Initial state fluctuations in heavy-ion collisions from SPS to LHC

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Outline

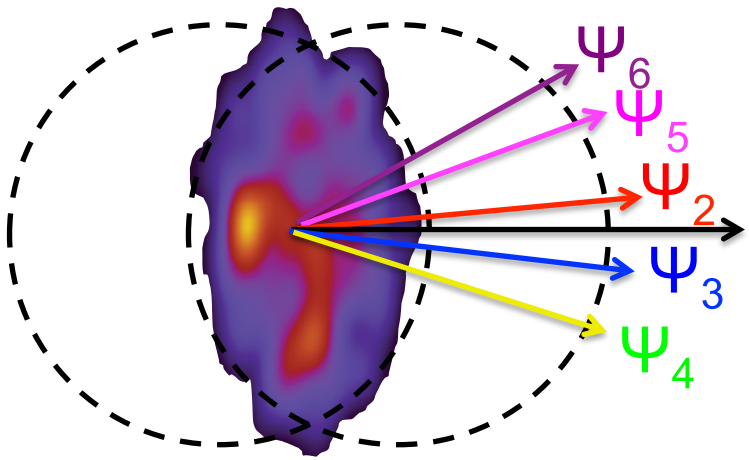
- Azimuthal anisotropy
- Initial-state fluctuations (ISF) and higher order Fourier harmonics
- Triangular flow at SPS, RHIC and LHC energies
- Collectivity over a wide p_T range in PbPb collisions
- Collectivity in small pPb and smallest pp collision systems
- ISF on sub-nucleonic level
- Factorization breaking – mechanism
  - – p_T dependent event plane
  - – η dependent event plane
- PCA method in flow physics – leading and sub-leading flow modes
- The PCA analysis in pPb and PbPb collisions at the LHC energy
- The PCA analysis in PbPb collisions from HYDJET++
- Conclusions

Role of initial state fluctuations (ISF) on anisotropy

Anisotropy harmonics with order higher than 2

geometry – v_2
ISF – v_3

Phys.Rev. C89 (2014) 044906
(*arXiv:1310.8651*)

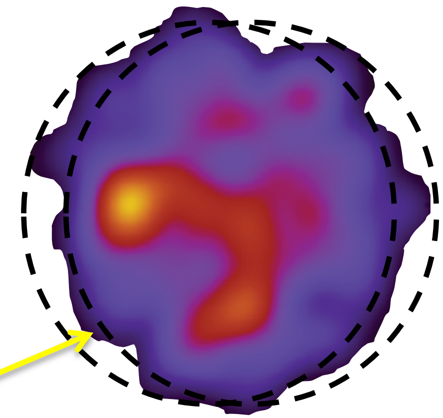


v_2, v_3, v_4, v_5 and v_6
using multiple methods

Simple, circle-like geometry does not describe the formed system precisely enough

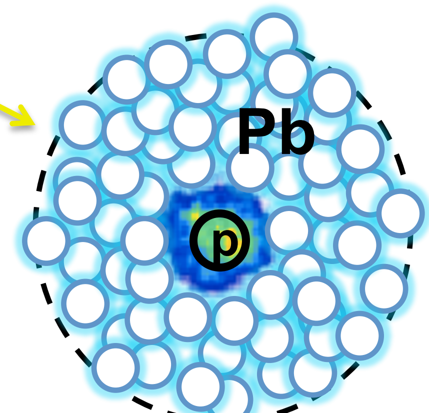
initial-state fluctuations dominates

Ultra-central collisions



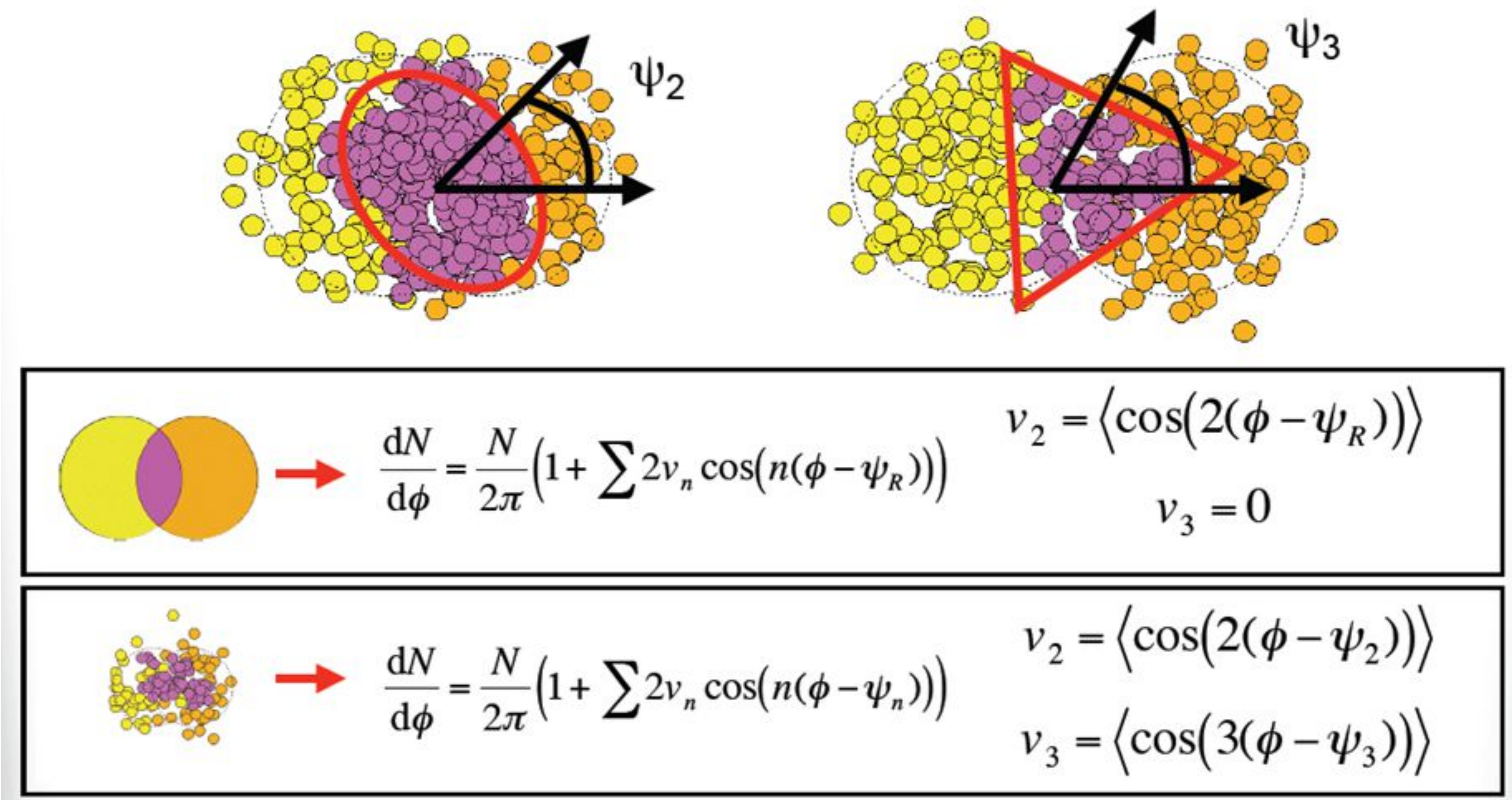
JHEP 1402 (2014) 088
(*arXiv:1312.1845*)

Asymmetric (pPb) high-multiplicity collisions



Phys.Lett. B724 (2013) 213
(*arXiv:1305.0609*)

Triangular flow – one of higher order Fourier harmonics



B. Alver and G. Roland
PRC 81(2010) 054905

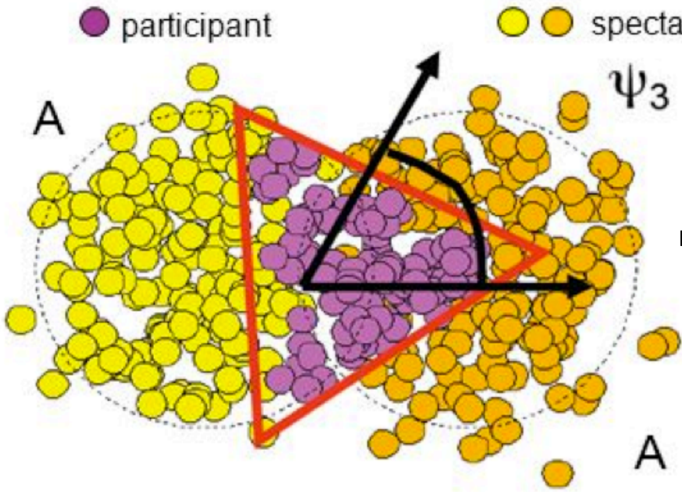
The triangular initial shape ➔ triangular hydrodynamic flow

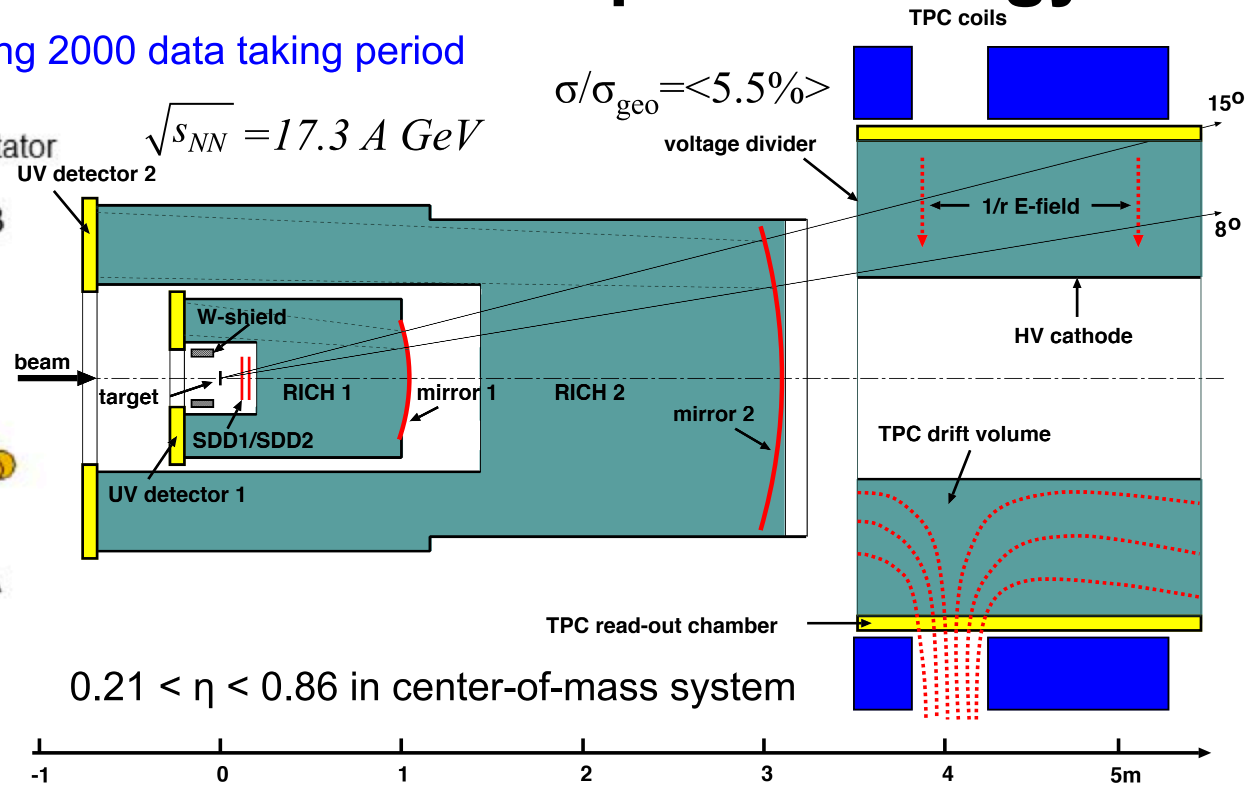
Triangular flow in PbAu at the top SPS energy

≈30M PbAu collisions collected during 2000 data taking period

$\sqrt{s_{NN}} = 17.3\ A\ GeV$

$\sigma/\sigma_{geo} = <5.5\%>$

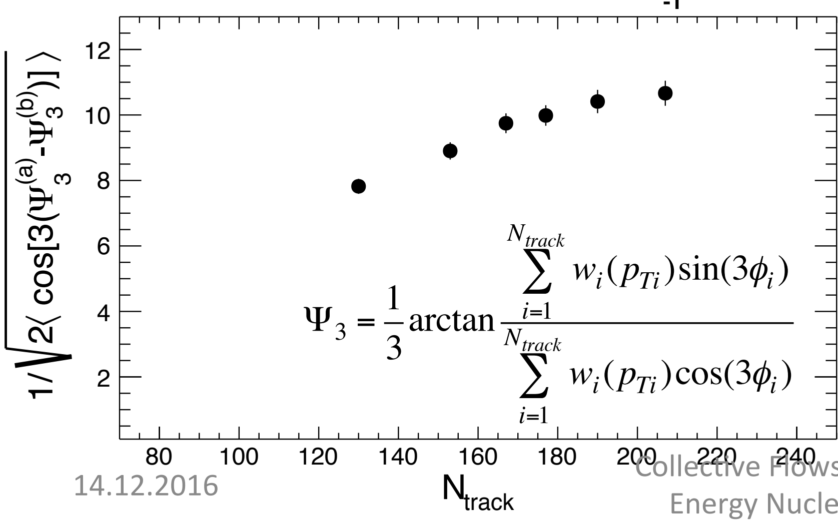




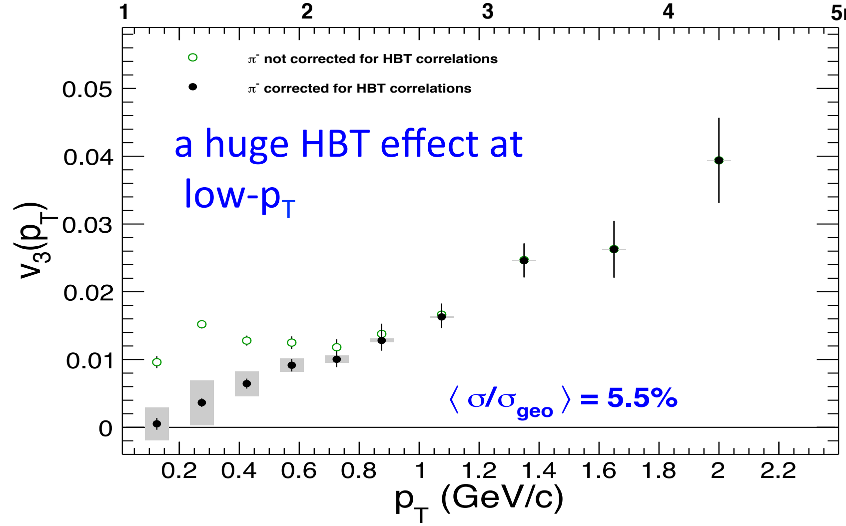
EP method is used

0.21 < η < 0.86 in center-of-mass system

Nucl.Phys.A957 (2017) 99



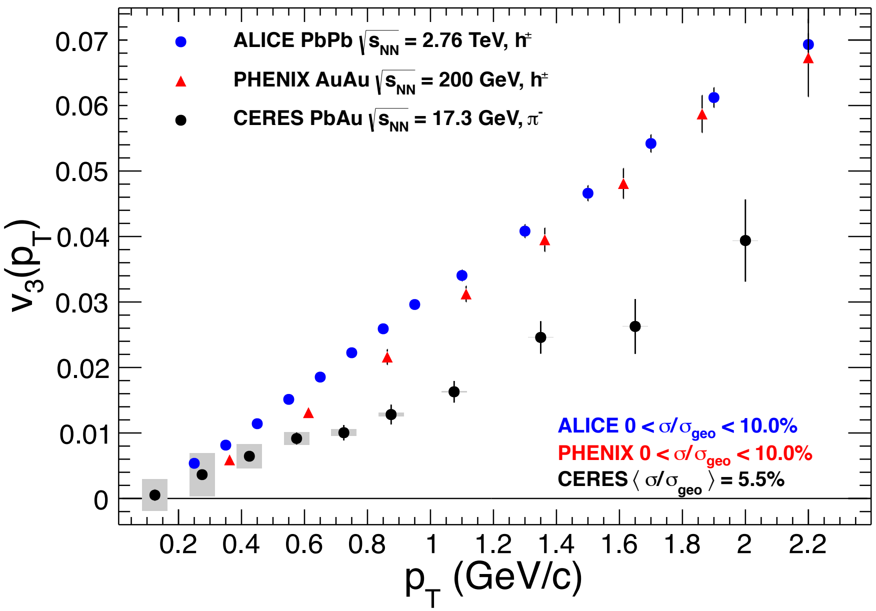
$$\Psi_3 = \frac{1}{3}\arctan\frac{\sum_{i=1}^{N_{track}} w_i(p_{Ti})\sin(3\phi_i)}{\sum_{i=1}^{N_{track}} w_i(p_{Ti})\cos(3\phi_i)}$$

