Initial Conditions for Hydrodynamics in Heavy Ion Collisions

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BROOKHAVEN NATIONAL LABORATORY

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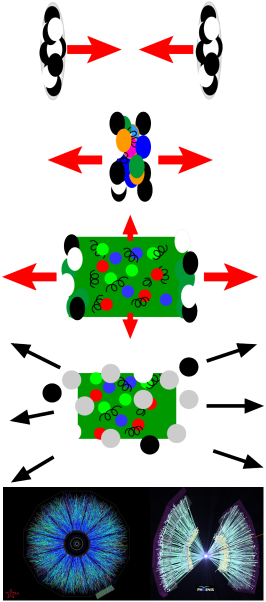
Collective Flows and Hydrodynamics in High Energy Nuclear Collisions
Department of Modern Physics, University of Science and Technology of China, Anhui Hefei

A heavy-ion collision



before collision — initial state (e.g. color glass condensate)

0 fm/c

pre-equilibrium — thermalization (glasma state)

~ 0.5 fm/c

quark-gluon-plasma — Hydrodynamics, Jet quenching, ..

$\sim 3-5$ fm/c

hadronization — Hydrodynamics

hadr.rescattering — Hadronic transport

~ 10 fm/c

freeze-out — compare theory to experiment

detection

Describing heavy-ion collisions with hydro

Hydrodynamics works for all systems with short mean free path.
(comparing to size scales of interest)

How do we incorporate the physics of heavy-ion collisions?

1. Equation of state $p(\varepsilon, \rho_B)$
2. Initial conditions
3. Freeze-out and conversion of energy densities into particles
4. Values of transport coefficients (e.g. shear viscosity)

Method

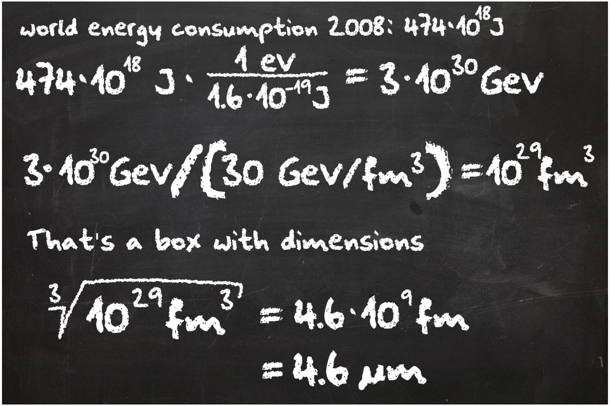
1. Use another model/calculation to determine these inputs:
   - e.g. take initial conditions from color glass condensate
   - Input equation of state from lattice QCD and hadron gas models

2. Use experimental data to fix parameters:
   - use one set of data to fix parameters:
     e.g. $\left.\frac{dN}{dyp_T dp_T}\right|_{b=0\,\mathrm{fm}}$ and $\frac{dN}{dy}(b)$

     Example parameters at RHIC:
     $\varepsilon_{0,\max} \approx 30\,\mathrm{GeV}/\mathrm{fm}^3$, $\tau_0 \approx 0.6\,\mathrm{fm}/c$, $T_{\mathrm{fo}} \approx 130\,\mathrm{MeV}$
   - predict another set of data:
     HBT, photons and dileptons, flow, ...

By the way: Initial energy density

The initial maximal energy densities needed to reproduce the experimental data are $\sim 30\,\mathrm{GeV/fm^3}$. How much is that?



Critical energy density to create quark-gluon-plasma: $1\,\mathrm{GeV/fm^3}$ (lattice QCD).

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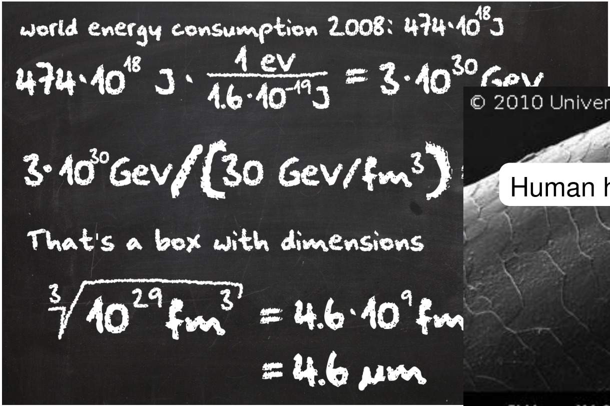
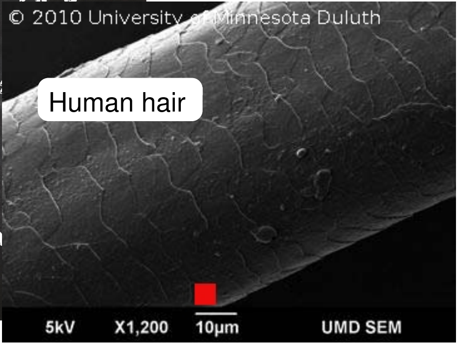




Critical energy density to create quark-gluon-plasma: $1\,\mathrm{GeV/fm^3}$ (lattice QCD).

Landau and Bjorken hydrodynamics

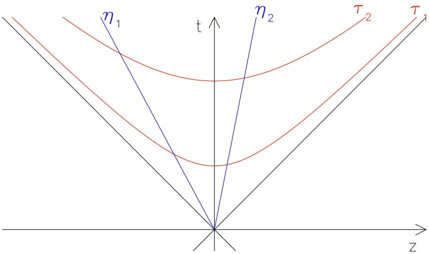
- Landau hydrodynamics
  - Initial fireball at rest: $u^\mu = (1, 0, 0, 0)$ everywhere
  - Start with a slab of radius r_{nucleus} and thickness $2r/\gamma$
    (γ is the γ-factor of the colliding nuclei)
  - Assumption of $v_z = 0$ seems unrealistic

- Bjorken hydrodynamics
  - At large energies $\gamma \to \infty$, Landau thickness $\to 0$
  - No longitudinal scale $\to$ scaling flow

$$v = \frac{z}{t}$$

Because all particles are assumed to have been produced at
$(t, z) = (0\,\text{fm}/c, 0\,\text{fm})$
a particle at point (z, t) must have had average $v = z/t$

Practical coords. for scaling flow expansion



- Longitudinal proper time τ:

$$\tau = \sqrt{t^2 - z^2}$$

- Space-time rapidity η_s:

$$\eta_s = \frac{1}{2} \ln \frac{t+z}{t-z}$$

Inversely: $t = \tau \cosh \eta_s$ and $z = \tau \sinh \eta_s$

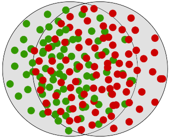
Initial conditions

Need to give hydro initial values for $T^{\mu\nu}(t_0, x, y, z)$ and the conserved currents. The latter are usually ignored at high energy.

Boost invariant initial condition get rid of one dimension (η-direction)

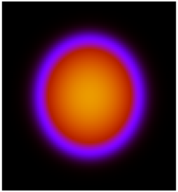
Need at a minimum the initial energy density and flow velocities if initial viscous corrections are ignored.

Fluctuating initial conditions

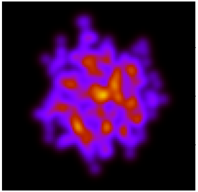


average, then evolve

evolve, then average

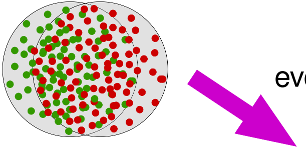


initial energy density

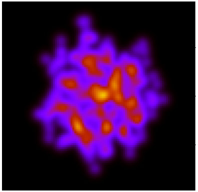


initial energy density

Fluctuating initial conditions



evolve, then average

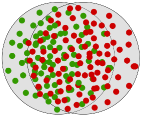


initial energy density

Fluctuating initial conditions

Typical procedure in "event-by-event" hydro ("MC-Glauber"):

- Sample Woods-Saxon distributions to determine nucleon positions
- Overlap those distributions using impact parameter b

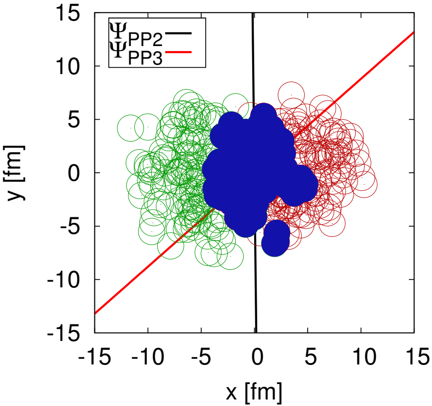


b is sampled from $P(b)db = 2bdb/(b^2_{\mathrm{max}} - b^2_{\mathrm{min}})$

- Nucleon-nucleon collision occurs if distance is $< \sqrt{\sigma_{NN}/\pi}$
- At position of collision add 2D-Gaussian energy density distribution with width σ_0.
  Width of the Gaussian is a free parameter $\sigma_0 \approx 0.5\,\mathrm{fm}$.

Single event initial conditions (simple MC-Glauber)

fixed impact parameter $b = 8\,\mathrm{fm}$



$$\psi_{PPn} = \frac{1}{n} \arctan \frac{\langle r^n \sin(n\phi) \rangle}{\langle r^n \cos(n\phi) \rangle}$$

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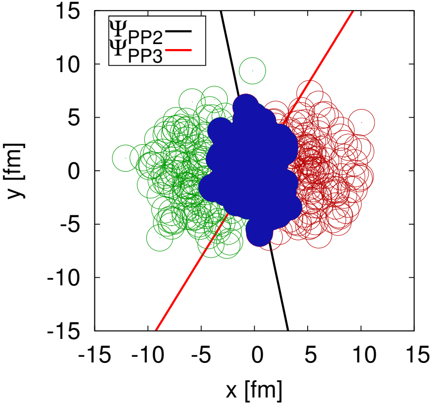

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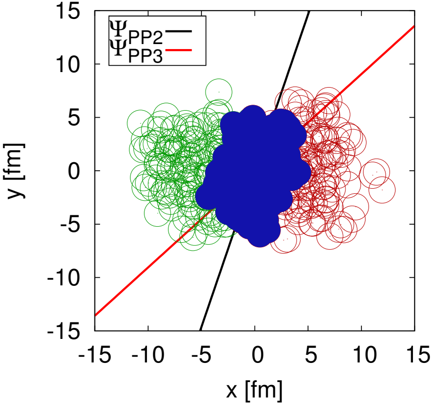

