

Charmonium spectroscopy

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Outline:

- 1. Charmonium spectroscopy**
- 2. EM transitions**
- 3. Observations**
- 4. Summary**

1. Charmonium Spectroscopy

Mass spectrum for S wave

LP:Linear potential

SP:Screened potential

$n^{2S+1}L_J$	name	J^{PC}	Exp. [6]	[7]	[9]	LP	SP	Difference
1^3S_1	J/ψ	1^{--}	3097 ^a	3090	3097	3097	3097	
1^1S_0	$\eta_c(1S)$	0^{-+}	2984 ^a	2982	2979	2983	2984	
2^3S_1	$\psi(2S)$	1^{--}	3686 ^a	3672	3673	<u>3679</u>	3679	$\Delta M=0$ MeV
2^1S_0	$\eta_c(2S)$	0^{-+}	3639 ^a	3630	3623	3635	3637	
3^3S_1	$\psi(3S)$	1^{--}	4040 ^a	4072	4022	<u>4078</u>	4030	$\Delta M=48$ MeV
3^1S_0	$\eta_c(3S)$	0^{-+}		4043	3991	4048	4004	
4^3S_1	$\psi(4S)$	1^{--}	4415?	4406	4273	<u>4412</u>	4281	$\Delta M=131$ MeV
4^1S_0	$\eta_c(4S)$	0^{-+}		4384	4250	4388	4264	
5^3S_1	$\psi(5S)$	1^{--}			4463	<u>4711</u>	4472	$\Delta M=239$ MeV
5^1S_0	$\eta_c(5S)$	0^{-+}			4446	4690	4459	

对于4S,5S..., 哪种模型有效? 或者都失效? $\Psi(4415)$ 真是4S态?

Mass spectrum for P wave

$n^{2S+1}L_J$	name	J^{PC}	Exp. [6]	[7]	[9]	LP	SP	Difference
1^3P_2	$\chi_{c2}(1P)$	2^{++}	3556^a	3556	3554	<u>3552</u>	3553	$\Delta M=1 \text{ MeV}$
1^3P_1	$\chi_{c1}(1P)$	1^{++}	3511^a	3505	3510	3516	3521	
1^3P_0	$\chi_{c0}(1P)$	0^{++}	3415^a	3424	3433	3415	3415	
1^1P_1	$h_c(1P)$	1^{+-}	3525^a	3516	3519	3522	3526	
2^3P_2	$\chi_{c2}(2P)$	2^{++}	3927^a	3972	3937	<u>3967</u>	3937	$\Delta M=30 \text{ MeV}$
2^3P_1	$\chi_{c1}(2P)$	1^{++}		3925	3901	3937	3914	
2^3P_0	$\chi_{c0}(2P)$	0^{++}	3918?	3852	3842	3869	3848	
2^1P_1	$h_c(2P)$	1^{+-}		3934	3908	3940	3916	
3^3P_2	$\chi_{c2}(3P)$	2^{++}		4317	4208	<u>4310</u>	4211	$\Delta M=99 \text{ MeV}$
3^3P_1	$\chi_{c1}(3P)$	1^{++}		4271	4178	4284	4192	
3^3P_0	$\chi_{c0}(3P)$	0^{++}		4202	4131	4230	4146	
3^1P_1	$h_c(3P)$	1^{+-}		4279	4184	4285	4193	

对于3P..., 哪种模型有效? 或者都失效? X(4140,4274)会是3P态?