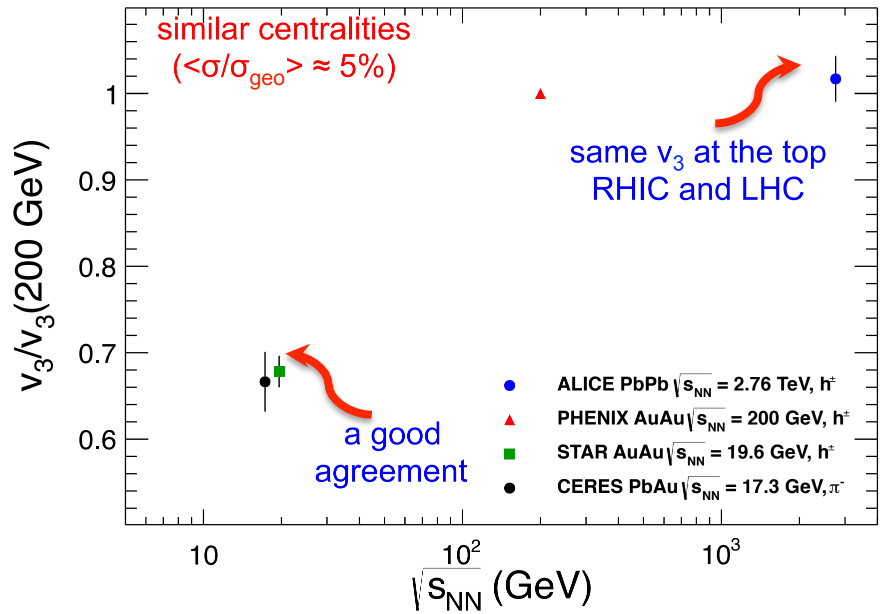


v_3 *vs* p_T – comparison to other experiments

- First p_T dependent measurement of the triangular flow at the top SPS energy
- Top RHIC and LHC energy gives very similar v_3 magnitudes
- The v_3 at the top SPS energy is about half of those at top RHIC and LHC
- Linear increase but with different slopes
- RHIC 19.6 GeV is quite close to the top SPS energy of 17.3 GeV

Nucl.Phys.A957 (2017) 99





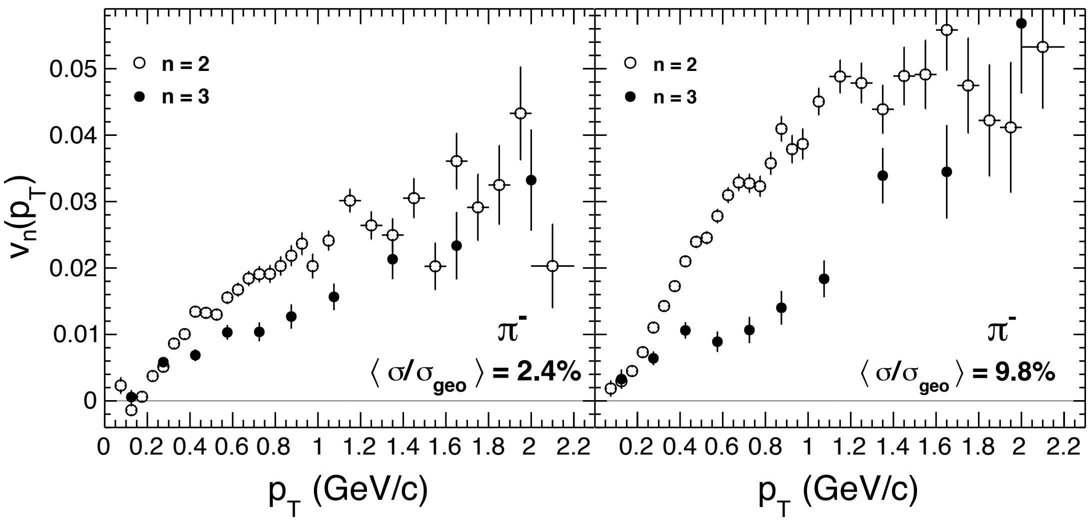
- As a referent level is taken v_3 value at the top RHIC energy
- **v_3 values integrated over 0.3 < p_T < 2.1 GeV/c !**
- Note limited p_T range restricted to the CERES acceptance
- ALICE uses large |Δη| gaps; No option to include |Δη| gap at CERES
- Jet yield is for more than one order of magnitude smaller at SPS

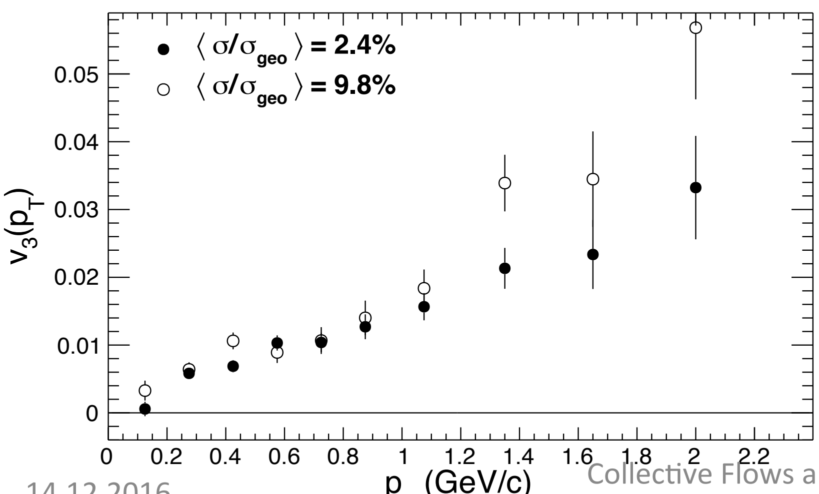
PHENIX
PRL 107 (2011) 252301
ALICE
PLB 719 (2013) 18
STAR
PRL 116 (2016) 112302

v_3 in comparison with v_2

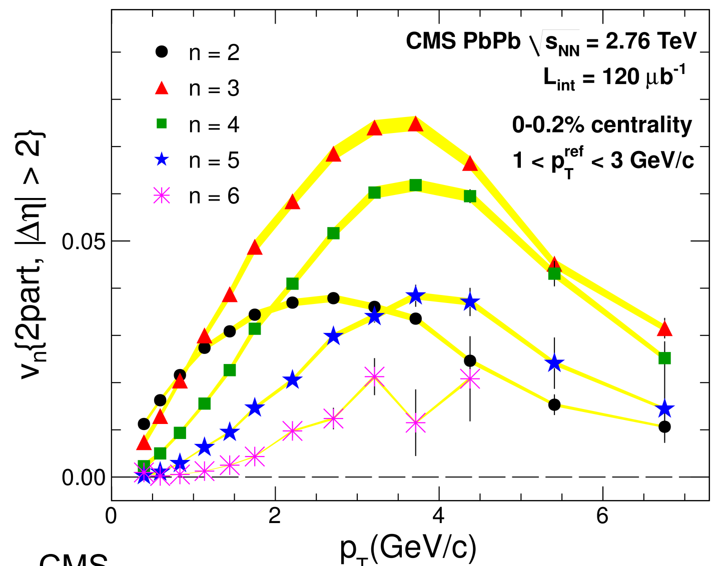
- Elliptic flow reflects the initial anisotropy and thus depends strongly on centrality
- Triangular flow comes from the ISF and weakly depends on centrality
- The different centrality behavior between v_2 and v_3
- For very central collisions ($\langle\sigma/\sigma_{geo}\rangle$ = 2.4%), v_3 becomes close to the v_2

Nucl.Phys.A957 (2017) 99





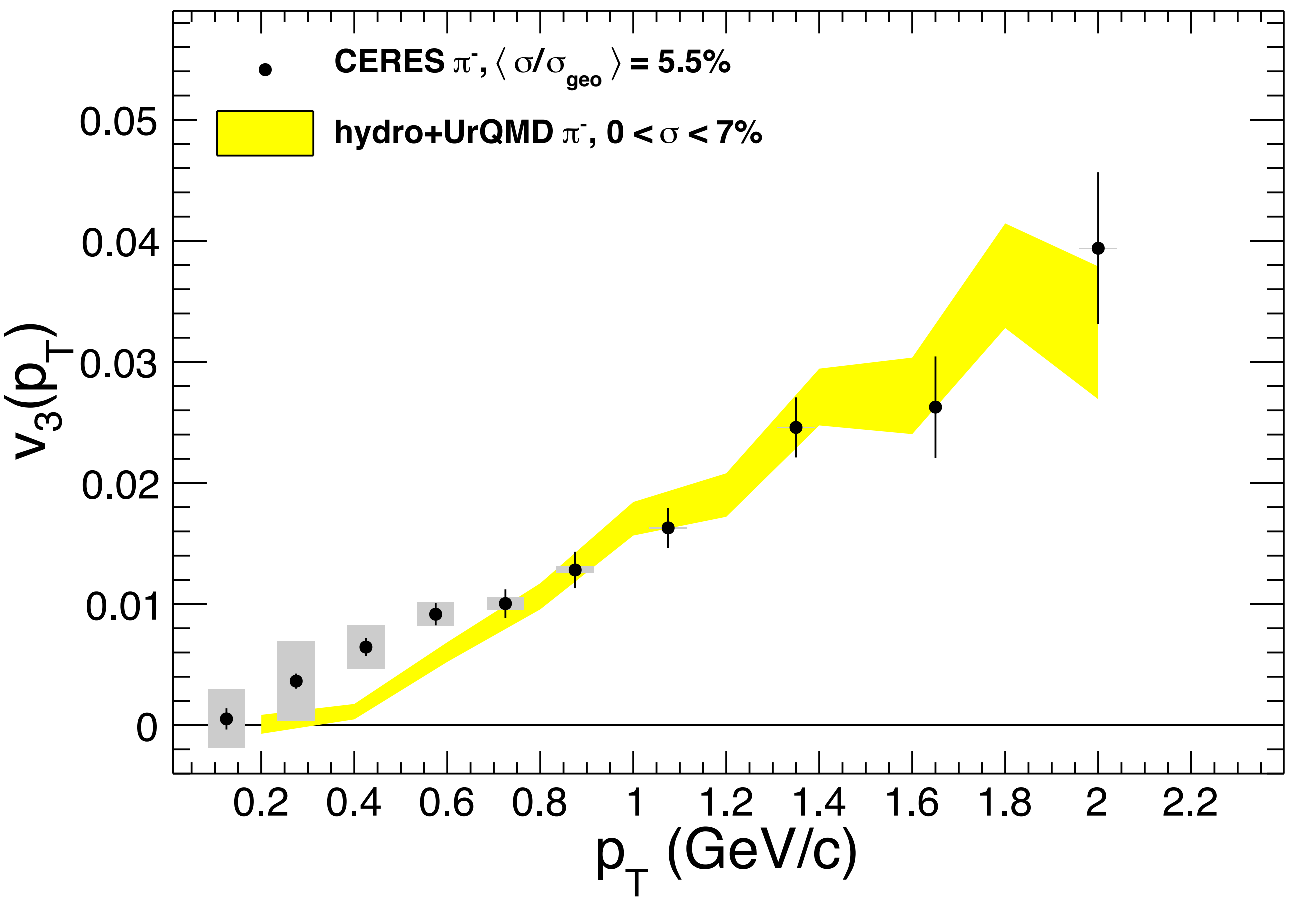
- Triangular flow is dominant anisotropy for ultra-central collisions at the LHC energies



CMS
JHEP 1402 (2014) 088

Comparision with hydro+UrQMD predictions

- Relativistic hydrodynamics + transport models (hybrid models)
  - vHLLE viscous hydrosolver + UrQMD hadron cascade (I. Karpenko, P. Huovinen, H. Petersen and M. Bleicher **PRC 91 (2015) 064901)**
- The model predictions for hadrons within $0.2 < p_T < 2.0$ GeV/c and $-1 < \eta < 1$
- Cerentrality samples roughly correspond to the experimental ones

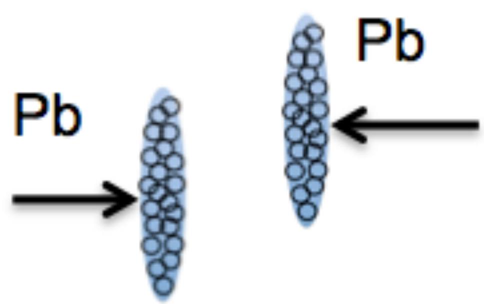


Nucl.Phys.A957 (2017) 99

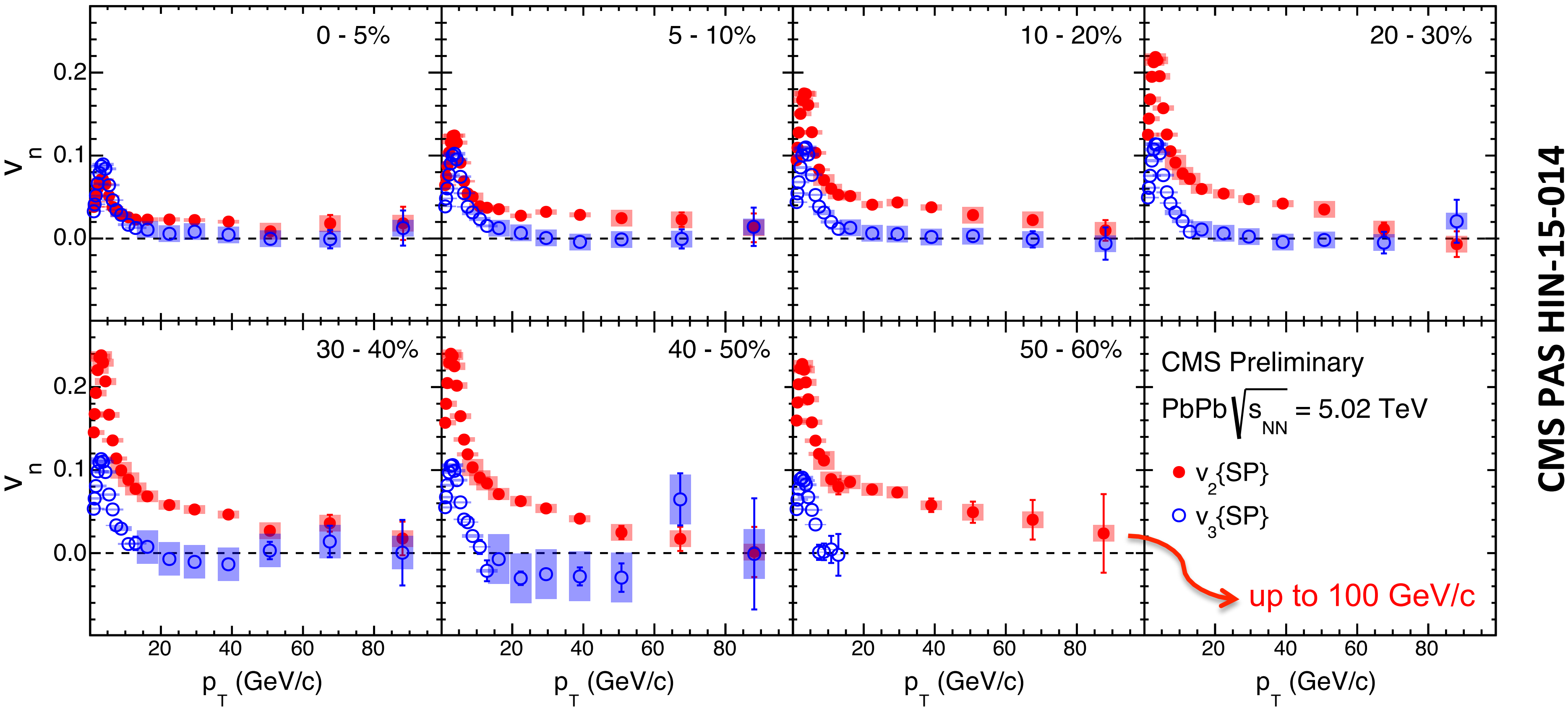
- Particlization at constant energy density 0.5 GeV fm^3
- Kinetic and chemical freeze-out are dynamical

- Model predictions in a very good agreement with the CERES results
- A small disagreement appears at low-p_T

