} = \sum_{ls} \left(\frac{2l+1}{2J+1}\right)^{1/2} < l0S\delta|J\delta >< s\lambda\sigma-\nu|S\delta > g_{ls} r^l B_l(r),$$

See Ref.

1 S. U. Chung, Phys. Rev. D57, 431 (1998);
2 S. U. Chung, Phys. Rev. D48, 1225 (1993).

(c): direct 3-body decay

Z. Phys. C **8**, 43
Prog. Part. Nucl. Phys., **61**, 455
Eur. Phys. J. C **71**, 1808

$$A_3(\lambda_0,\lambda_3) = \frac{F}{f_\pi^2}\epsilon_Y(\lambda_0)\cdot\epsilon_{J/\psi}(\lambda_3)\left\{\left[q^2+\kappa(\Delta M)^2(1+\frac{2m_\pi^2}{q^2})\right]_{\text{S-wave}}\right.$$
$$\left. + \left[\frac{3}{2}\kappa((\Delta M)^2(1-\frac{4m_\pi^2}{q^2})(\cos^2\theta-\frac{1}{3})\right]_{\text{D-wave}}\right\},$$

- Resonance lineshape

$$BW(m) = \frac{1}{m^2 - m_0^2 - im\Gamma},$$

For $f_0(980)$, using Flatte formula

$$f = \frac{1}{M^2 - s - i(g_1\rho_{\pi\pi}(s) + g_2\rho_{K\bar{K}}(s))}, \quad \text{where } \rho(s) = 2k/\sqrt{s}$$

For σ_0, there are many types of lineshape in the market, using

$$\Gamma_X(s) = \rho\Gamma = \sqrt{1-\frac{4m_\pi^2}{s}}\Gamma. \quad \text{(E791)}$$

Other types need to check consistency in results.

• Total amplitude and differential cross section

$$A(\lambda_0,\lambda_3)=\sum_{i=1}^{3} g_i A_i(\lambda_0,\lambda_3), \qquad d\Gamma=\left(\frac{3}{8\pi^2}\right)\sum_{\lambda_0,\lambda_3} A(\lambda_0,\lambda_3)A^*(\lambda_0,\lambda_3)d\phi_3,$$

• Fit method

The joint probability density

$$\mathcal{L}=\prod_{i=1}^{N} P(x_i), \text{ where: } P(x_i)=\frac{(d\sigma/d\Phi)_i}{\sigma_{MC}}, \qquad \sigma_{MC}=\frac{1}{N_{MC}}\sum_{i=1}^{N_{MC}}\left(\frac{d\sigma}{d\Phi}\right)_i.$$

$S=-\ln\mathcal{L}$ is minimized using the package MINUIT.

$$\ln\mathcal{L}=\ln\mathcal{L}_{\rm data}-\ln\mathcal{L}_{\rm bg}.$$

Signal yields: $N_i=R_i*(N_{\rm obs}-N_{\rm bg})$, with $R_i=\dfrac{\sigma_i}{\sigma_{\rm tot}}$,

Stati. error: $$\delta N_i^2=\sum_{m=1}^{N_{\rm pars}}\sum_{n=1}^{N_{\rm pars}}\left(\frac{\partial N_i}{\partial X_m}\frac{\partial N_i}{\partial X_n}\right)_{\mathbf{X}=\mu} V_{mn}(\mathbf{X}),$$

Amplitude fitting

In the process $e^+e^- \rightarrow \gamma^* \rightarrow \pi^+\pi^- J/\psi$

- **The helicity value of** γ^* **is taken as** $\lambda_0 = \pm 1$ **due to from e+e- annihination**
- $\gamma^* \rightarrow Z_c^{\pm}\pi^{\mp}, Z_c^{\pm} \rightarrow J/\psi\pi^{\pm}$, **we try** J^P **for X:** $0^-, 1^-, 1^+, 2^-, 2^+$, **and** 0^+ **is not allowed**
- Z_c^+ **and** Z_c^- **states are assumed as isospin partner, share the same mass and coupling constants**
- **Six resonances are inclued in fitting to data:**

$\sigma_0, f_0(980), f_2(1270), f_0(1370), Z_c^{\pm}$, **and** $\pi^+\pi^- J/\psi$

Z_c is taken as 1^+.