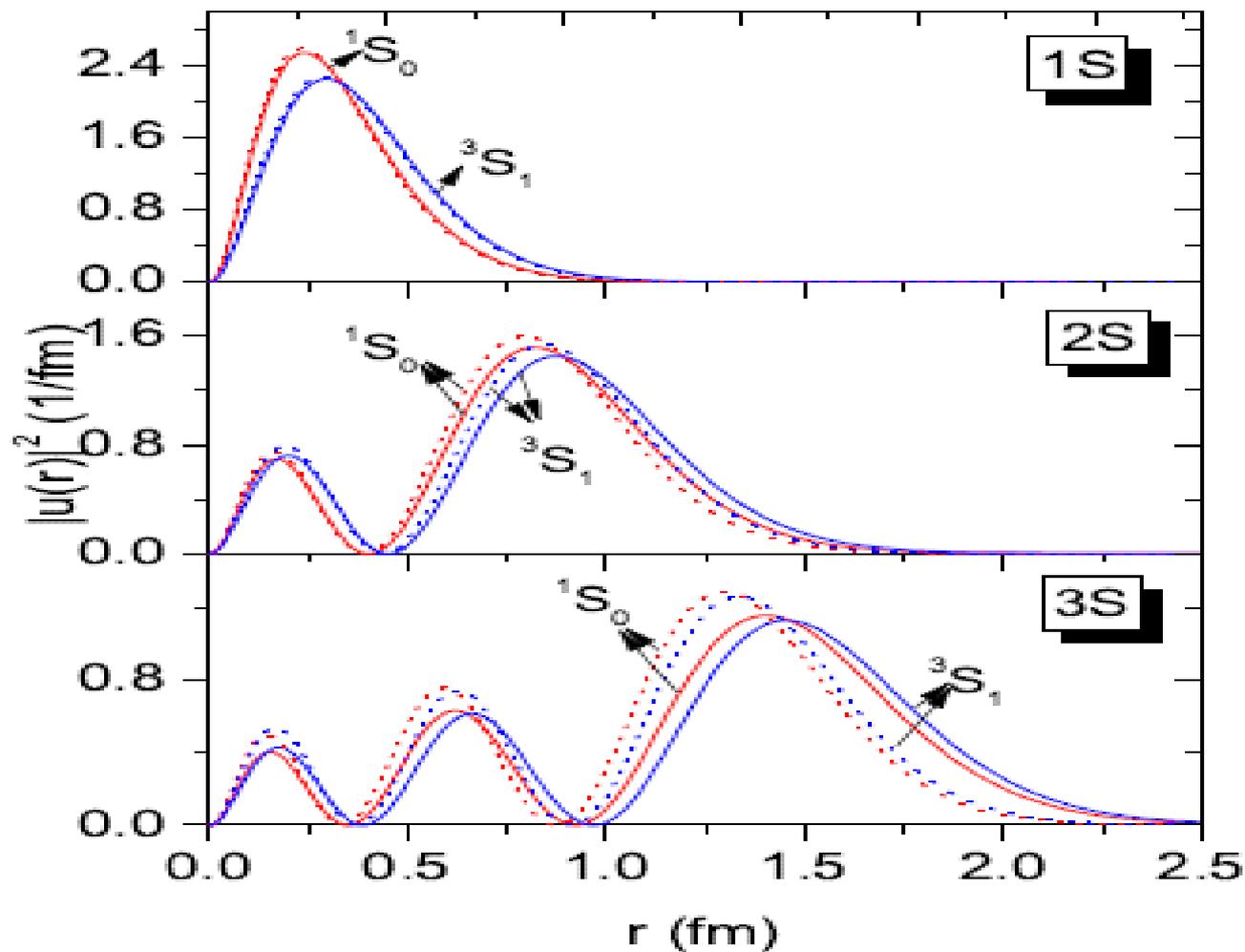
Mass spectrum for D wave

$n^{2S+1}L_J$	name	J^{PC}	Exp. [6]	[7]	[9]	LP	SP	Difference
1^3D_3	$\psi_3(1D)$	3^{--}		3806	3799	<u>3811</u>	3808	$\Delta M=3 \text{ MeV}$
1^3D_2	$\psi_2(1D)$	2^{--}	3823 ^a	3800	3798	3807	3807	
1^3D_1	$\psi_1(1D)$	1^{--}	3778 ^a	3785	3787	3787	3792	
1^1D_2	$\eta_{c2}(1D)$	2^{-+}		3799	3796	3806	3805	
2^3D_3	$\psi_3(2D)$	3^{--}		4167	4103	<u>4172</u>	4112	$\Delta M=60 \text{ MeV}$
2^3D_2	$\psi_2(2D)$	2^{--}		4158	4100	4165	4109	
2^3D_1	$\psi_1(2D)$	1^{--}	4191?	4142	4089	4144	4095	
2^1D_2	$\eta_{c2}(2D)$	2^{-+}		4158	4099	4164	4108	
3^3D_3	$\psi_3(3D)$	3^{--}			4331	<u>4486</u>	4340	$\Delta M=146 \text{ MeV}$
3^3D_2	$\psi_2(3D)$	2^{--}			4327	4478	4337	
3^3D_1	$\psi_1(3D)$	1^{--}			4317	4456	4324	
3^1D_2	$\eta_{c2}(3D)$	2^{-+}			4326	4478	4336	

对于2D..., 模型相关性就非常明显了! SP模型对 $\psi(4191)$ 的解释并不理想!