Collectivity over a wide p_T range in PbPb

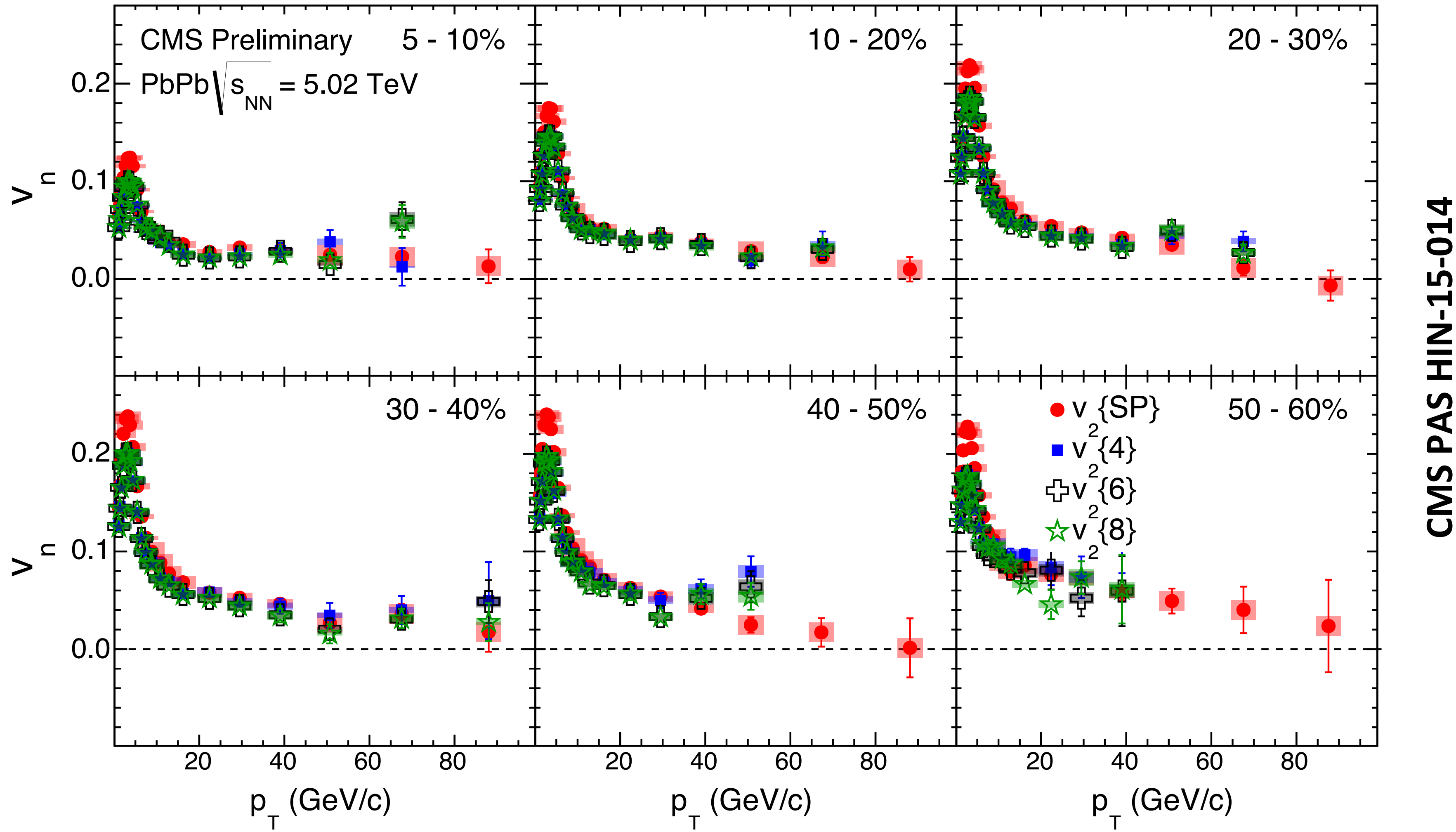


$v_n\{SP\}$ over a wide p_T range



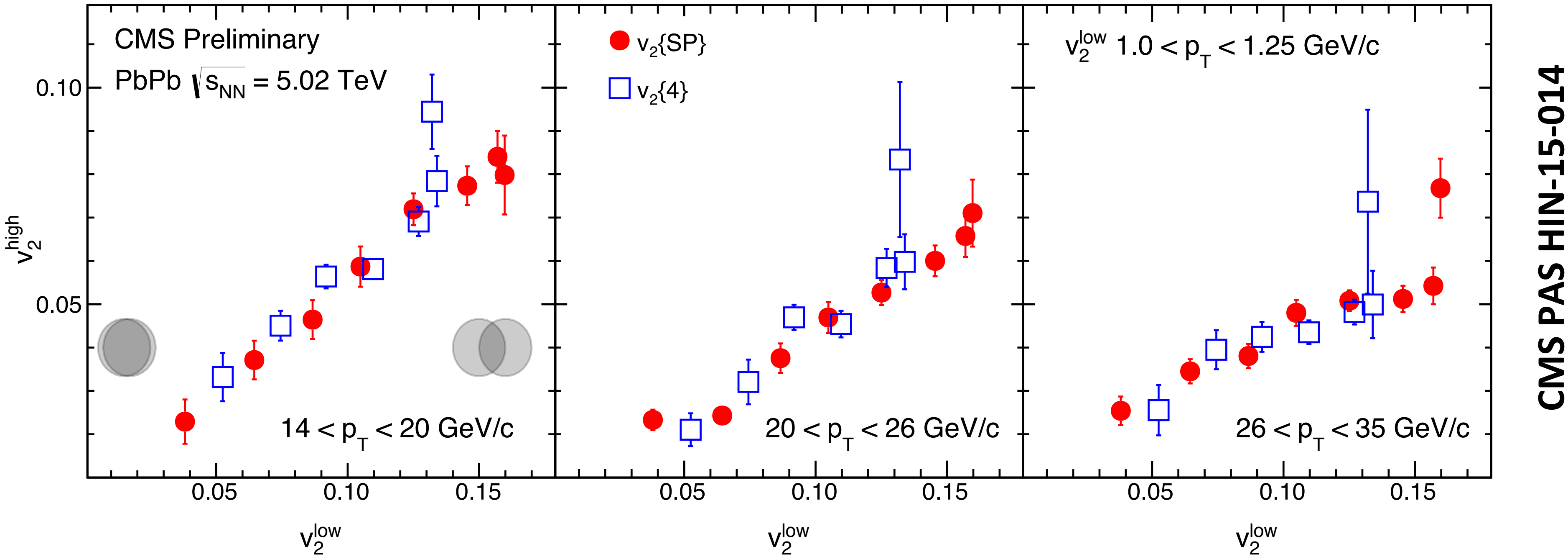
- low-p_T - hydrodynamic flow (v_2 – geometry, v_3 – ISF on nucleonic level)
- v_2 non-zero up to very high p_T
- high-p_T - may reflect the path-length dependence of parton energy loss
- v_2 is complementary to R_{AA} measurements
- v_3 mainly consistent with zero at high-p_T

Collectivity over a wide p_T range



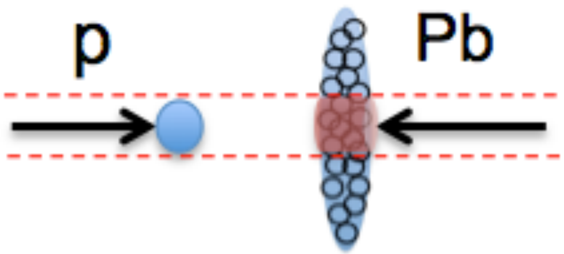
- low-p_T – ratio $v_2\{2k\}/v_2\{SP\} \approx 0.8$ and $v_2\{4\} \approx v_2\{6\} \approx v_2\{8\}$ ← hydrodynamics
- high-p_T – SP and multi-particle correlation tend to converge to the same value
- $v_2\{4\} \approx v_2\{6\} \approx v_2\{8\} \neq 0$ ← collectivity (likely to be related to jet quenching)

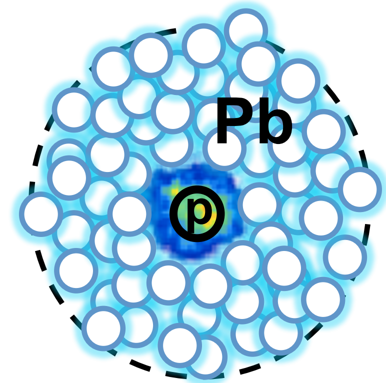
Collectivity over a wide p_T and centrality range



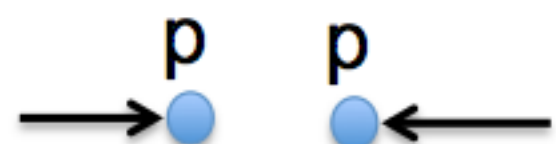
Soft and hard correlation

- Correlation between low-p_T v_2 and high-p_T v_2 over a wide centrality range
- Each point represents one centrality bin
- Strong correlation may indicate that low-p_T v_2 and high-p_T v_2 may have the same origin
- Within uncertainties, slopes between v_2{SP} and v_2{2k} are compatible
- Extrapolations compatible to 0 within uncertainties

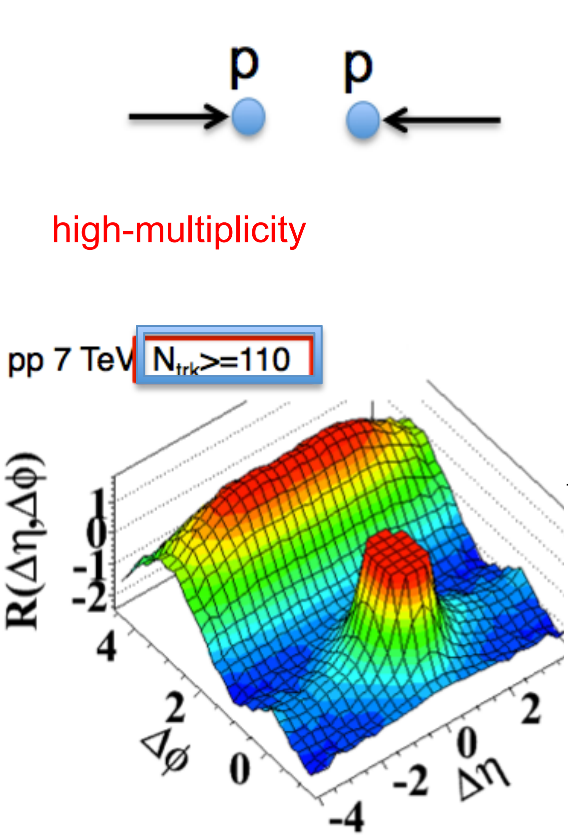




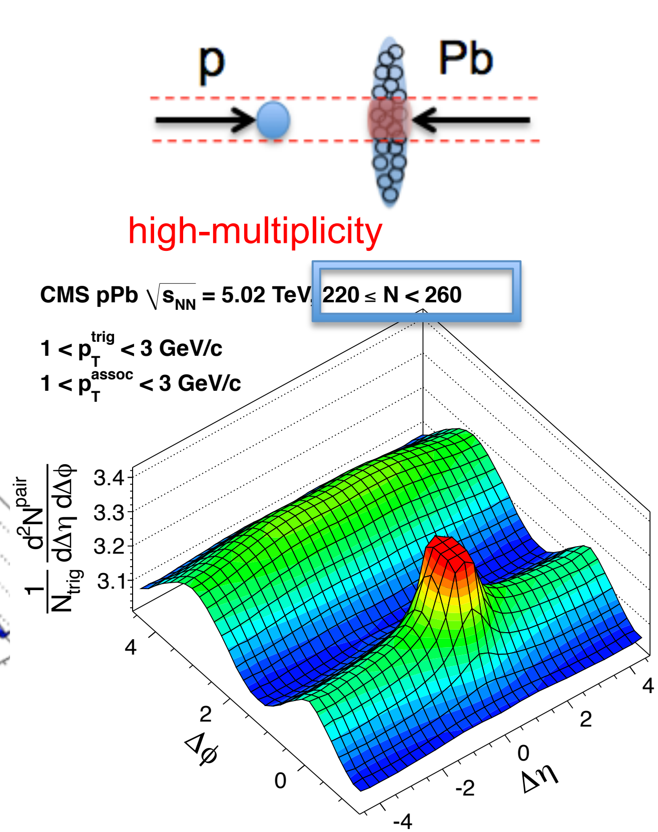
Collectivity in small pPb and smallest pp systems?



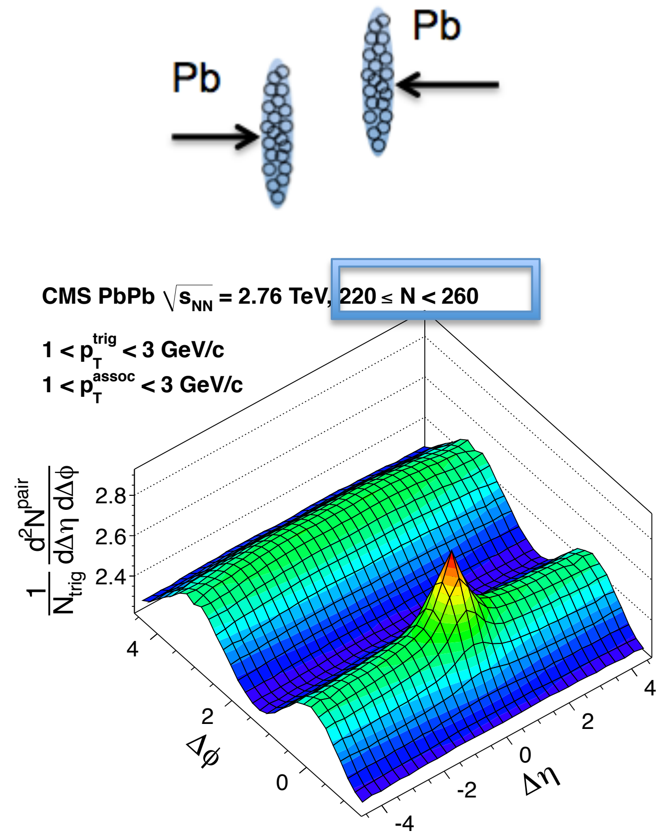
The ridge seen in all colliding systems at LHC



JHEP 09 (2010) 091



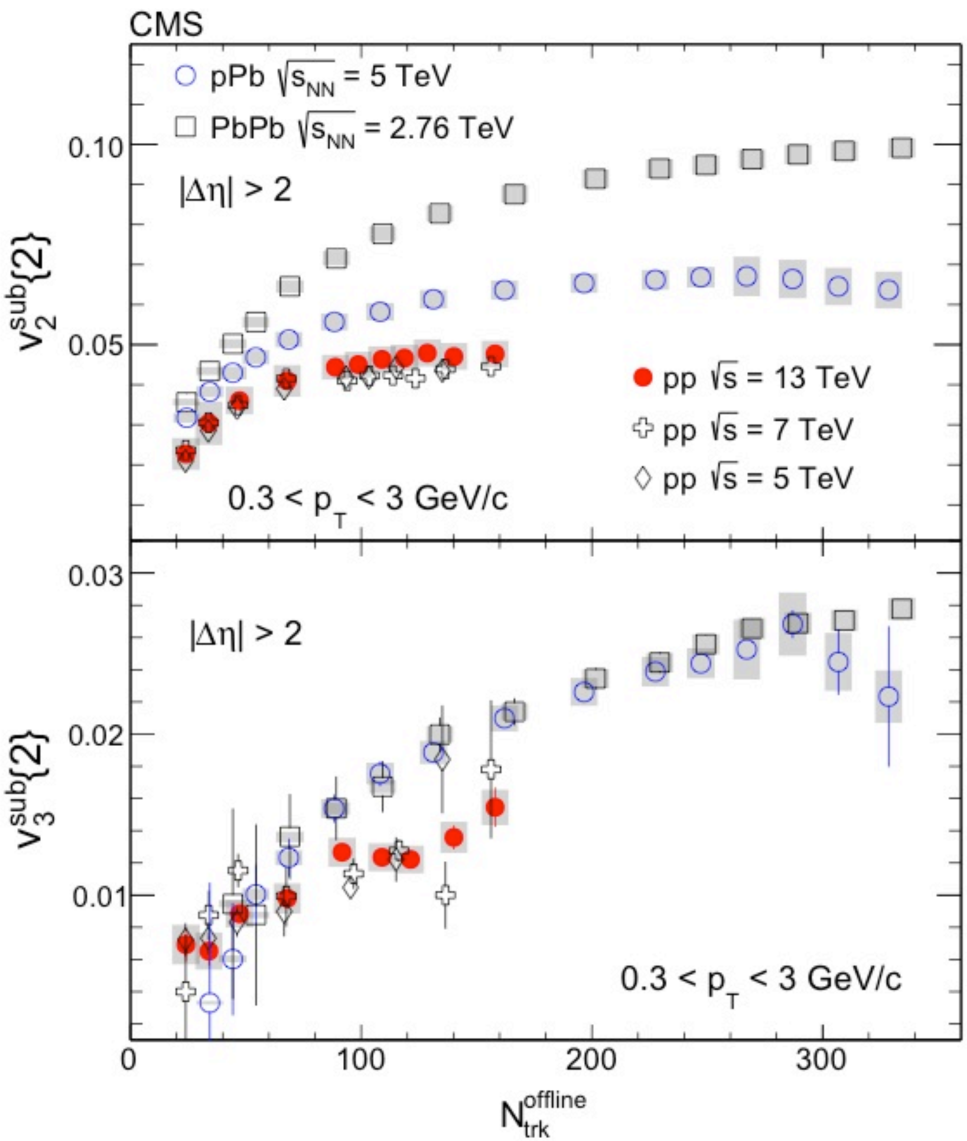
PLB 718 (2013) 795



PLB 724 (2013) 213

- Does the ridge in *pp* and *pPb* collisions originate from hydrodynamics flow like in PbPb collisions or it is connected with color-glass condensate (CGC)