Resonance	σ	$f_0(980)$	$f_2(1270)$	$f_0(1370)$	Z_c^+	Z_c^-
Significanceσ	13	25	5	11	22	22

Study Zc as $J^P=1^+$ state

- $f_0(980)$ line shape parameterized with Flatte formula

  Mass fixed to the PDG value, and g_1,g_2 determined with data

BESII analysis $J/\psi \to \omega\pi^+\pi^-$ Phys. Lett., B598, 149(2004).

$$g_1 = 0.138 \pm 0.010 \text{ GeV}^2 \text{ and } g_2/g_1 = 4.45 \pm 0.25.$$

- Zc line shape parameterized with Flatte-like formula

$$BW(s) = \frac{1}{s - M^2 + i(g_1' \rho_{\pi J/\psi}(s) + g_2' \rho_{D^* D}(s))},$$

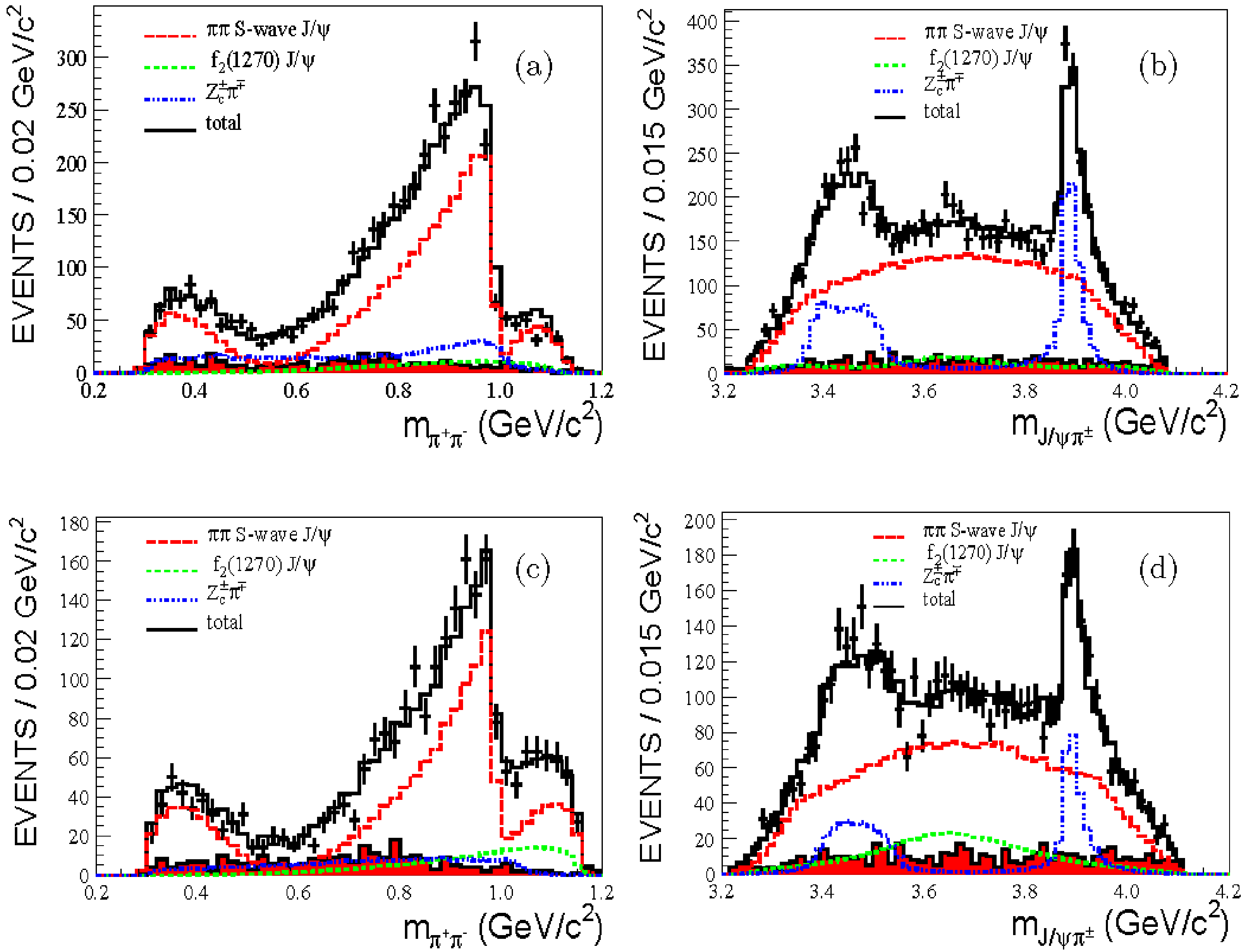
$g_2'/g_1' = 27.1 \pm 13.1$ according to the measurement

