Methods used to measure azimuthal anisotropies

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Outline

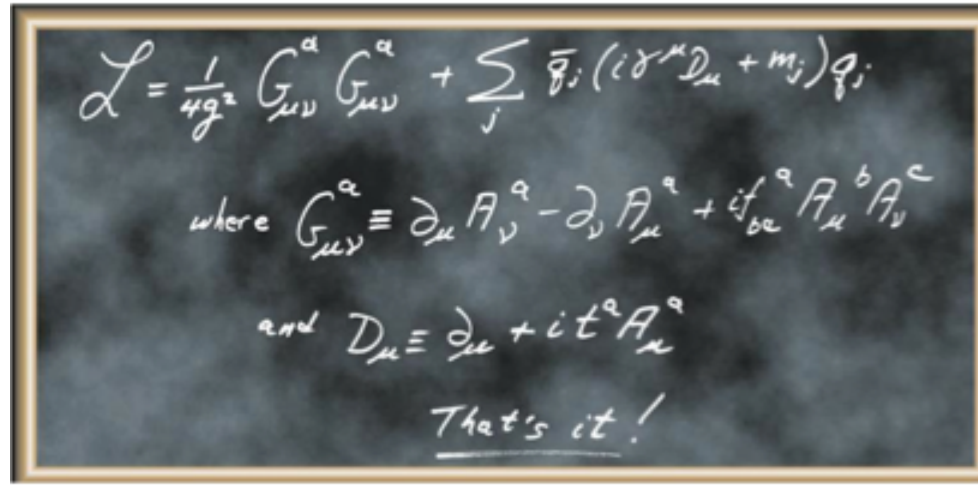
- Motivation to study collective phenomena in the nuclear collisions
- Little history of anisotropic transverse flow
  - Shock waves
  - Sphericity
  - Different kind of flows
  - Directed flow – transverse momentum analysis
  - Squeeze-out
  - Fourier analysis
- Contemporary methods for studying anisotropies
  - Event plane method
  - Two-particle correlations
  - Many-particle correlations – cumulants
  - Method od Lee-Yang zeros
  - Scalar product method
  - Principal component analysis
- Distinguishing between collective motion and fake flows
  - HBT effect from Bose-Einstein correlations
  - Correlations arising from jet fragmentation
- Conclusions

Collective flow - motivation

- Collective properties of nuclear matter
- Contemporary nuclear physics and bulk properties nuclear medium
- Equation of state
- Extraction of pressure build up during the collisions of nuclei
- Constituents at early time of the collision system
- Partonic matter
- Study of little Bang in the laboratory
- Cosmological aspect

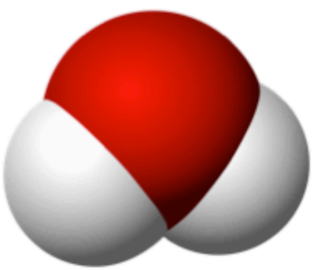
QCD is a fundamental theory of strong interaction

"The study of the strong interaction is now a mature subject – we have a theory of fundamentals that is correct and complete"
- **Frank Wilczek,** 2014



Different phenomena, like many body QCD system, emerge

"More is different" P.W. Anderson



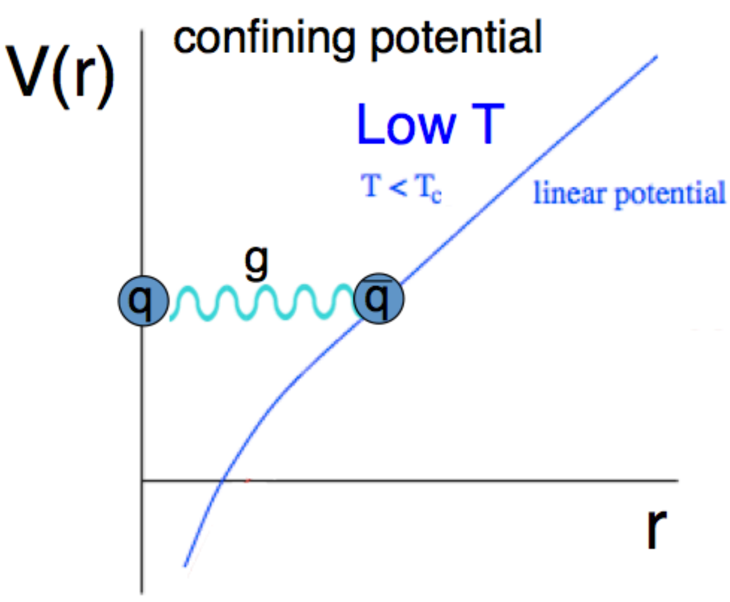
x
1,000,000,000,000,
000,000,000,000 =

QCD phenomena – from hadrons to QGP

- Deconfinement

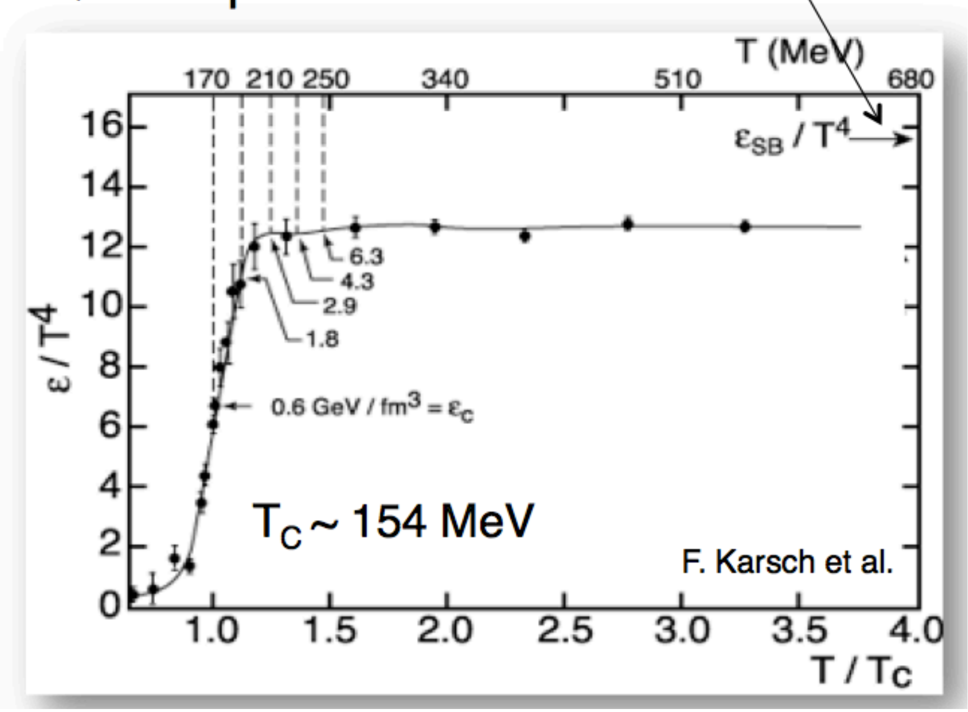
  At high enough T, quarks and gluons are liberated from their 'home' hadrons and can move at distances significantly bigger than the size of the nucleons

Lattice QCD calculation



"ideal gas" limit

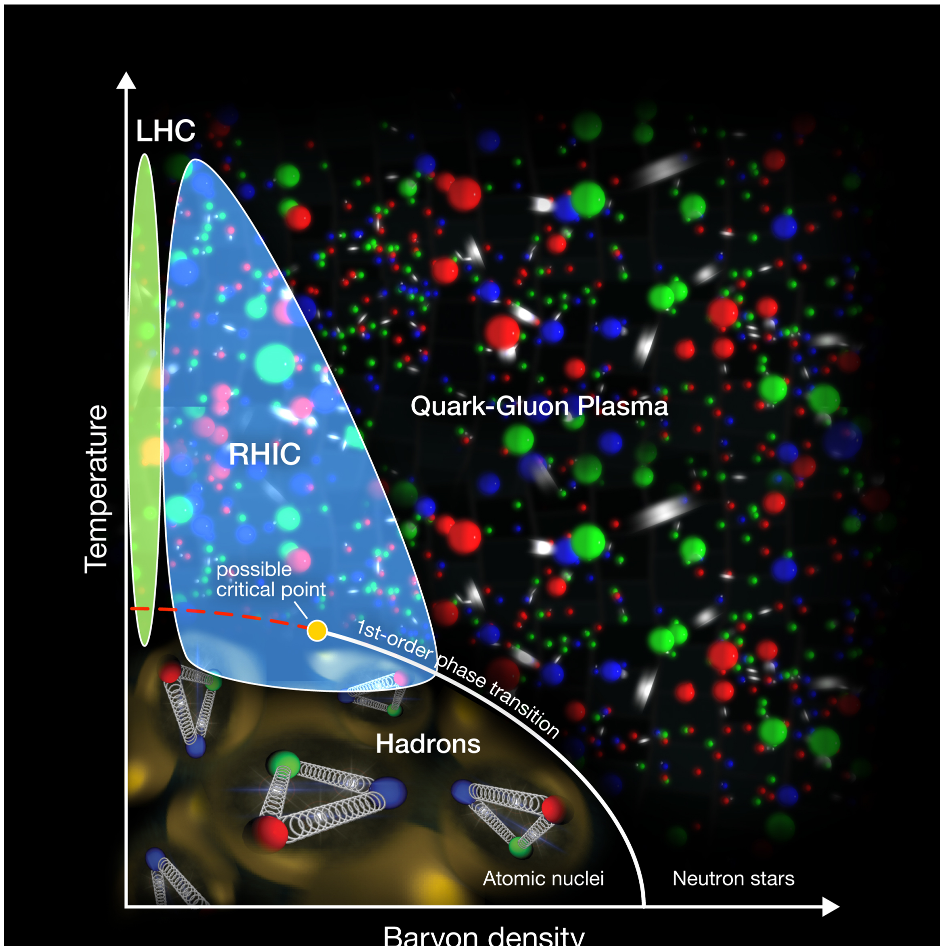
QCD equation of state

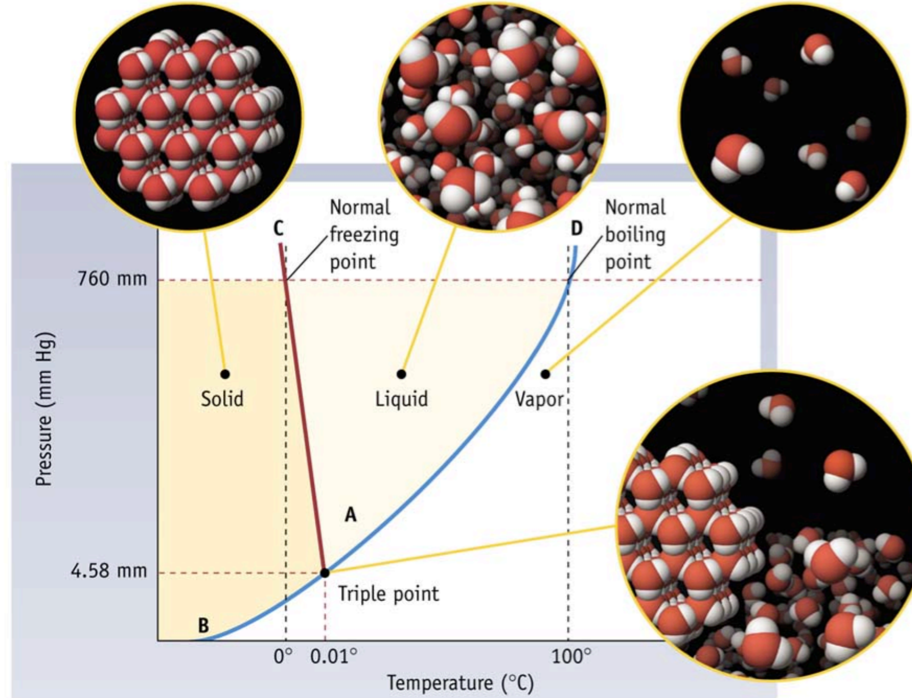


QCD phase diagram

New phase(s) of QCD matter – QGP

- ✧ explore its properties
- ✧ phase transition
- ✧ critical point?
- ✧ Equation of state