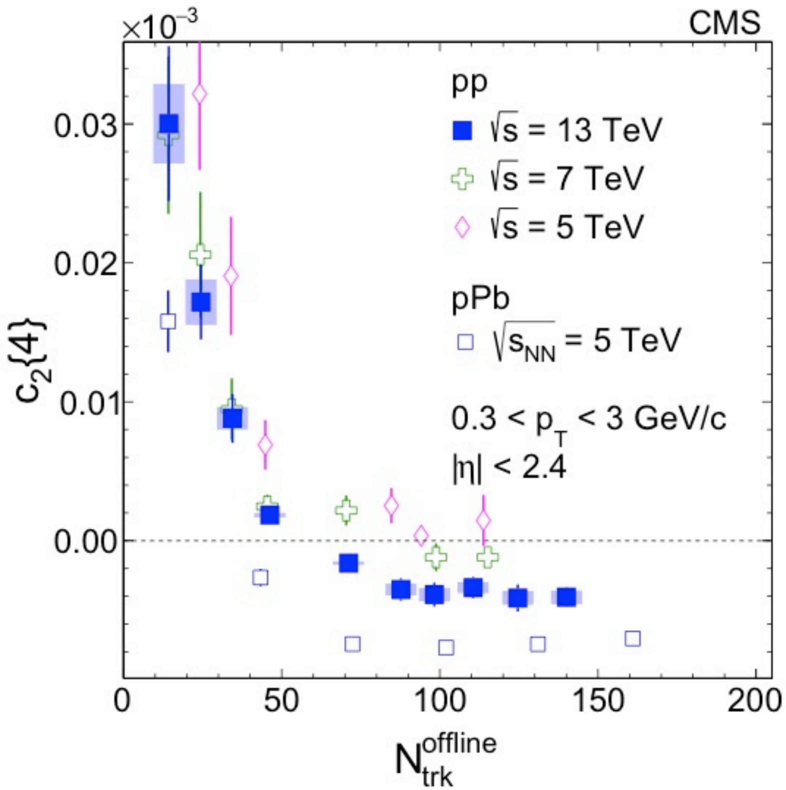
$v_2\{2\}$ and $v_3\{2\}$ in pp at different collision energies

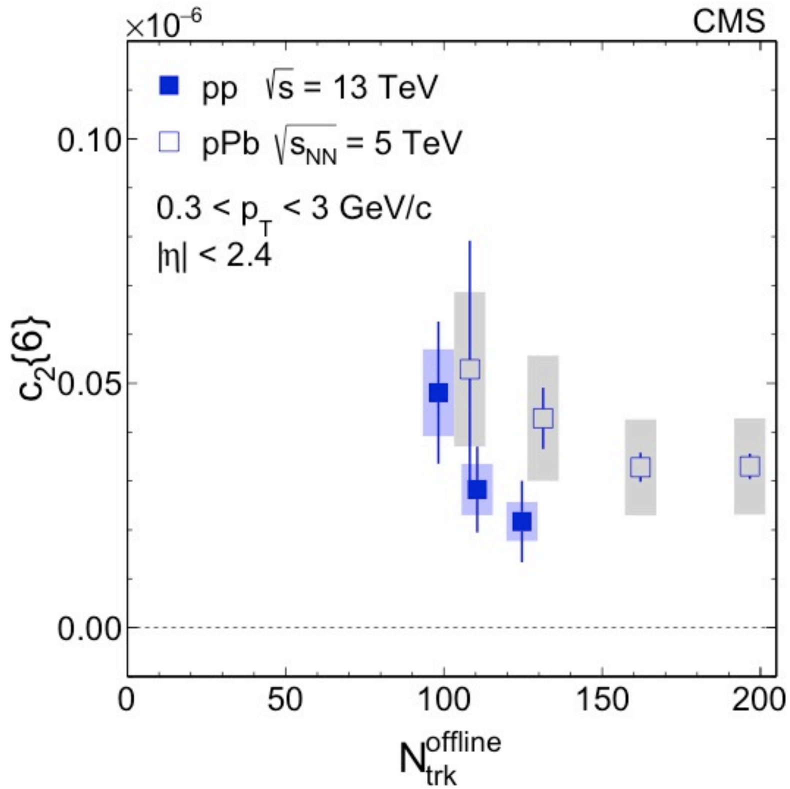


arXiv:1606.16198, CMS-HIN-16-010

- There is no or a very weak energy dependence of v_2 in pp collisions
- $v_2\{2\}$ in pp collisions shows a similar pattern as the one seen in pPb collisions (gets flat at the highest multiplicities)
- The $v_2\{2\}$ magnitude is ordered: it is highest in PbPb, gets smaller in pPb and become smallest in pp collisions
- In difference of the v_2, the v_3 magnitude is comparable to those in pPb and PbPb collisions
- At low multiplicities, the systematic uncertainties are large for all the three systems
- At high multiplicities, v_3 in pp increases at a slower rate than in pPb and PbPb systems

$c_2\{4\}$ and $c_2\{6\}$ in pp at different collision energies

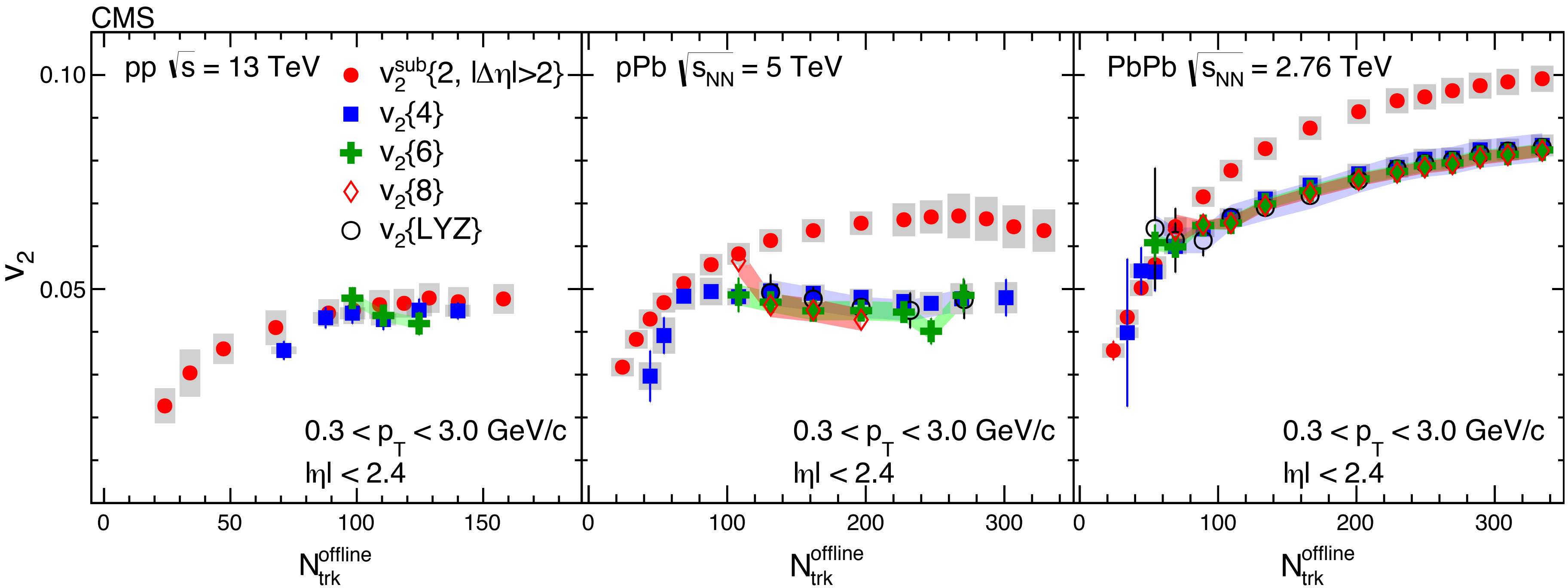




arXiv:1606.06198, CMS-HIN-16-010

- Multi-particle correlations are used to reduce jet correlations from the away side and to explore collective nature of the long-range correlations in pp. $v_2\{4\}$ and $v_2\{6\}$ are extracted
- Clear negative $c_2\{4\}$ at high multiplicities in pp at 13 TeV is seen $v_n\{4\} = \sqrt[4]{-c_n\{4\}}$
- and positive $c_2\{6\}$ $v_n\{6\} = \sqrt[6]{\frac{1}{4} c_n\{6\}}$
- Statistical limitations

v_2 in pp compared to pPb and PbPb results

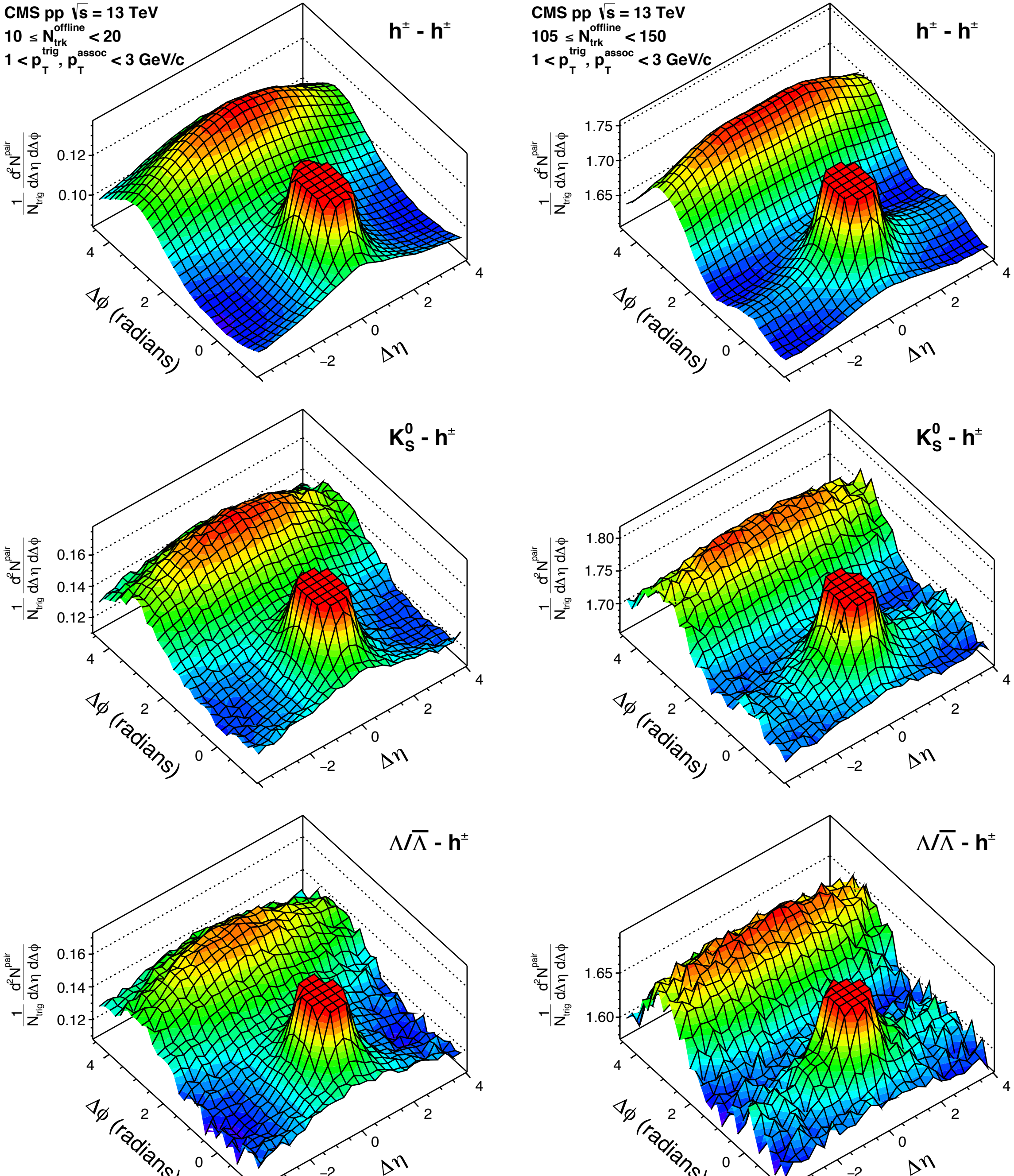


$v_2\{2\} \geq v_2\{4\} \approx v_2\{6\}$

collectivity!

arXiv:1606.06198, CMS-HIN-16-010

- Elliptic flow in pp measured using 2- and multi-particle correlations – compared to pPb and PbPb results
- $v_2\{2\}/v_2\{4\}(pp) \leq v_2\{2\}/v_2\{4\}(pPb)$ ← related to initial-state (IS) fluctuations
- smaller $v_2\{2\}/v_2\{4\}$ ← less IS fluctuating sources (PRL **112** (2014) 082301)



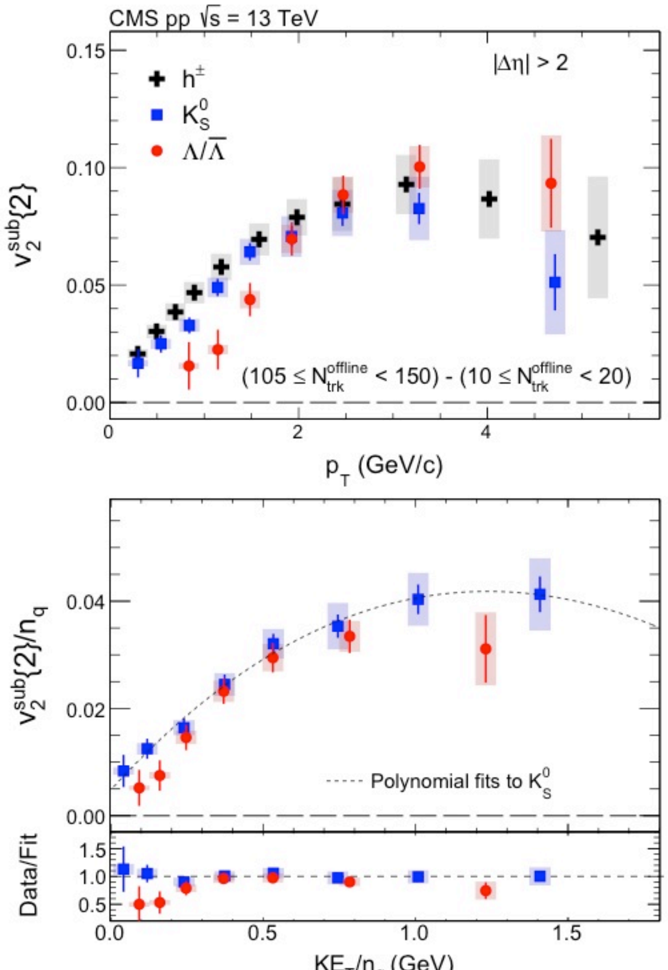
2D 2-particle corr. function in low- and high-multiplicity pp

arXiv:1606.06198, CMS-HIN-16-010

- charged-charged or charged-strange (K_S^0 and $\Lambda/\bar{\Lambda}$) particles
- particles are correlated within given multiplicity bin
- The ridge, at $\Delta\phi \approx 0$ and elongated at $\Delta\eta$, is seen only in high-multiplicity pp events
- The ridge is present not only for charged, but also for strange particles
- What is the origin of the ridge in the smallest pp system?

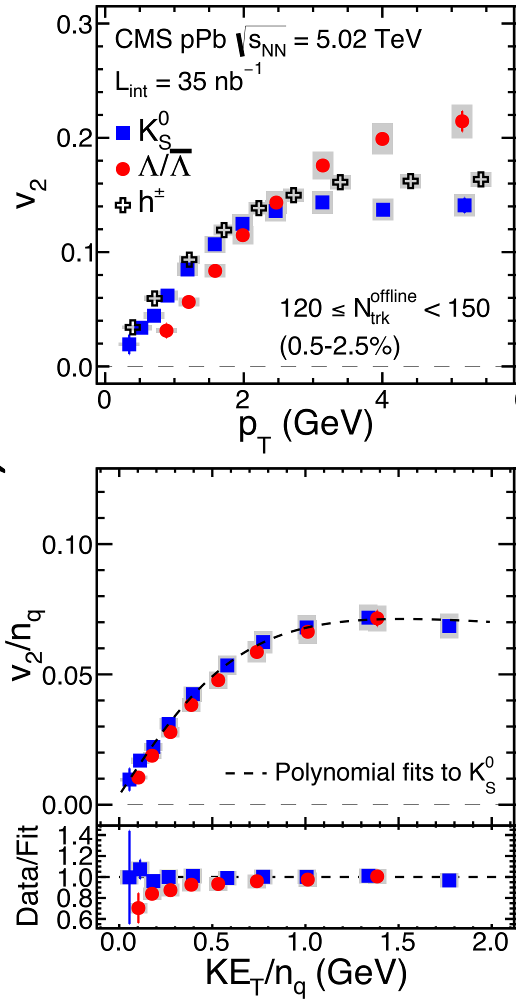
collective behavior in pp?