$\Gamma(Z_c^\pm \to (D\bar{D}^*)^\pm)/\Gamma(Z_c^\pm \to J/\psi\pi^\pm) = 6.2 \pm 2.9$

The fitted mass, g_1', g_2'/g_1' and $-\ln L$ for the Z_c resonance.

$Z_c : J^P$	M (MeV)	g_1' (GeV^2)	g_2'/g_1'	$-\ln L$
1^+	3900.2 ± 1.5	0.075 ± 0.006	21.8 ± 1.7	-1569.8

• Fit results with Zc(1^+)



Z_c pole mass and with:

$$M_{\text{pole}} = 3887.0 \pm 0.8 \pm 10.0 \text{ MeV}, \ \Gamma_{\text{pole}} = 45.2 \pm 4.8 \pm 16.8 \text{ MeV}$$

Helicity amplitudes for Zc production and decays

$$F_{\lambda,\nu} = \sum_{lS} g_{lS} \sqrt{\frac{2l+1}{2J+1}} \langle l0S\delta | J\delta \rangle \langle s\lambda\sigma - \nu | S\delta \rangle r^l B_l(r)$$

g_{lS} : coupling constant, $B_l(r)$: barrier factor

For $e^+e^- \to Z_c^{\pm}\pi^{\mp}$, we measured

$$|F_{1,0}^{Zc}|^2 / |F_{0,0}^{Zc}|^2 = 0.3 \pm 0.2_{\text{stat}} \text{ at } 4.23 \text{ GeV}$$

$$= 0.9 \pm 0.7_{\text{stat}} \text{ at } 4.26 \text{ GeV}$$

For $Z_c^{\pm} \to J/\psi\pi^{\pm}$:

$$|F_{1,0}^{\psi}|^2 / |F_{0,0}^{\psi}|^2 = 0.6 \pm 0.3_{\text{stat}}$$

Signal yields with Zc(1^+)

$\sqrt{s}$ (GeV)	$\pi\pi$-S wave	$Zc^{\pm}$
4.23	$2814.8 \pm 190.4_{stat}$	$875.2 \pm 84.4_{stat}$
4.26	$1450.7 \pm 119.6_{stat}$	$314.2 \pm 21.2_{stat}$

Born cross section for $e^+e^- \to Z_c^+ \pi^- + c.c \to \pi^+\pi^- J/\psi$

$20.3 \pm 2.0_{stat}$ (pb) at 4.23 GeV

$10.1 \pm 0.7_{stat}$ (pb) at 4.26 GeV

Significance for $e^+e^- \to Z_c^+(4020)\ \pi^- + c.c \to \pi^+\pi^- J/\psi$ is ~3σ.

Upper limits at 90% C.L.:

$$\frac{\sigma(e^+e^- \to Z_c^+(4020)\ \pi^- + c.c \to \pi^+\pi^- J/\psi)}{\sigma(e^+e^- \to Z_c^+(3900)\ \pi^- + c.c \to \pi^+\pi^- J/\psi)} < 3.3\% \text{ at 4.23 GeV}$$