phases of water

- ✧ gas
- ✧ solid
- ✧ liquid

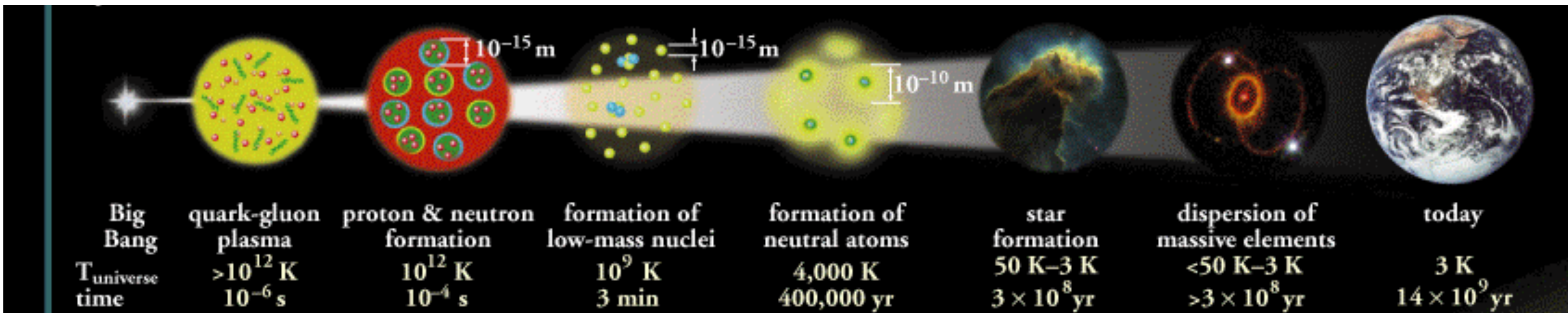
phase transitions

- ✧ gas ←→ solid
- ✧ gas ←→ liquid
- ✧ liquid ←→ solid

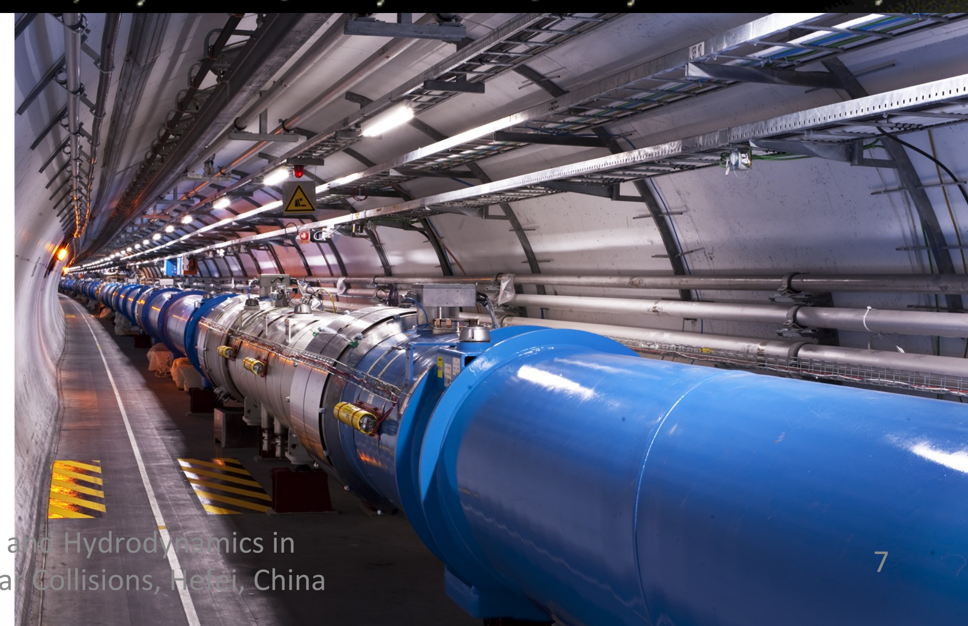
critical point

Part of the Universe history and little Bang

- Expanding, the Universe passed through different phases
- Temperature and energy density was dropping
- ≈ 1μs after Big Bang quarks, antiquarks and gluons (partons) formed QGP
- When, T dropped to 170 MeV, partons are glued together to form hadrons
- Universe was not transparent
- After ≈ 3×10^5y neutral atoms are formed and Universe became transparent



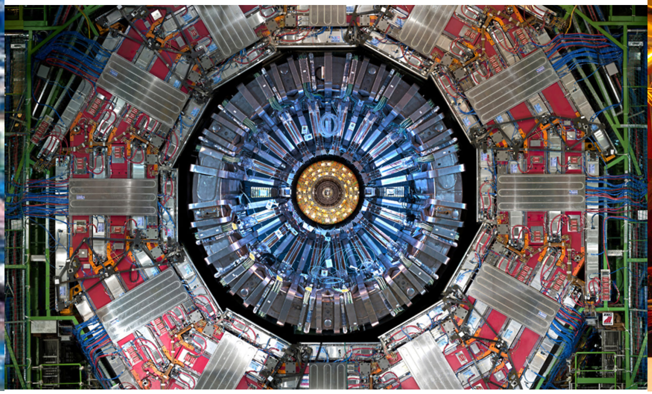
- How to 'see' QGP?
- One can recreate such a state by placing enough energy into a given volume
- Can be done by ultrarelativistic heavy ion collisions (HIC)
- With high enough incident energy maybe even in proton-proton collisions
- In a laboratory created a little Big Bang
- For it, one needs accelerators



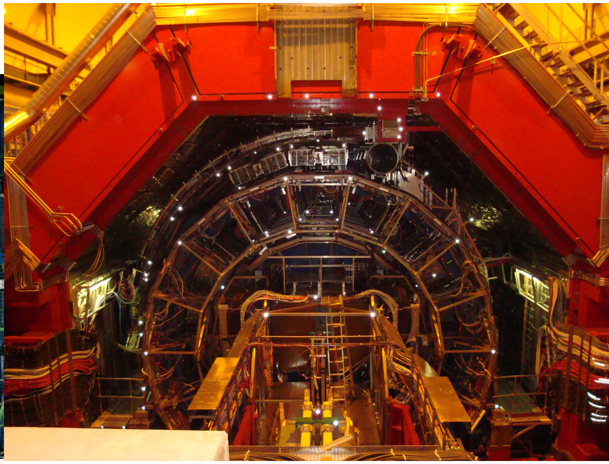
HIC experiments

ATLAS

CMS

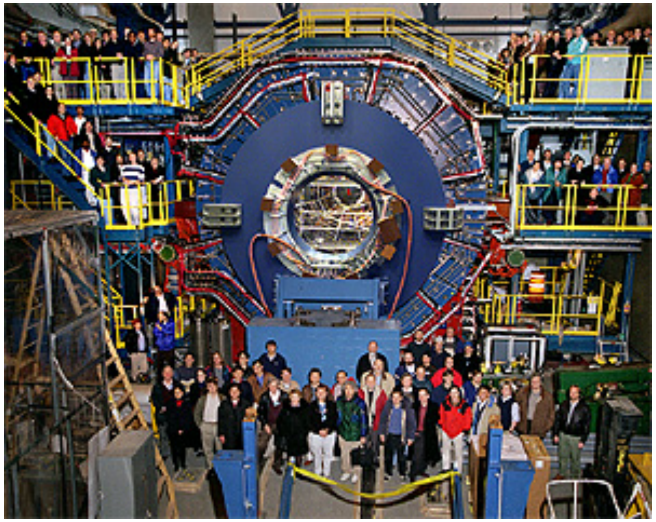


ALICE



LHC

STAR



PHENIX

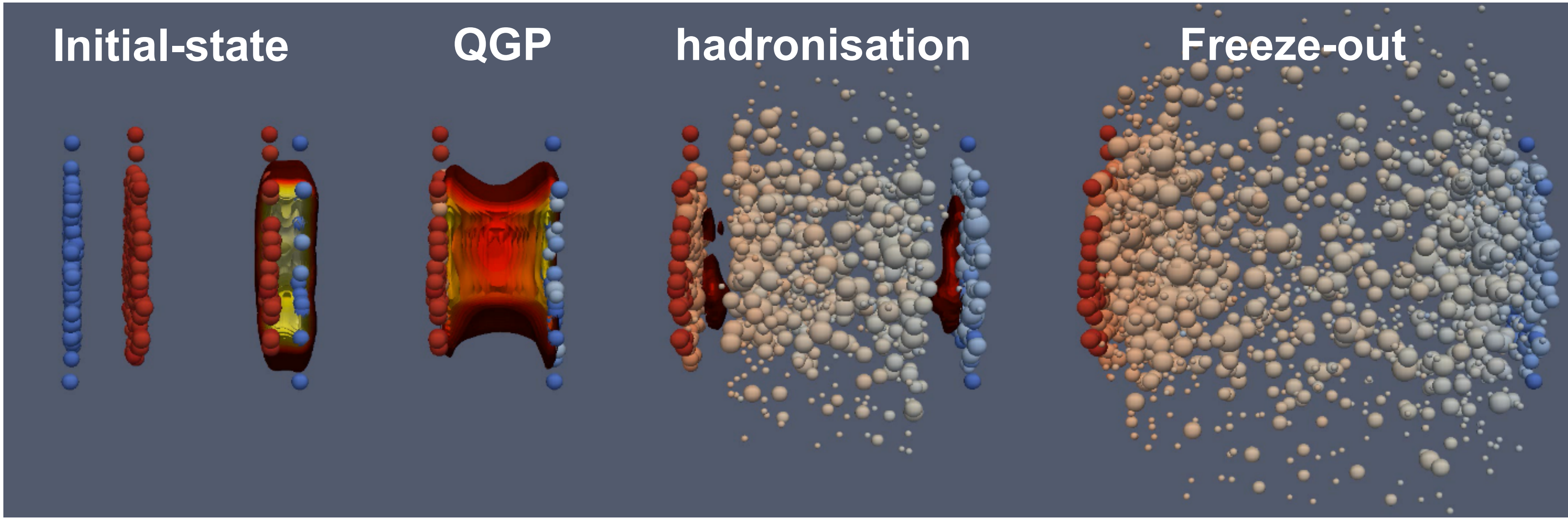
CERES/NA45

RHIC

SPS

… as well as other detectors

Evolution of a heavy ion collision



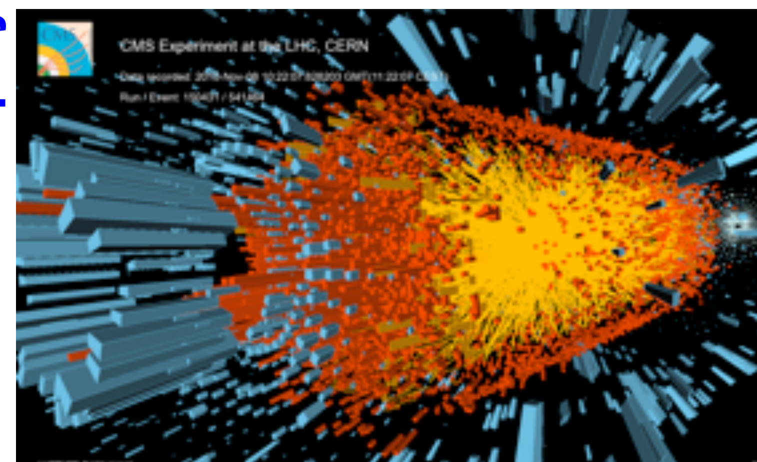
0 ≈ 1 ≈ 10 ≈ 20 fm/c

There are different observables to study QGP

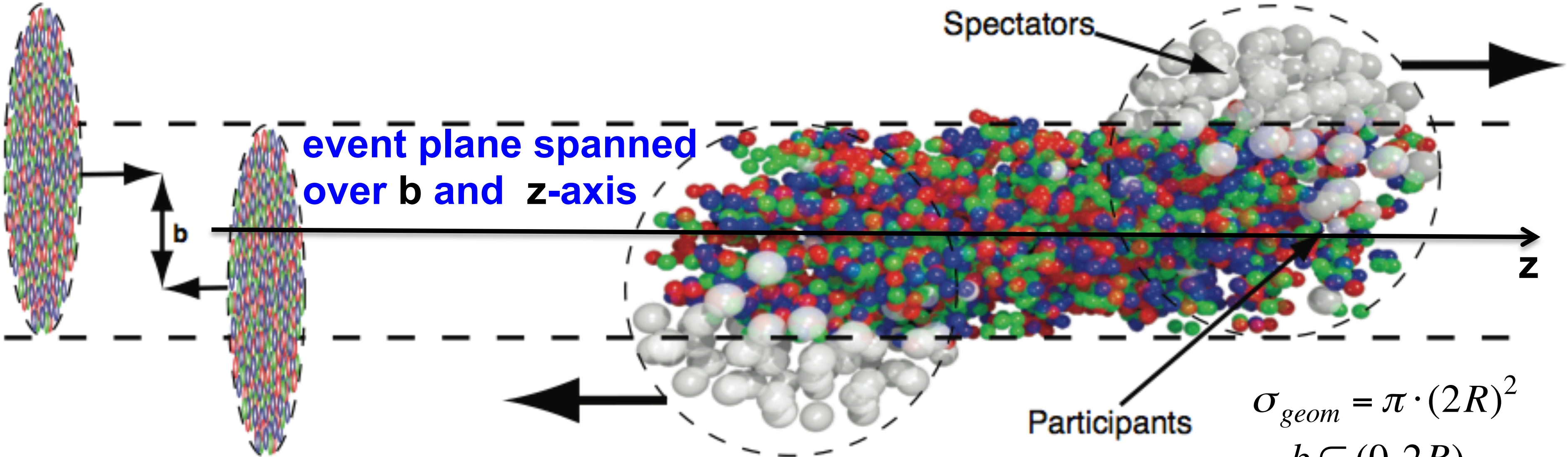
- ✧ particle spectra
- ✧ particle ratios
- ✧ particle interferometry
- ✧ direct photons
- ✧ thermal leptons
- ✧ jet quenching

and anisotropic flows

CMS event display



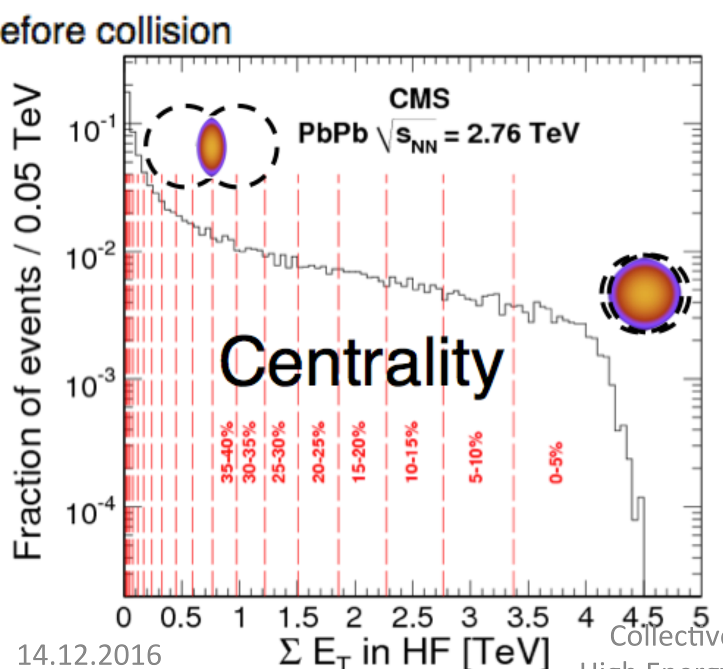
Some basic denotations



$$\sigma_{geom} = \pi \cdot (2R)^2$$

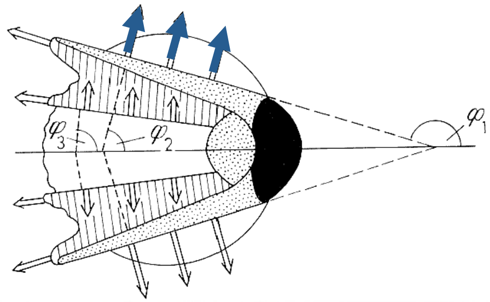
$$b \in (0, 2R)$$

$$\sigma / \sigma_{geom} \in (0, 100)\%$$

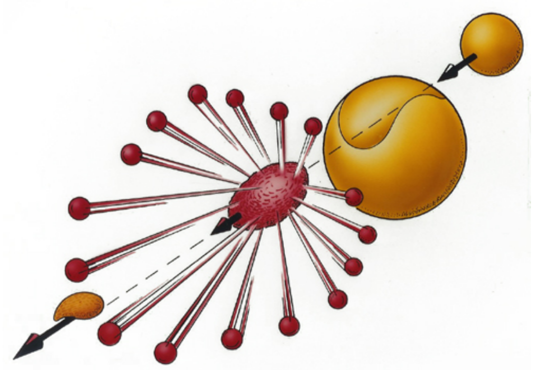


- ✧ N_{part} – number of wounded nucleons – participants
- ✧ In a full head-on PbPb collisions N_{part}=2x208
- ✧ **b** impact parameter vector – conects 'centres' of the colliding nuclei
- ✧ **z**-axis – the beam direction
- ✧ 0% means most central collisions
- ✧ 100% means most peripheral collisions