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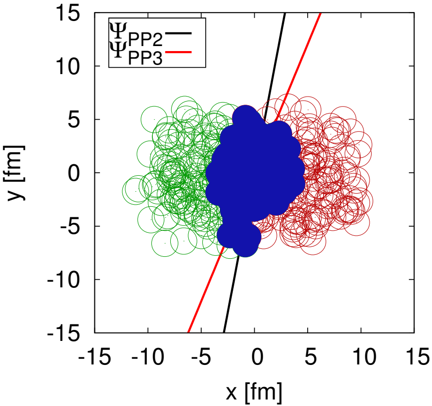

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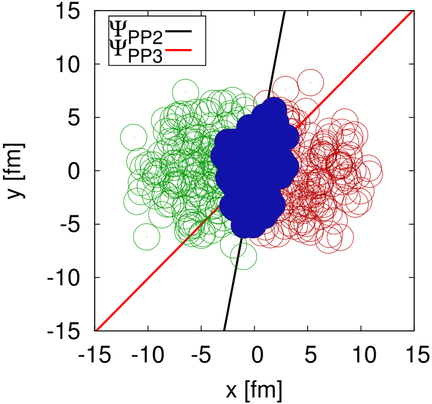

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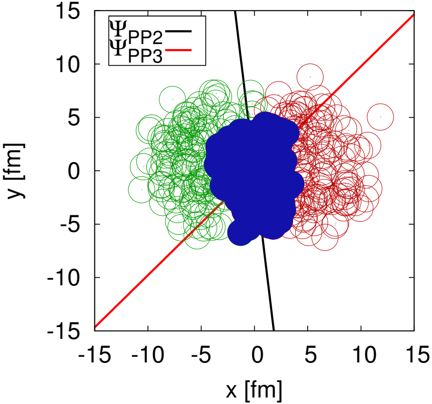

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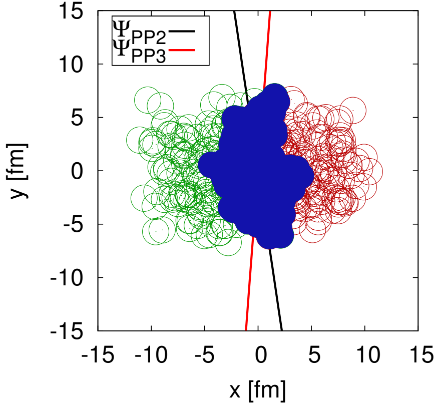

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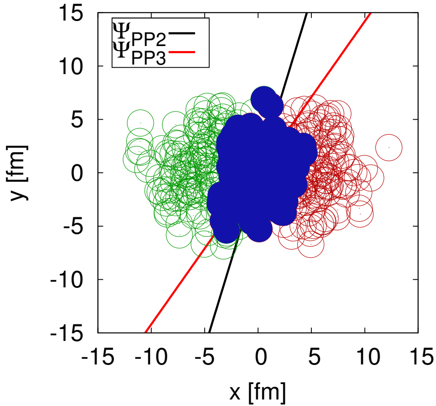

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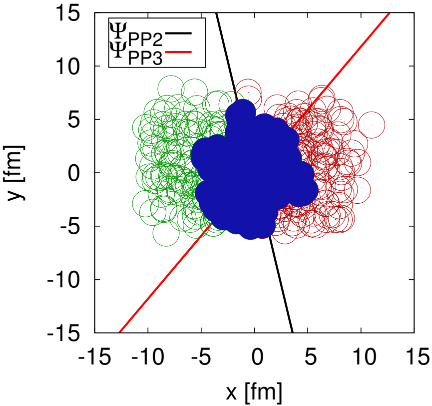

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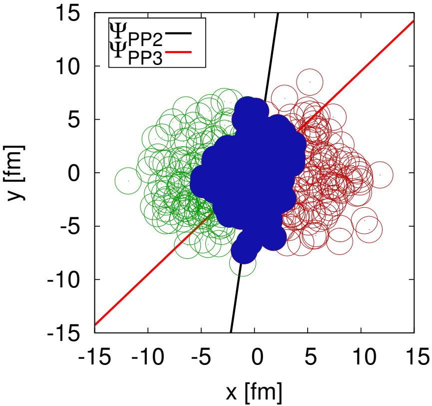

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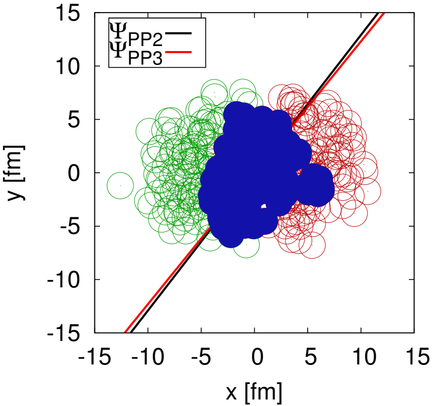

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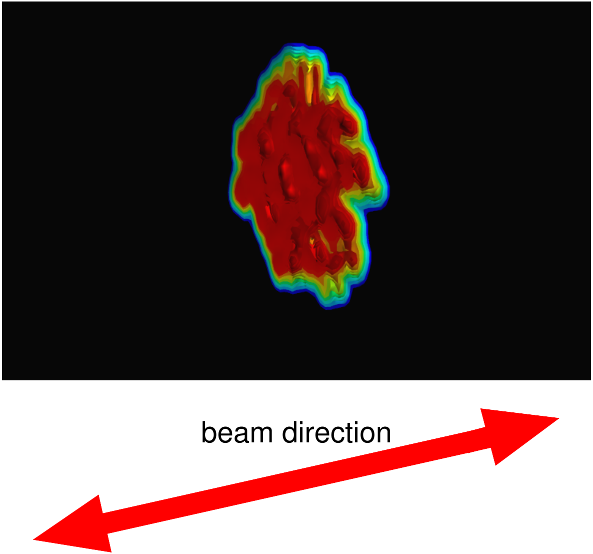
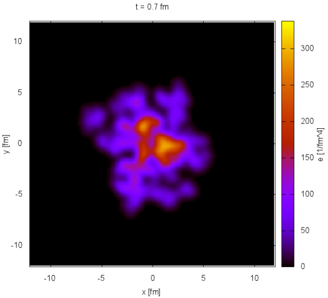
Single event initial conditions

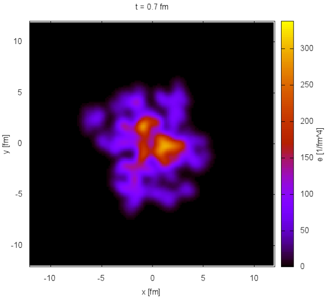


(scale in 1/fm^4):

ideal

$\eta/s = 0.16$



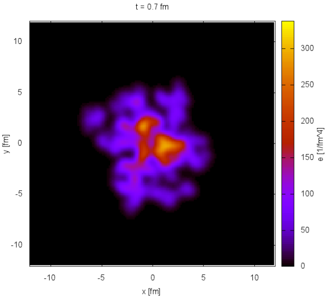


B. Schenke, S. Jeon, and C. Gale, PRL 106, 042301 (2011)

Viscosity in event-by-event simulations

(scale in $1/\text{fm}^4$):

ideal



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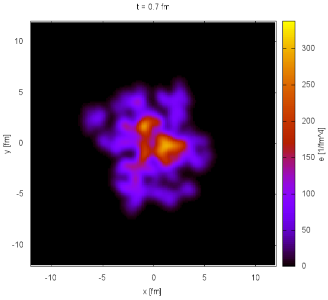


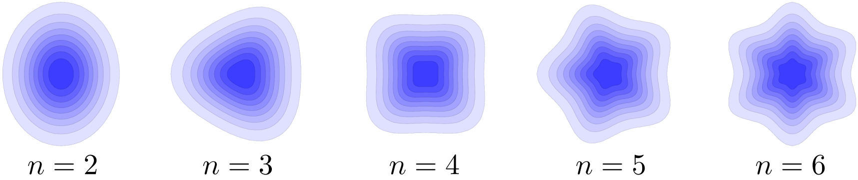
(note that the scale is dynamically adjusted)

B. Schenke, S. Jeon, and C. Gale, PRL 106, 042301 (2011)

Higher harmonics

$$\frac{dN}{d\phi} = \frac{N}{2\pi}\left(1 + \sum_n (2v_n \cos(n\phi))\right)$$

When including fluctuations, all moments appear:



also v_1 and $n > 6$.

Event plane

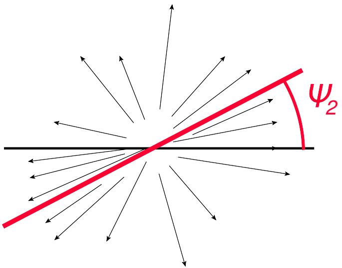
To get non-zero odd moments, we rotate the event plane in each event.

Event plane is defined by the angle:

$$\psi_n = \frac{1}{n} \arctan \frac{\langle \sin(n\phi) \rangle}{\langle \cos(n\phi) \rangle}$$

using particle momenta.

A.Poskanzer and S.Voloshin, Phys.Rev.C58:1671-1678 (1998)



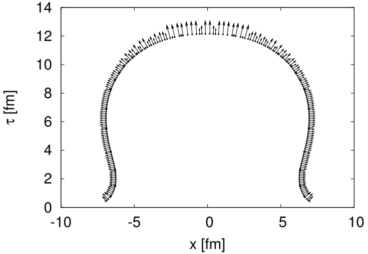
$$v_n = \langle \cos(n(\phi - \psi_n)) \rangle$$

... different angle for every flow coefficient.

The end of hydro

Well - the end of the hydrodynamic evolution.

- Particles are observed. Not a fluid.
- How to convert fluid into particles?
- So how far is hydro valid - when to switch to a particle description?



- Kinetic equilibrium requires scattering rate $\gg$ expansion rate
- scattering rate $\tau_{\rm sc}^{-1} \sim \sigma n \sim \sigma T^3$
- expansion rate $\theta = \partial_\mu u^\mu$ $= \tau^{-1}$ in 1+1D

Fluid description breaks down when $\tau_{\rm sc}^{-1} \approx \theta$
$\rightarrow$ momentum distributions freeze out

Note: need additional T-dependence of σ to freeze-out...