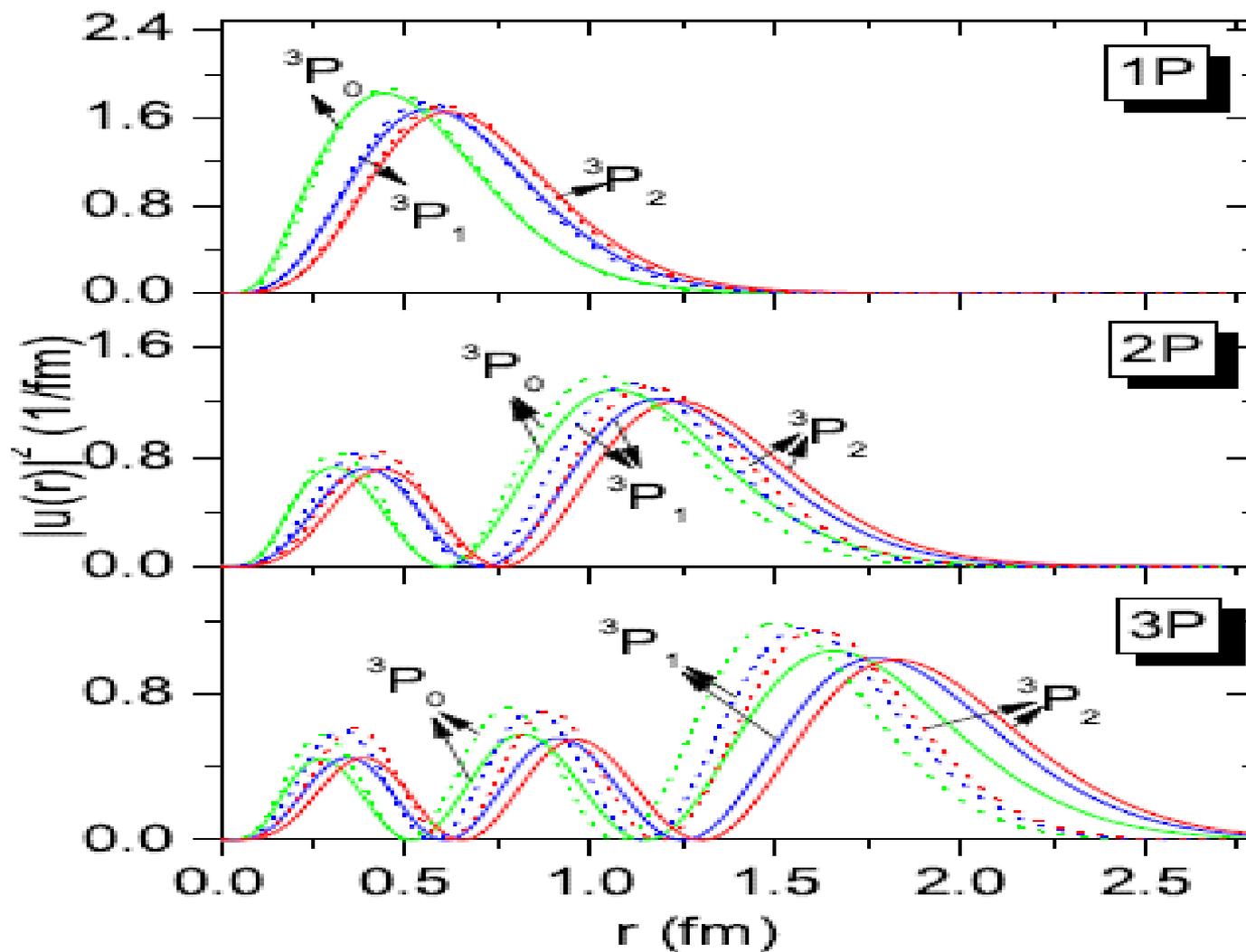
Wave function for S wave

虚线:LP模型结果
实线:SP模型结果



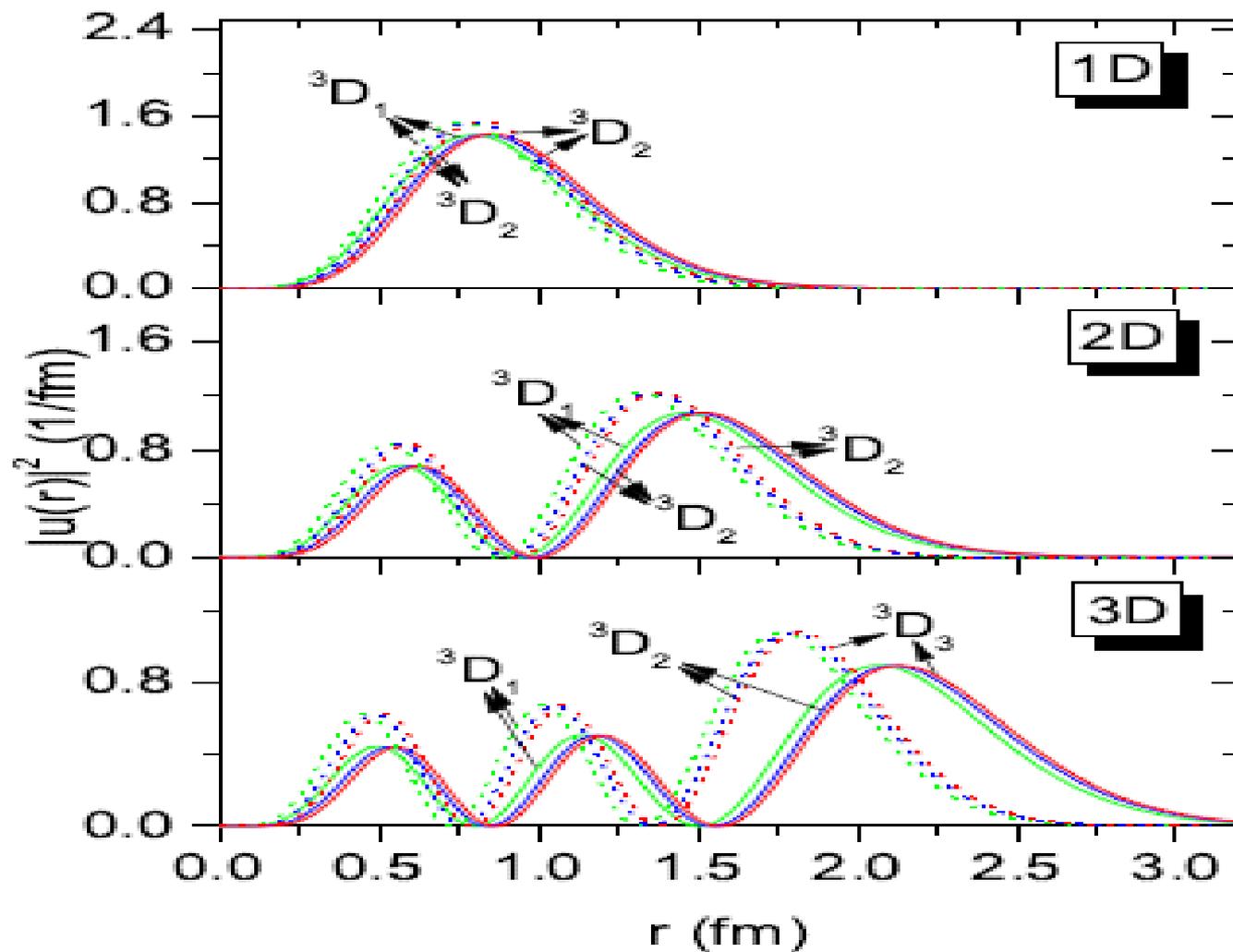
对于**3S**及以上分波两种模型差别明显!

Wave function for P wave



对于2P及以上分波两种模型差别明显!

Wave function for D wave



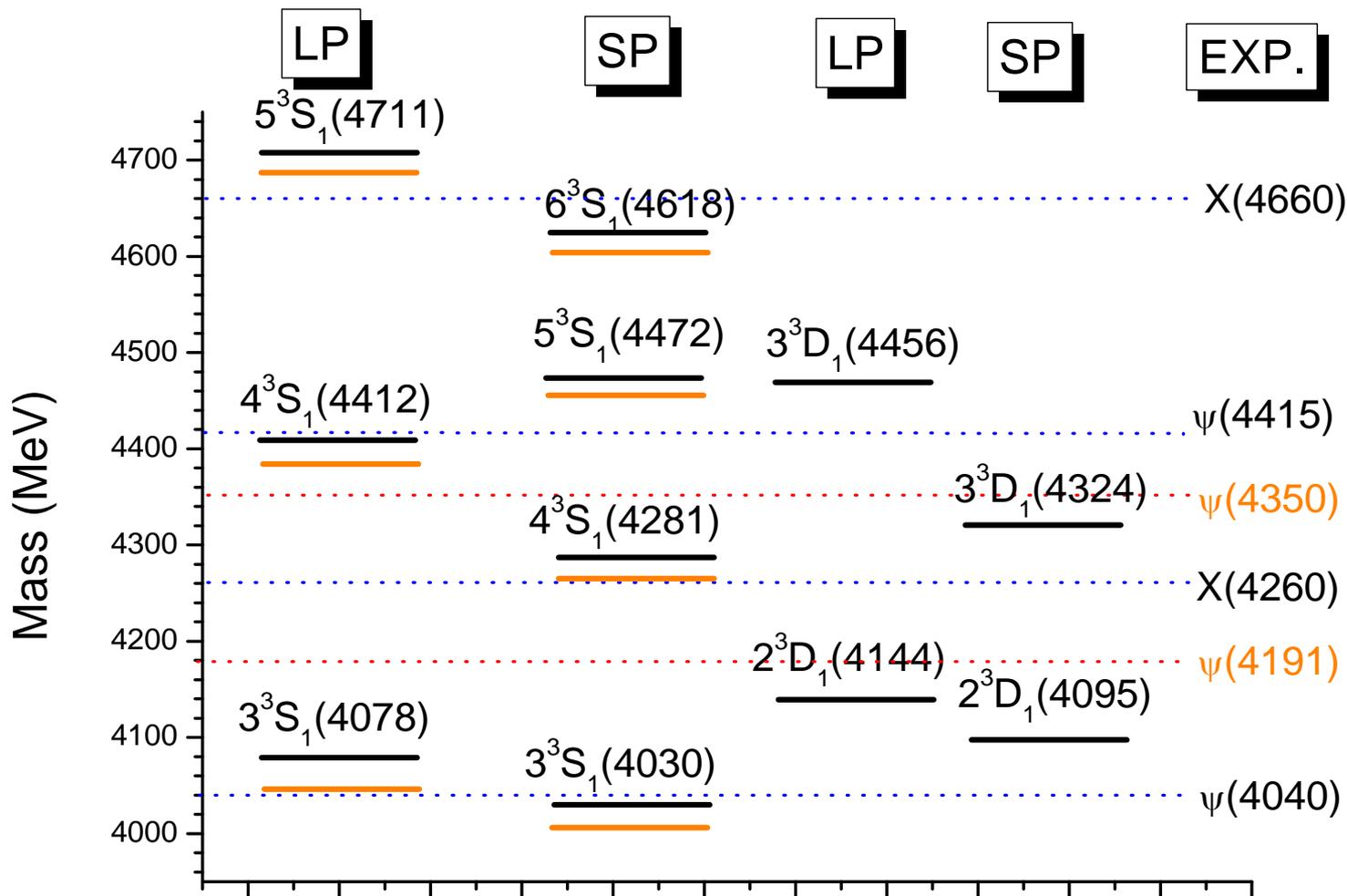
2D及以上分波两种模型差别明显! 模型只能可靠理解 $r=1\text{fm}$ 以内的物理