NCQ scaled v_2 in pp collisions compared to pPb and PbPb

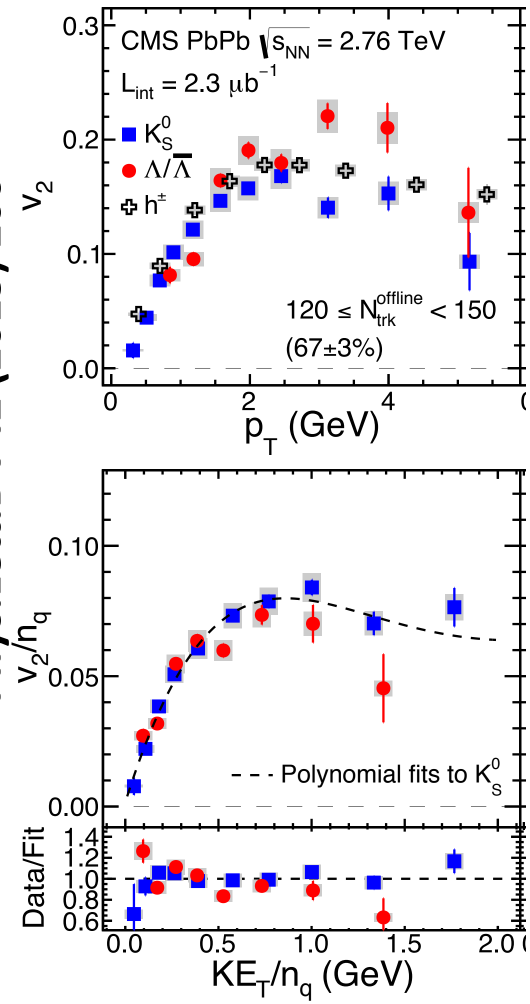


collectivity!

arXiv:1606.06198, CMS-HIN-16-010



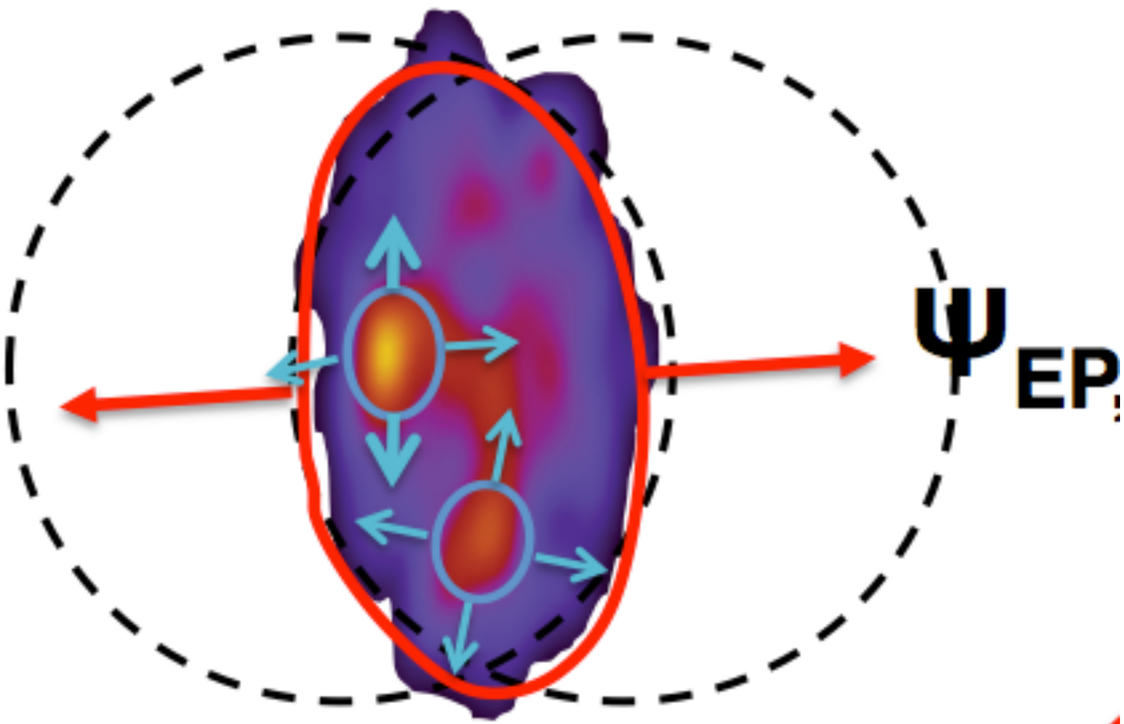
Phys.Lett.B 742 (2015) 200



Phys.Lett.B 742 (2015) 200

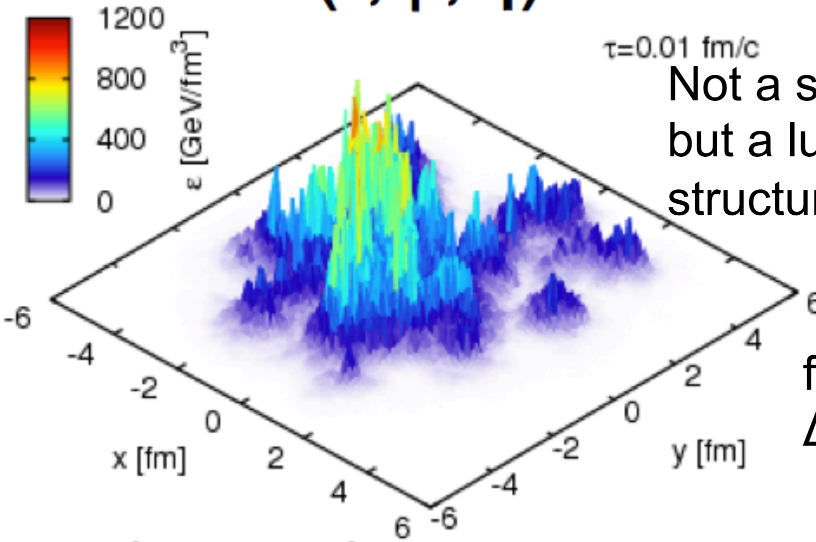
- ❖ Significant magnitude of the NCQ scaled v_2 in *pp,* comparable to the ones seen in pPb and PbPb collisions

Factorization breaking –
p_T dependent event plane fluctuations



Initial-state inhomogeneity

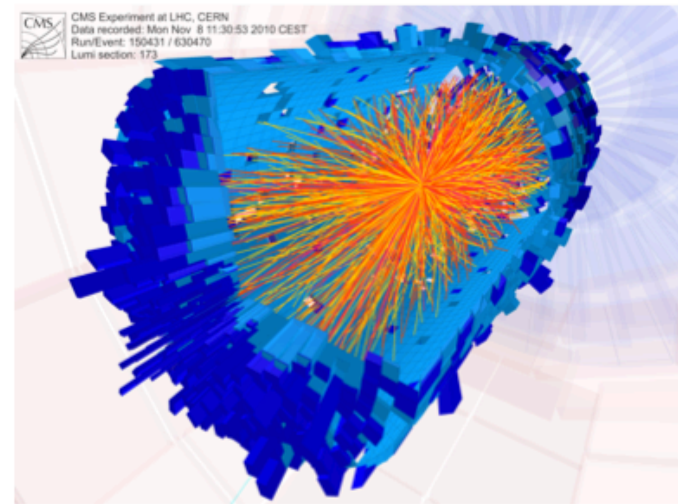
Initial state $\varepsilon(r,\varphi,\eta)$ —— EbE hydro. ——→ Final state $f(p_T,\varphi,\eta)$



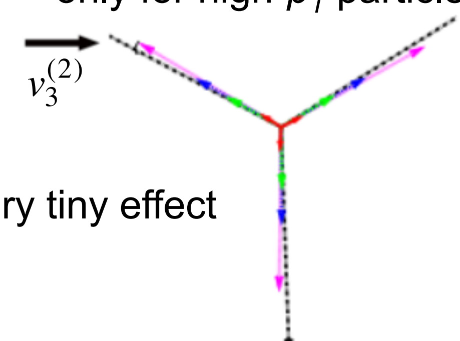
Not a smooth but a lumpy structure

fluctuations $\Delta\varepsilon(r,\phi,\eta)$

overlap zone in x-y



only for high-p_T particles



a very tiny effect

Example: sub-leading triangular flow

- ✧ The goal is to map initial-state and its fluctuations in 3D
- ✧ Local hotspots perturb the EP of a smooth medium, so $\Psi_n(p_T)$ contains information about initial-state fluctuations Phys.Rev.C **92** (2015) 034911
- ✧ Within hydrodynamics, initial-state fluctuations could appear as (sub-leading) flows

Factorization breaking

- How to connect $v_n(p_T)$ *and* $V_{n\Delta}(p_T)$?
- Usual assumption that EP angle Ψ_n does not depend on p_T leads to factorization

$$V_{n\Delta}(p_{T1},p_{T2}) = \sqrt{V_{n\Delta}(p_{T1},p_{T1})} \times \sqrt{V_{n\Delta}(p_{T2},p_{T2})} = v_n(p_{T1}) \times v_n(p_{T2})$$

- *Gardim et al.*, PRC 87 (2013) 031901 and *Heinz et al.*, PRC 87 (2013) 034913 proposed that not only v_n depends on p_T, but also Ψ_n could depends on p_T due to event-by-event (EbE) fluctuating initial state

- then:

$$V_{n\Delta}(p_{T1},p_{T2}) = \left\langle v_n(p_{T1}) v_n(p_{T2}) \cos\left[n(\Psi_n(p_{T1}) - \Psi_n(p_{T2}))\right]\right\rangle$$
$$\neq \sqrt{V_{n\Delta}(p_{T1},p_{T1})} \times \sqrt{V_{n\Delta}(p_{T2},p_{T2})}$$

even if hydro flow is the only source of the correlation

initial state fluctuations → $\Psi_n(p_T)$ → factorization breaking

Factorization breaking

❖ new observable: $$r_n = \frac{V_{n\Delta}(p_T^{trig}, p_T^{assoc})}{\sqrt{V_{n\Delta}(p_T^{trig}, p_T^{trig})}\sqrt{V_{n\Delta}(p_T^{assoc}, p_T^{assoc})}} =$$

$$\frac{\left\langle v_n(p_T^{trig}) v_n(p_T^{assoc}) \cos\left[n(\Psi_n(p_T^{trig}) - \Psi_n(p_T^{assoc})) \right] \right\rangle}{\sqrt{v_n^2(p_T^{trig}) v_n^2(p_T^{assoc})}} = \left\{ \begin{array}{c} 1 \\ <1 \\ >1 \end{array} \right\} \begin{array}{l} \text{fact. holds} \\ \text{fact. breaks} \\ \text{non-flow} \end{array}$$

❖ Large effect is expected and confirmed in ultra central PbPb collisions
***CMS collaboration**: Studies of azimuthal dihadron correlations in ultra-central PbPb collisions at $\sqrt{s_{NN}}$ = 2.76 TeV , JHEP **1402** (2014) 088*

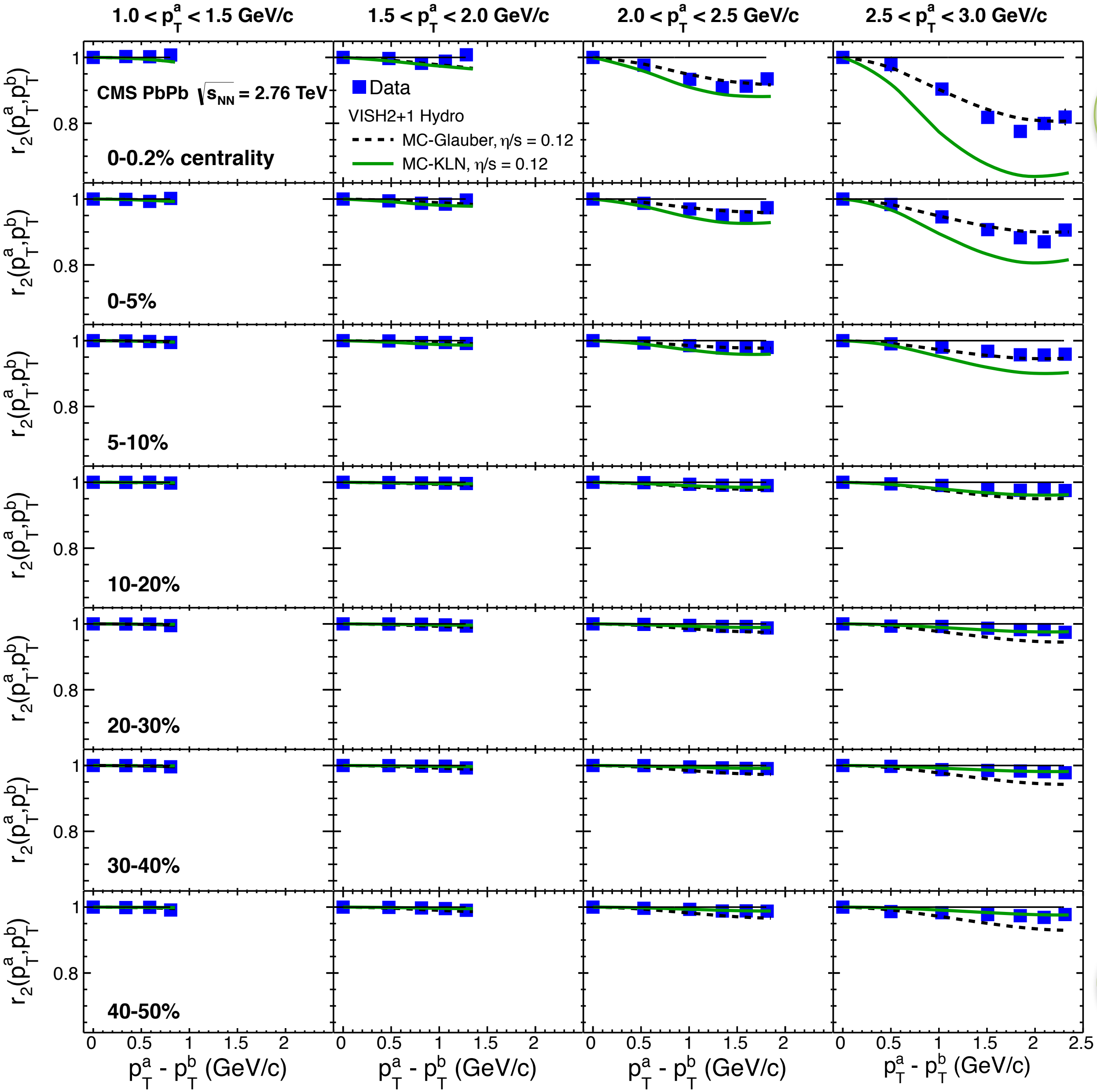
❖ As in pPb collisions initial-state fluctuations play a dominant role could we expect a similar (in size) effect?

❖ Two hydro models with different initial conditions and *η/s* were developed:
- ✧ *Heinz-Shen VISH2+1: PRC **87** (2013) 034913*
- ✧ *Kozlov et. al.: arXiv:1405.3976*

❖ Constraining of initial conditions and *η/s* by comparing to the exp. data?

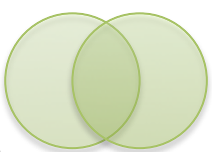
PbPb case

p_T^{trig}



0-0.2%

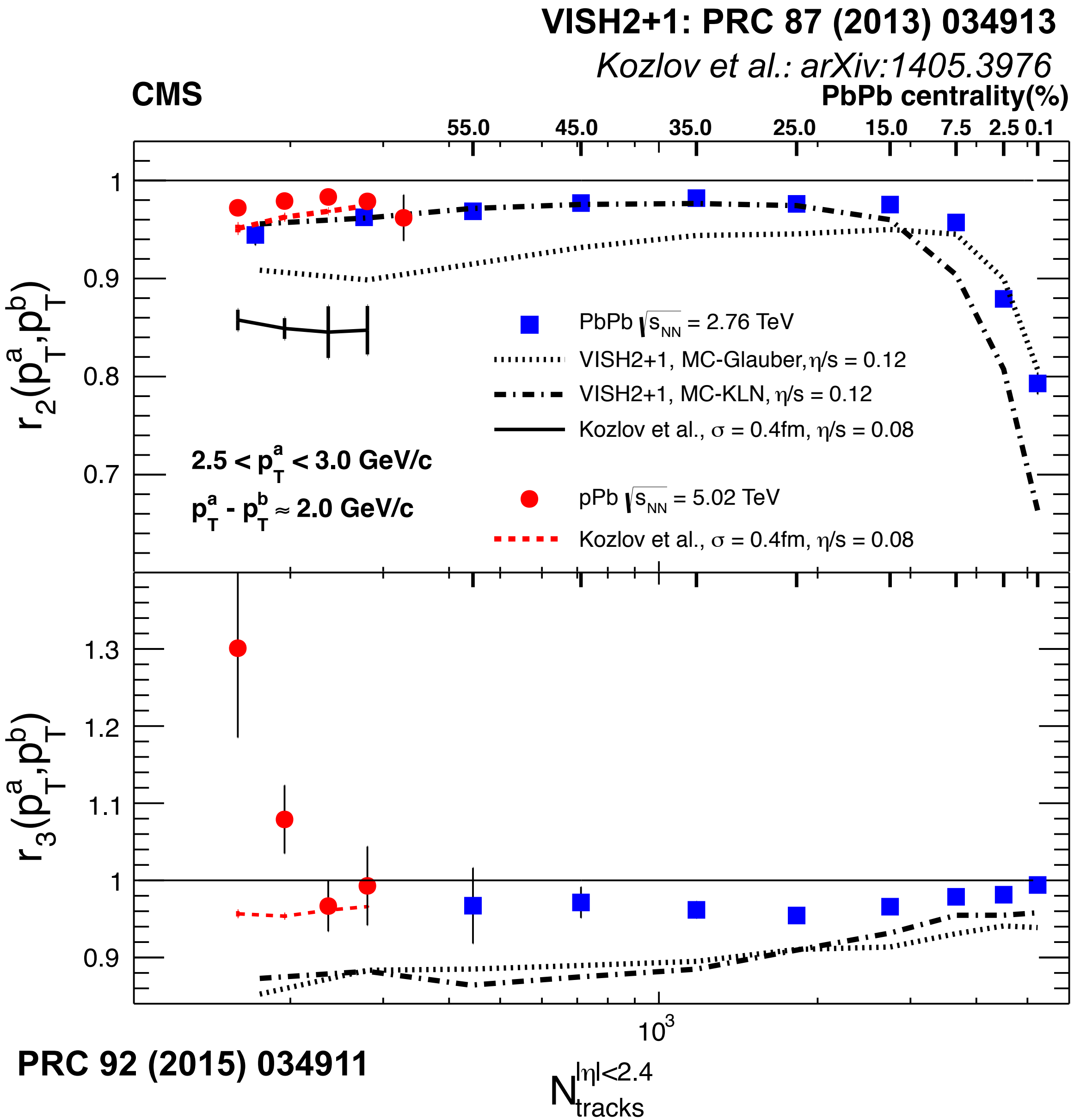
40-50%



- ❖ The effect increases with rise of p_T^{trig} and p_T^{trig}-p_T^{assoc}
- ❖ Approaching the central collisions, the effect dramatically increases achieving value over 20%
- ❖ For semi-central collisions, the effect achieves only a size of 2−3%