A little history of methods used to study flows



- ✧ Shock waves – 1959 (angle depends on speed of sound which depends on the EOS) – first prediction of flow at high energy

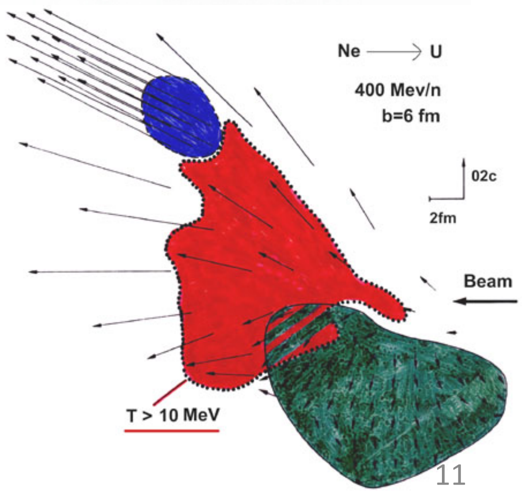


- ✧ Relativistic collisions – 1977

-fireball

Coalescence

"At still higher densities it is possible that the nucleons might break up into their constituents to produce quark matter" 1979, A. M. Poskanzer

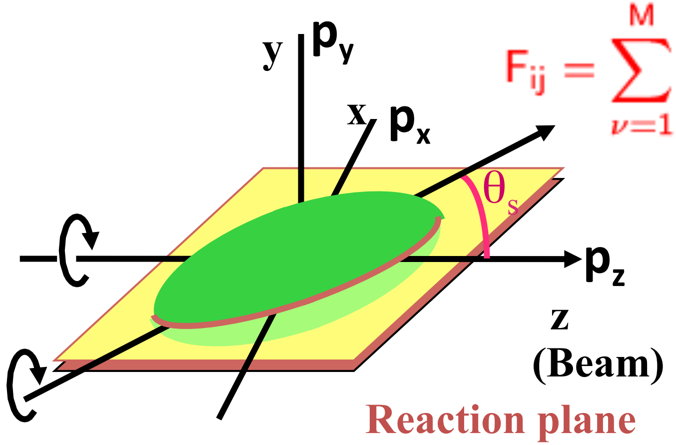


- ✧ Inspiration from hydrodynamics – 1995

H. Stöcker, J.A. Maruhn, and W. Greiner, PRL **44**, 725 (1980)

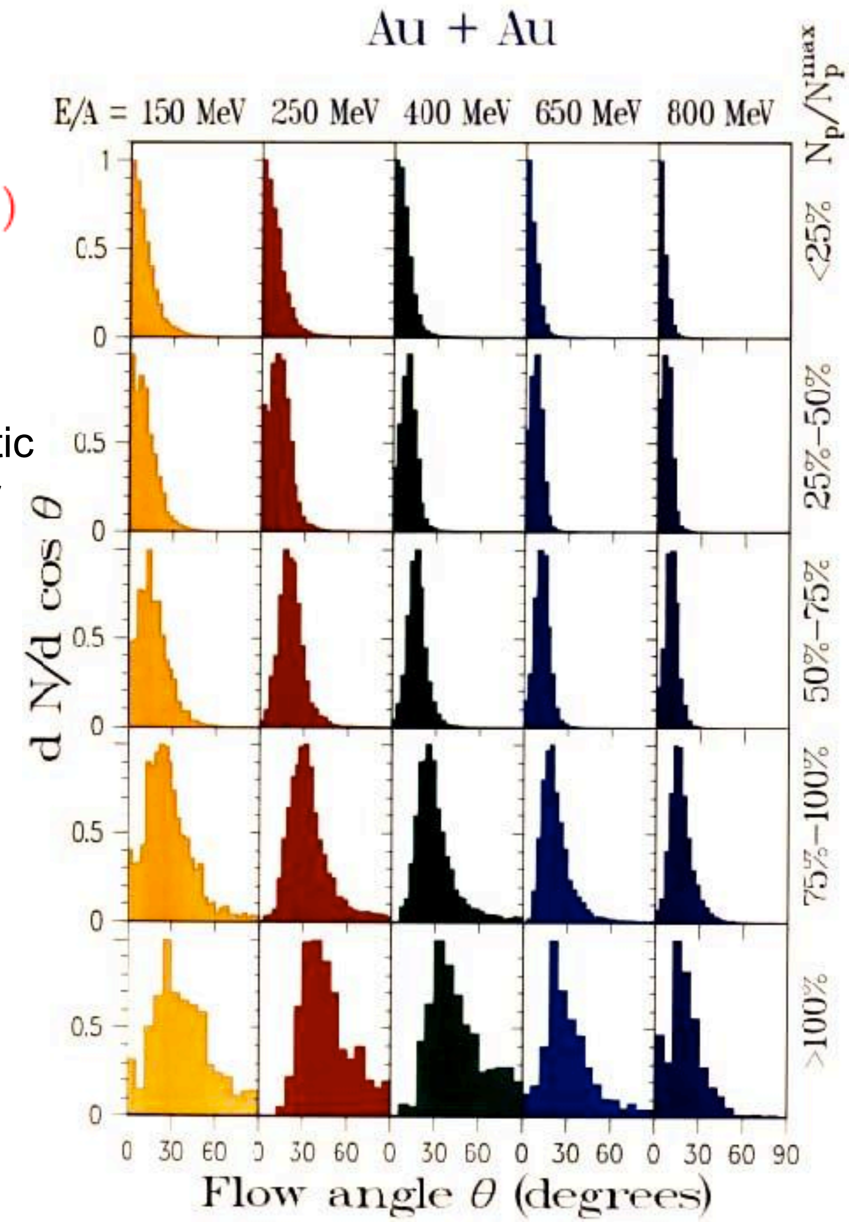
A little history of methods used to study flows

Sphericity



$$F_{ij} = \sum_{\nu=1}^{M} p_i(\nu) p_j(\nu) / (2m(\nu))$$

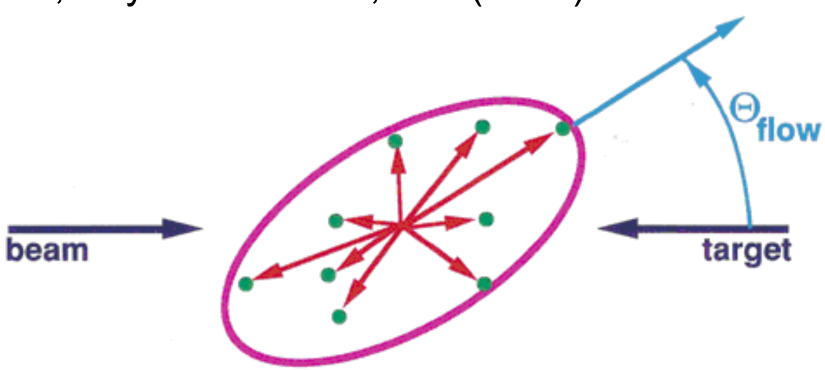
Best ellipsoid for each event by diagonalizing kinetic energy flow tensor



Plastic Ball, H.G. Ritter et al., Nucl. Phys. **A447**, 3c (1985)

"The only true signature of collective flow is a clear maximum of dN/d cosθ away from θ = 0"

M. Gyulassy, K.A. Frankel, and H. Stocker, Phys. Lett. **110B**, 185 (1982)



A little history of methods used to study flows

participant-spectator picture

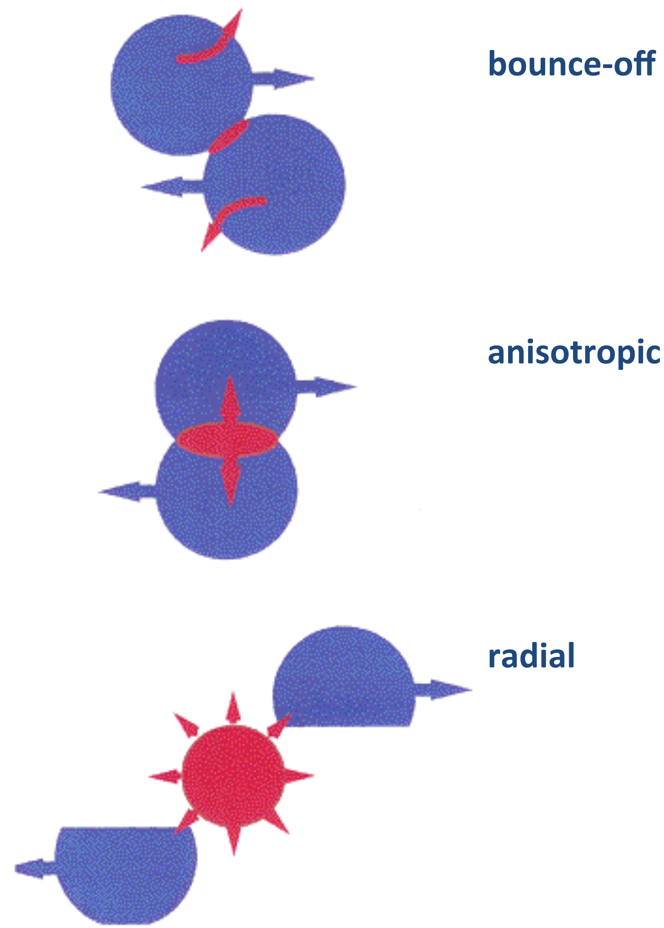
J.D. Bowman, W.J. Swiatecki, and C.F. Tsang, LBL-2908 (1973)

Kinds of Flow

At low colliding energies – bounce-off of the colliding niclei

Anisotropic flow – strong dependence on colliding energy

Radial flow – used to measure temperature of the formed fireball

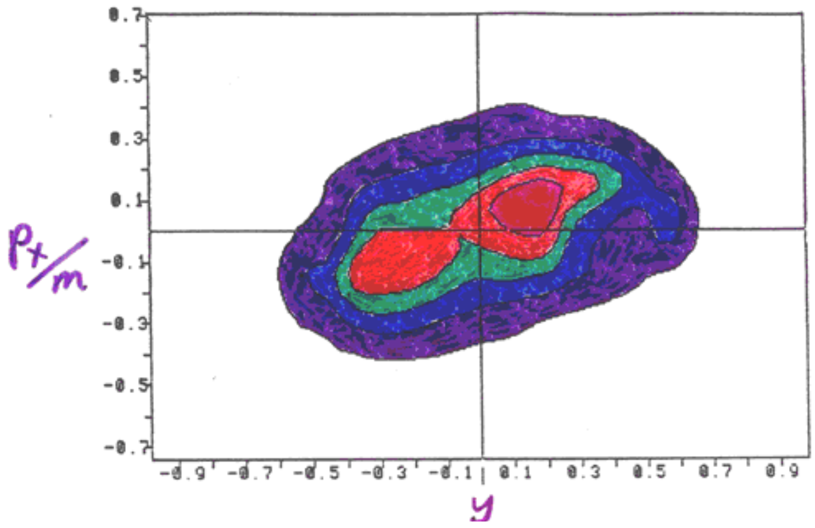


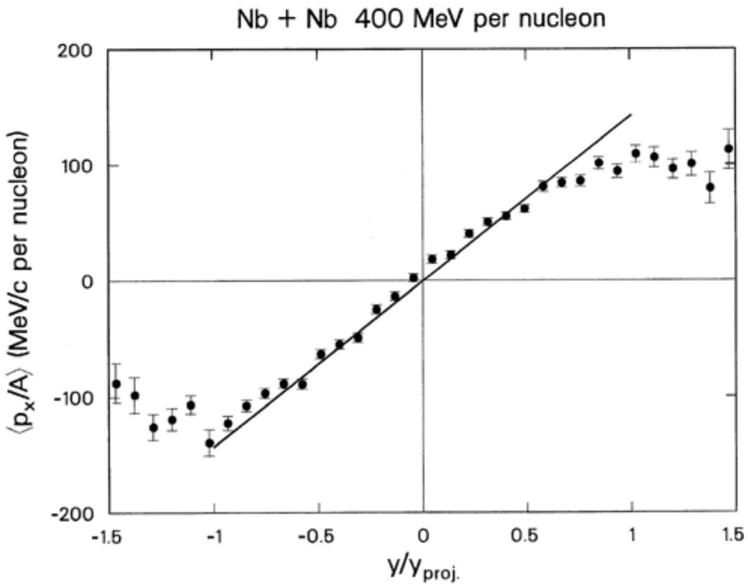
A little history of methods used to study flows