Hyperfine and fine splittings

Splitting	LP	SP	SNR [9]	NR [7]	GI [7]	Exp. [6]
$m(1^3S_1)-m(1^1S_0)$	114	113	118	108	113	113.3 ± 0.7
$m(2^3S_1)-m(2^1S_0)$	44	43	50	42	53	46.7 ± 1.3
$m(3^3S_1)-m(3^1S_0)$	30	26	31	29	36	
$m(4^3S_1)-m(4^1S_0)$	22	17		22	25	
$m(5^3S_1)-m(5^1S_0)$	21	13				
$m(1^3P_2)-m(1^3P_1)$	36	32	44	51	40	45.5 ± 0.2
$m(1^3P_1)-m(1^3P_0)$	101	106	77	81	65	95.9 ± 0.4
$m(2^3P_2)-m(2^3P_1)$	30	23	36	47	26	
$m(2^3P_1)-m(2^3P_0)$	68	66	59	53	37	
$m(3^3P_2)-m(3^3P_1)$	25	19	30	46	20	
$m(3^3P_1)-m(3^3P_0)$	51	46	47	69	25	

相同分波之间的质量劈裂模型相关性相对较小！可用以用来确定尚未找到态的质量范围。

$J^{PC}=1^-$ states in LP and SP models



2. EM transitions

The model : EM operators

The quark-photon EM coupling at the tree level is described by

$$H_e = - \sum_j e_j \bar{\psi}_j \gamma_\mu^j A^\mu(\mathbf{k}, \mathbf{r}) \psi_j, \quad (14)$$

The non-relativistic expansion form is

$$h_e \simeq \sum_j \left[e_j \mathbf{r}_j \cdot \boldsymbol{\epsilon} - \frac{e_j}{2m_j} \boldsymbol{\sigma}_j \cdot (\boldsymbol{\epsilon} \times \hat{\mathbf{k}}) \right] e^{-i\mathbf{k} \cdot \mathbf{r}_j}, \quad (15)$$

保留两项

电跃迁算符

磁跃迁算符

The model : Multipole expansion

$$\begin{aligned} \mathcal{A}_\lambda^{\text{El}} &= \sqrt{\frac{\omega_\gamma}{2}} \left\langle J' \lambda' \left| \sum_j (-i)^l B_l e_j j_{l+1}(kr_j) r_j Y_{l1} \right| J \lambda \right\rangle \\ &+ \sqrt{\frac{\omega_\gamma}{2}} \left\langle J' \lambda' \left| \sum_j (-i)^l B_l e_j j_{l-1}(kr_j) r_j Y_{l1} \right| J \lambda \right\rangle, \quad (20) \end{aligned}$$

$$\begin{aligned} \mathcal{A}_\lambda^{\text{Ml}} &= \sqrt{\frac{\omega_\gamma}{2}} \left\langle J' \lambda' \left| \sum_j (-i)^l C_l \frac{e_j}{2m_j} j_{l-1}(kr_j) \sigma_j^+ Y_{l-1 0} \right| J \lambda \right\rangle \\ &= \sqrt{\frac{\omega_\gamma}{2}} \left\langle J' \lambda' \left| \sum_j (-i)^l C_l \frac{e_j}{2m_j} j_{l-1}(kr_j) [\sigma_j^+ \otimes Y_{l-1 0}]_1^l \right| J \lambda \right\rangle \\ &+ \sqrt{\frac{\omega_\gamma}{2}} \left\langle J' \lambda' \left| \sum_j (-i)^l C_l \frac{e_j}{2m_j} j_{l-1}(kr_j) [\sigma_j^+ \otimes Y_{l-1 0}]_1^{l-1} \right| J \lambda \right\rangle \quad (21) \end{aligned}$$

$$\mathcal{A}_\lambda = \sum_l \left\{ \frac{1 + (-1)^{\pi_i \pi_f + l}}{2} \mathcal{A}_\lambda^{\text{El}} + \frac{1 - (-1)^{\pi_i \pi_f + l}}{2} \mathcal{A}_\lambda^{\text{Ml}} \right\}. \quad (22)$$

The model : Multipole expansion

The most general decomposition of the helicity amplitudes:

$$\mathcal{A}_\lambda = \sum_{k>1} (-1)^{k+1} \sqrt{\frac{2k+1}{2J+1}} a_k \langle k-1; J' \lambda+1 | J \lambda \rangle, \quad (23)$$

with a_k corresponding to the multipole amplitude of the EM tensor operators with a rank k . The second term of Eq. (21) is called as “extra” electric-multipole term, E_R , by F. E. Close *et al.* [76]. Specifically, for ${}^3S_1 \leftrightarrow {}^3P_1$:

$$\begin{aligned} a_1 &= E_1 + E_R = -\frac{\sqrt{2}}{2} (\mathcal{A}_0 + \mathcal{A}_{-1}), \\ a_2 &= M_2 = -\frac{\sqrt{2}}{2} (\mathcal{A}_0 - \mathcal{A}_{-1}); \end{aligned} \quad (24)$$

Results : M1 transitions

Initial state	Final state	E_γ (MeV)				Γ_{M1} (keV)					Γ_{M1} (keV) Exp.
		[36]	NR[7]	GI [7]	Ours	[36]	NR[7]	GI[7]	LP	SP	
J/ψ	$\eta_c(1S)$	115	116	115	111	1.05	2.9	2.4	2.39	2.44	1.58 ± 0.37
$\psi(2S)$	$\eta_c(2S)$	32	48	48	47	0.043	0.21	0.17	0.19	0.19	0.21 ± 0.15
	$\eta_c(1S)$	639	639	638	635	0.95	4.6	9.6	8.08	7.80	1.24 ± 0.29
$\eta_c(2S)$	J/ψ	514	501	501	502	1.53	7.9	5.6	2.64	2.29	
$\psi(3S)$	$\eta_c(3S)$		29	35	30/36		0.046	0.067	0.051	0.088	
	$\eta_c(2S)$		382	436	381		0.61	2.6	1.65	1.78	
	$\eta_c(1S)$		922	967	918		3.5	9.0	6.66	6.76	

[36] Ebert et al., PRD67(03)014027

[7] Godfrey et al., PRD72(05)054026

我们的模型，包括NR和GI模型均给出相比实验明显偏大的结果，需要进一步研究。

Results : E1 dominant transitions

Initial state	Final state	E_γ (MeV)	Γ_{E1} (keV)					Γ_{EM} (keV)		
			Ours	[36]	NR/GI [7]	SNR _{0/1} [9]	LP	SP	LP	SP
$\psi(2S)$	$\chi_{c2}(1P)$	128	18.2	38 / 24	43/34	36	44	38	46	25.2 ± 2.9
	$\chi_{c1}(1P)$	171	22.9	54 / 29	62/36	45	48	42	45	25.5 ± 2.8
	$\chi_{c0}(1P)$	261	26.3	63 / 26	74/25	27	26	22	22	26.3 ± 2.6
$\eta_c(2S)$	$h_c(1P)$	112	41	49 / 36	146/104	49	52	49	52	
$\chi_{c2}(1P)$	J/ψ	429	327	424 / 313	473/309	327	338	284	292	371 ± 34
$\chi_{c1}(1P)$		390	265	314 / 239	354/244	269	278	306	319	285 ± 14
$\chi_{c0}(1P)$		303	121	152 / 114	167/117	141	146	172	179	133 ± 8
$h_c(1P)$	$\eta_c(1S)$	499	560	498 / 352	764/323	361	373	361	373	357 ± 280
$\psi_1(1D)$	$\chi_{c2}(1P)$	215	6.9	4.9/3.3	5.8/4.6	5.4	5.7	7.1	8.1	< 24.8
	$\chi_{c1}(1P)$	258	135	125/77	150/93	115	111	138	135	81 ± 27
	$\chi_{c0}(1P)$	346	355	403/213	486/197	243	232	272	261	202 ± 42
$\psi_2(1D)$	$\chi_{c2}(1P)$	258	59	64/66	70/55	79	82	91	96	
	$\chi_{c1}(1P)$	299	215	307/268	342/208	281	291	285	296	