Cooper-Frye

Approximation: Decoupling takes place on constant temperature hypersurface Σ at $T = T_{\rm fo}$ (some use energy density or even time)

- Number of particles emitted = number of particles crossing Σ:

$$N = \int_\Sigma d\Sigma_\mu N^\mu$$

- We can compute the particle current:

$$\Rightarrow N^\mu = \int \frac{d^3p}{E} p^\mu f(x, p_\mu u^\mu, T)$$

$$\Rightarrow N = \int \frac{d^3p}{E} \int_\Sigma d\Sigma_\mu p^\mu f(x, p_\mu u^\mu, T)$$

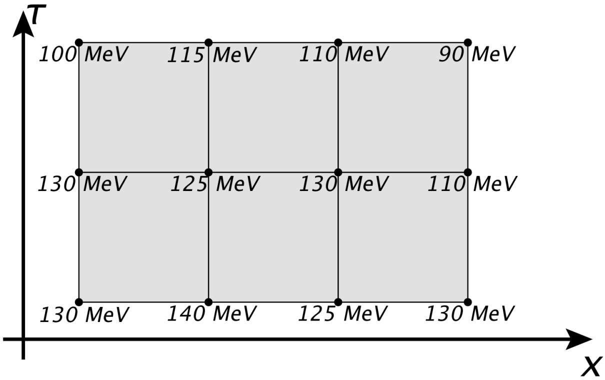
So we get the invariant inclusive momentum spectrum (Cooper-Frye formula):

$$E\frac{dN}{d^3p} = \int_\Sigma d\Sigma_\mu p^\mu f(x, p_\mu u^\mu, T)$$

Cooper and Frye, Phys.Rev.D10, 186 (1974)

Determining the freeze-out surface

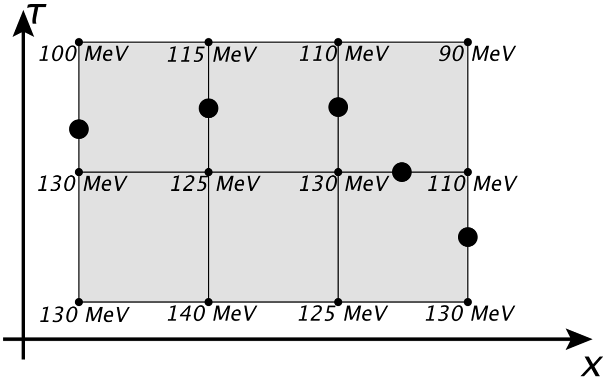
One can do it geometrically
In 1+1 dimensions:



given a temperature distribution at a given time step

Determining the freeze-out surface

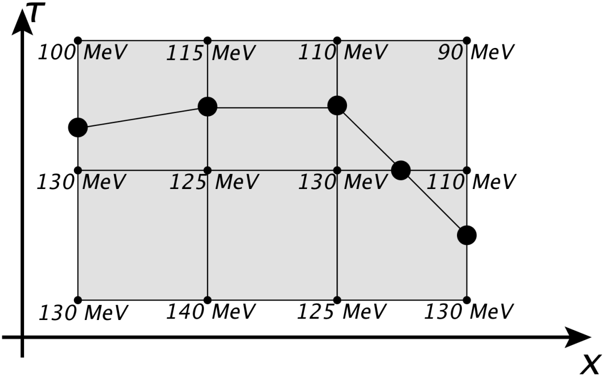
One can do it geometrically
In 1+1 dimensions:



determine where freeze-out temperature is crossed:
example $T_{\mathrm{freeze}} = 120 MeV$
Find crossing point by linear interpolation

Determining the freeze-out surface

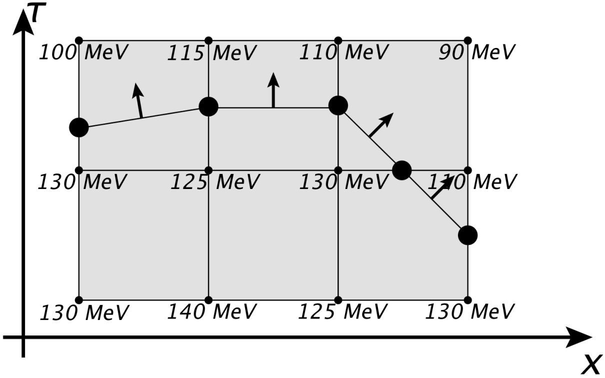
One can do it geometrically
In 1+1 dimensions:



Construct surface elements by connecting the crossing points
Non-trivial in more than 1+1 dimensions!

Determining the freeze-out surface

One can do it geometrically
In 1+1 dimensions:



Determine direction of the surface element by requiring it to point towards lower T

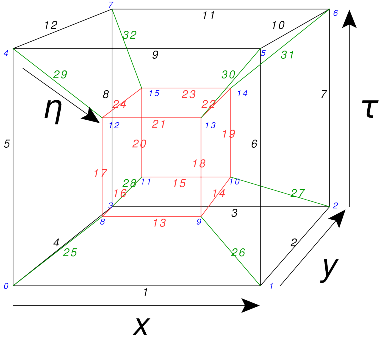
Determining the freeze-out surface

In 2+1 dimensions:

Same, but construct surface using triangles.

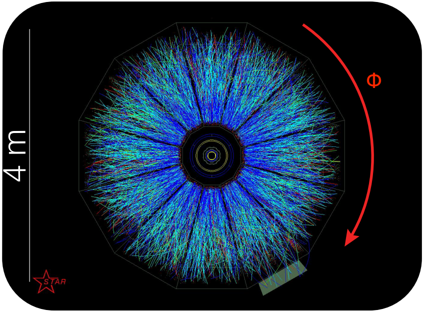
In 3+1 dimensions:

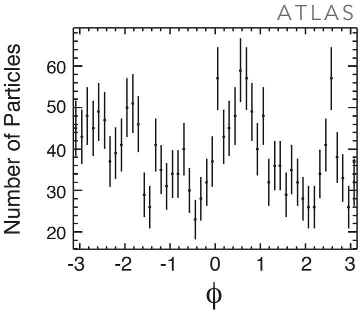
Same, but construct surface using tetrahedra.
"Find a 3D surface in a 4D volume"



Anisotropic Flow

Measure the anisotropy in the transverse particle spectra





Quantify anisotropy using Fourier expansion:

$$\frac{dN}{d\phi} = \frac{N}{2\pi}\left(1 + \sum_n 2v_n \cos[n(\phi - \psi_n)]\right)$$

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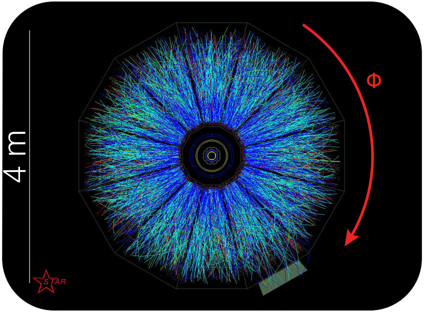
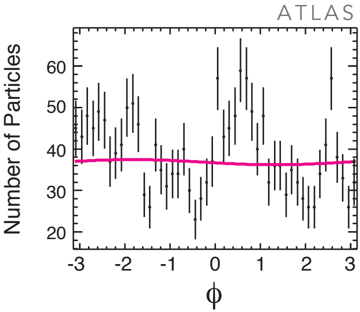
Anisotropic Flow

Measure the anisotropy in the transverse particle spectra





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Anisotropic Flow

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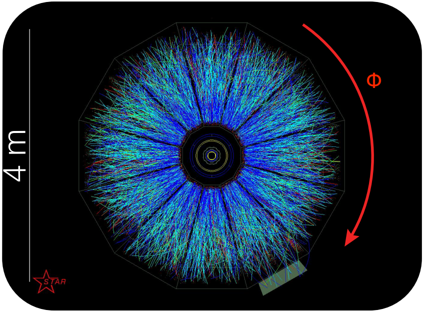

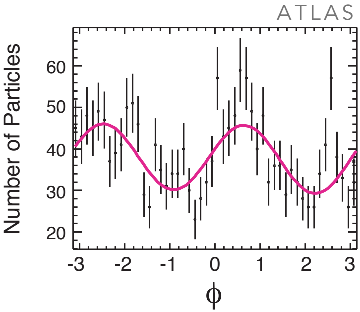

Quantify anisotropy using Fourier expansion:

$$\frac{dN}{d\phi} = \frac{N}{2\pi}\left(1 + 2(v_1 \cos(\phi) + v_2 \cos(2\phi))\right)$$

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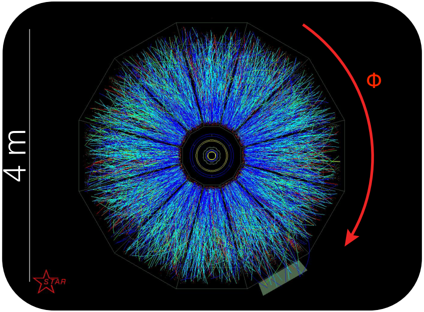

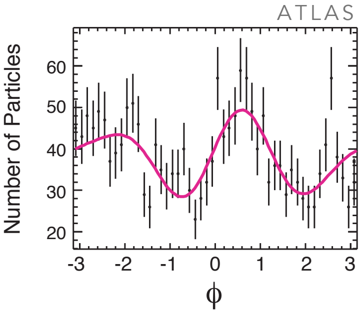

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Anisotropic Flow

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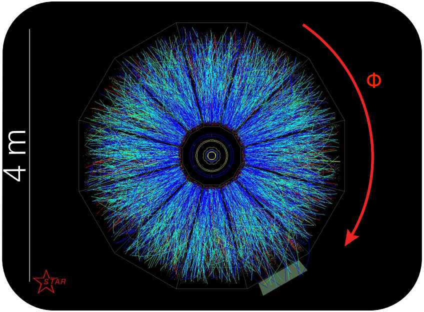

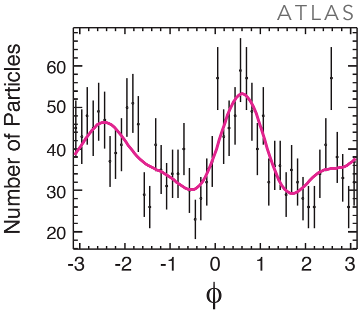

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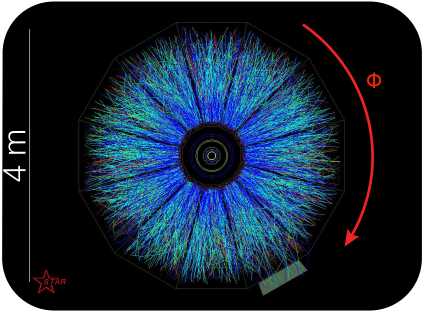
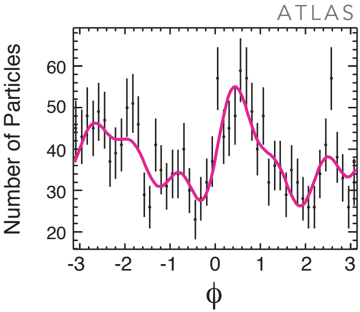
$$\frac{dN}{d\phi} = \frac{N}{2\pi}\Big(1 + 2(v_1 \cos(\phi) + v_2 \cos(2\phi) + v_3 \cos(3\phi) + v_4 \cos(4\phi))\Big)$$