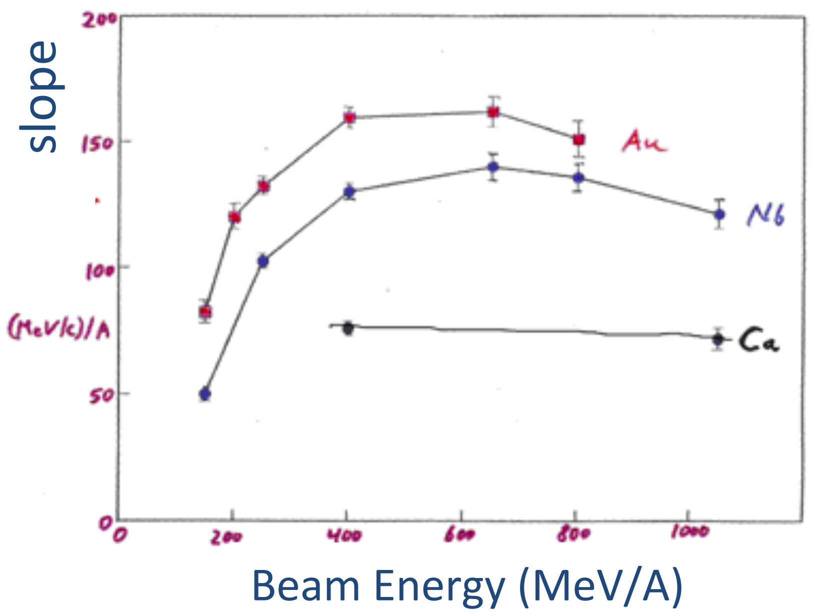
Directed Flow

$$F = \frac{1}{A}\frac{d\langle p_x\rangle}{dy}\Big|_{y=y_0}$$

- F is the slope of the p_x distribution at midrapidity
- Describes collective transverse momentum transfer
- Mass dependence
- Incident energy dependence







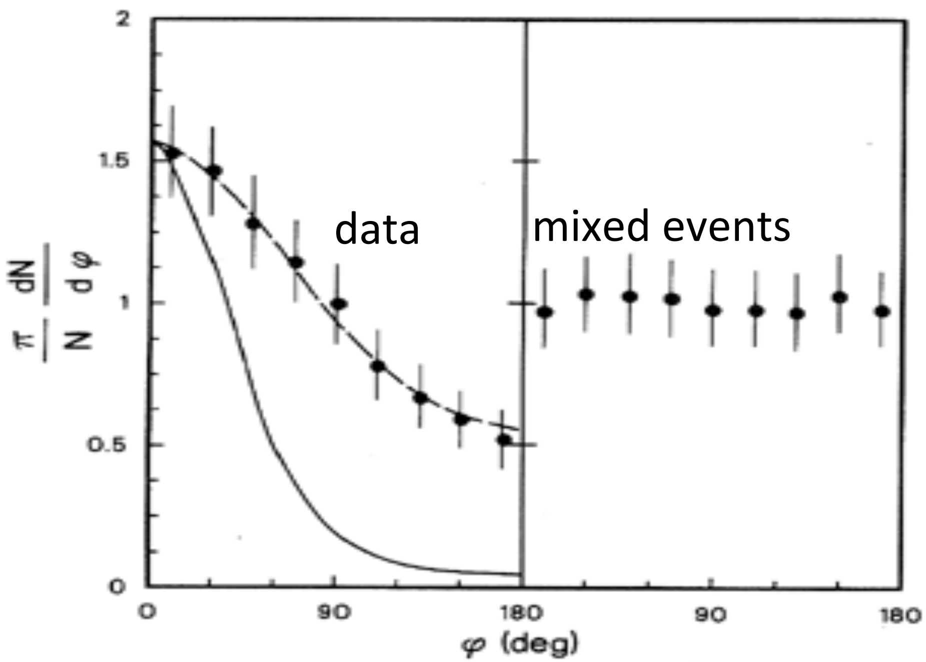
Plastic Ball, K.G.R. Doss et al., PRL **57**, 302 (1986)

A little history of methods used to study flows

Directed Flow – Transverse momentum analysis

$$Q_1 = \sum_{j=1}^{N} w_j u_j$$

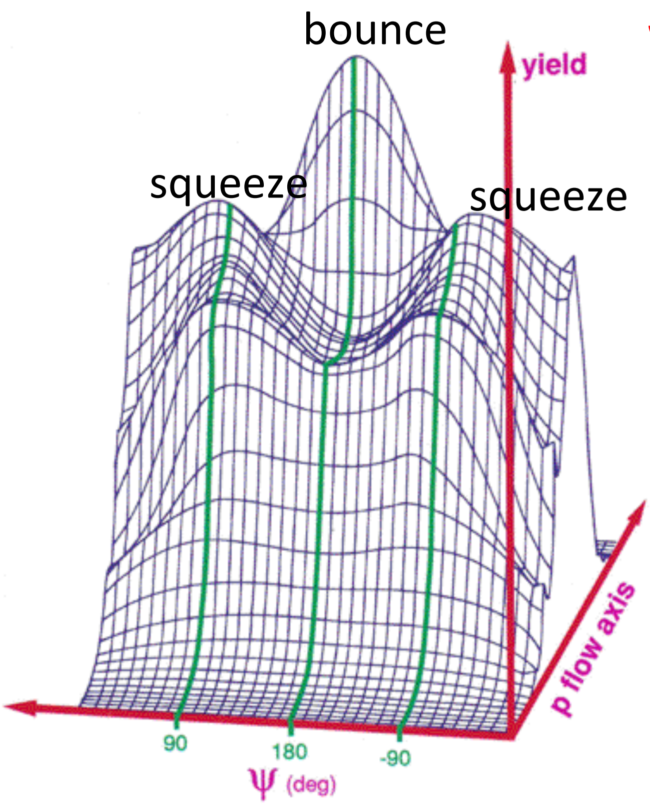
- Analysis in the transverse plane
- Definition of the 1st harmonic Q-vector
- Using of weights w_j
- Weights are negative in the backward hemishere
- Using of sub-events
- Removing auto-correlations
- Reaction plane resolution

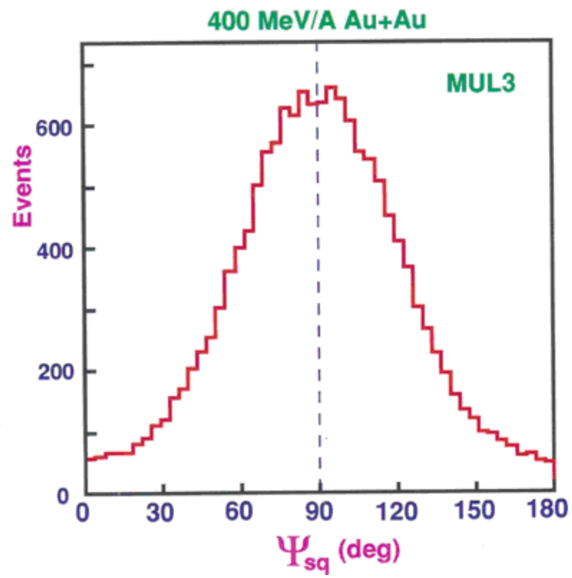


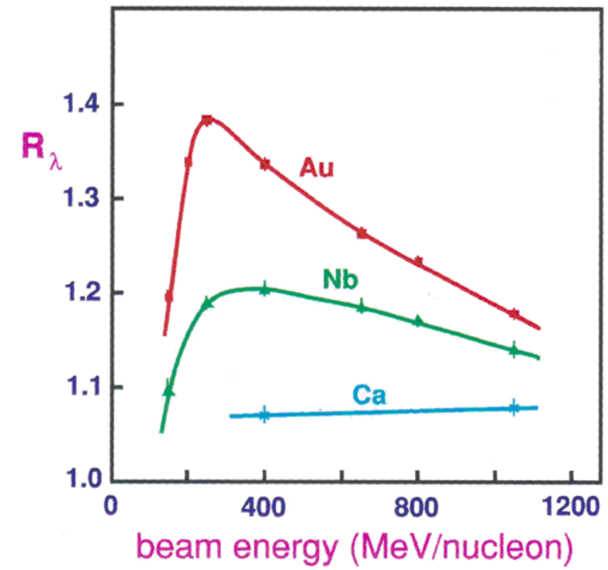
P. Danielewicz and G. Odyniec, Phys. Lett. **157B**, 146 (1985)

A little history of methods used to study flows

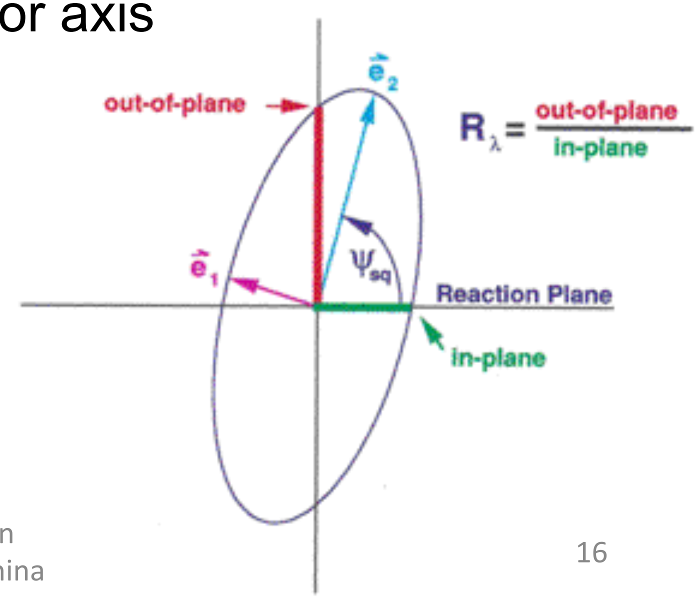
Squeeze-out







around the major axis



Diogene, M. Demoulins et al., Phys. Lett. **B241**, 476 (1990)

Plastic Ball, H.H. Gutbrod et al., Phys. Lett. **B216**, 267 (1989)

Plastic Ball, H.H. Gutbrod et al., PRC **42**, 640 (1991)

A little history of methods used to study flows

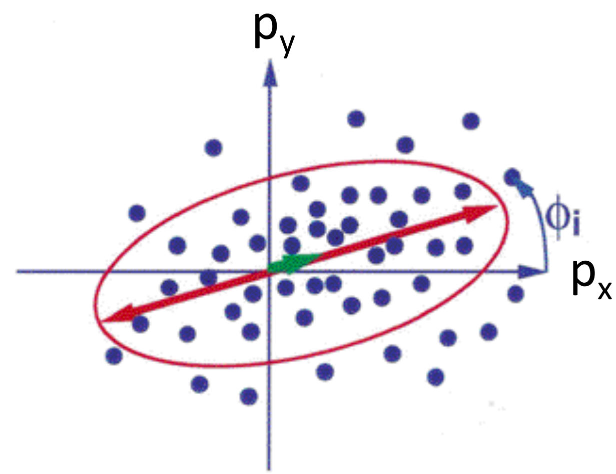
Fourier harmonics

$$\frac{dN}{d\phi} \propto 1 + 2v_1\cos(\phi - \Psi_{RP}) + 2v_2\cos[2(\phi - \Psi_{RP})] + \ldots$$

$$v_n = \langle \cos[n(\phi - \Psi_{RP})] \rangle$$

- event plane resolution used for each harmonic
- Weights negative in backward hemisphere for odd harmonics

Transverse Plane



$$Q_{n,x} = \sum_{j=1}^{N} w_j \cos(n\phi_j) \quad \text{and} \quad Q_{n,y} = \sum_{j=1}^{N} w_j \sin(n\phi_j) \rightarrow$$