PRC 92 (2015) 034911

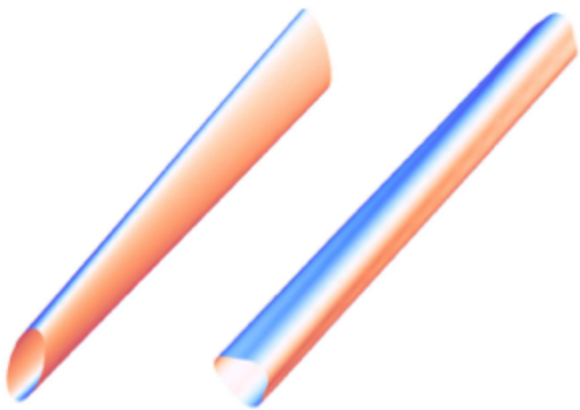
r_n multiplicity dependence at the highest Δp_T



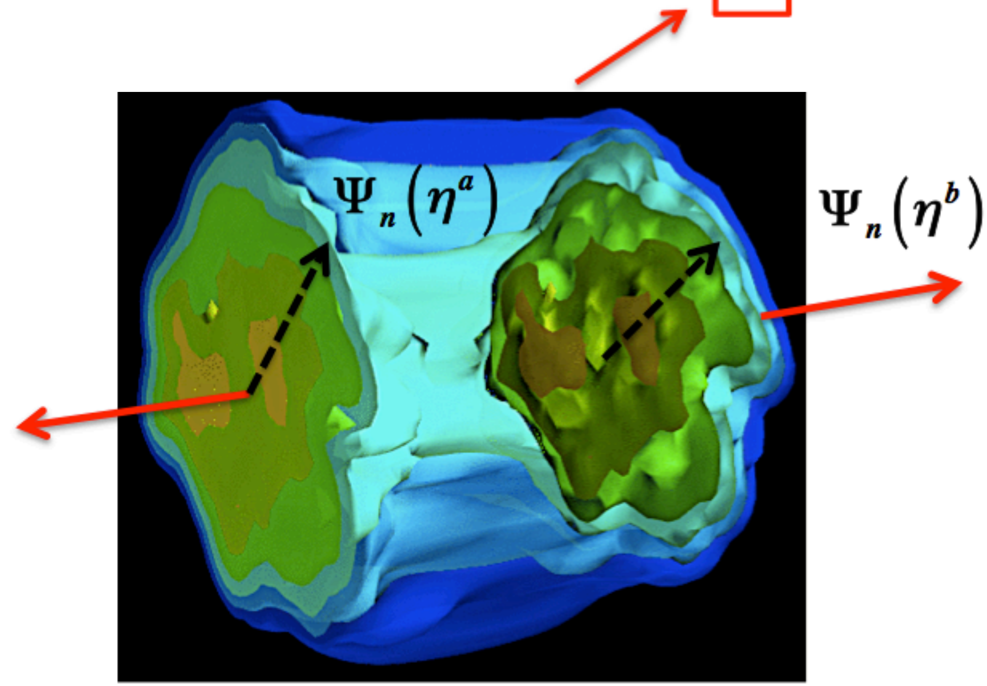
- Dramatic increase at ultra-central PbPb. For small centralities (>5%) ≈ few %
- The r_2 in pPb is a bit smaller than in PbPb
- Strong r_3 multiplicity dependence in pPb, but very weak in PbPb
- A non-flow effect in pPb for the highest p_T^{trig} in lower multiplicities
- VISH2+1 qualitatively describes CMS data
- Kozlov et al. hydro model describes pPb. Gives stronger effect for PbPb and fails for r_3 at lower multiplicity

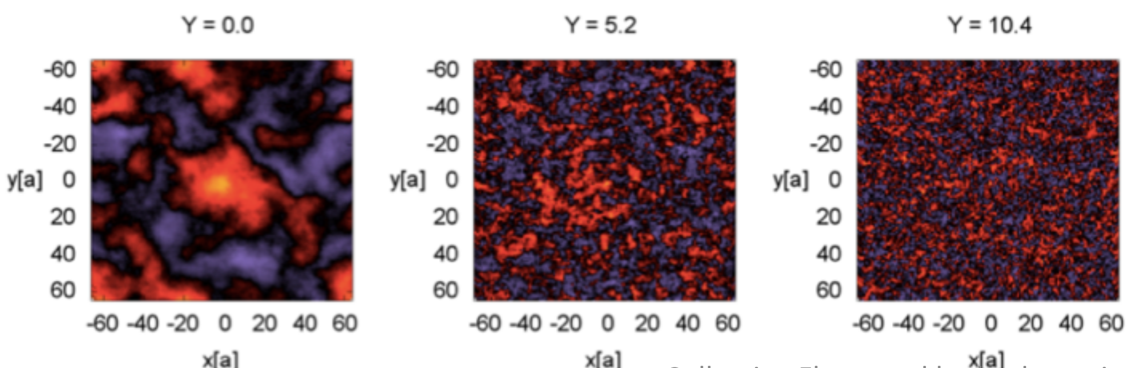
Factorization breaking – η dependence

$$f(p_T,\phi,\eta) \sim 1 + 2\sum_{n=1}^{\infty} v_n(p_T,\eta)\cos\left[n\left(\phi - \Psi_n(p_T,\eta)\right)\right]$$



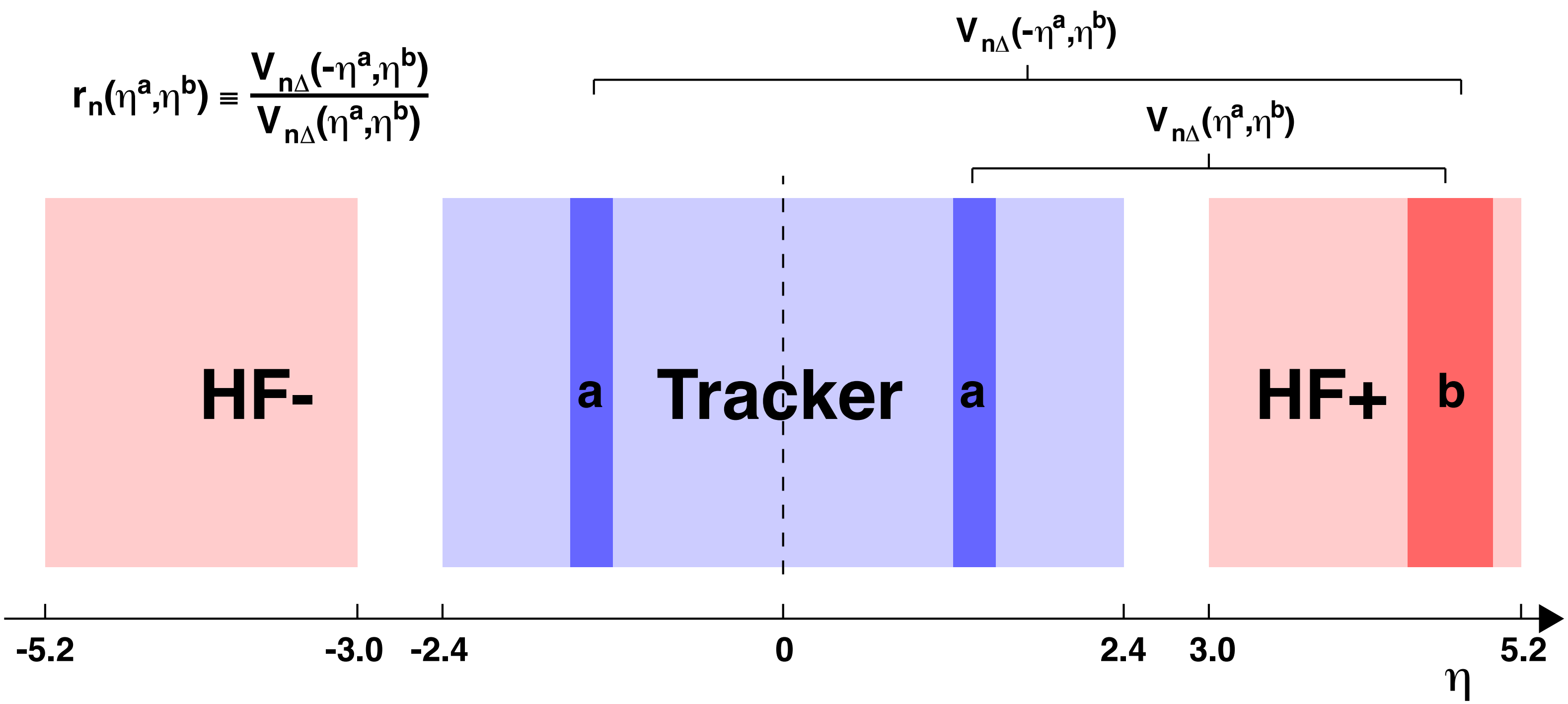
Bozek et al., arXiv: 1011.3354
Global twist





Dumitru et al., arXiv: 1108.4764

η-dependent r_n using Hadronic Forward (HF)



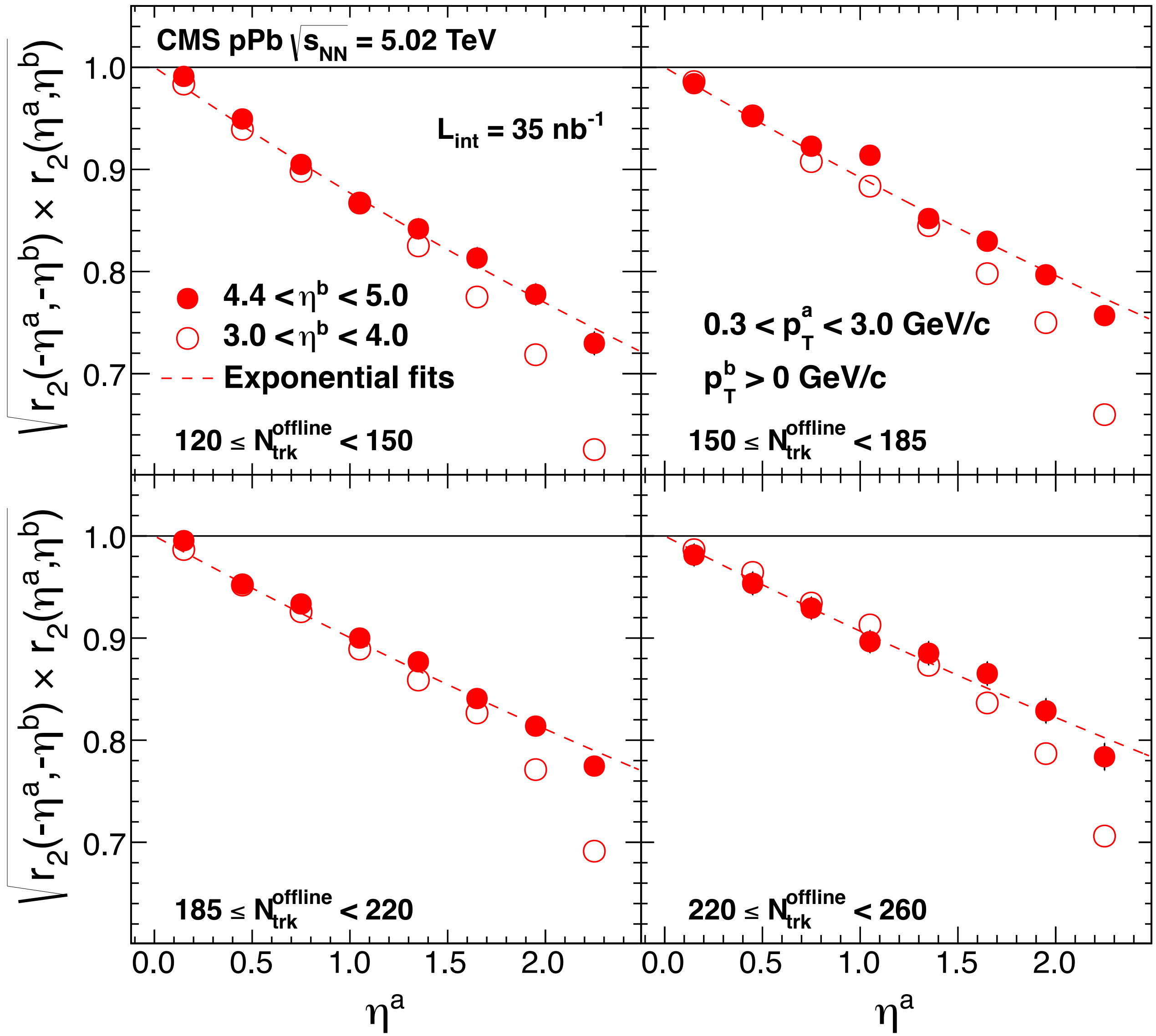
For symmetric collision:

$$r_n\left(\eta^a,\eta^b\right) \approx \frac{\left\langle \cos\left[n\left(\Psi_n\left(-\eta^a\right)-\Psi_n\left(\eta^b\right)\right)\right]\right\rangle}{\left\langle \cos\left[n\left(\Psi_n\left(\eta^a\right)-\Psi_n\left(\eta^b\right)\right)\right]\right\rangle}$$

For asymmetric collision:

$$\sqrt{r_n\left(\eta^a,\eta^b\right)\times r_n\left(-\eta^a,-\eta^b\right)} \approx \sqrt{\frac{\left\langle \cos\left[n\left(\Psi_n\left(-\eta^a\right)-\Psi_n\left(\eta^b\right)\right)\right]\right\rangle}{\left\langle \cos\left[n\left(\Psi_n\left(\eta^a\right)-\Psi_n\left(\eta^b\right)\right)\right]\right\rangle}\frac{\left\langle \cos\left[n\left(\Psi_n\left(\eta^a\right)-\Psi_n\left(-\eta^b\right)\right)\right]\right\rangle}{\left\langle \cos\left[n\left(\Psi_n\left(-\eta^a\right)-\Psi_n\left(-\eta^b\right)\right)\right]\right\rangle}}$$