12

Anisotropic Flow

Measure the anisotropy in the transverse particle spectra





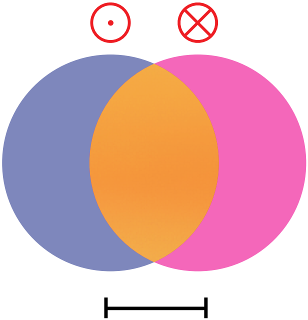
Quantify anisotropy using Fourier expansion:

$$\frac{dN}{d\phi} = \frac{N}{2\pi}\left(1 + 2(v_1 \cos(\phi) + v_2 \cos(2\phi) + v_3 \cos(3\phi) + v_4 \cos(4\phi) + \dots)\right)$$

13

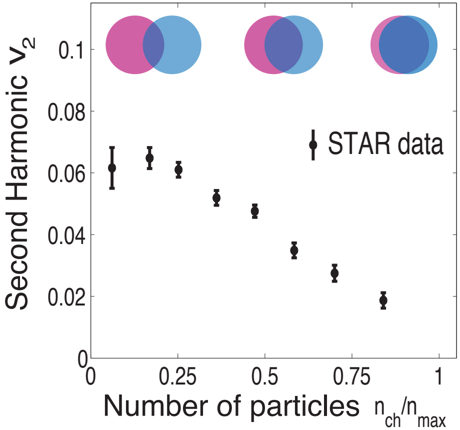
Fluid dynamic response to the initial shape

beam directions



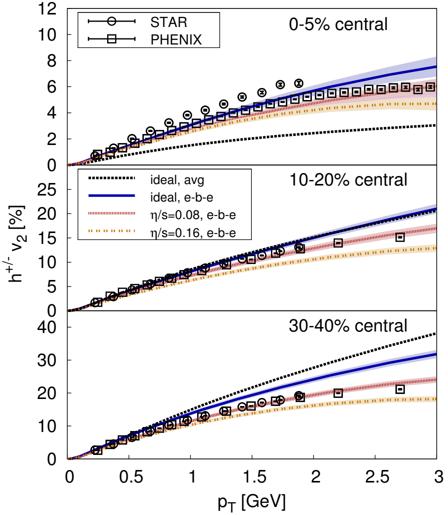
impact parameter b

Second Fourier Coefficient v_2



14

Flow results from e-b-e viscous MUSIC

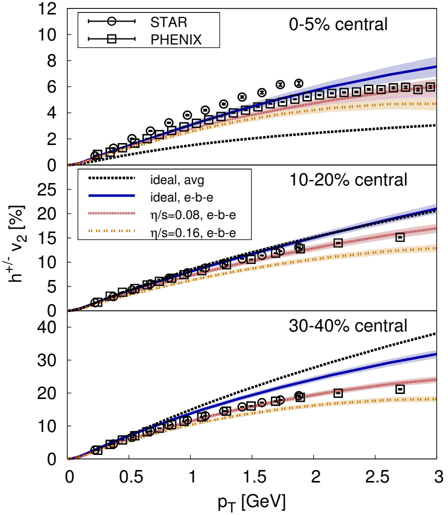


B. Schenke, S. Jeon, and C. Gale, PRL 106, 042301 (2011)

Ideal and shear viscosities:
$\eta/s = 0.08$ and
$\eta/s = 0.16$

Experimental data: J. Adams et al. (STAR), Phys.Rev.C72, 014904 (2005)
A. Adare et al. (PHENIX), Phys.Rev.Lett.105, 062301 (2010)

Flow results from e-b-e viscous MUSIC

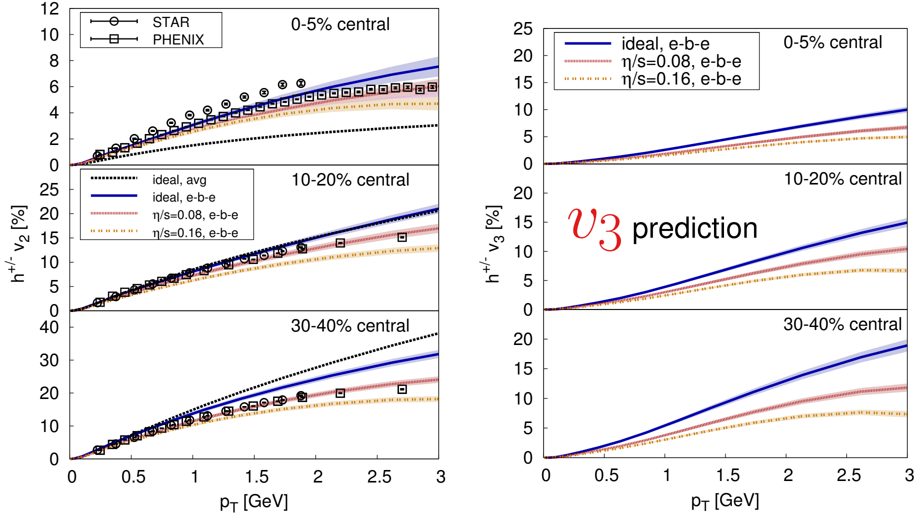


B. Schenke, S. Jeon, and C. Gale, PRL 106, 042301 (2011)

- event-by-event fluctuations important!
- Viscosity is **very low.** The lower bound of viscosity/entropy density conjectured from AdS/CFT duality is $\eta/s = 1/4\pi \approx 0.08$ (red curves)

Experimental data: J. Adams et al. (STAR), Phys.Rev.C72, 014904 (2005)
A. Adare et al. (PHENIX), Phys.Rev.Lett.105, 062301 (2010)

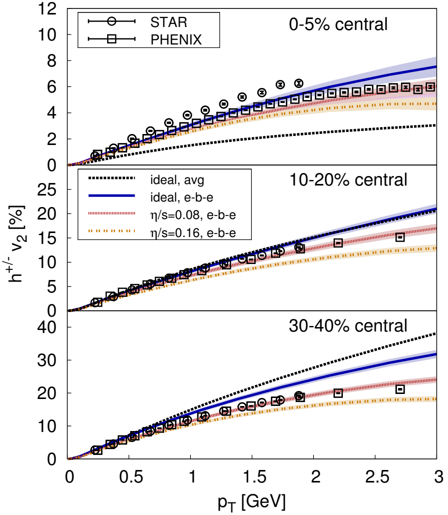
Flow results from e-b-e viscous MUSIC

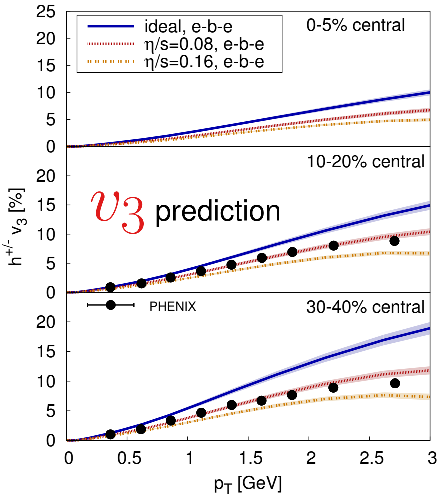


B. Schenke, S. Jeon, and C. Gale, PRL 106, 042301 (2011)

Experimental data: J. Adams et al. (STAR), Phys.Rev.C72, 014904 (2005)
A. Adare et al. (PHENIX), Phys.Rev.Lett.105, 062301 (2010)

Flow results from e-b-e viscous MUSIC



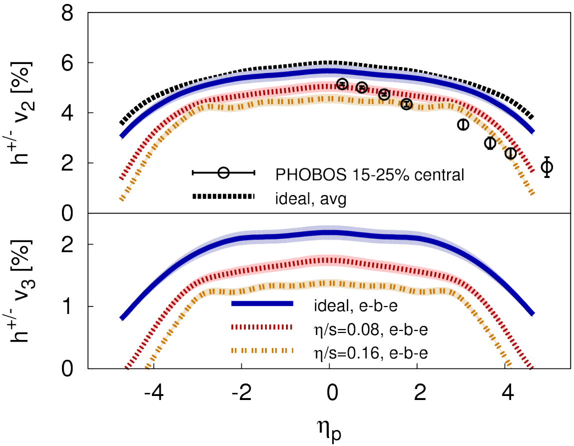


B. Schenke, S. Jeon, and C. Gale, PRL 106, 042301 (2011)

Experimental data: J. Adams et al. (STAR), Phys.Rev.C72, 014904 (2005)
A. Adare et al. (PHENIX), Phys.Rev.Lett.105, 062301 (2010) , A. Adare et al. (PHENIX), arXiv:1105.3928 (2011)

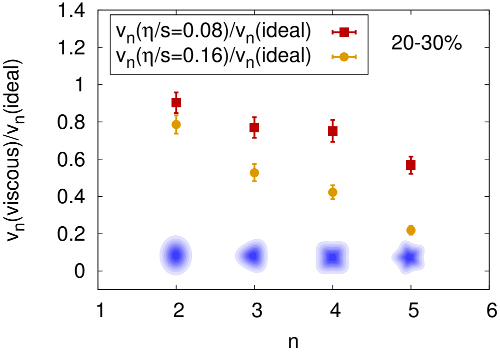
Flow results from e-b-e viscous MUSIC

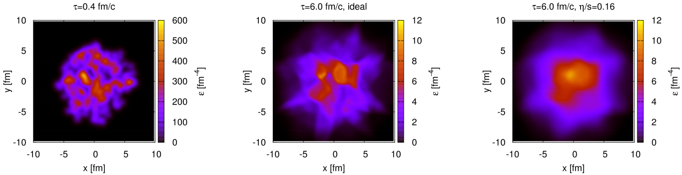
v_2 and v_3 as functions of pseudo-rapidity η_p



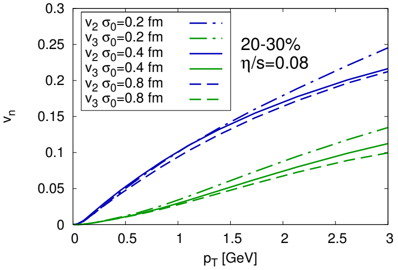
B. Schenke, S. Jeon, and C. Gale, PRL 106, 042301 (2011)
Experimental data: B.B. Back et al. (PHOBOS), Phys.Rev.C72:051901 (2005)

Higher Fourier coefficients are suppressed more by viscosity.