$$\Psi_{plane} = \tan^{-1}\frac{\sum \sin(\phi_i)}{\sum \cos(\phi_i)}$$

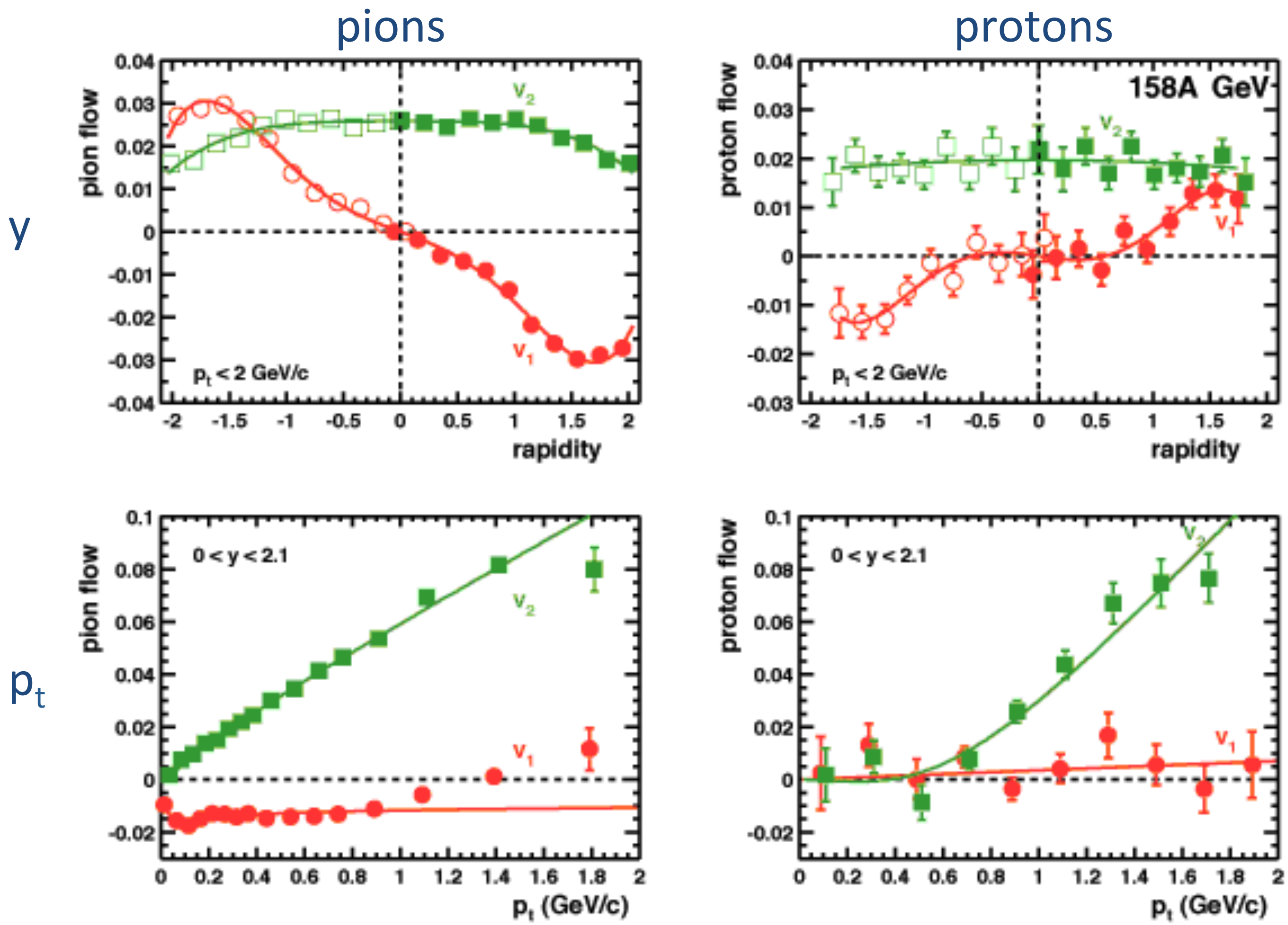
$$2\,\Psi_{ellipse} = \tan^{-1}\frac{\sum \sin(2\phi_i)}{\sum \cos(2\phi_i)}$$

S. Voloshin and Y. Zhang, hep-ph/940782; Z. Phys. C **70**, 665 (1996)

J.-Y. Ollitrault, arXiv nucl-ex/9711003 (1997)

J.-Y. Ollitrault, Nucl. Phys. **A590**, 561c (1995)

Directed and elliptic flow at SPS

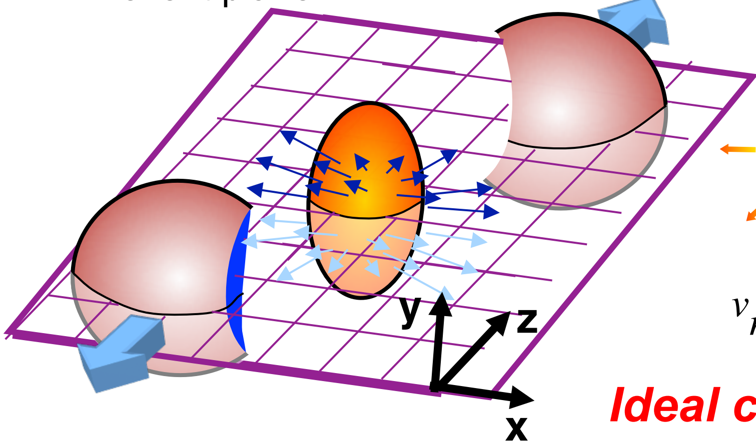


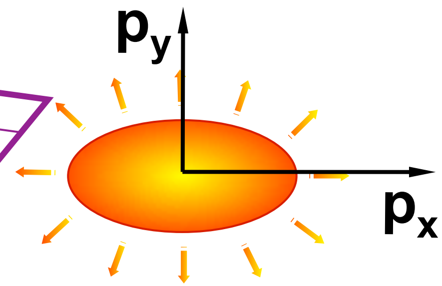
NA49, C. Alt et al., PRC **68**, 034903 (2003)

Anisotropy harmonics v_n – conventional methods

Event Plane (EP) method

event plane x-z





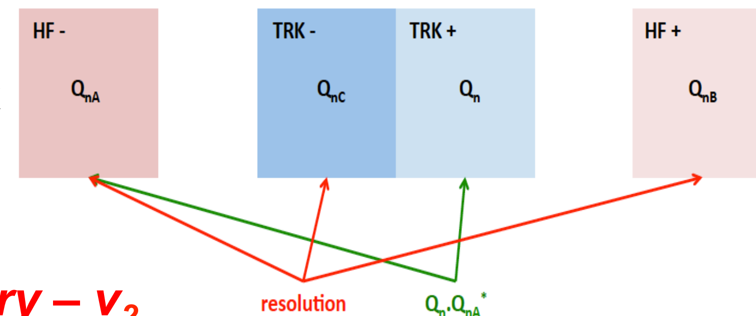
$$v_n = \left\langle \cos n(\phi - \Psi_n) \right\rangle$$

Ideal circle-like geometry – v_2

Scalar Product (SP) method

$|\Delta\eta| > 3$

$$v_n\{SP\} = \frac{\langle Q_n \cdot Q_{nA}^* \rangle}{\sqrt{\frac{\langle Q_{nA} \cdot Q_{nB}^* \rangle \cdot \langle Q_{nA} \cdot Q_{nC}^* \rangle}{\langle Q_{nB} \cdot Q_{nC}^* \rangle}}}$$



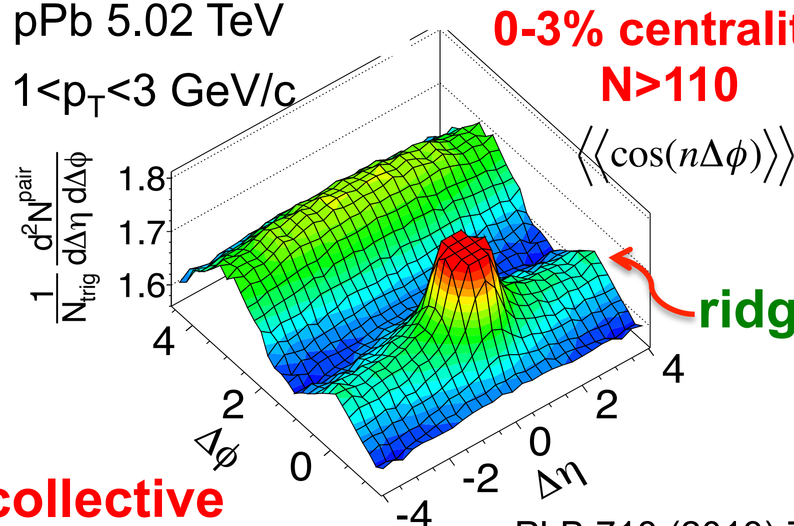
two-particle correlation method

pPb 5.02 TeV

$1 < p_T < 3$ GeV/c

0-3% centrality
N>110

$$\left\langle\left\langle \cos(n\Delta\phi) \right\rangle\right\rangle$$

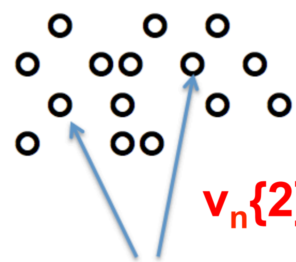


ridge

collective behavior?

PLB 718 (2013) 795

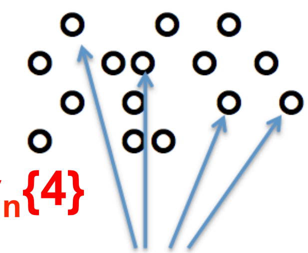
four-particle cumulant method



$v_n\{2\}$

$$\left\langle e^{in(\varphi_1 - \varphi_2)} \right\rangle$$

Advantage wrt 2-part.corr.: removes two- and three-particle non-flow correlation



$v_n\{4\}$

$$\left\langle e^{in(\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4)} \right\rangle$$

- **v_n from even higher order cumulants: $v_n\{6\}$, $v_n\{8\}$, ...**

Lee-Yang zero method

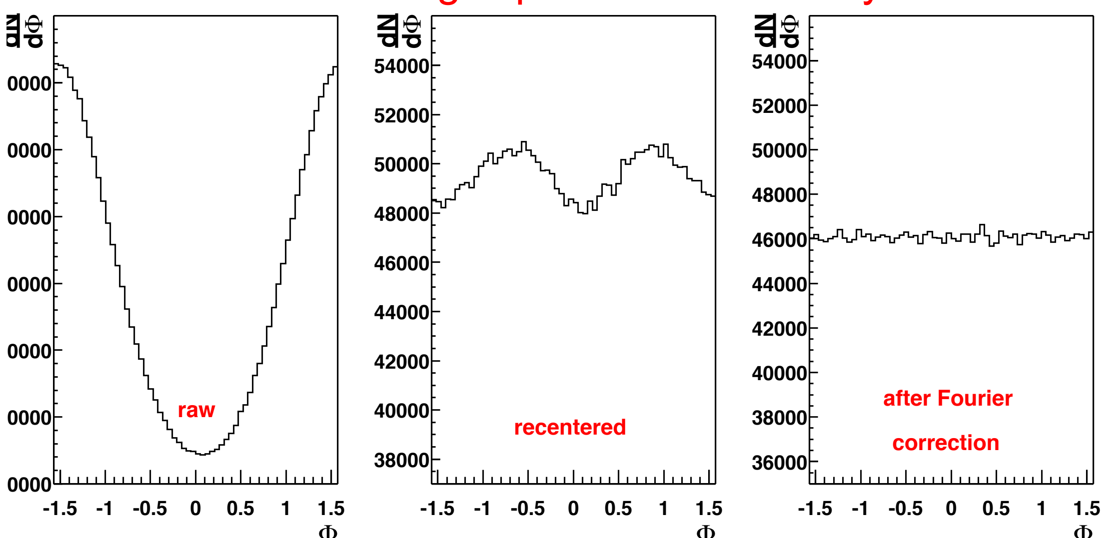
correlates all particles of interest

Event plane method – event plane Ψ_n

- Event plane is defined by the vector **b** (connects centers of the colliding nuclei) and the beam axis. A mathematical simplification. Event plane is not known apriori
- From Q_n vector $Q_{n,x} = \sum_{j=1}^{N} w_j \cos(n\phi_j)$ and $Q_{n,y} = \sum_{j=1}^{N} w_j \sin(n\phi_j)$ ➔ event plane angle Ψ_n
  - The beam can be shifted or a bit titled form the z-axis. One corrects it by shifting (or recentering) method: $Q_{n,x} \to Q_{n,x} - \langle Q_{n,x} \rangle$ and $Q_{n,y} \to Q_{n,y} - \langle Q_{n,y} \rangle$ averaging is performed over many events and then event plane angle is recalculated
  - If the resulting $dN/d\Psi_n$ is not flat then additionally a Fourier flattening can be applied:

$$\Delta\Psi_n = \frac{1}{n}\sum_{m=1}^{4}\frac{2}{m}\left[-\langle\sin(mn\Psi_n)\rangle\cos(mn\Psi_n) + \langle\cos(mn\Psi_n)\rangle\sin(mn\Psi_n)\right]$$

the flattening is performed event by event



- After applying recentering and Fourier flattening, the final $dN/d\Psi_n$ distribution must be flat (up to the statistical fluctuations)
- In opposite, non-zero $\langle \sin[n(\phi - \Psi_n)] \rangle$ will appears

Event plane method - resolution

- Due to the finite multiplicity, the resolution of the reconstructed event plane is finite. So, the observed flow magnitude has to be corrected for the event plane resolution
- The event plane resolution is given by $\left\langle \cos\left[n(\Psi_n - \Psi)\right]\right\rangle$ where **Ψ** is the true event plane angle
- The event plane resolution can be determined using the sub-event techniques
- Sub-events **a** and **b** are correlated: $\left\langle \cos\left[n(\Psi_n^a - \Psi_n^b)\right]\right\rangle$
- From the simple relation: $\left\langle \cos\left[n(\Psi_n^a - \Psi_n^b)\right]\right\rangle = \left\langle \cos\left[n(\Psi_n^a - \Psi)\right]\right\rangle \left\langle \cos\left[n(\Psi_n^a - \Psi)\right]\right\rangle$

  follows: $\Re^{sub-event} = \left\langle \cos\left[n(\Psi_n^a - \Psi)\right]\right\rangle = \sqrt{\left\langle \cos\left[n(\Psi_n^a - \Psi_n^b)\right]\right\rangle}$
- As the multiplicity of the sub-event is a half of the event multiplicity

$$\Re = \sqrt{2\left\langle \cos\left[n(\Psi_n^a - \Psi_n^b)\right]\right\rangle}$$

- There could be more than 2 sub-events. In case of 3 sub-events resolution is given with:

$$\Re = \sqrt{\frac{\left\langle \cos\left[n(\Psi_n^a - \Psi_n^b)\right]\right\rangle \left\langle \cos\left[n(\Psi_n^a - \Psi_n^c)\right]\right\rangle}{\left\langle \cos\left[n(\Psi_n^b - \Psi_n^c)\right]\right\rangle}}$$

Event plane method – Fourier harmonics

- Beside the directed and elliptic flow there are other anisotropic effects in heavy-ion collisions. All of them are present at the same time – it makes the picture more complicated