理论值
偏大

较好符合

理论值
偏大

理解得并不够！理论需要考虑更多的修正？或者实验还需要进一步提高精度？

Results : X(3872)

Considering $X(3872)$ as a pure $\chi_{c1}(2P)$ state, we calculate the radiative decays $X(3872) \rightarrow J/\psi\gamma, \psi(2S)\gamma$. With the linear potential model, we predict that

$$\Gamma[X(3872) \rightarrow J/\psi\gamma] \simeq 72 \text{ keV}, \quad (53)$$

$$\Gamma[X(3872) \rightarrow \psi(2S)\gamma] \simeq 94 \text{ keV}. \quad (54)$$

With these predicted partial widths, we can easily obtain the ratio

$$R_{\psi'\gamma/\psi\gamma}^{\text{th}} = \frac{\Gamma[X(3872) \rightarrow \psi(2S)\gamma]}{\Gamma[X(3872) \rightarrow J/\psi\gamma]} \simeq 1.3, \quad (55)$$

比值与实验值不矛盾，结合理论预言质量：**M=3900 MeV**，有可能是传统**2P**为主的态。

Results : X(3823)

Considering X(3823) as the $\psi_2(1D)$ state:

$$\Gamma[X(3823) \rightarrow \chi_{c1}(1P)\gamma] \simeq 300 \text{ keV}, \quad (28)$$

$$\Gamma[X(3823) \rightarrow \chi_{c2}(1P)\gamma] \simeq 90 \text{ keV}. \quad (29)$$

Our prediction of $\Gamma[X(3823) \rightarrow \chi_{c1}(1P)\gamma]$ is close to the predictions in Refs. [7, 9, 36, 83], while our prediction for $\Gamma[X(3823) \rightarrow \chi_{c2}(1P)\gamma]$ is about a factor of 1.4~1.8 larger than the predictions in these works. Furthermore, our predicted partial width ratio,

$$\frac{\Gamma[X(3823) \rightarrow \chi_{c2}(1P)\gamma]}{\Gamma[X(3823) \rightarrow \chi_{c1}(1P)\gamma]} \simeq 30\%, \quad (30)$$

比值与实验值不矛盾，结合理论预言质量：**M=3807 MeV**，有可能是传统**2P**为主的态。

Results : $X(4140)$

观测 $\psi(1S,2S,3S)$ γ 衰变道

$X(4140)$ seems to favor the $\chi_{c1}(3P)$ state. If the $X(4140)$ state is assigned as $\chi_{c1}(3P)$, within the screened potential model the partial radiative decay widths of the dominant channels are predicted to be

$$\Gamma[\psi(4140) \rightarrow J/\psi\gamma] \simeq 38 \text{ keV}, \quad (80)$$

$$\Gamma[\psi(4140) \rightarrow \psi(2S)\gamma] \simeq 51 \text{ keV}, \quad (81)$$

$$\Gamma[\psi(4140) \rightarrow \psi(3S)\gamma] \simeq 36 \text{ keV}, \quad (82)$$

Combined the average measured width with the predicted partial radiative decay widths of $X(4140)$, the branching ratios are estimated to be

$$Br[\psi(4140) \rightarrow J/\psi\gamma] \simeq 2.4 \times 10^{-3}, \quad (83)$$

$$Br[\psi(4140) \rightarrow \psi(2S)\gamma] \simeq 3.2 \times 10^{-3}, \quad (84)$$

$$Br[\psi(4140) \rightarrow \psi(3S)\gamma] \simeq 2.3 \times 10^{-3}. \quad (85)$$

Results : $X(4274)$

观测 $\psi(1S,2S,3S)$ γ 衰变道

While, if the $X(4274)$ state is assigned as $\chi_{c1}(3P)$, within the linear potential model the partial radiative decay widths of the dominant channels are predicted to be

$$\Gamma[\psi(4274) \rightarrow J/\psi\gamma] \simeq 48 \text{ keV}, \quad (86)$$

$$\Gamma[\psi(4274) \rightarrow \psi(2S)\gamma] \simeq 88 \text{ keV}, \quad (87)$$

$$\Gamma[\psi(4274) \rightarrow \psi(3S)\gamma] \simeq 297 \text{ keV}, \quad (88)$$

Combined the measured width with the predicted partial radiative decay widths of $X(4274)$, the branching ratios are estimated to be

$$Br[\psi(4274) \rightarrow J/\psi\gamma] \simeq 0.9 \times 10^{-3}, \quad (89)$$

$$Br[\psi(4274) \rightarrow \psi(2S)\gamma] \simeq 1.6 \times 10^{-3}, \quad (90)$$

$$Br[\psi(4274) \rightarrow \psi(3S)\gamma] \simeq 5.3 \times 10^{-3}. \quad (91)$$