$<25.1\%$ at 4.26 GeV

Study Zc with different spin-parity numbers

decay	helicity angules	helicity amplitudes
$\gamma^*(1,\lambda_0) \to \mathbf{Z_c^\pm}(\mathbf{J},\lambda_1)\pi^\mp$	$\theta_{1,}\phi_1$	A_{λ_1}
$\mathbf{Z_c^\pm}(\mathbf{J},\lambda_1) \to \mathbf{J}/\psi(1,\lambda_2)\pi^\pm$	$\theta_{2,}\phi_2$	B_{λ_2}

$$|M(\theta_i,\phi_i)|^2 \propto \sum_{\lambda_1,\lambda_1',\lambda_2} A_{\lambda_1}A^*_{\lambda_1'}\rho^{(\lambda_1,\lambda_1')}(\theta_1,\phi_1)B_{\lambda_2}B^*_{\lambda_2}D^J_{\lambda_1,\lambda_2}(\theta_2,\phi_2)D^{J*}_{\lambda_1',\lambda_2}(\theta_2,\phi_2),$$

Where the spin density matrix $\rho^{(i,j)}$ describing the $Y(4260)$ production rate, which is

$$\rho^{(i,j)}(\theta_1,\phi_1) = \sum_{k=\pm1} D^1_{i,k}(\theta_1,\phi_1)D^{1*}_{j,k}(\theta_1,\phi_1).$$

Table 1: The helicity angular distributions of Z_c for different quantum assignment.

J^P	A_i, B_i	$d\|M\|^2/d\cos\theta_1$	$d\|M\|^2/d\cos\theta_2$
0^-	—	$1-\cos^2\theta_1$	$1+0*\cos\theta_2^2$
1^-	$A_{-1}=-A_1, A_0=0$	$1+\cos^2\theta_1$	$1+\cos\theta_2^2$
	$B_{-1}=-B_1, B_0=0$		
1^+	$A_{-1}=A_1$	$1+\alpha\cos^2\theta_1$, with	$1+\alpha\cos\theta_2^2$, with
	$B_{-1}=B_1$	$\alpha=\frac{\|A_1\|^2-\|A_0\|^2}{\|A_1\|^2+\|A_0\|^2}$	$\alpha=\frac{(\|A_1\|^2-\|A_0\|^2)(\|B_0\|^2-\|B_1\|^2)}{\|A_0\|^2\|B_1\|^2+\|A_1\|^2(\|B_0\|^2+\|B_1\|^2)}$
2^-	$A_{-1}=A_1$	$1+\alpha\cos^2\theta_1$, with	$\|A_0\|^2[(1-3\cos^2\theta_2)^2\|B_0\|^2-12\cos^2\theta_2\sin^2\theta_2\|B_1\|^2]$
	$B_{-1}=B_1$	$\alpha=\frac{\|A_1\|^2-\|A_0\|^2}{\|A_1\|^2+\|A_0\|^2}$	$+4\|A_1\|^2[3\cos^2\theta_2\sin^2\theta_2\|B_0\|^2+(1-3\cos^2\theta_2+4\cos^4\theta_2)\|B_1\|^2]$
2^+	same as 1^-	$1+\cos^2\theta_1$	$1-3\cos^2\theta_2+4\cos^4\theta_2$

Comparison of fit results with different J^P for Zc

- Mass, g_1' and Log-likelihood

$Z_c : J^P$	M (MeV)	$g_1'(\text{GeV}^2)$	g_2'/g_1'	$-\ln L$
0^-	3906.3 ± 2.3	0.079 ± 0.007	25.8 ± 2.9	-1528.8
1^-	3903.1 ± 1.9	0.063 ± 0.005	26.5 ± 2.6	-1457.7
1^+	3900.2 ± 1.5	0.075 ± 0.006	21.8 ± 1.7	-1569.8
2^-	3905.2 ± 2.1	0.060 ± 0.004	28.7 ± 2.7	-1516.5
2^+	3894.3 ± 1.9	0.051 ± 0.005	23.4 ± 3.3	-1316.2

- Zc favors the quantum number $J^P=1^+$

If Zc is assigned as 0^-, the fit quality gets worse by about $\Delta(LnL) = 41$. To figure out the Zc quantum numbers, the information on the statistical significance is desirable.

Statistical significance for the Zc as 1^+ state

Null hypothesis H_0 : **data described with** $(\sigma_0, f_0(980), f_2(1270), f_0(1370), Zc(J^P))$

Alternative hypothesis H_1: data described with $(\sigma_0, f_0(980), f_2(1270), f_0(1370), Zc(1^+), \text{other } Zc(J^P))$

$$t \equiv -2\ln\lambda = 2[\ln L_{\max}(H_1) - \ln L_{\max}(H_0)],$$

See Ref.