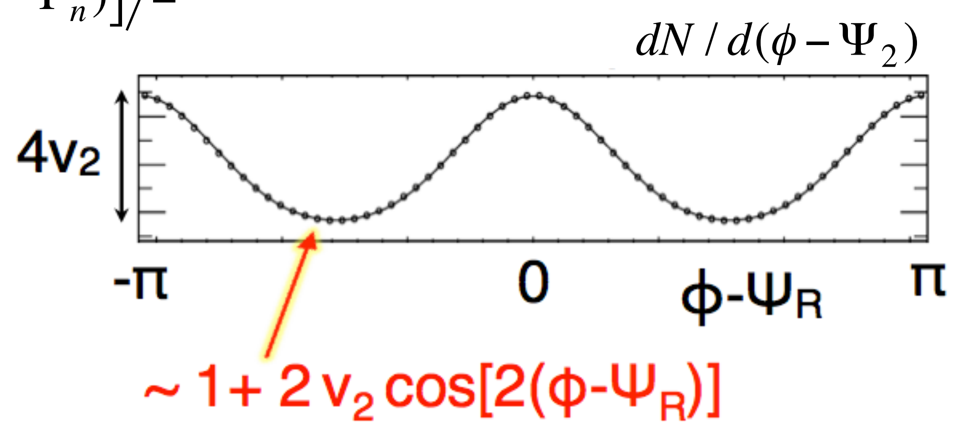
- A way to characterize them is to use a Fourier expansion of the invariant particle distribution:
$$E\frac{d^3N}{d^3p} = \frac{1}{2\pi}\frac{d^2N}{p_T dp_T}\left(1+\sum_{n=1}^{\infty} 2v_n \cos\left[n(\phi-\Psi_n)\right]\right)$$

- One correlate particles with the event plane $v_n^{obs}(p_T,\eta) = \left\langle \cos\left[n(\phi-\Psi_n)\right]\right\rangle$

  where $\langle \ldots \rangle$ denotes averaging over all particles and over all events of interest

- Due to the finite event plane resolution, the observed Fourier coefficients must be corrected for it:
$$v_n^{obs} = \left\langle \cos\left[n(\phi-\Psi_n)\right]\right\rangle = \left\langle \cos\left[n(\phi-\Psi+\Psi-\Psi_n)\right]\right\rangle =$$
$$= \left\langle \cos\left[n(\phi-\Psi)\right]\right\rangle \left\langle \cos\left[n(\Psi_n^a-\Psi)\right]\right\rangle = v_n \Re$$

- Finally
$$v_n(p_T,\eta) = \frac{v_n^{obs}}{\Re}$$



Non-flow contributions

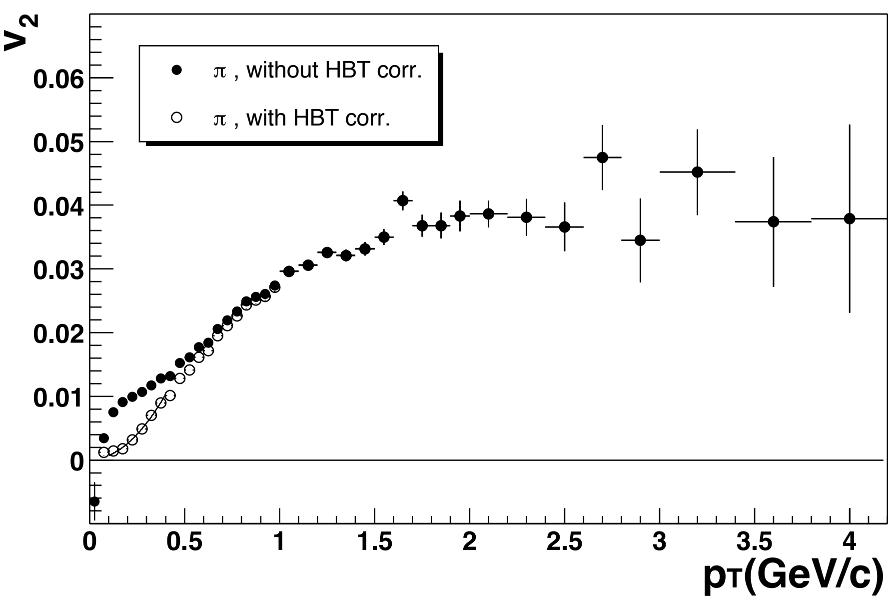
- There are physical correlations which look like flow
- The Hanbury-Brown & Twiss (HBT) effect produce a spurious flow
- The HBT effect between, let's say, two identical pions with momenta p_1 and p_2 apperas only if:

$$|\vec{p}_2 - \vec{p}_1| \leq \hbar / R$$

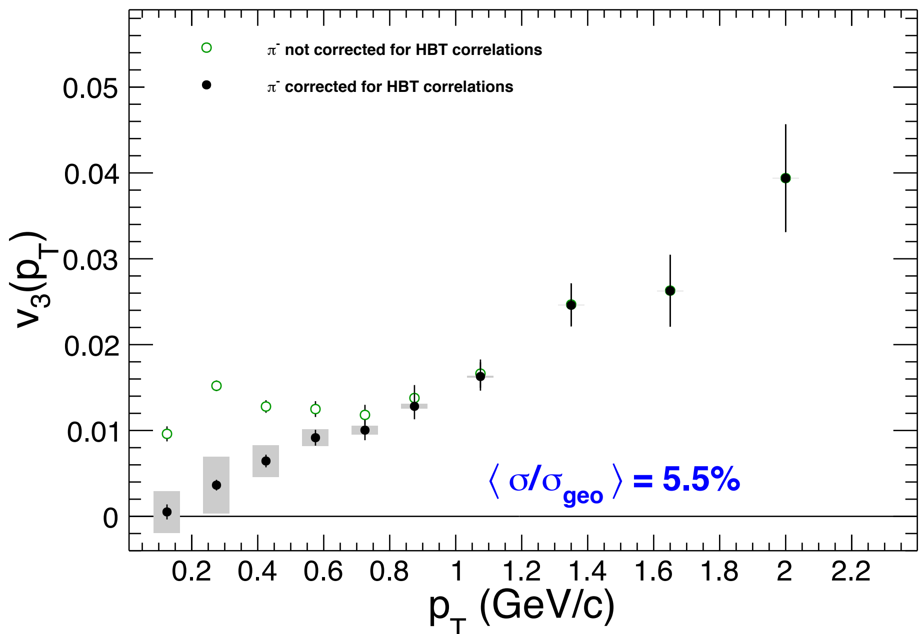
where R is a typical size of the interacting zone

- The HBT affects only pairs with quite low momenta
- In P. M. Dinh, N. Borghini, and J.-Y. Ollitrault, Phys. Lett. **B477,** 51 (2000) was shown it was shown procedure which should be applied in order to correct the contribution form the HBT effect

elliptic flow



triangular flow

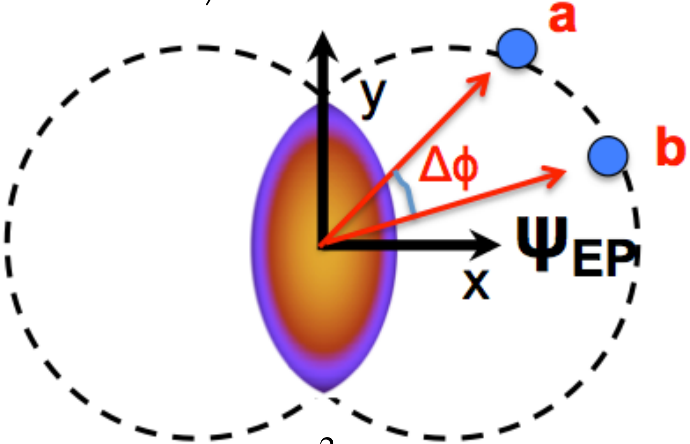


Two-particle correlation method

- If particles are correlated with the event plane then they are also mutually correlated
- Thus, distribution formed from pairs of particles could be also Fourier decomposed as:

$$\frac{dN^{pairs}}{d\Delta\phi} = C_0\left(1+\sum_{n=1}^{\infty} 2v_n^2 e^{in\Delta\phi}\right) = C_0\left(1+\sum_{n=1}^{\infty} 2v_n^2 \cos(n\Delta\phi)\right)$$

where $\Delta\phi = \phi^a - \phi^b$ and $v_n^2 = \left\langle \cos\left[n(\phi^a - \phi^b)\right]\right\rangle$



- The main advantage tis that one does not need the event plane
- Now, usually, one constructs two-dimensional, in $\Delta\phi$ and in $\Delta\eta$, correlation by pairing particles from the same event. In order to increase the statistics, the procedure is repeated over many events of interest. This we call, signal ***S*** distribution
- In order to avoid the finite acceptance effect, one construct the background ***B*** distribution using the mixed-event technique
- The two-dimensional of associated particles per trigger particle is then defined as

$$S(\Delta\phi,\Delta\eta) = \frac{1}{N^{trigg}}\frac{d^2N^{same}}{d\Delta\phi d\Delta\eta}$$

$$B(\Delta\phi,\Delta\eta) = \frac{1}{N^{trigg}}\frac{d^2N^{mix}}{d\Delta\phi d\Delta\eta}$$

$$\frac{1}{N^{trigg}}\frac{d^2N^{pair}}{d\Delta\phi d\Delta\eta} = B(0,0)\frac{S(\Delta\phi,\Delta\eta)}{B(\Delta\phi,\Delta\eta)}$$

Two-particle correlation method

- The N^{trigg} is the total number of trigger particles, wile $B(0,0)/B(\Delta\phi,\Delta\eta)$ accounts for the pair-acceptance effect
- One-dimensional projection onto $\Delta\phi$-axis is then used to extract two-particle Fourier coefficients $V_{n\Delta}$ from the following Fourier decomposition

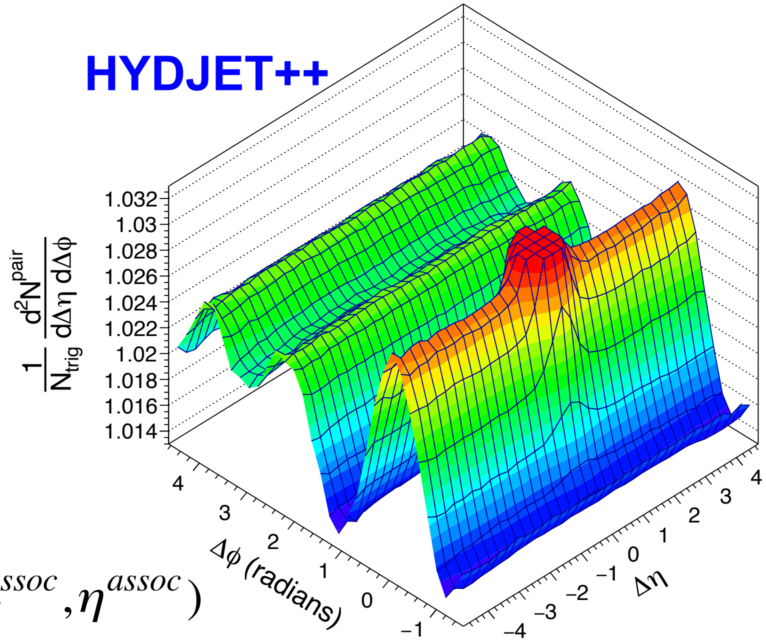
$$\frac{1}{N^{trig}}\frac{dN^{pairs}}{d\Delta\phi}=\frac{N^{assoc}}{2\pi}\left(1+\sum_{n=1}^{\infty}2V_{n\Delta}\cos(n\Delta\phi)\right)$$

In order to avoid of the short range correlations arising from fragmentation of jets and resonance decays, a cut $|\Delta\eta|\geq 2$ is applied

- The $V_{n\Delta}$ can be directly calculated as

$$V_{n\Delta}=\langle\langle\cos(n\Delta\phi)\rangle\rangle_S-\langle\langle\cos(n\Delta\phi)\rangle\rangle_B$$

where $\langle\langle ... \rangle\rangle$ denotes averaging over all particles pairs and over all events of interest



- It was thought that if the correlation is purely driven by hydrodynamics then $V_{n\Delta}$ factorizes into a product of single-particle anisotropies

$$V_{n\Delta}(p_T^{trig},p_T^{assoc};\eta^{trig},\eta^{assoc})=v_n(p_T^{trig},\eta^{trig})\times v_n(p_T^{assoc},\eta^{assoc})$$

factorization!