Results : 3S,4S EM transitions

Initial state	Final state	E_γ (MeV)				Γ_{E1} (keV)				Γ_{EM} (keV)		Br	
		NR [7]	GI [7]	LP	SP	NR [7]	GI [7]	LP	SP	LP	SP	LP	SP
$\psi(3S)$	$\chi_{c2}(2P)$	67	119	111	111	14	48	65	79	67	82	8.4×10^{-4}	1.0×10^{-3}
80 ± 10^a	$\chi_{c1}(2P)$	113	145	138	138	39	43	58	71	55	67	6.9×10^{-4}	8.4×10^{-4}
	$\chi_{c0}(2P)$	184	180	167	187	54	22	21	31	19	27	2.4×10^{-4}	3.4×10^{-4}
	$\chi_{c2}(1P)$	455	508	455	455	0.7	13	0.21	2.1	0.25	2.5	3.1×10^{-6}	3.1×10^{-5}
	$\chi_{c1}(1P)$	494	547	494	494	0.53	0.85	4.8	8.0	4.0	6.7	5.0×10^{-5}	8.4×10^{-5}
	$\chi_{c0}(1P)$	577	628	577	577	0.27	0.63	9.1	10.6	5.9	6.7	7.4×10^{-5}	8.4×10^{-5}
$\eta_c(3S)$	$h_c(2P)$	108	108	108	108	105	64	104	128	104	128	1.3×10^{-3}	1.6×10^{-3}
80^b	$h_c(1P)$	485	511	456	456	9.1	28	0.045	1.4	0.045	1.4	5.6×10^{-7}	1.8×10^{-5}
$\psi(4S)$	$\chi_{c2}(1P)$	775	804	773	664	0.61	5.2	0.13	0.66	0.17	0.84	2.2×10^{-6}	1.1×10^{-5}
78^b	$\chi_{c1}(1P)$	811	841	809	701	0.41	0.53	3.8	3.9	2.9	3.0	3.7×10^{-5}	3.8×10^{-5}
	$\chi_{c0}(1P)$	887	915	884	778	0.18	0.13	7.5	6.2	3.7	2.7	4.7×10^{-5}	4.2×10^{-5}
	$\chi_{c2}(2P)$	421	446	458	339	0.62	15	11	4.7	13	5.3	1.7×10^{-4}	6.8×10^{-5}
	$\chi_{c1}(2P)$	423	469	482	364	0.49	0.92	24	12	20	11	2.6×10^{-4}	1.4×10^{-4}
	$\chi_{c0}(2P)$	527	502	510	411	0.24	0.39	17	12	12	8.7	1.5×10^{-4}	1.1×10^{-4}
	$\chi_{c2}(3P)$	97	112	101	69	68	66	80	39	82	40	1.1×10^{-3}	5.1×10^{-4}
	$\chi_{c1}(3P)$	142	131	126	88	126	54	74	38	71	37	9.1×10^{-4}	4.7×10^{-4}
	$\chi_{c0}(3P)$	208	155	178	133	0.003	25	40	23	36	21	4.6×10^{-4}	2.7×10^{-4}
$\eta_c(4S)$	$h_c(1P)$	782	808	778	675	5.2	9.6	0.29	0.63	0.29	0.63	4.8×10^{-6}	1.0×10^{-5}
61^b	$h_c(2P)$	427	444	461	348	10.1	31.3	20	7.9	20	7.9	3.3×10^{-4}	1.3×10^{-4}
	$h_c(3P)$	104	106	142	70	159	101	102	70	102	70	1.7×10^{-3}	1.1×10^{-3}

Results : D wave EM transitions

Initial state	Final state	E_γ (MeV)				Γ_{E1} (keV)					Γ_{EM} (keV)		Br	
		[36]	NR/GI [7]	SNR [9]	LP/SP	[36]	NR/GI [7]	SNR _{0/1} [9]	LP	SP	LP	SP	LP	SP
$\psi_3(1D)$	$\chi_{c2}(1P)$	250	242/282	236	264/264	156	272/296	284/223	377	393	350	364	12%	12%
$\eta_{c2}(1D)$	$h_c(1P)$	275	264/307	260	284/284	245	339/344	575/375	362	376	362	376	72%	75%
$\psi_3(2D)$	$\chi_{c2}(1P)$	566/609			571/518	29/16			83	78	72	67	4.9×10^{-4}	4.5×10^{-4}
148 ^a	$\chi_{c2}(2P)$	190/231			238/181	239/272			457	256	427	243	2.9×10^{-3}	1.6×10^{-3}
$\psi_2(2D)$	$\chi_{c2}(1P)$	558/602			564/516	7.1/0.62			16	16	20	20	1.7×10^{-4}	2.2×10^{-4}
92 ^a	$\chi_{c1}(1P)$	597/640			603/554	26/23			64	64	68	68	7.4×10^{-4}	7.4×10^{-4}
	$\chi_{c2}(2P)$	182/223			231/178	52/65			101	57	115	64	1.3×10^{-3}	7.0×10^{-4}
	$\chi_{c1}(2P)$	226/247			222/204	298/225			220	186	223	188	2.4×10^{-3}	2.0×10^{-3}
$\psi_1(2D)$	$\chi_{c2}(1P)$	559/590			587	0.79/0.027			16	16	17	20	2.3×10^{-4}	2.7×10^{-4}
74 ^a	$\chi_{c1}(1P)$	598/628			625	14/3.4			25	42	37	63	5.0×10^{-4}	8.5×10^{-4}
	$\chi_{c0}(1P)$	677/707			704	27/35			120	149	150	189	2.0×10^{-3}	2.6×10^{-3}
	$\chi_{c2}(2P)$	183/210			256	5.9/6.3			18	21	24	29	3.2×10^{-4}	3.9×10^{-4}
	$\chi_{c1}(2P)$	227/234			281/281	168/114			253	280	309	347	4.2×10^{-3}	4.7×10^{-3}
	$\chi_{c0}(2P)$	296/269			312/329	483/191			299	321	332	360	4.5×10^{-3}	4.9×10^{-3}
$\eta_{c2}(2D)$	$h_c(1P)$	585/634			590/542	40/25			96	92	96	92	1.3×10^{-3}	1.2×10^{-3}
111 ^a	$h_c(2P)$	218/244			256/203	336/296			438	271	438	271	3.9×10^{-3}	2.4×10^{-3}

寻找 1^3D_3 , $1^3D_3 \rightarrow 1^3P_2$ 值得观测; $\psi(4160)$ 有可能用来产生丢失的2P态。

Results : 2P wave EM transitions

观测1S γ , 2S γ 衰变道

Initial state	Final state	E_γ (MeV)			Γ_{E1} (keV)				Γ_{EM} (keV)		Br	
		NR/GI [7]	SNR [9]	LP/SP	NR/GI [7]	SNR _{0/1} [9]	LP	SP	LP	SP	LP	SP
$\chi_{c2}(2P)$ 24±6 ^a	$\psi_3(1D)$	163/128		96/96	88/29		20	24	20	24	8.3×10^{-4}	1.0×10^{-3}
	$\psi_2(1D)$	168/139		103	17/5.6		3.3	4.1	3.2	4.0	1.3×10^{-4}	1.7×10^{-4}
	$\psi_1(1D)$	197/204		146	1.9/1.0		0.47	0.62	0.36	0.46	1.5×10^{-5}	1.9×10^{-5}
	$\psi(2S)$	276/282	235	234	304/207	225/100	146	163	135	150	5.6×10^{-3}	6.3×10^{-3}
	J/ψ	779/784	744	742	81/53	101/109	118	119	93	93	3.9×10^{-3}	3.9×10^{-3}
$\chi_{c1}(2P)$ 165 ^b	$\psi_2(1D)$	123/113		76/76	35/18		2.8	3.4	2.9	3.5	1.8×10^{-5}	2.1×10^{-5}
	$\psi_1(1D)$	152/179		120/120	22/21		8.6	10.8	7.9	9.8	4.9×10^{-5}	5.9×10^{-5}
	$\psi(2S)$	232/258	182	208/208	183/183	103/60	129	145	139	155	8.4×10^{-4}	9.4×10^{-4}
	J/ψ	741/763	697	720/720	71/14	83/45	64	68	81	88	4.9×10^{-4}	5.3×10^{-4}
$\chi_{c0}(2P)$ 30 ^b	$\psi_1(1D)$	81/143		90/69	13/51		21	12	20	12	6.7×10^{-4}	4.0×10^{-4}
	$\psi(2S)$	162/223	152	179/159	64/135	61/44	108	89	121	99	4.0×10^{-3}	3.3×10^{-3}
	J/ψ	681/733	672	695/678	56/1.3	74/9.3	4.0	1.5	6.1	2.3	2.0×10^{-4}	7.7×10^{-5}
$h_c(2P)$ 87 ^b	$\eta_{c2}(1D)$	133/117		100/100	60/27		25	25	25	25	2.9×10^{-4}	2.9×10^{-4}
	$\eta_c(2S)$	285/305	261	252/252	280/218	309/108	160	176	160	176	1.8×10^{-3}	2.0×10^{-3}
	$\eta_c(1S)$	839/856	818	808/808	140/85	134/250	135	134	135	134	1.6×10^{-3}	1.6×10^{-3}