Ilya Narsky, Nucl. Instr. Meth., A **450**, 444 (2000); Zhu Yong-Sheng, High Energy Physics and Nuclear Physics, **30**, 331 (2006).

$$p(t_{\mathrm{obs}}) = \int_{t_{\mathrm{obs}}}^{\infty} \chi^2(t;r)dt.$$

$$\int_{-S}^{S} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx = 1 - p(t_{\mathrm{obs}}) = \int_0^{t_{\mathrm{obs}}} \chi^2(t;r)dt.$$

Significance to distinguish the quantum number 1^+ over other quantum numbers.

Hypothesis	$\Delta(-\ln L)$	$\Delta(ndf)$	significance
1^+ over 0^-	44.5	$4\times2+5$	7.3σ
1^+ over 1^-	107.0	$4\times2+5$	$> 8.0\sigma$
1^+ over 2^-	51.8	$4\times2+5$	$> 8.0\sigma$
1^+ over 2^+	193.5	$4\times2+5$	$> 8.0\sigma$

Systematic uncertainties

- Luminosity, tracking, lineshape, kinematic fit and branching fraction

Source	$\mu^+\mu^-$	e^+e^-
Luminosity	1.0	1.0
Tracking	4.0	4.0
$Y(4260)$ line shape	0.6	0.6
Kinematic fit	2.2	2.2
$\mathrm{Br}(J/\psi \to l^+l^-)$	1.0	1.0
Total	4.8	4.8

The uncertainty of mass calibration is estimated with J/ψ and D^0 mass. We quote 1.8 MeV.

- Line shape of σ_0 parameterization

PKU ansatz:

$$\Gamma_X(s) = \sqrt{1 - \frac{4m_\pi^2}{s}} \frac{s}{m_X^2} \Gamma,$$

Zou and Bugg's approach:

$$\Gamma_X(s) = g_1 \frac{\rho_{\pi\pi}(s)}{\rho_{\pi\pi}(M_\sigma^2)} + g_2 \frac{\rho_{4\pi}(s)}{\rho_{4\pi}(M_\sigma^2)},$$

$$g_1 = f(s) \frac{s - m_\pi^2/2}{M_\sigma^2 - m_\pi^2/2} e^{-\frac{s - M_\sigma^2}{a}},$$

- Zc line shape Breit-Wigner parameterization

$$M = 3890.1 \pm 0.9 \text{ MeV} \qquad \Gamma = 35.4 \pm 7.1 \text{ MeV}$$

Compared to the Flatte-like formula, the Breit-Wigner parameterization make the fit quality worse by about Δ(ln L) =25.

- Backgrounds

  637 background events ➔ 662 events (1 σ deviation)

- $f_0(980)$ Flatte formula

  BESII analysis $J/\psi \to \omega\pi^+\pi^-$: $g_1 = 0.138 \pm 0.010\ \mathrm{GeV}^2$

  $g_2 / g_1 = 4.45 \pm 0.25$

  Uncertainty estimated with: $g_2 / g_1 = 4.7$ for conservative case

- $f_0(1370)$ mass and width

  (M,Γ): (1.35,0.265)➔ (1.2,0.2) GeV

- Barrier radius

  For meson decays: $r \in (0.25, 0.76)\,\mathrm{fm},$ both ends are checked.

  Uncertaity is estimated conservatively with r=0.76 fm

- Mass resolution for Zc coupling constant

$$\delta g_1'/g_1' \propto \delta\Gamma_{Z_c}/\Gamma_{Z_c}, \text{ is about } 1.8\%$$

- Uncertainty due to nonresonant decay

$$\begin{aligned} A_3(\lambda_0,\lambda_3) &= \frac{F}{f_\pi^2}\epsilon_Y(\lambda_0)\cdot\epsilon_{J/\psi}(\lambda_3)\left\{\left[q^2+\kappa(\Delta M)^2(1+\frac{2m_\pi^2}{q^2})\right]_{\text{S-wave}}\right. \\ &+ \left.\left[\frac{3}{2}\kappa((\Delta M)^2(1-\frac{4m_\pi^2}{q^2})(\cos^2\theta-\frac{1}{3})\right]_{\text{D-wave}}\right\}, \end{aligned}$$

Uncertainty is estimated with κ=0

- Summary of systematic errors

Summary of uncertainties the Z_c ($J^P = 1^+$) resonance parameters (%).