η-dependent r_n in pPb

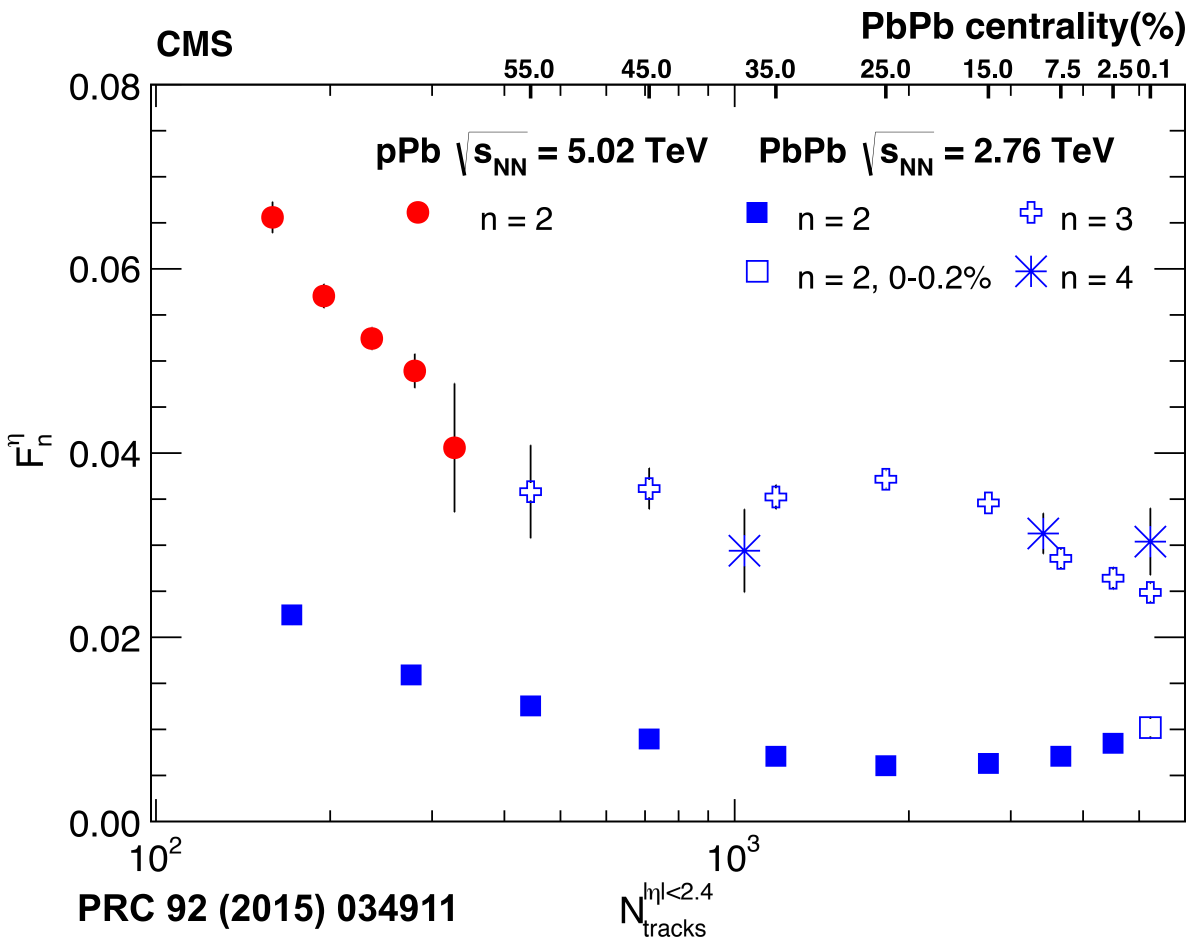


- A significant factorization breakdown in η found in pPb collisions with increase of η^a
- The effect increases approximately linearly with η^a
- Parameterization with F_n^η is purely empirical introduced just to quantify behavior of the data

$$r_n\left(\eta^a,\eta^b\right) \approx e^{-2F_n^\eta \eta^a}$$

PRC 92 (2015) 034911

η-dependent r_n vs multiplicity

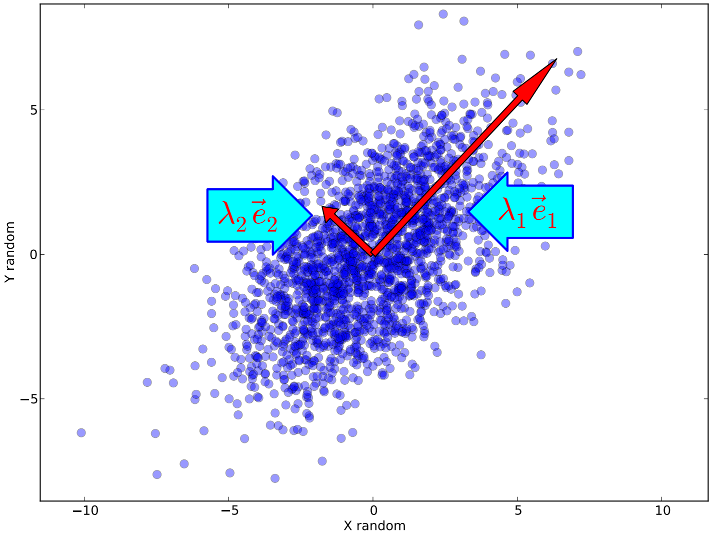


PRC 92 (2015) 034911

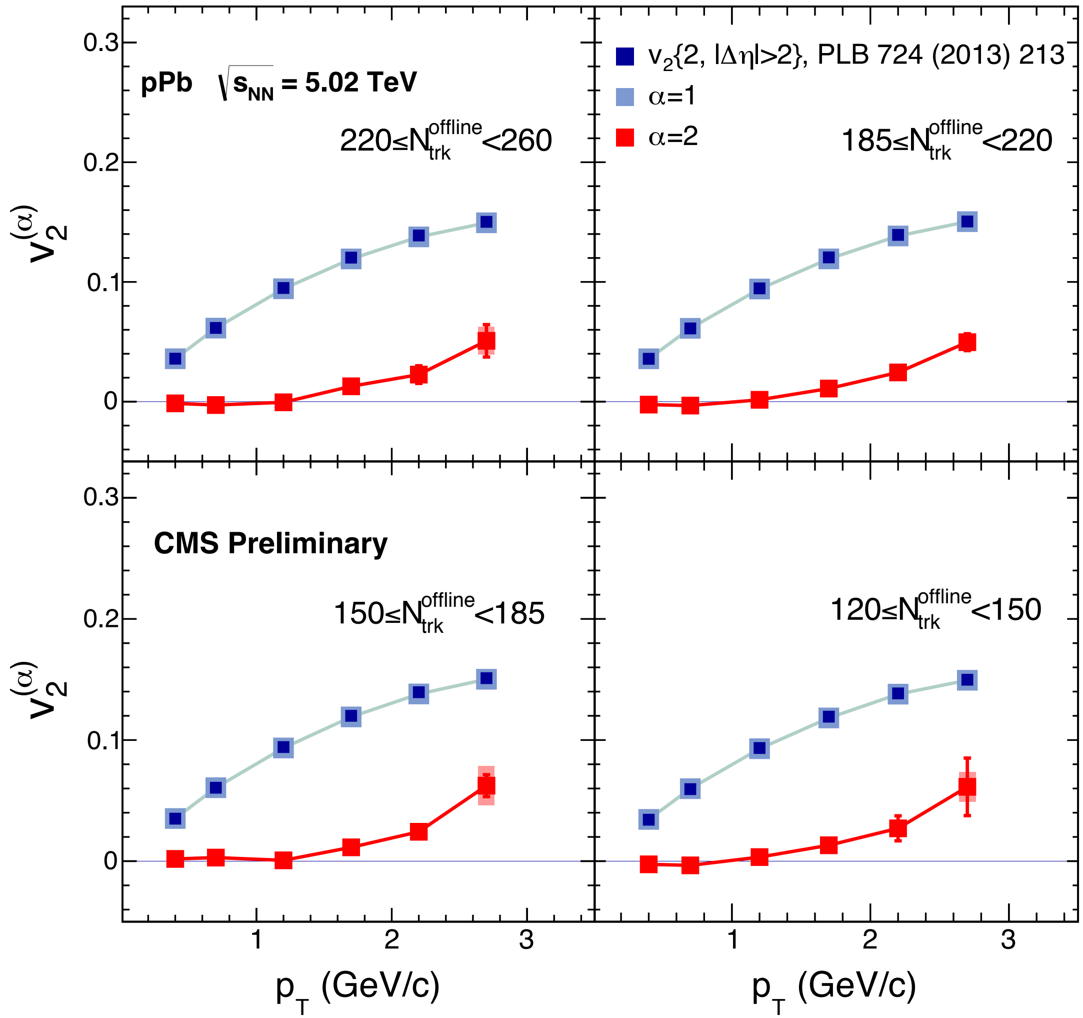
- The F_2^η has a minimum around midcentral PbPb and increases for peripheral and most central collisions
- At similar multiplicity, F_2^η in pPb larger than the one in PbPb
- Except for the most central PbPb, there is a very weak centrality dependence of F_3^η

- In PbPb, higher-orders F_3^η and F_4^η, show much stronger factorization breaking than for the second order

Principal Component Analysis as a new tool to study flow



Results – elliptic flows in pPb collisions

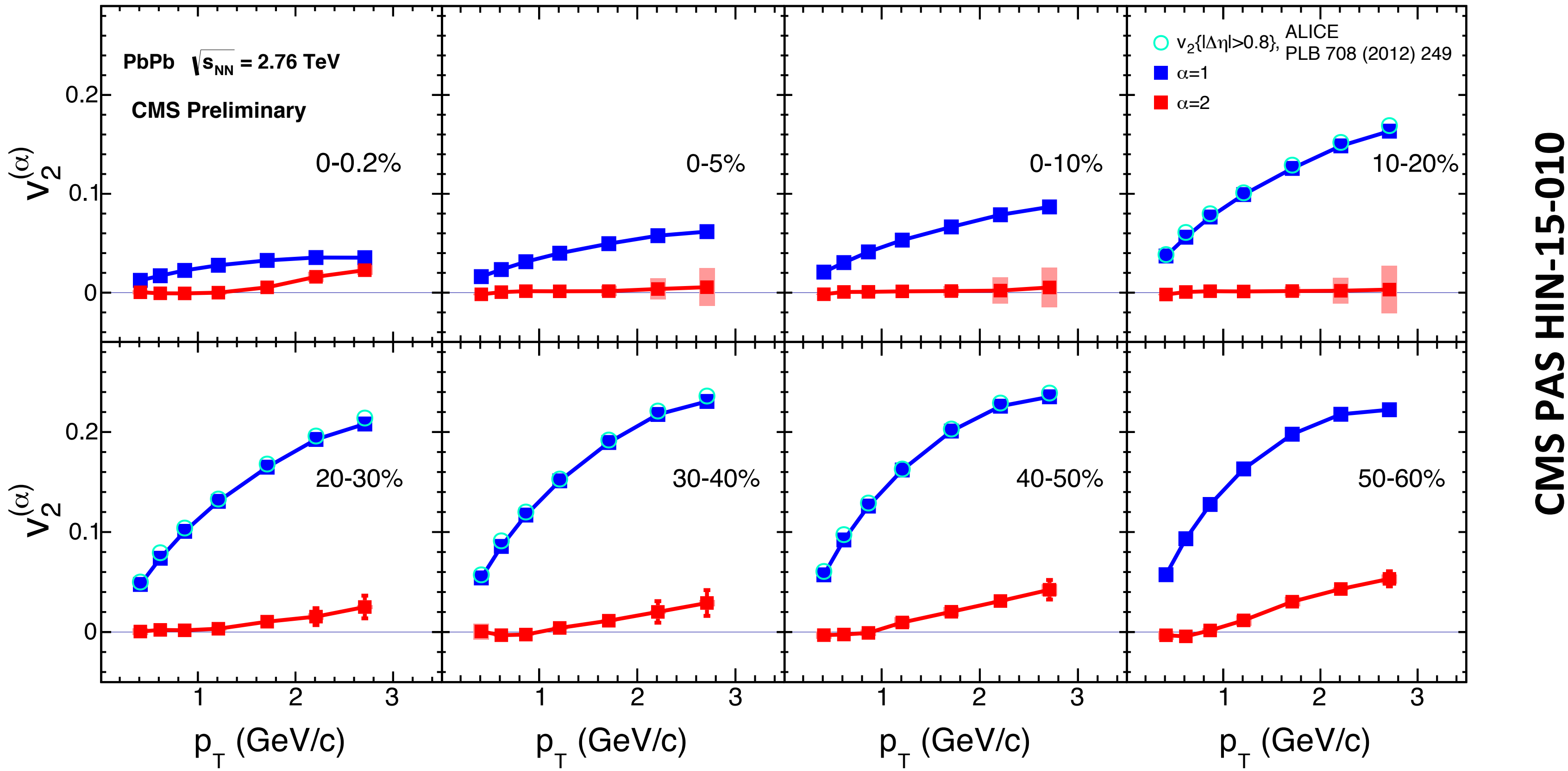


- The leading flow mode, *α=1*, practically identical to the v_2 measured using two-particle correlations
- The sub-leading flow mode, *α=2*, is essentially equal to zero at small p_T and increases up to 4-5% going to the high-p_T

CMS PAS HIN-15-010

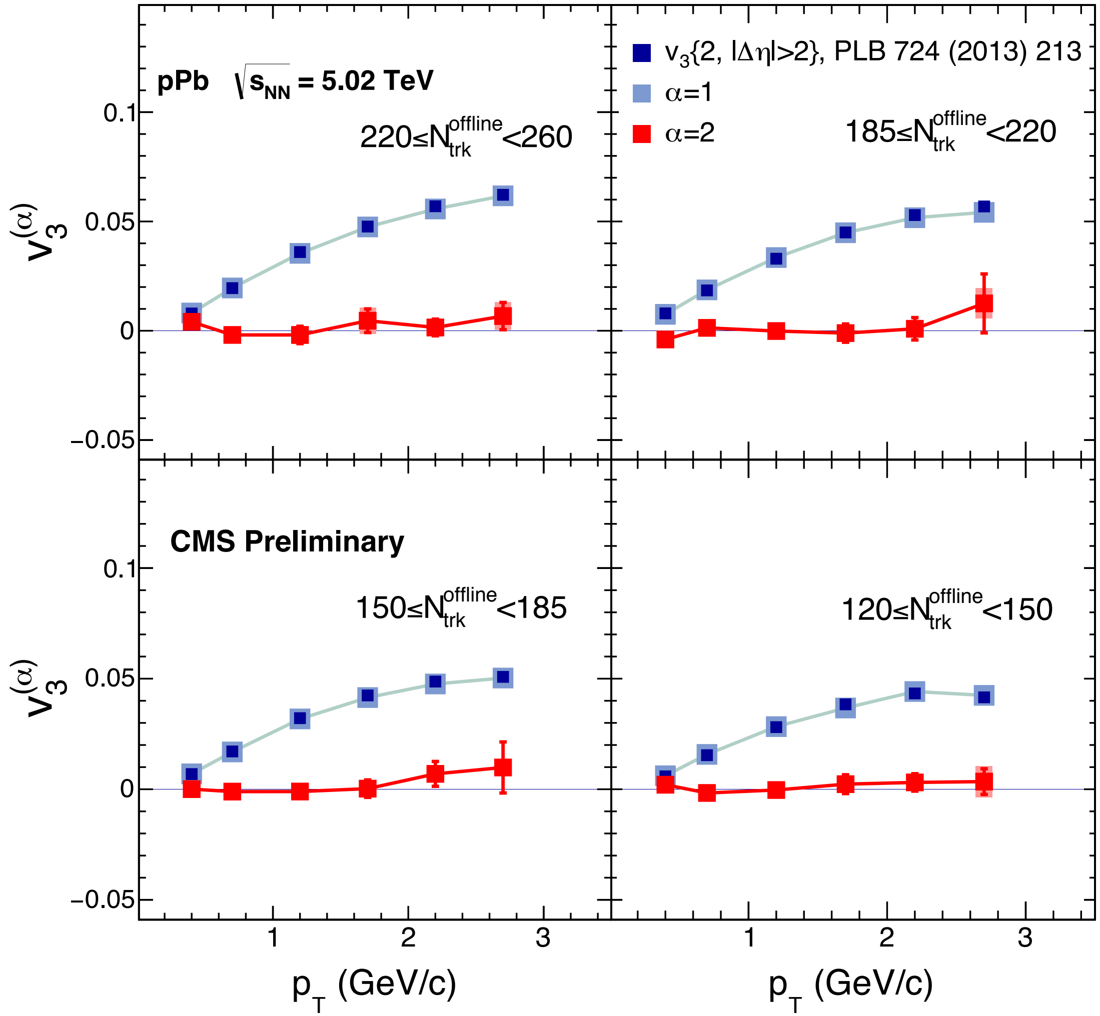
- The first experimental measurement of the elliptic sub-leading flow
- Systematical uncertainties small or comparable to statistical ones only at high-p_T

Results – elliptic flows in PbPb collisions



- The leading flow mode, *α=1,* essentially equal to the v_2 measured by ALICE using two-particle correlations
- The sub-leading flow mode, *α=2,* is positive at UCC and for collisions with centralities above 20%
- In the region 0-20% centrality comparable with zero
- Similar behavior wrt the r_2 results (10.1103/PhysRevC.92.034911, *arXiv: 1503.01692)*

Results – triangular flows in pPb collisions

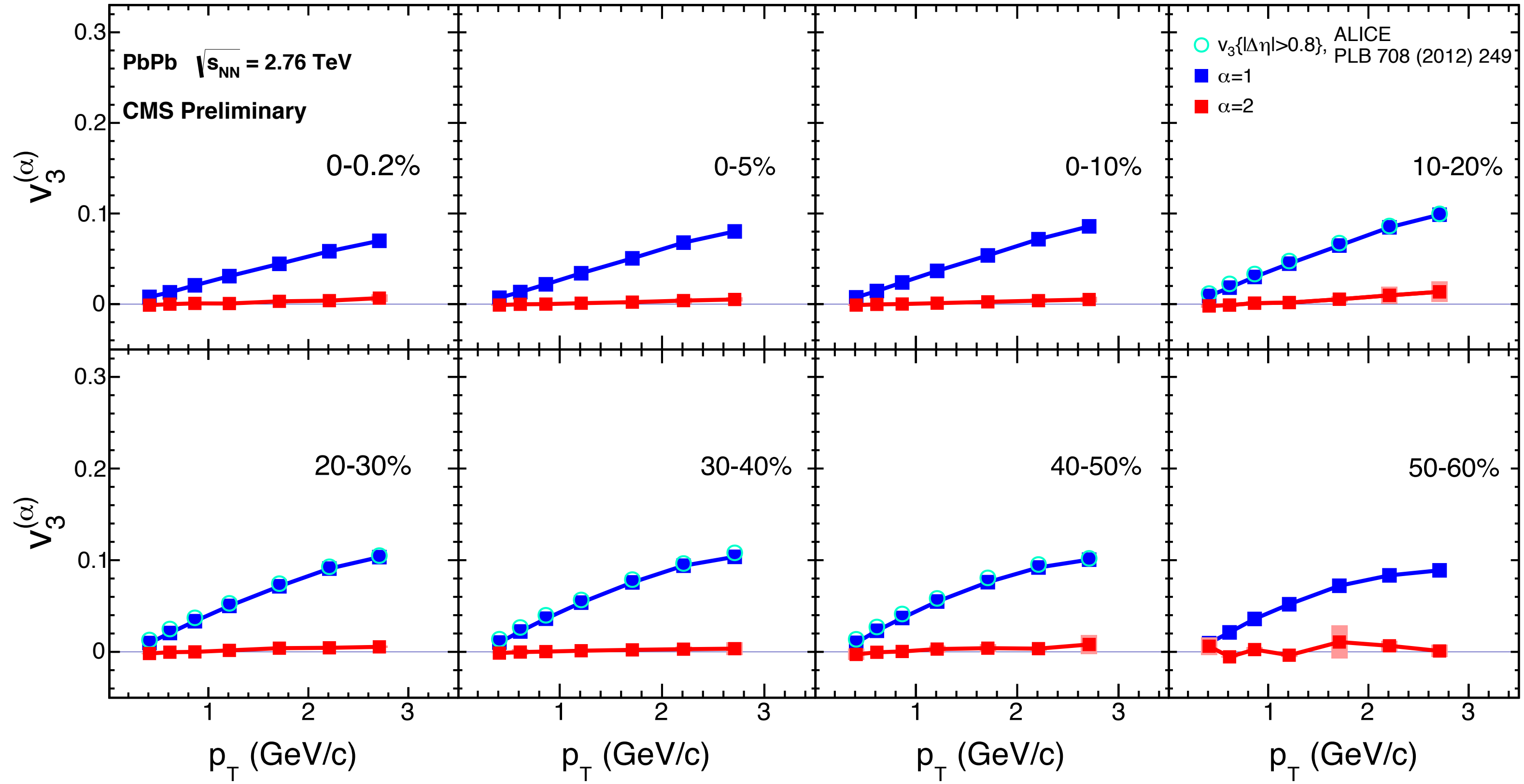


- The leading triangular flow mode, *α=1*, nearly identical to the v_3 measured using two-particle correlations
- The sub-leading flow mode, *α=2*, is comparable with zero within the given uncertainties.