Results : 3P wave EM transitions

Initial state	Final state	E_γ (MeV)			Γ_{E1} (keV)				Γ_{EM} (keV)		Br	
		NR/GI [7]	SNR [9]	LP/SP	NR/GI [7]	SNR _{0/1} [9]	LP	SP	LP	SP	LP	SP
$\chi_{c2}(3P)$ 66^b	$\psi_3(2D)$	147/118		136/98	148/51		116	64	121	66	1.8×10^{-3}	1.0×10^{-3}
	$\psi_2(2D)$	156/127		143/101	31/10		18	10	18	10	2.7×10^{-4}	1.5×10^{-4}
	$\psi_1(2D)$	155/141		117/20	2.1/0.77		0.55	0.004	0.44	0.004	6.7×10^{-6}	6.0×10^{-8}
	$\psi_3(1D)$	481/461		453/364	0.049/6.8		15	10	17	12	1.1×10^{-4}	1.8×10^{-4}
	$\psi_2(1D)$	486/470		459/370	0.01/0.13		4.6	2.5	4.6	2.4	7.0×10^{-5}	3.6×10^{-5}
	$\psi_1(1D)$	512/530		495/411	0.00/0.00		1.9	1.0	1.5	0.79	2.2×10^{-5}	1.2×10^{-5}
	$\psi(3S)$	268/231		261/168	509/199		306	121	281	114	4.3×10^{-3}	1.7×10^{-3}
	$\psi(2S)$	585/602		574/492	55/30		116	90	97	76	1.5×10^{-3}	1.2×10^{-3}
	J/ψ	1048/1063		1042/967	34/19		83	69	61	51	9.2×10^{-4}	7.7×10^{-4}
$\chi_{c1}(3P)$ 39^b	$\psi_2(2D)$	112/108		117/82	58/35		22	11	23	11	5.9×10^{-4}	2.8×10^{-4}
	$\psi_1(2D)$	111/121		92/1	19/15		8.6	0	8.1	0	2.1×10^{-4}	0
	$\psi_2(1D)$	445/452		436/353	0.035/4.6		0.13	0.09	0.12	0.09	3.1×10^{-6}	2.3×10^{-6}
	$\psi_1(1D)$	472/512		476/394	0.014/0.39		4.4	2.7	3.2	2.0	6.1×10^{-5}	4.1×10^{-5}
	$\psi(3S)$	225/212		237/149	303/181		305	111	331	117	8.5×10^{-3}	3.0×10^{-3}
	$\psi(2S)$	545/585		556/475	45/8.9		78	63	94	74	2.4×10^{-3}	1.9×10^{-3}
$\chi_{c0}(3P)$ 51^b	J/ψ	1013/1048		1023/952	31/2.2		36	33	50	45	1.3×10^{-3}	1.2×10^{-3}
	$\psi_1(2D)$	43/97		39/45	4.4/35		3.8	9.3	3.8	9.1	7.5×10^{-5}	1.8×10^{-4}
	$\psi_1(1D)$	410/490		427/352	0.037/9.7		0.31	0.44	0.27	0.39	5.3×10^{-6}	7.6×10^{-6}
	$\psi(3S)$	159/188		186/105	109/145		214	56	241	61	4.7×10^{-3}	1.2×10^{-3}
	$\psi(2S)$	484/563		509/434	32/0.045		13	6.9	17	9.1	3.3×10^{-4}	1.8×10^{-4}
	J/ψ	960/1029		981/916	27/1.5		0.14	0.08	0.24	0.13	4.7×10^{-6}	2.5×10^{-6}

3. observation

3P_2 decay chains

Decay chain	Br_1	Br_2	Br_3	Br
$2^3P_2 \rightarrow 1^3D_1 \rightarrow 1^3P_0 \rightarrow J/\psi$	1.5×10^{-5}	9.9×10^{-3}	1.6%	2.4×10^{-9}
$2^3P_2 \rightarrow 1^3D_1 \rightarrow 1^3P_1 \rightarrow J/\psi$	1.5×10^{-5}	5.5×10^{-3}	34.8%	2.9×10^{-8}
$2^3P_2 \rightarrow 1^3D_1 \rightarrow 1^3P_2 \rightarrow J/\psi$	1.5×10^{-5}	4.8×10^{-4}	14.6%	1.0×10^{-8}
$2^3P_2 \rightarrow 1^3D_2 \rightarrow 1^3P_1 \rightarrow J/\psi$	1.3×10^{-4}	42%	34.8%	1.9×10^{-5}
$2^3P_2 \rightarrow 1^3D_2 \rightarrow 1^3P_2 \rightarrow J/\psi$	1.3×10^{-4}	13%	14.6%	2.5×10^{-6}
$2^3P_2 \rightarrow 1^3D_3 \rightarrow 1^3P_2 \rightarrow J/\psi$	8.3×10^{-4}	12%	14.6%	1.4×10^{-5}

红颜色标出的是重要的两个衰变链，值得实验观测

3 3S_1 decay chains

Decay chain	$Br_1(10^{-4})$	$Br_2(10^{-4})$	$Br(10^{-7})$
$3^3S_1 \rightarrow 2^3P_2 \rightarrow 2^3S_1$	8.4	56	47
$3^3S_1 \rightarrow 2^3P_1 \rightarrow 2^3S_1$	6.9	8.4	5.8
$3^3S_1 \rightarrow 2^3P_0 \rightarrow 2^3S_1$	2.4	40	9.6
$3^3S_1 \rightarrow 2^3P_2 \rightarrow J/\psi$	8.4	39	33
$3^3S_1 \rightarrow 2^3P_1 \rightarrow J/\psi$	6.9	4.9	3.4
$3^3S_1 \rightarrow 2^3P_0 \rightarrow J/\psi$	2.4	2.0	0.5