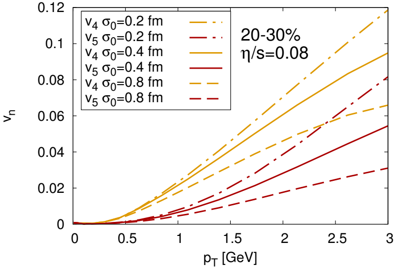




Dependence on initial granularity



v_2: weak dependence



Higher harmonics:
much stronger dependence

Initial conditions - all including fluctuations

You will see different initial conditions being used:

- MC-Glauber: geometric model determining wounded nucleons based on the inelastic cross section (different implementations)
- MC-KLN: Color-Glass-Condensate (CGC) based model using k_T-factorization
  Same fluctuations in the wounded nucleon positions as MC-Glauber
- MCrcBK: Similar to MC-KLN but with improved energy/rapidity dependence following from solutions to the running coupling Balitsky Kovchegov equation
- IP-Glasma: CGC based model using classical Yang-Mills evolution of early-time gluon fields, including additional fluctuations in the particle production
- Also CGC based EKRT and hadronic cascades UrQMD or NEXUS and partonic cascades (e.g. BAMPS) can provide initial conditions.

Glasma initial state

To see how one can use CGC calculations and the Glasma as initial conditions for hydro, I will briefly explain the IP-Glasma model

B. Schenke, P. Tribedy, R. Venugopalan, Phys. Rev. Lett. 108, 252301 (2012)
B. Schenke, P. Tribedy, R. Venugopalan, arXiv:1206.6805 (2012)

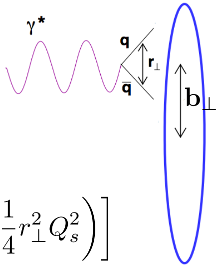
Modeling x and b dependence: IP-Sat model

Energy dependence (x-dependence) can be computed by starting from MV at given x_0 and using BK/JIMWLK evolution to get the distributions at smaller x.

Here, however, we use the IP-Sat model to extend the MV model to include

- x-dependence
- Impact parameter dependence (IP)

  Kowalski, Teaney, Phys.Rev. D68 (2003) 114005



Proton dipole cross section in DIS:
MV model:

$$\frac{\mathrm{d}\sigma^{\mathrm{p}}_{\mathrm{dip}}}{\mathrm{d}^2\mathbf{b}_\perp}(\mathbf{r}_\perp, \mathbf{b}_\perp) = 2\mathcal{N}(\mathbf{r}_\perp, \mathbf{b}_\perp) = 2\left[1 - \exp\left(-\frac{1}{4}r_\perp^2 Q_s^2\right)\right]$$

IP-Sat:

$$\frac{\mathrm{d}\sigma^{\mathrm{p}}_{\mathrm{dip}}}{\mathrm{d}^2\mathbf{b}_\perp}(\mathbf{r}_\perp, x, \mathbf{b}_\perp) = 2\mathcal{N}(\mathbf{r}_\perp, x, \mathbf{b}_\perp) = 2\left[1 - \exp\left(-\frac{\pi^2}{2N_c}{\mathbf{r}_\perp}^2 \alpha_s(\tilde{\mu}^2) xg(x, \tilde{\mu}^2) T_p(\mathbf{b}_\perp)\right)\right]$$

with $\tilde{\mu}^2$ an energy scale related to the dipole radius $\mathbf{r}_\perp$ and $xg(x, \tilde{\mu}^2)$ is the gluon density.

Modeling x and b dependence of Q_s: IP-Sat model

The important feature of the IP-Sat model is its impact parameter dependence encoded in the gluon density profile function

$$T_p(\mathbf{b}_\perp) = \frac{1}{2\pi B_G} \exp\left(\frac{-\mathbf{b}_\perp{}^2}{2B_G}\right)$$

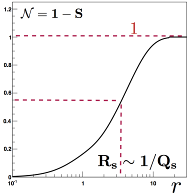
where B_G is an energy independent parameter, fit to HERA diffractive data.

$\langle b^2 \rangle = 2B_G$ is the average squared *gluonic* radius of the proton.

see P. Tribedy and R. Venugopalan, Nucl.Phys. A850 (2011) 136-156

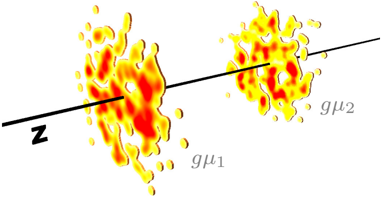
After fitting parameters to HERA DIS data the model provides a distribution of $Q_s^2(x, \mathbf{b}_\perp)$, which will be our input.
It is determined self consistently from the requirement that

$$\mathcal{N}(R_s, x, \mathbf{b}_\perp) = 1 - e^{-1/2}, \quad \text{with } Q_s^2 = \frac{2}{R_s^2}$$



IP-Glasma: 1. Color charge densities of incoming nuclei

- Sample positions of nucleons from Woods-Saxon distributions in nucleus A and B.
- IP-Sat provides $Q_s^2(x, \mathbf{b}_\perp)$ for each nucleon.
  The color charge density squared $g^2\mu^2$ is proportional to Q_s^2.
  (proportionality factor depends on details of calculation - see Lappi, arXiv:0711.3039)
- We add all $g^2\mu^2(\mathbf{x}_\perp)$ in each nucleus to obtain $g^2\mu_1^2(\mathbf{x}_\perp)$ and $g^2\mu_2^2(\mathbf{x}_\perp)$.

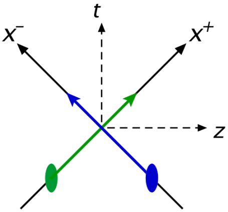


- Sample ρ^a from local Gaussian distribution for each nucleus

$$\langle \rho^a(\mathbf{x}_\perp)\rho^b(\mathbf{y}_\perp)\rangle = \delta^{ab}\delta^2(\mathbf{x}_\perp - \mathbf{y}_\perp)g^2\mu^2(\mathbf{x}_\perp)$$

Color currents:

$J_1^\nu = \delta^{\mu+}\rho_1(x^-, \mathbf{x}_\perp)$
$[D_\mu, F^{\mu\nu}] = J_1^\nu$



$J_2^\nu = \delta^{\mu-}\rho_2(x^+, \mathbf{x}_\perp)$
$[D_\mu, F^{\mu\nu}] = J_2^\nu$

Solution in covariant gauge:

$$A^+_{\mathrm{cov}(1,2)}(x^-, \mathbf{x}_\perp) = -\frac{g\rho_{(1,2)}(x^-, \mathbf{x}_\perp)}{\nabla_\perp^2 + m^2}$$

with infrared cutoff m of order Λ_{QCD}.

Solution in light cone gauge:

$$A^+_{(1,2)}(\mathbf{x}_\perp) = A^-_{(1,2)}(\mathbf{x}_\perp) = 0$$
$$A^i_{(1,2)}(\mathbf{x}_\perp) = \frac{i}{g} V_{(1,2)}(\mathbf{x}_\perp)\partial_i V^\dagger_{(1,2)}(\mathbf{x}_\perp)$$

V is the path-ordered exponetial of $A^+_{\mathrm{cov}(1,2)}$

IP-Glasma: 2. Gauge fields before the collision

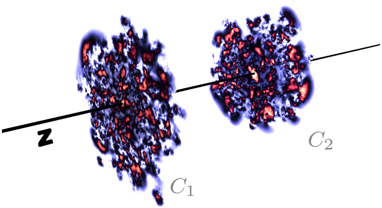
The correlator of the Wilson lines

$$C_{(1,2)}(\mathbf{x}_\perp) = \frac{1}{N_c}\mathrm{Re}[\mathrm{tr}(V(1,2)^\dagger(0,0)V(1,2)(x,y))]$$

with

$$V_{(1,2)}(\mathbf{x}_\perp) = P\exp\left(-ig\int dx^- \frac{\rho_{(1,2)}(x^-,\mathbf{x}_\perp)}{\nabla_\perp^2 + m^2}\right)$$

shows the degree of correlations and fluctuations in the gluon fields.



The length scale of fluctuations is $1/Q_s$. Not the nucleon size.

IP-Glasma: 3. Gauge fields after the collision (Glasma)

Initial condition on the lightcone: require that fields match smoothly on the lightcone.

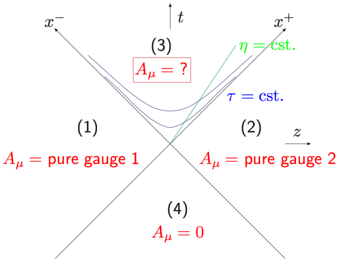


figure from Lappi, arXiv:1003.1852

Solution:

$$A^i_{(3)}|_{\tau=0} = A^i_{(1)} + A^i_{(2)}$$

$$A^\eta_{(3)}|_{\tau=0} = \frac{ig}{2}[A^i_{(1)}, A^i_{(2)}]$$

On the lattice the Wilson lines in the future lightcone are obtained from the condition:

$$\mathrm{tr}\left\{t^a\left[\left(U^i_{(1)} + U^i_{(2)}\right)(1 + U^{i\dagger}_{(3)}) - (1 + U^i_{(3)})\left(U^{i\dagger}_{(1)} + U^{i\dagger}_{(2)}\right)\right]\right\} = 0$$

where t^a are the generators of $SU(N_c)$ in the fundamental representation. Solve iteratively.

Krasnitz, Venugopalan, Nucl.Phys. B557 (1999) 237

$$U^i_{(1,2),j} = V_{(1,2),j} V^\dagger_{(1,2),j+\hat{e}_i} \qquad \text{(gauge transform of } \mathbf{1}\text{: pure gauge)}$$

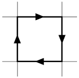
IP-Glasma: Initial energy density

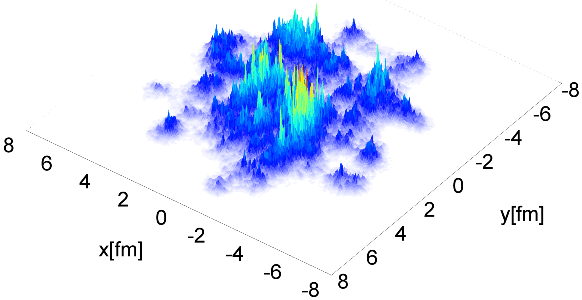
Initial energy density at $\tau = 0$:

$$\varepsilon(\tau = 0) = \frac{2}{g^2 a^4}(N_c - \mathrm{Re}\,\mathrm{tr}\,U_\square) + \frac{1}{a^4}\mathrm{tr}\,E_\eta^2$$

with the longitudinal magnetic and electric energy density.