Two-particle correlation method

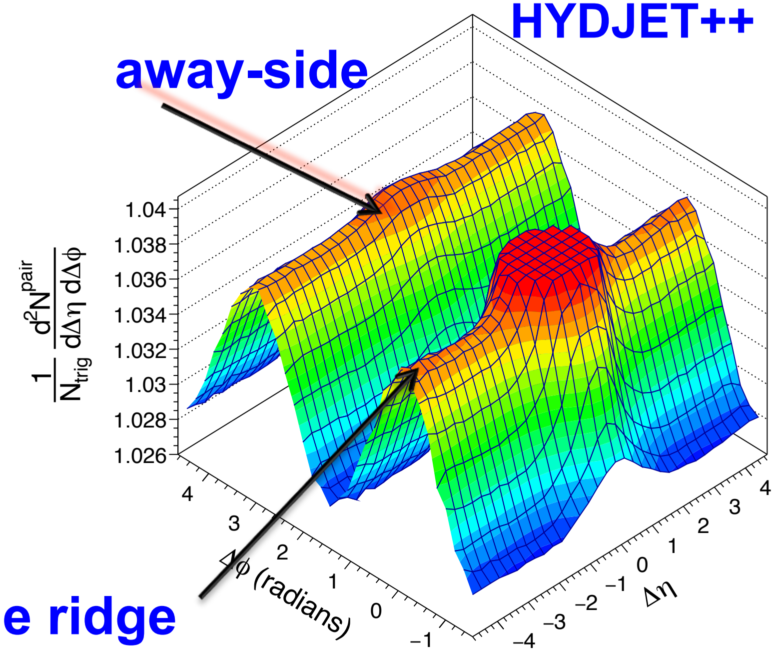
- The single-particle anisotropy harmonics can be then extracted as:

$$v_n(p_T) = \frac{V_{n\Delta}(p_T, p_T^{ref})}{\sqrt{V_{n\Delta}(p_T^{ref}, p_T^{ref})}}$$

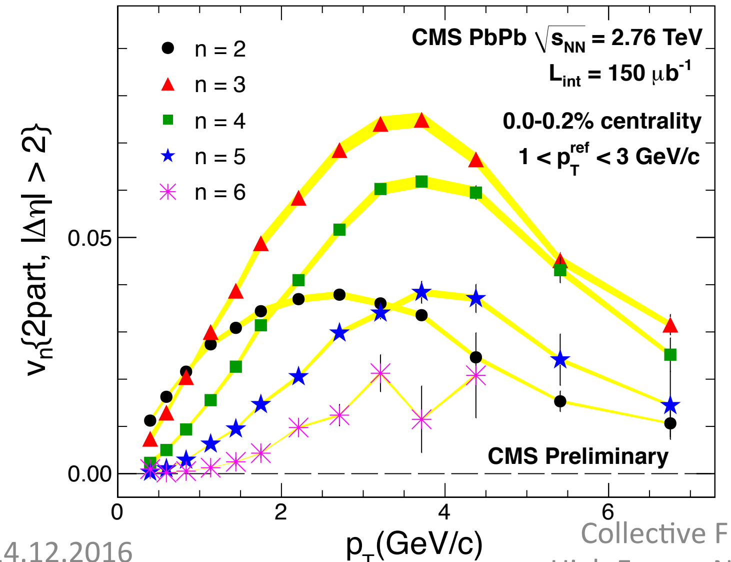
where p_T^{ref} is a wide range of the 'referent particles' bin

- The near-side jet as well as the big part of the away-side jet remnants could be excluded from the flow study by applying the $|\Delta\eta| \geq 2$
- The $V_{n\Delta}$ calculation: $V_{n\Delta} = \langle\langle \cos(n\Delta\phi) \rangle\rangle_S - \langle\langle \cos(n\Delta\phi) \rangle\rangle_B$

is performed using only pairs which satisfy condition $|\Delta\eta| \geq 2$
This, up to some extent removes also contribution from the resonance decays



CMS, JHEP **1402**, 088 (2014)



results

Multi-particle cumulant method

- Multi-particle method based on Q-cumulants by correlating no less then 4 particles has the advantage of suppressing short-range two or three-particle correlations arising from jet fragmentation and resonance decays.
- Single-event average 2- and 4-particle correlations are defined as:

$$\langle 2\rangle = \left\langle e^{in(\phi_1-\phi_2)}\right\rangle = \frac{2!(n-2)!}{n!}\sum_{i,j} e^{in(\phi_i-\phi_j)} \quad \text{and} \quad \langle 4\rangle = \left\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)}\right\rangle = \frac{4!(n-4)!}{n!}\sum_{i,j,k,l} e^{in(\phi_i+\phi_j-\phi_k-\phi_l)}$$

- The next step is to average over events of interest

$$\langle\langle 2\rangle\rangle = \left\langle\left\langle e^{in(\phi_1-\phi_2)}\right\rangle\right\rangle = \frac{\sum_{events} \varpi_i^{(2)} \langle\langle 2\rangle\rangle_i}{\sum_{events} \varpi_i^{(2)}} \quad \text{and} \quad \langle\langle 4\rangle\rangle = \left\langle\left\langle e^{in(\phi_1+\phi_2-\phi_3-\phi_4)}\right\rangle\right\rangle = \frac{\sum_{events} \varpi_i^{(4)} \langle\langle 4\rangle\rangle_i}{\sum_{events} \varpi_i^{(4)}}$$

- The second order cumulant is 2-particle correlation: $c_n\{2\} = \langle\langle 2\rangle\rangle$
- The 4-th order cumulant is *genuine* 4-particle correlation: $c_n\{4\} = \langle\langle 4\rangle\rangle - 2\cdot\langle\langle 2\rangle\rangle^2$
- The 6-th order cumulant is *genuine* 6-particle correlation: $c_n\{6\} = \langle\langle 6\rangle\rangle - 9\cdot\langle\langle 4\rangle\rangle\langle\langle 2\rangle\rangle + 12\cdot\langle\langle 2\rangle\rangle^3$
- And there are more …..
- The next step is to calculate integrated Fourier coefficients

more details in A. Bilandzic et al., PRC**83**, 044913 (2011) and references in

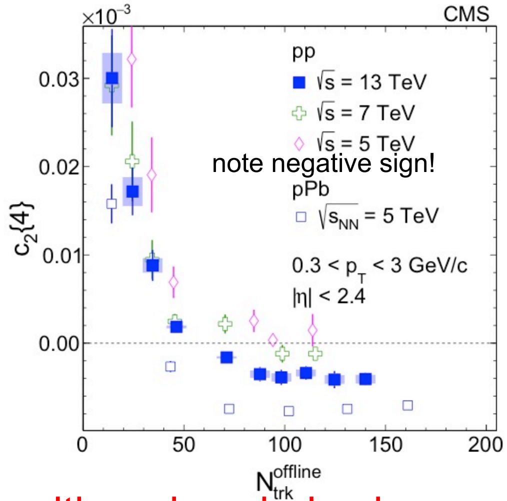
Multi-particle cumulant method

- ✧ The integrated second order cumulant v_n harmonic: $v_n\{2\} = \sqrt{c_n\{2\}}$

- ✧ The integrated 4-th order cumulant v_n harmonic: $v_n\{4\} = \sqrt[4]{-c_n\{4\}}$

- ✧ The integrated 6-th order cumulant v_n harmonic: $v_n\{6\} = \sqrt[6]{\frac{1}{4} c_n\{6\}}$

- ✧ And there are more …..



❖ To get single-particle differential cumulant v_n harmonic, one of particle with a given index is required to belong to a given p_T bin (denoted with a prime). The corresponding equations are:

$$d_n\{4\} = \langle\langle 4' \rangle\rangle - \langle\langle 2' \rangle\rangle \langle\langle 2 \rangle\rangle$$

$$d_n\{4\} = \langle\langle 6' \rangle\rangle - 6 \cdot \langle\langle 4' \rangle\rangle \langle\langle 2 \rangle\rangle - 3 \cdot \langle\langle 2' \rangle\rangle \langle\langle 4 \rangle\rangle + 12 \cdot \langle\langle 2' \rangle\rangle \langle\langle 2 \rangle\rangle^2$$

- ✧ And there are more ….. and finally, 4-, 6- ,…. differential v_n (p_T,η) coefficients are given as:

$$v_n\{4\}(p_T,\eta) = -d_n\{4\} \cdot \left(-c_n\{4\}\right)^{-3/4}$$

$$v_n\{6\}(p_T,\eta) = d_n\{6\} \cdot \left(c_n\{6\}\right)^{-5/6} \cdot 4^{-1/6}$$

more details in A. Bilandzic et al., PRC**83**, 044913 (2011) and references in

Lee-Yang Zero (LYZ) method

- First, one should calculate the integrated flow magnitude. To perform it, a complex-valued function:

$$g^{\theta}(ir) = \prod_{i=1}^{N}\left[1 + irw_i \cos[n(\phi_i - \theta)]\right]$$

for various values of the real variable *r* and the angle θ ($0 \le \theta < \pi/n$)

- In each event one compute: $Q_x = \sum_{i=1}^{N} w_i \cos(n\phi_i)$ and $Q_y = \sum_{i=1}^{N} w_i \sin(n\phi_i)$
- Averaging over events for each *r* and θ value: $G^{\theta}(ir) = \left\langle g^{\theta}(ir) \right\rangle_{events} = \frac{1}{N_{events}} \sum_{events} g^{\theta}(ir)$
- For each θ value find the first minimum r_0^{θ} of the $|G^{\theta}(ir)|$:
- The integrated flow magnitude is then given with: $V_n^{\theta}\{\infty\} = \frac{j_{01}}{r_0^{\theta}}$

where $j_{01} = 2.40483$ is the first zero of the Bessel function J_0

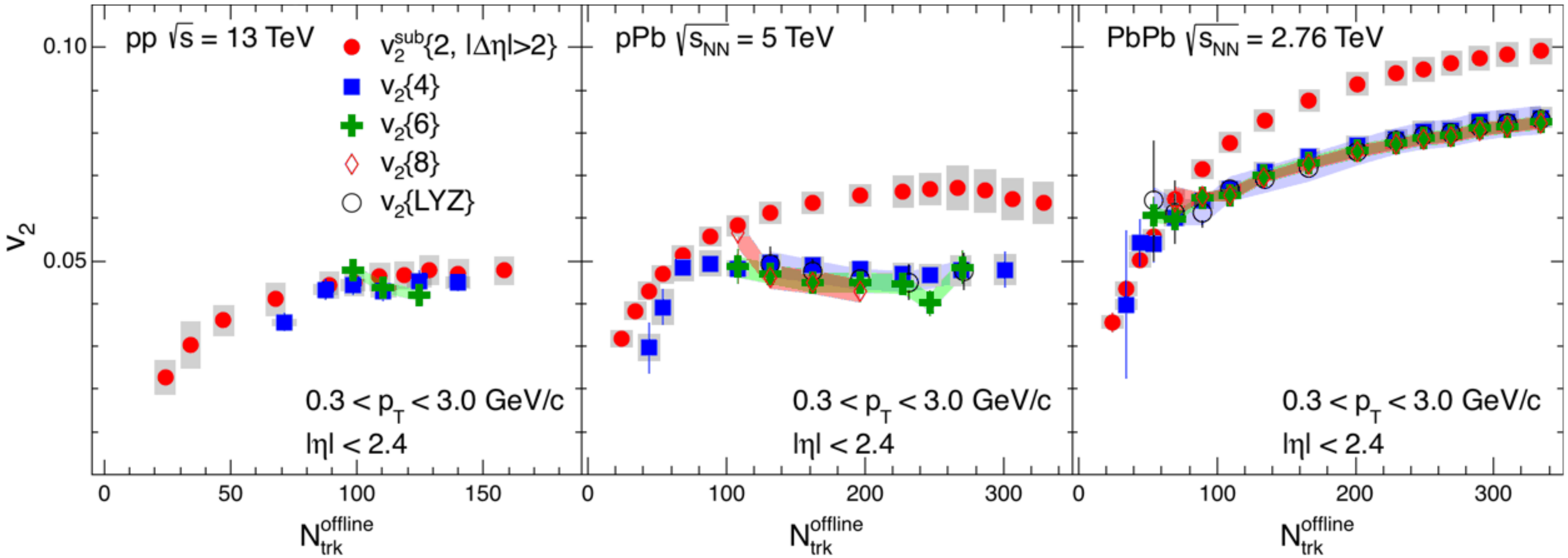
- If the detector acceptance is azimuthally symmetric, then $\boldsymbol{V_n^{\theta}}$ does not depends on $\boldsymbol{\theta}$. One can average over θ to get smaller statistical errors
- The differential $\boldsymbol{v_n}$ are then calculated as:

$$\frac{v_{mn}^{\theta}\{\infty\}}{V_n^{\theta}\{\infty\}} = \frac{J_1(j_{01})}{J_m(j_{01})} \mathrm{Re}\left(\frac{\left\langle g^{\theta}(ir_0^{\theta}) \frac{\cos[mn(\psi - \theta)]}{1 + ir_0^{\theta} w_{\psi} \cos[n(\psi - \theta)]} \right\rangle_{\psi}}{i^{m-1} \left\langle g^{\theta}(ir_0^{\theta}) \sum_j \frac{w_j \cos[n(\phi_j - \theta)]}{1 + ir_0^{\theta} w_{\psi} \cos[n(\phi_j - \theta)]} \right\rangle_{events}} \right)$$

more details in N. Borghini et al., JPG **30**, S1213 (2004) and in R.S. Bhalerao et al., NPA **727**, 373 (2003)

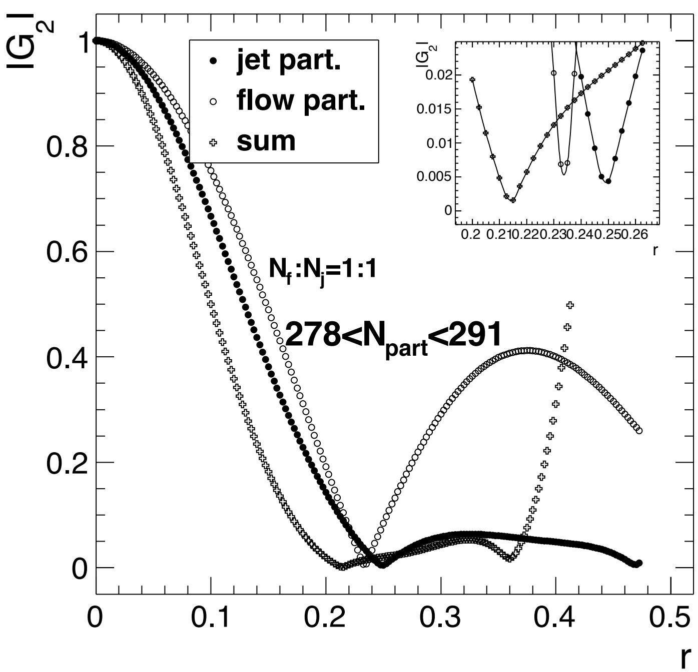
Lee-Yang Zero (LYZ) and cumulants

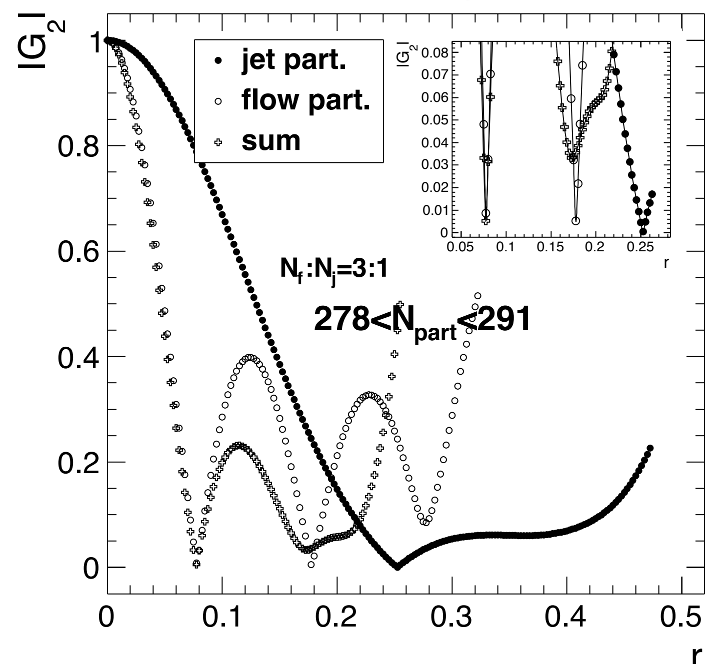
CMS HIN-16-010 (2016)