	Z_c resonance parameters		
Sources	Mass	g_1'	g_2'/g_1'
Event selection	0.04	...	
σ-PKU	0.01	4.00	0.00
σ-ZB	0.01	1.33	0.45
Z_c parametrization	0.26	...	...
Backgrounds	0.01	1.33	0.46
$f_0(980), g_1, g_2/g_1$	0.00	2.67	0.00
$f_0(1370)$	0.00	10.67	0.00
Barrier radius	0.07	4.00	6.88
Z_c Mass resolution	0.00	2.67	2.30
non-resonance	0.09	5.33	6.00
Total	0.28	13.86	9.41

Summary of uncertainties of signal yields for $Z_c^{\pm}\pi^{\mp}$ mode at $\sqrt{s}$ =4.23 GeV and 4.26 GeV (%).

$\sqrt{s}$	4.23 (GeV)	4.26 (GeV)
Event selection	4.8	4.8
σ PKU line shape	5.5	2.2
σ ZB line shape	6.2	3.9
Z_c parametrization	19.8	2.8
Backgrounds	4.4	0.3
$f_0(980), g_1, g_2/g_1$	6.6	1.8
$f_0(1370)$	4.7	6.2
Barrier radius	3.2	9.9
Z_c-mass resolution	0.2	1.9
non-resonance	0.7	0.6
Total	23.6	13.1

Summary

- Z_c spin parity are studied with 1.92fb^{-1} data taken at 4.23 and 4.26 GeV, the data suggests $J^P=1^+$ with statistical significance larger than 7.3σ over other quantum numbers, e.g. $0^-, 1^-, 2^+$ and 2^-.
- If Z_c is parameterized with a Flatte-like formula

  $M_{\text{pole}} = 3887.0 \pm 0.8 \pm 10.0$ MeV, $\Gamma_{\text{pole}} = 45.2 \pm 4.8 \pm 16.8$ MeV
- Born cross section for $e^+e^- \to Z_c^+ \pi^- + c.c \to \pi^+\pi^- J/\psi$

  $20.3 \pm 2.0_{\text{stat}}$ pb at 4.23 GeV

  $10.1 \pm 0.7_{\text{stat}}$ pb at 4.26 GeV
- Significance for $e^+e^- \to Z_c^+(4020)\ \pi^- + c.c \to \pi^+\pi^- J/\psi$ is $\sim 3\sigma$.

Upper limits at 90% C.L.:

$$\frac{\sigma(e^+e^- \to Z_c^+(4020)\ \pi^- + c.c \to \pi^+\pi^- J/\psi)}{\sigma(e^+e^- \to Z_c^+(3900)\ \pi^- + c.c \to \pi^+\pi^- J/\psi)} < 3.3\% \text{ at 4.23 GeV}$$

<25.1% at 4.26 GeV

Backup slides

Table 2. Flatté parameters for the $f_0(980)$-meson taken from the literature. The values of m_R, $\Gamma_{\pi\eta}$ and $E_{\rm BW}$ are given in MeV. Values for the references labeled with the superscript ($^{\rm a}$) are based on Achasov's parametrization [13], cf. also the appendix.

Ref.	m_R	$\Gamma_{\pi\pi}$	$\bar{g}_\pi$	$\bar{g}_K$	R	E_R	α
[14][(a)]	969.8	196	0.417	2.51	6.02	−21.5	−1.35
[15][(a)]	975	149	0.317	1.51	4.76	−16.3	−1.00
[16][(a)]	973	256	0.538	2.84	5.28	−18.3	−1.07
[17]	977	42.3	0.09	0.02	0.22	−14.3	−0.66
[18]	–	90	0.19	0.40	2.11	–	–
[19]	957	42.3	0.09	0.97	10.78	−34.3	−1.60

[14]: NOVOSIBIRSK-SND

[15]: CMD-2

[16]: CLOE

[17]: E791

[18]: WA102

[19]: OPAL

g_K/g_π=0.22 ~ 10.8

BESII: g_K/g_π =4.5