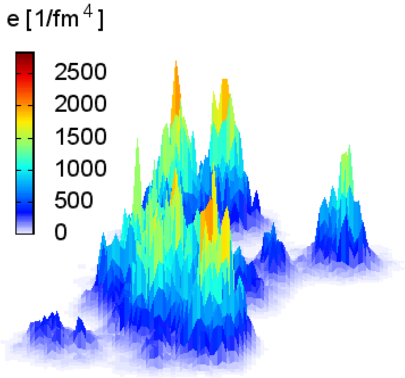
The plaquette is given by

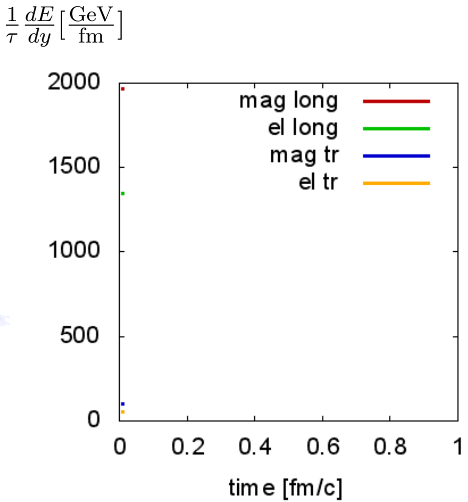
$$U_\square^j = U_j^x \, U_{j+\hat{x}}^y \, U_{j+\hat{y}}^{x\dagger} \, U_j^{y\dagger} =$$ 



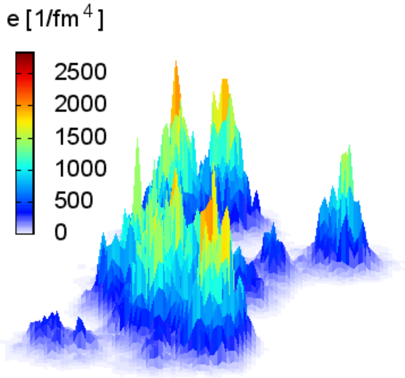
arbitrary units

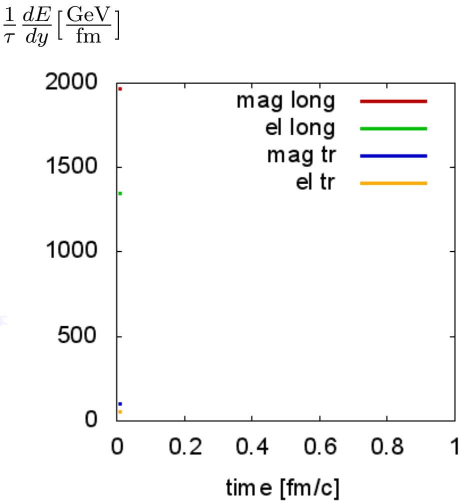
Classical Yang-Mills evolution



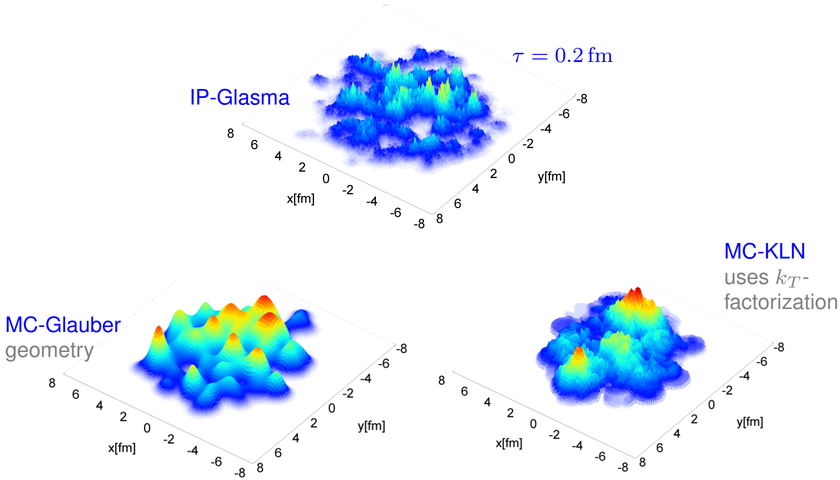


Classical Yang-Mills evolution





Initial energy densities

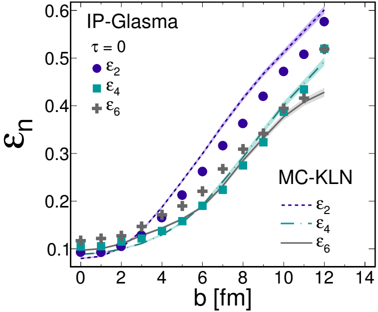


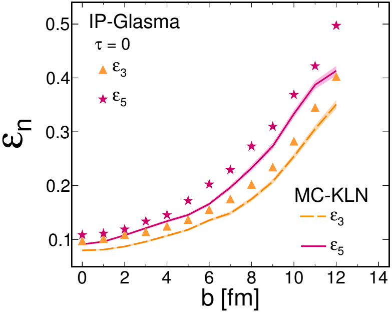
MC-KLN: Drescher, Nara, nucl-th/0611017

mckln-3.52 from `http://physics.baruch.cuny.edu/files/CGC/CGC_IC.html` with defaults, energy density scaling

$$\varepsilon_n = \frac{\sqrt{\langle r^n \cos(n\phi)\rangle^2 + \langle r^n \sin(n\phi)\rangle^2}}{\langle r^n \rangle}$$

Averages are weighted by the energy density.



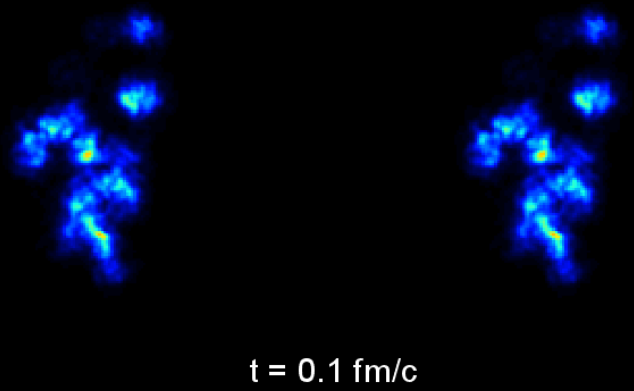


As we discussed for ε_2, a larger eccentricity will cause a larger flow due to the hydrodynamic response.

To extract η/s for example, the initial state needs to be under control - looking at all v_n will give better control.

The IP-Glasma initial condition includes evolution

shown are energy density distributions

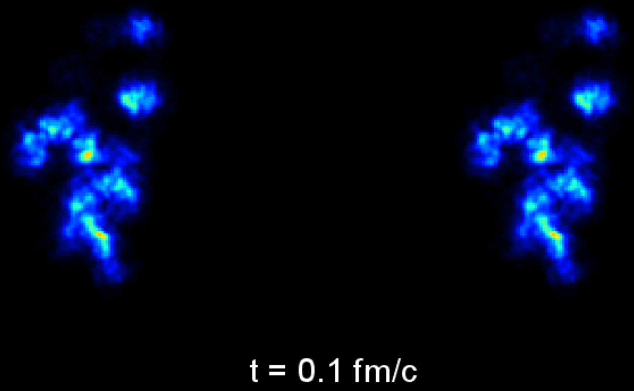


2+1D CYM
(weakly coupled at late times)

Hydro
after $\tau = 0.2\,\mathrm{fm}/c$ (CYM before)

This is the movie from the first lecture...

shown are energy density distributions

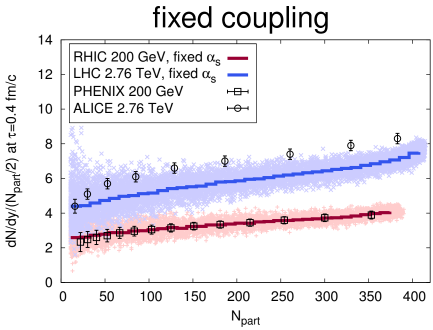


2+1D CYM
(weakly coupled at late times)

Hydro
after $\tau = 0.2\,\mathrm{fm}/c$ (CYM before)

Multiplicity B.Schenke, P.Tribedy, R.Venugopalan, arXiv:1206.6805

$dN_{\rm g}/dy$ at finite time $\tau = 0.4\,{\rm fm}$ in transverse Coulomb gauge $\partial_i A^i = 0$
$N_{\rm part}$ from MC-Glauber with $\sigma_{NN} = 42\,{\rm mb}$ and $64\,{\rm mb}$ respectively

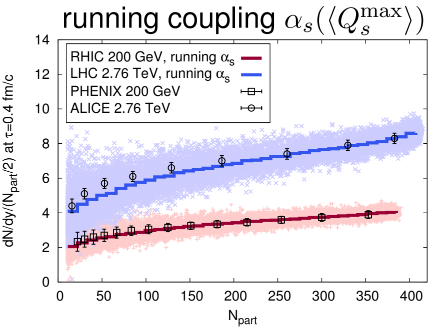


Experimental data: PHENIX, Phys.Rev.C71 034908 (2004) and ALICE, Phys.Rev.Lett. 106, 032301 (2011)

Scaled by $2/3$ to compare to charged particles.
Some freedom in normalization - will need to account for entropy production.