Some results as examples

O. Dordic et al., PS **7**, 075301 (2014)



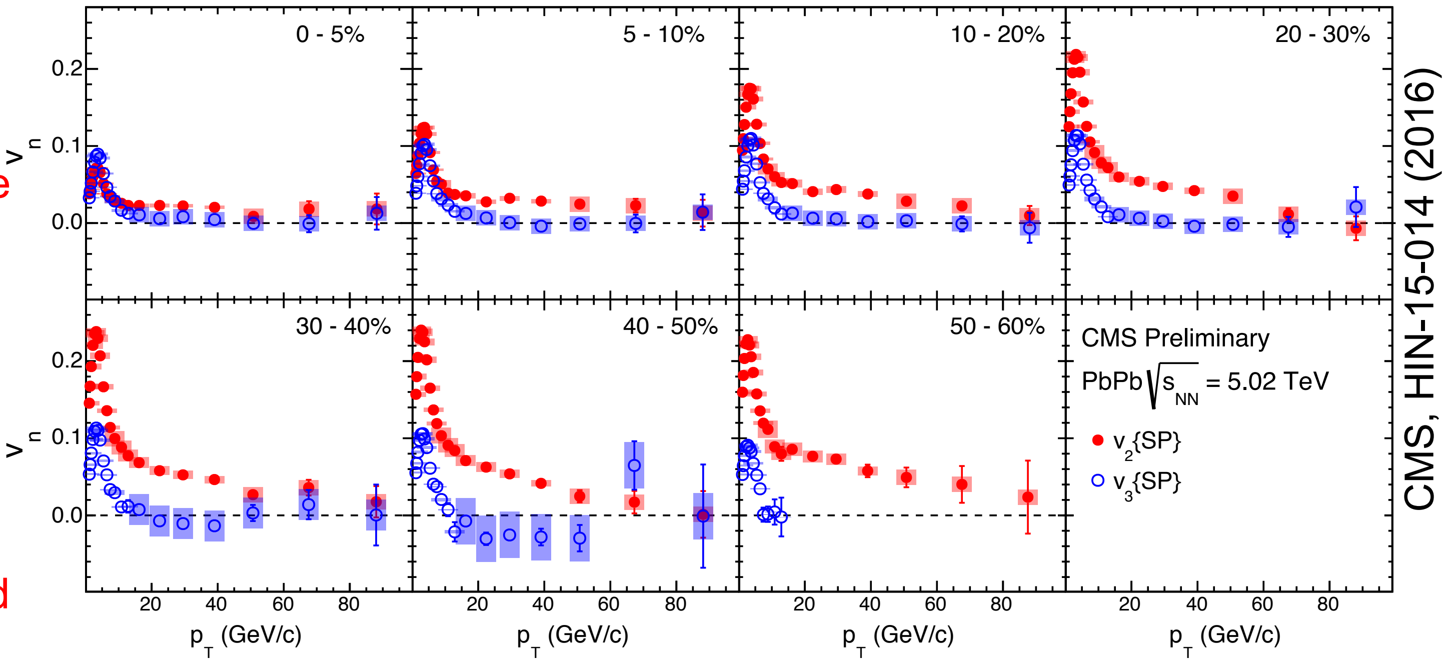


Scalar Product (SP) method

- The SP method is used to measure anisotropy that are long-range in pseudorapidity

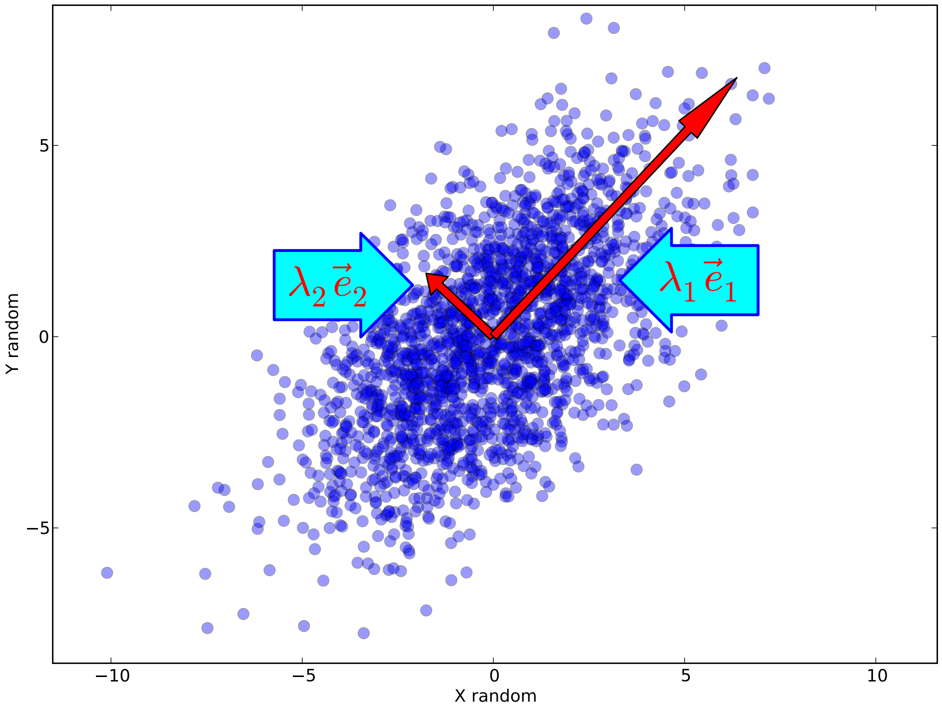
$$v_n\{SP\} = \frac{\langle Q_n Q_{nA}^* \rangle}{\sqrt{\frac{\langle Q_{nA} Q_{nb}^* \rangle \langle Q_{nA} Q_{nC}^* \rangle}{\langle Q_{nB} Q_{nC}^* \rangle}}} \quad \text{where} \quad Q_n = \sum_{i=1}^{N} \omega_i e^{in\phi_i}$$

- The SP method excellently suits in the experiments with huge pseudorapidity coverage
- Forming sub-events from different detector parts separated with a large ***η*** gap suppresses few-particle non-flow correlations, such as those induced by dijets fragmentation and
- Similarly as in the EP method, in order to account for asymmetries that arise from the acceptance and other detector-related effects, a two-step process can be used:
  - the ***Q****-vector* is first recentered
  - and then flattened



Principal Component Analysis (PCA) method

A simple 2D example



- Random data generated by 2D multivariate Gauss distribution

$$\vec{X}_n = (x_1, x_2, \ldots, x_n)$$
$$\vec{Y}_n = (y_1, y_2, \ldots, y_n)$$

- a matrix

$$\Sigma = \begin{bmatrix} \mathrm{var}(X) & \mathrm{cov}(X,Y) \\ \mathrm{cov}(X,Y) & \mathrm{var}(Y) \end{bmatrix}$$

- eigenvectors e_i and eigenvalues λ_i by diagonalization Σ

$$[e]^T \Sigma [e] = diag(\lambda_1, \lambda_2)$$

- First Principal Component: eigenvector e_1 points to maximum variance of data cloud. Its magnitude is $\sqrt{\lambda_1} e_1$

- Second Principal Component: eigenvector e_2 points to the next maximum variance of data cloud. Its magnitude is $\sqrt{\lambda_2} e_2$

PCA method in hydrodynamic flow - prescription

Two very recent theoretical papers: R.S.Bhalerao, J-Y. Ollitrault, S.Pal and D.Teaney, Phys.Rev.Lett. 114 (2015) 152301 and A.Mazeliauskas and D.Teaney, Phys.Rev. C91 (2015) 044902 introduced the PCA as a new method to study hydrodynamics flows

- *"The simplest correlations are pairs. The **principal component analysis** is a method which extracts all the information from pair correlations in a way which facilitates comparison between theory and experiment." J.-Y. Ollitrault*

In this analysis:

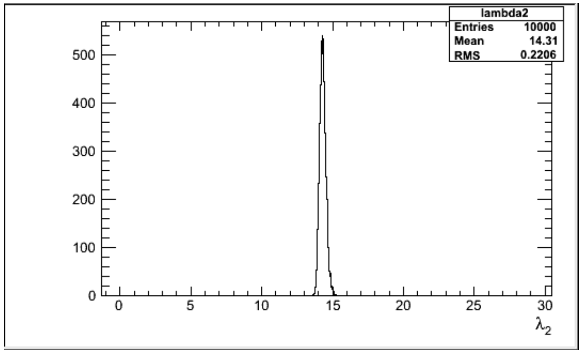
PhysRev.C **92** (2015) 034911
arXiv:1503.01692
and other CMS analyses

- **Input**: two-particle Fourier coefficients measured as
$V_{n\Delta} = \langle\langle \cos(n\Delta\phi)\rangle\rangle_S - \langle\langle \cos(n\Delta\phi)\rangle\rangle_B$ where
$\langle\langle \cos(n\Delta\phi)\rangle\rangle_S$ and $\langle\langle \cos(n\Delta\phi)\rangle\rangle_B$ are calculated for pairs with $|\Delta\eta|>2$
- 7 p_T bins ($0.3<p_T<3.0$ GeV/c); the eigenvalue problem of a matrix $[V_{n\Delta}(p_i,p_j)]$

$$\begin{pmatrix} e^{(1)} & e^{(2)} & . & . & . & . & e^{(7)} \end{pmatrix} \begin{bmatrix} V_{n\Delta}(p_1,p_1) & V_{n\Delta}(p_2,p_1) & V_{n\Delta}(p_3,p_1) & . & . & . & . \\ V_{n\Delta}(p_1,p_2) & V_{n\Delta}(p_2,p_2) & V_{n\Delta}(p_3,p_2) & . & . & . & . \\ V_{n\Delta}(p_1,p_3) & V_{n\Delta}(p_2,p_3) & V_{n\Delta}(p_3,p_3) & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & V_{n\Delta}(p_7,p_7) \end{bmatrix} \begin{pmatrix} e^{(1)} \\ e^{(2)} \\ . \\ . \\ . \\ . \\ e^{(7)} \end{pmatrix} = diag\begin{pmatrix} \lambda^{(1)} & \lambda^{(2)} & . & . & . & . & \lambda^{(7)} \end{pmatrix}$$

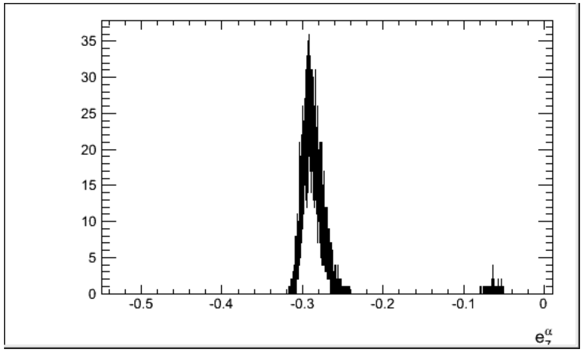
PCA method in hydrodynamic flow - prescription

λ distribution, α=2

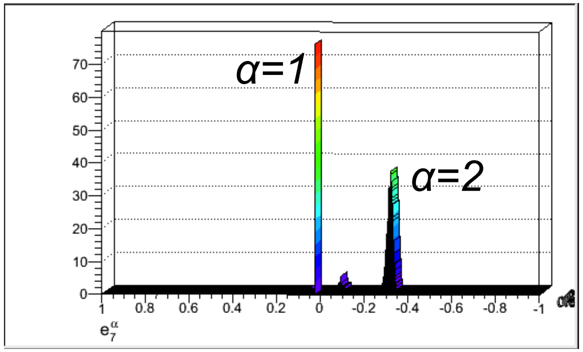


CMS Preliminary

e distribution, α=2



α=2 signal 200 times smaller wrt α=1



2.5<p_T<3.0 GeV/c

- The new introduced p_T dependent variable, **flow mode**, is defined as

$$V_n^{(\alpha)}(p_i) = \sqrt{\lambda^{(\alpha)}} e^{(\alpha)}(p_i)$$ where α=1,…,7

- corresponding single-particle flow mode $$v_n^{(\alpha)}(p) = \frac{V_n^{(\alpha)}(p)}{\langle M(p) \rangle}$$
- experimental data ➔ $V_{n\Delta}(p_i,p_j)$ ➔ it has its own statistical error $\Delta V_{n\Delta}(p_i,p_j)$
- The error propagation through $V_n^{(\alpha)}$ up to $v_n^{(\alpha)}$
- $\Delta\lambda^\alpha$ and $\Delta \mathbf{e}^\alpha$ as RMS of the distributions like ones shown above. Matrix elements $V_{n\Delta}$ were perturbed (10k times) within its $\Delta V_{n\Delta}$ → matrix $[V_{n\Delta}]$ **nonlinearly** perturbed

Conclusions

- We presented motivation why to study QGP
- One of observables used to study QGP is azimuthal anisotropy
- Some of the first, now old and mainly outdated methods for studying azimuthal anisotropies are presented
- We presented several methods which are now extensively used in measuring of the azimuthal anisotropies
- We also shortly mentioned some techniques which could be used to correct or to avoid of some non-flow effects