BESIII 是否有机会通过上述衰变链寻找丢失的**2P**态呢？

2 3D_1 decay chains

Decay chain	$Br_1(10^{-3})$	$Br_2(10^{-4})$	$Br(10^{-5})$
$2^3D_1 \rightarrow 2^3P_2 \rightarrow 2^3S_1$	0.32	56	0.18
$2^3D_1 \rightarrow 2^3P_1 \rightarrow 2^3S_1$	4.2	8.4	0.35
$2^3D_1 \rightarrow 2^3P_0 \rightarrow 2^3S_1$	4.5	40	1.8
$2^3D_1 \rightarrow 2^3P_2 \rightarrow J/\psi$	0.32	39	0.13
$2^3D_1 \rightarrow 2^3P_1 \rightarrow J/\psi$	4.2	4.9	0.2
$2^3D_1 \rightarrow 2^3P_0 \rightarrow J/\psi$	4.5	2.0	0.09
$2^3D_1 \rightarrow 1^3P_2 \rightarrow J/\psi$	0.23	1460	3.3
$2^3D_1 \rightarrow 1^3P_1 \rightarrow J/\psi$	0.50	3480	17
$2^3D_1 \rightarrow 1^3P_0 \rightarrow J/\psi$	2.0	160	3.2

BESIII 是否有机会通过上述衰变链寻找丢失的**2P**态呢？

4 3S_1 decay chains

Decay chain	$Br_1(10^{-4})$	$Br_2(10^{-4})$	$Br(10^{-7})$
$4^3S_1 \rightarrow 2^3P_2 \rightarrow 2^3S_1$	1.7	56	9.5
$4^3S_1 \rightarrow 2^3P_1 \rightarrow 2^3S_1$	2.6	8.4	2.1
$4^3S_1 \rightarrow 2^3P_0 \rightarrow 2^3S_1$	1.5	40	5.9
$4^3S_1 \rightarrow 2^3P_2 \rightarrow J/\psi$	1.7	39	6.6
$4^3S_1 \rightarrow 2^3P_1 \rightarrow J/\psi$	2.6	4.9	1.3
$4^3S_1 \rightarrow 2^3P_0 \rightarrow J/\psi$	1.5	2.0	0.31
$4^3S_1 \rightarrow 3^3P_2 \rightarrow 2^3S_1$	11	15	16
$4^3S_1 \rightarrow 3^3P_1 \rightarrow 2^3S_1$	9.1	24	21
$4^3S_1 \rightarrow 3^3P_0 \rightarrow 2^3S_1$	4.6	33	15
$4^3S_1 \rightarrow 3^3P_2 \rightarrow J/\psi$	1.7	9.2	1.6
$4^3S_1 \rightarrow 3^3P_1 \rightarrow J/\psi$	2.6	13	3.4
$4^3S_1 \rightarrow 3^3P_0 \rightarrow J/\psi$	1.5	0.047	0.007

BESIII 是否有机会通过上述衰变链寻找丢失的2P,3P态呢?

4. Higher multipole

Higher multipole contributions

process	$\frac{E_R}{a_1}$	$\frac{E_R}{a_1}$	$\frac{a_2}{a_1}$	$\frac{a_2}{a_1}$	$\frac{a_2}{a_1}$	$\frac{a_2}{a_1}$	$\frac{a_2}{a_1}$
	LP	SP	SP	LP	Lat. [19]	CLEO [105]	BESII [104]
$\chi_{c1}(1P) \rightarrow J/\psi\gamma$	+0.062	+0.065	-0.065	-0.062	-0.09(7)	-0.0626(87)	
$\chi_{c2}(1P) \rightarrow J/\psi\gamma$	-0.078	-0.082	-0.110	-0.105	-0.39(7)	-0.093(19)	
$\psi(2S) \rightarrow \chi_{c1}(1P)\gamma$	-0.030	+0.031	-0.031	-0.030		0.0276(96)	
$\psi(2S) \rightarrow \chi_{c2}(1P)\gamma$	+0.021	+0.022	-0.030	-0.028		0.010(16)	-0.051 ^{+0.054} _{-0.036}

Specifically, for ${}^3S_1 \leftrightarrow {}^3P_1$:

for ${}^3P_2 \rightarrow {}^3S_1$:

$$a_1 = E_1 + E_R = -\frac{\sqrt{2}}{2}(\mathcal{A}_0 + \mathcal{A}_{-1}),$$

$$a_1 = E_1 + E_R = \frac{\sqrt{10}}{2}(\sqrt{3}\mathcal{A}_{-1} - \mathcal{A}_0),$$

$$a_2 = M_2 = -\frac{\sqrt{2}}{2}(\mathcal{A}_0 - \mathcal{A}_{-1});$$

$$a_2 = M_2 = \frac{\sqrt{6}}{2}(\sqrt{3}\mathcal{A}_0 - \mathcal{A}_{-1});$$

磁酸符贡献ER对E1跃迁的修正对某些过程很明显，M2/E1与实验测量一致。

Other predictions

process	$\frac{E_R}{a_1}$ (LP)	$\frac{E_R}{a_1}$ (SP)	$\frac{a_2}{a_1}$ (SP)	$\frac{a_2}{a_1}$ (LP)
$\psi_1(1D) \rightarrow \chi_{c1}(1P)\gamma$	+0.088	+0.092	+0.041	+0.040
$\psi_1(1D) \rightarrow \chi_{c2}(1P)\gamma$	+0.214	+0.224	+0.074	+0.066
$\chi_{c1}(2P) \rightarrow J/\psi\gamma$	+0.108	+0.113	-0.113	-0.108
$\chi_{c2}(2P) \rightarrow J/\psi\gamma$	-0.143	-0.151	-0.203	-0.192
$\chi_{c1}(2P) \rightarrow \psi(2S)\gamma$	+0.034	+0.036	-0.036	-0.034
$\chi_{c2}(2P) \rightarrow \psi(2S)\gamma$	-0.041	-0.043	-0.058	-0.055
$\chi_{c1}(3P) \rightarrow J/\psi\gamma$	+0.147	+0.144	-0.144	-0.147
$\chi_{c2}(3P) \rightarrow J/\psi\gamma$	-0.213	-0.207	-0.277	-0.286
$\chi_{c1}(3P) \rightarrow \psi(2S)\gamma$	+0.086	+0.078	-0.078	-0.086
$\chi_{c2}(3P) \rightarrow \psi(2S)\gamma$	-0.107	-0.096	-0.128	-0.144
$\chi_{c1}(3P) \rightarrow \psi(3S)\gamma$	+0.038	+0.026	-0.026	-0.038
$\chi_{c2}(3P) \rightarrow \psi(3S)\gamma$	-0.046	-0.031	-0.041	-0.062

5. Summary

- 面对高激发态，势模型需要作改进；
- 电磁衰变并没有很好的测量和理解；
- 磁跃迁算符对E1跃迁有较大修正；
- 电磁辐射跃迁可以帮助寻找一些丢失的共振态；
- 一些有较大分支比的常见的电磁衰变道急需测量

Thanks !