Multiplicity B.Schenke, P.Tribedy, R.Venugopalan, arXiv:1206.6805

dN_g/dy at finite time $\tau = 0.4\,\mathrm{fm}$ in transverse Coulomb gauge $\partial_i A^i = 0$
N_{part} from MC-Glauber with $\sigma_{NN} = 42\,\mathrm{mb}$ and $64\,\mathrm{mb}$ respectively



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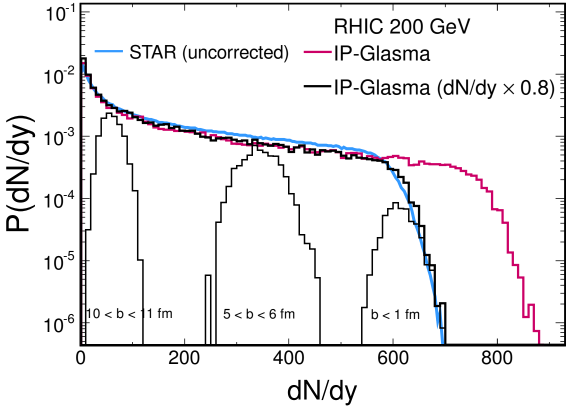
Multiplicity

B.Schenke, P.Tribedy, R.Venugopalan, arXiv:1206.6805

$P(dN_{\rm g}/dy)$ at time $\tau = 0.4\,{\rm fm}$ with $P(b)$ from a Glauber model

Experimental data: STAR, Phys. Rev. C79, 034909 (2009)

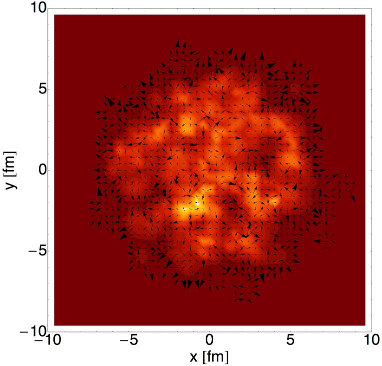


Glasma model gives a convolution of negative binomial distributions

No need to put them in by hand

$T^{\mu\nu}$ and flow velocities

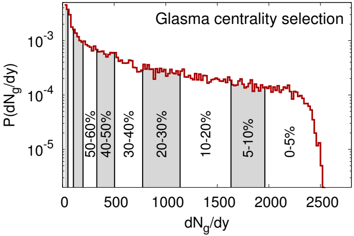
Compute all components of $T^{\mu\nu}$
Determine energy density and (u^x, u^y) at $\tau > 0\,\mathrm{fm}$ from $u_\mu T^{\mu\nu} = \varepsilon u^\nu$
as input for hydrodynamic simulations



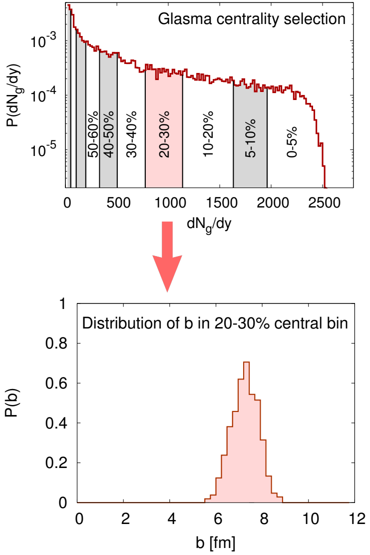
Energy density
and (u_x, u_y)
at $\tau = 0.4\,\mathrm{fm}/c$

No instabilities (need full 3+1D Yang-Mills for that):
system is far from equilibrium - cannot yet match $\Pi^{\mu\nu}$

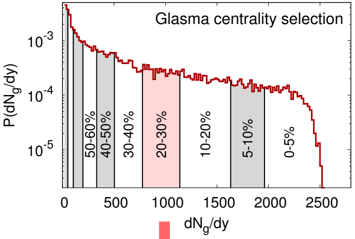
Centrality selection and flow

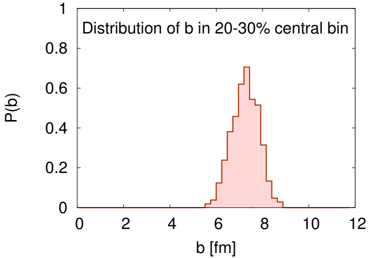


Centrality selection and flow



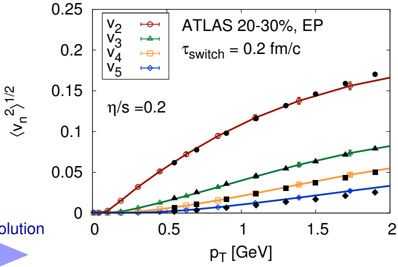
Centrality selection and flow



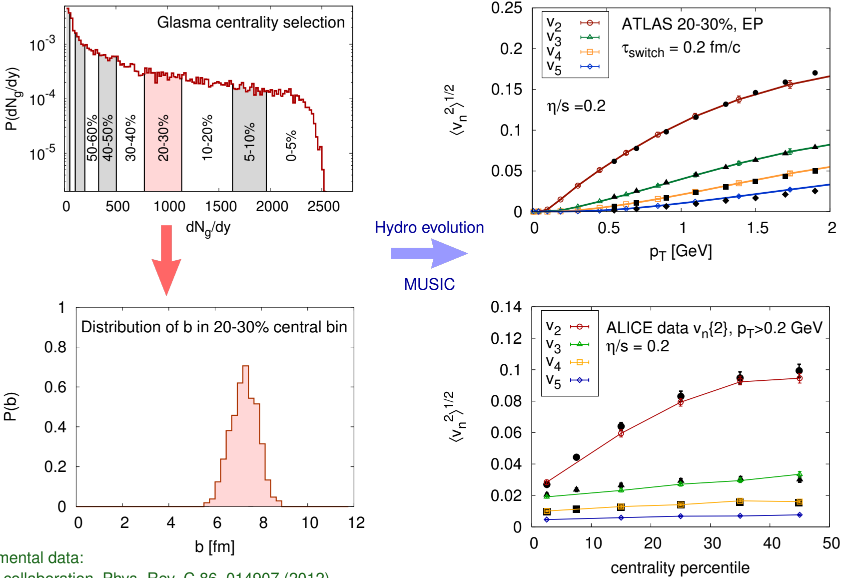


Hydro evolution

MUSIC



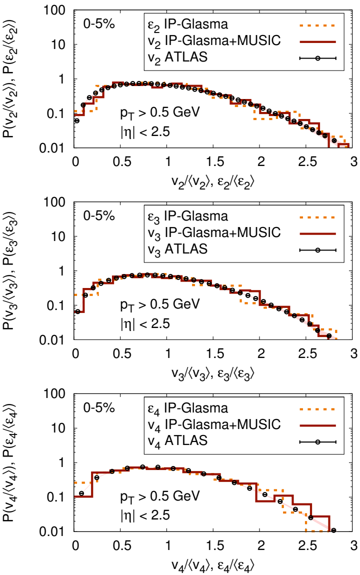
Centrality selection and flow



Experimental data:
ATLAS collaboration, Phys. Rev. C 86, 014907 (2012)
ALICE collaboration, Phys. Rev. Lett. 107, 032301 (2011)

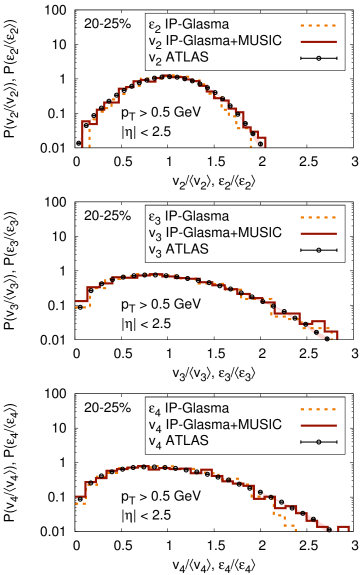
Event-by-event distributions of v_n

Experimental data: ATLAS collaboration, JHEP 11 (2013) 183



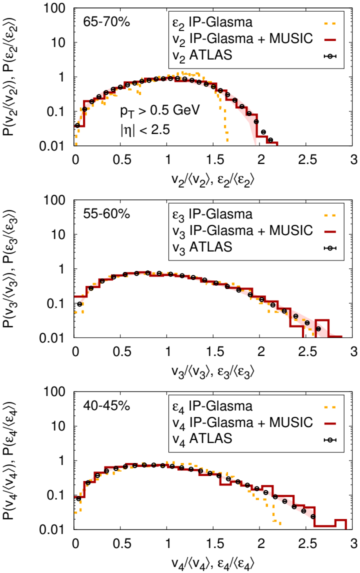
Event-by-event distributions of v_n

Experimental data: ATLAS collaboration, JHEP 11 (2013) 183



Event-by-event distributions of v_n

Experimental data: ATLAS collaboration, JHEP 11 (2013) 183



Event-by-event distributions of v_n

Responsible to get v_n distributions right: Correct energy deposition mechanism in the transverse plane

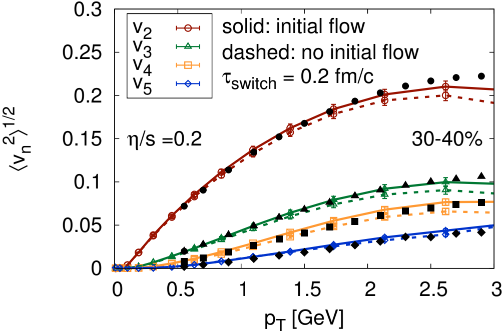
see J. Scott Moreland, Jonah E. Bernhard, Steffen A. Bass,Phys.Rev. C92 (2015) 1, 011901

Only IP-Glasma, EKRT H. Niemi, K.J. Eskola, R. Paatelainen, e-Print: arXiv:1505.02677, and Glauber like model with the initial energy density $\sim T_A \times T_B$ get it right

The scaled e-by-e distributions do not depend on the transport properties (like shear viscosity)
So they give direct information on the initial state

Now 3D initial states

Effect of initial flow



Weak effect of initial flow on hadron $v_n(p_T)$

Expect stronger effect for photon v_n:
Photons are mostly produced early at high temperatures

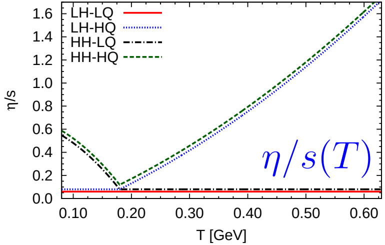
Effect of different switching time $0.4\,\mathrm{fm}/c$ is very weak

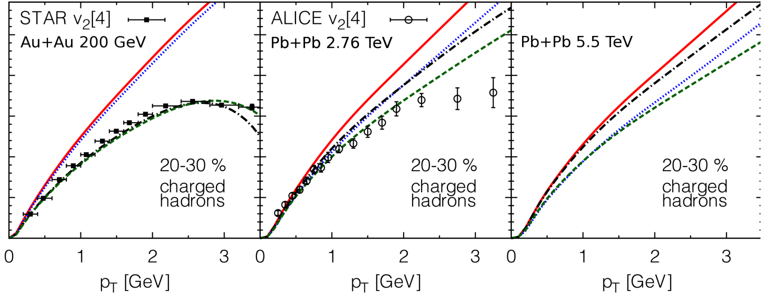
Experimental data:
ATLAS collaboration, Phys. Rev. C 86, 014907 (2012)

Beyond constant η/s

Determine dependence of v_2 on modeled $\eta/s(T)$.
L=low, H="high"
H=hadronic phase, Q=QGP

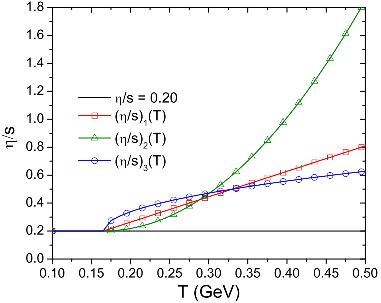
H. Niemi at al, arXiv:1101.2442

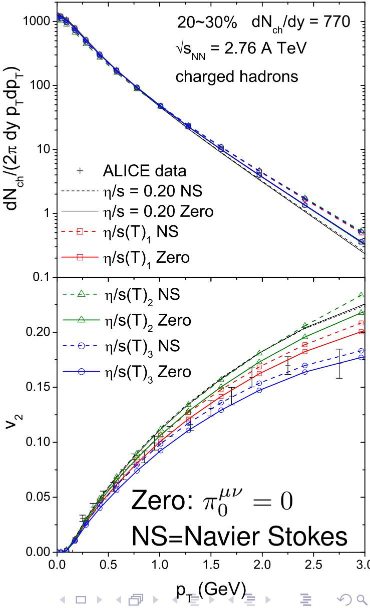




Weak dependence on QGP $\eta/s(T)$ at RHIC. Dependent on minimum.
Different at LHC energies (longer QGP phase, smaller gradients in the hadronic phase)

Beyond constant η/s





Need better understanding of the pre-thermal evolution and its matching to viscous hydrodynamics.

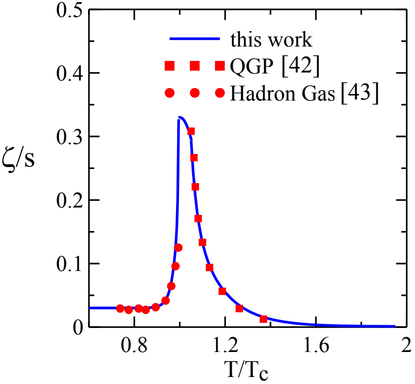
C. Shen, U. Heinz, P. Huovinen, H. Song, arXiv:1105.3226

Navier Stokes: $\pi_0^{\mu\nu} = \eta_0(\nabla^\mu u_0^\nu + \nabla^\nu u_0^\mu - \frac{2}{3}\Delta^{\mu\nu}\nabla_\alpha u_0^\alpha)$

Importance of bulk viscosity

S. Ryu, J.-F. Paquet, C. Shen, G.S. Denicol, B. Schenke, S. Jeon, C. Gale, Phys.Rev.Lett. 115 (2015) 13, 132301

Bulk viscosity is expected to peak around the transition temperature
The QCD matter is least conformal there (e.g. large trace anomaly)



For the relation of scale invariance (conformality) and vanishing bulk viscosity see e.g.

Lecture Notes in Physics, Volume 836. ISBN 978-3-642-21977-1, Castin, Yvan; Werner, Felix
D.T. Son, Phys.Rev.Lett. 98 (2007) 020604

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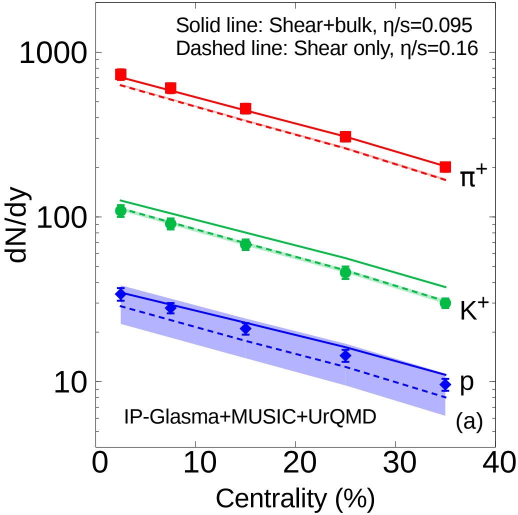

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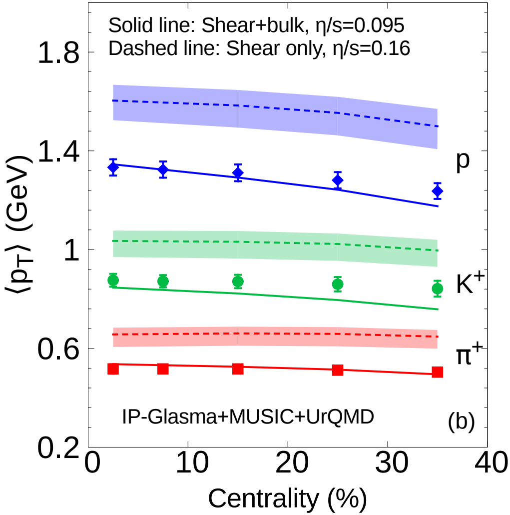

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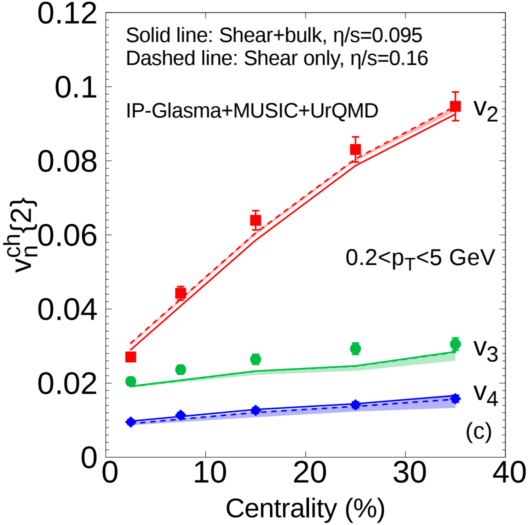
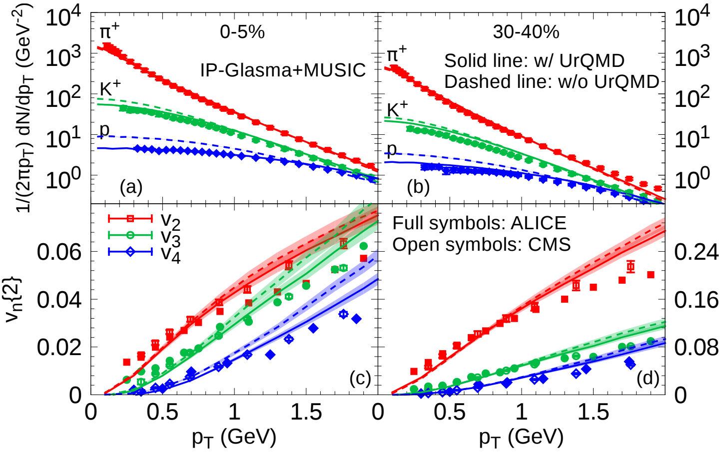


Importance of bulk viscosity

S. Ryu, J.-F. Paquet, C. Shen, G.S. Denicol, B. Schenke, S. Jeon, C. Gale, Phys.Rev.Lett. 115 (2015) 13, 132301



Conclusions

- A lot of progress has been made in the last 5-6 years, including viscosity, event-by-event simulations, 3+1D viscous simulations, lattice+HRG equations of state, ...
- IP-Glasma is well constrained initial state based on color glass effective theory of QCD
- Ongoing efforts to extend this to 3D (non-boost invariant)
- Event-by-event hydro allows for more precise comparison to experimentally measured $v_n\{2\}$, $v_n\{4\}$, ...
  allows for more insight into initial state and medium properties
- Uncertainties remain for the moment.

How to continue the progress

- Pre-equilibrium physics! Initial conditions for viscous hydro and transition from non-equilibrium to equilibrium
- Improve transition from hydrodynamics to particles (δf)
- Better understand temperature dep. of transport coefficients
- Finite baryon number, baryon diffusion
- Include all the improvements in a single calculation
  Try to describe as many observables as possible

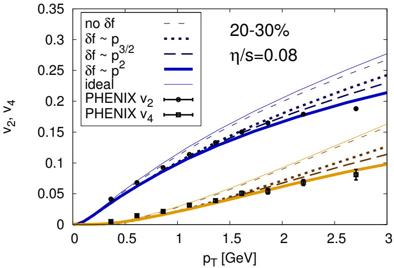
Remember: $$\delta f = f_0(1 \pm f_0) p^\mu p^\nu \pi_{\mu\nu} \frac{1}{2(\epsilon + \mathcal{P})T^2}$$

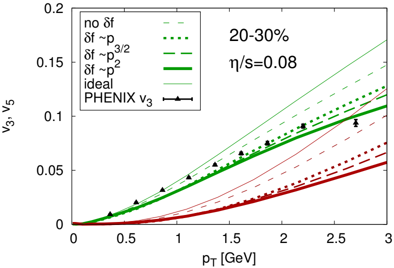
The choice $\delta f \sim p^2$ is not unique.

More generally: from using different energy dep. of relaxation time

$$\delta f = \frac{120}{\Gamma(6-\alpha)} f_0(1 \pm f_0) \left(\frac{T}{E}\right)^\alpha p^\mu p^\nu \pi_{\mu\nu} \frac{1}{2(\epsilon + \mathcal{P})T^2}$$

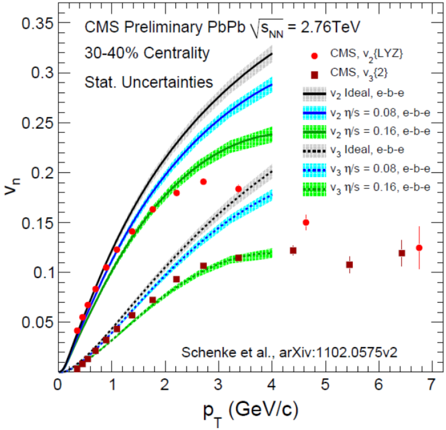
with $\alpha \in [0, 1]$.





below $2\,\mathrm{GeV}$: weak dependence

LHC $v_2(p_T)$ and $v_3(p_T)$



No perfect agreement. But not a lot of tuning.
This is $\langle v_n \rangle$, experiment is not.

Experimental data: The CMS collaboration 2011
B. Schenke, S. Jeon, C. Gale, Phys. Lett. B702, 59-63 (2011)