CMS PAS HIN-15-010

- The first experimental measurement of the triangular sub-leading flow

Results – triangular flows in PbPb collisions

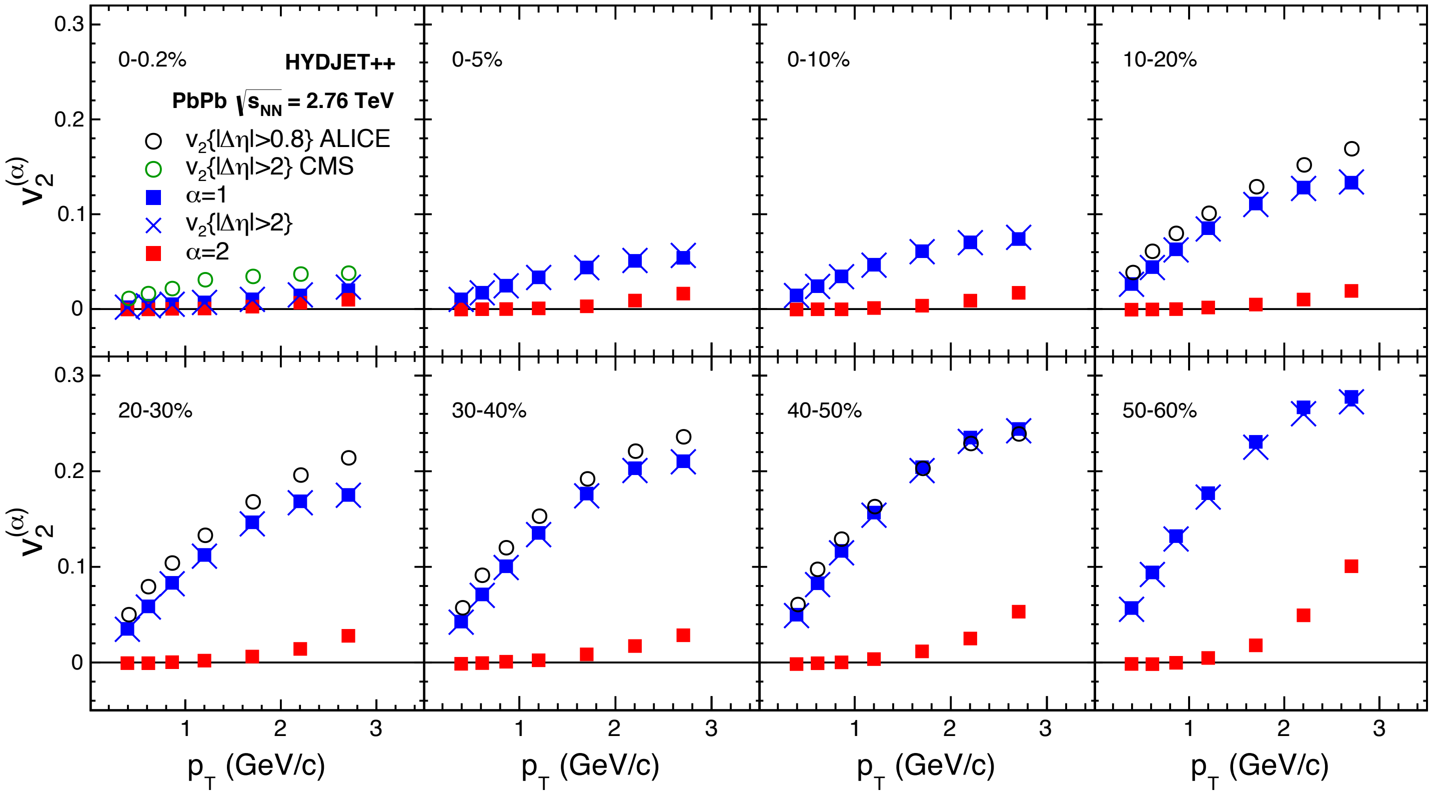


CMS PAS HIN-15-010

- Again, the leading flow mode, *α=1,* essentially equal to the v_3 measured by ALICE using two-particle correlations
- The sub-leading flow mode, *α=2,* is, within the uncertainties, equal to zero
- Results have a similar centrality dependence to that observed for r_3 (Phys. Rev C **92** (2015) 034911, *arXiv: 1503.01692)*

Elliptic flows in HYDJET++ PbPb collisions

Data generated under *'flow'+'quenched jet'* switch

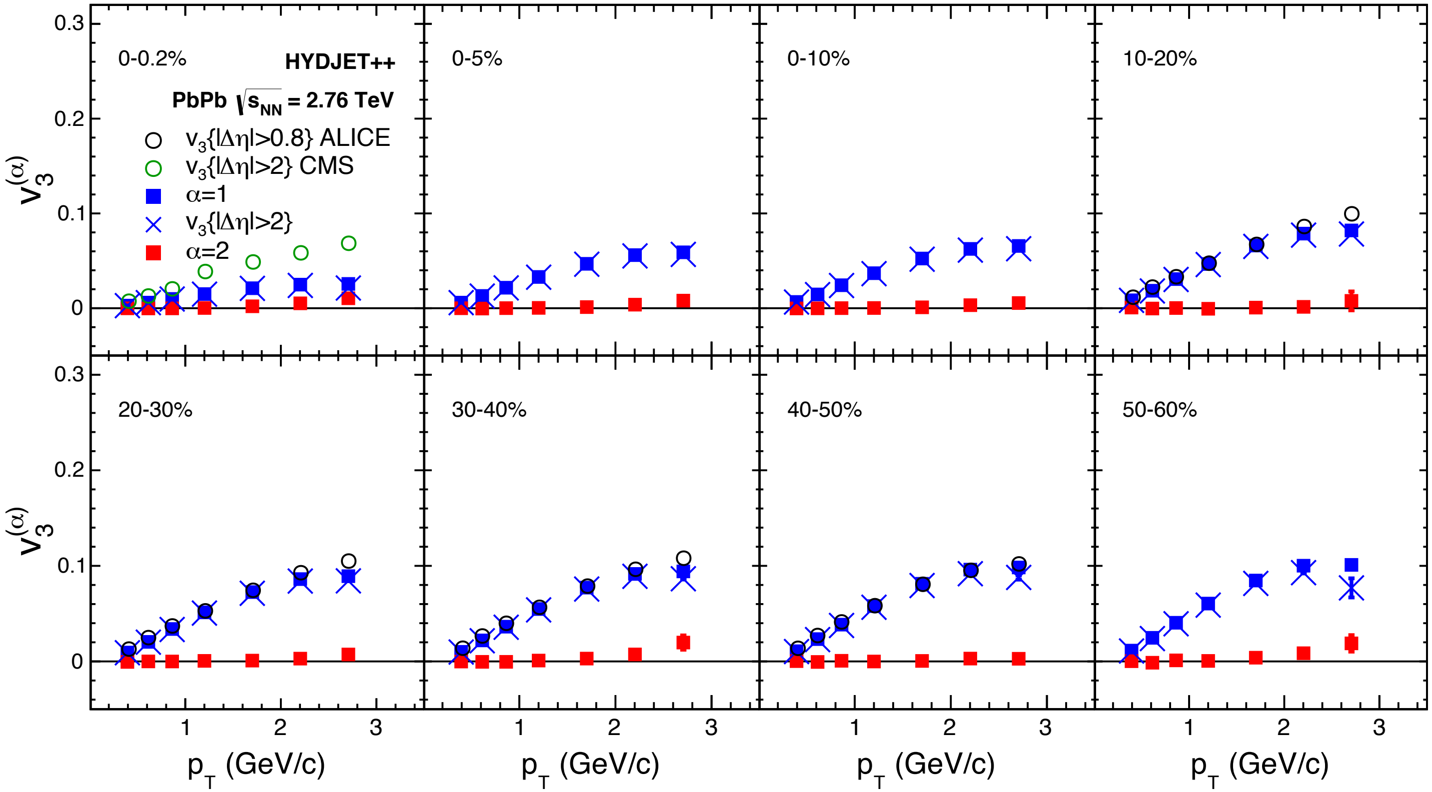


Submitted to Eur.Phys.J. C

- The HYDJET++ leading flow mode, *α=1*, essentially equal to the v_2 extracted from two-particle correlations. There is a rather good agreement with the v_2 experimentally measured by ALICE and CMS
- The sub-leading flow mode, *α=2*, is seen for all centralities in HYDJET++ PbPb collisions generated at 2.76 TeV
- Similar behavior wrt the r_2 results (Phys. Rev. C **92** (2015) 034911, *arXiv: 1503.01692)*

Triangular flows in HYDJET++ PbPb collisions

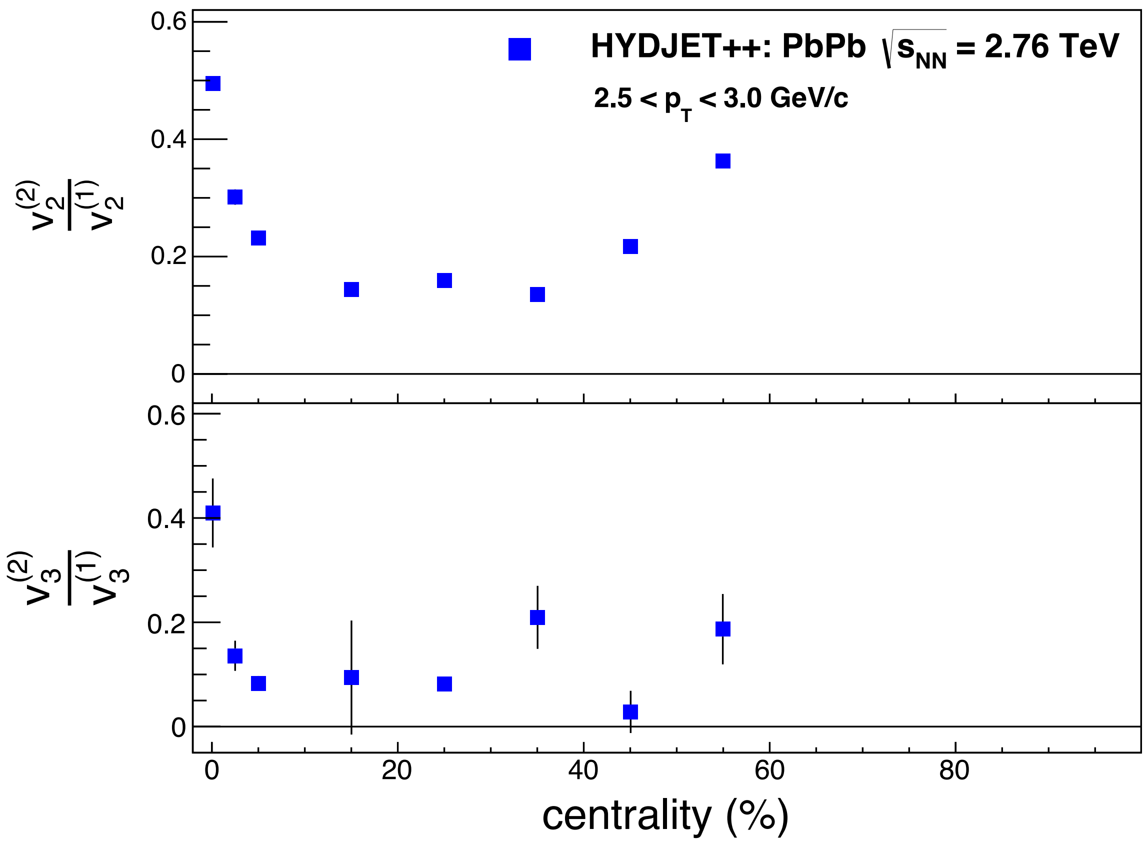
Data generated under *'flow'+'quenched jet'* switch



Submitted to Eur.Phys.J. C

- The HYDJET++ leading flow mode, *α=1,* essentially equal to the v_3 extracted from two-particle correlations. There is a rather good agreement with the v_3 experimentally measured by ALICE and CMS
- The sub-leading flow mode, *α=2,* is almost equal to zero ➔ the v_3 factorizes much better than the v_2
- Again, similar behavior wrt the r_3 results (Phys. Rev. C **92** (2015) 034911, *arXiv: 1503.01692)*

$v_n^{(2)}/v_n^{(1)}$ centality dependence at the highest p_T

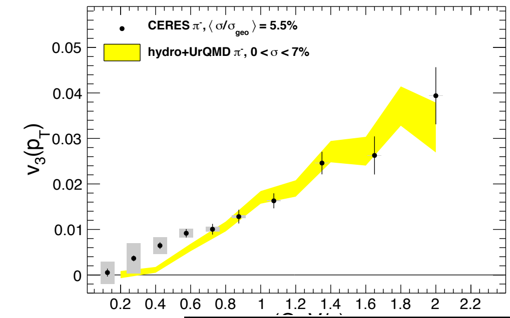


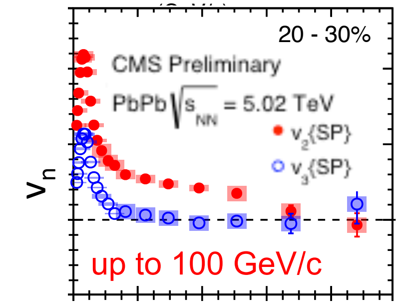
- Dramatic increase at ultra-central PbPb. Smallest effect for mid-central collisions. Increase for peripheral PbPb
- Qualitatively simiilar with the experimental r_2 in PbPb collisions (Phys.Rev. C92.034911, *(arXiv: 1503.01692)*
- A weak centrality dependence for *3-rd* harmonic. Also in an agreement with the experimental r_3 results

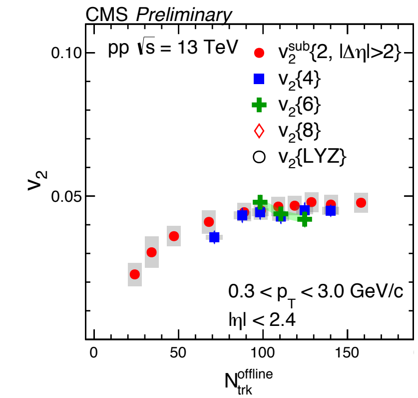
Submitted to Eur.Phys.J. C

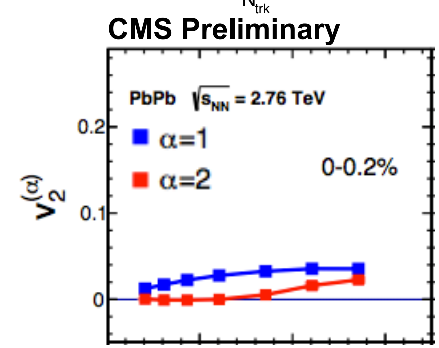
Conclusions

- The first $v_3(p_T)$ measurement at the top SPS energy with CERES using the event plane method
- The v_2 and v_3 measured up to 100 GeV/c in PbPb at 5 TeV
- The v_2 and v_3 in small pPb and smallest pp system formed in collisions at the LHC energies
- A strong factorization breaking effect for n=2 appears approaching UCC PbPb collisions
- The sub-leading flow modes are for the first time experimentally measured
- The sub-leading elliptic flow modes is in a qualitative agreement with the r_2 factorization-breaking results
- The sub-leading triangular flow modes in both collision system is small if not zero showing that the triangular flow factorizes much better than the elliptic flow
- Subleading flow mode from HYDJET++ are in qualitative agreement with the experimental findings
- These results could help in better understanding of the initial-state fluctuations



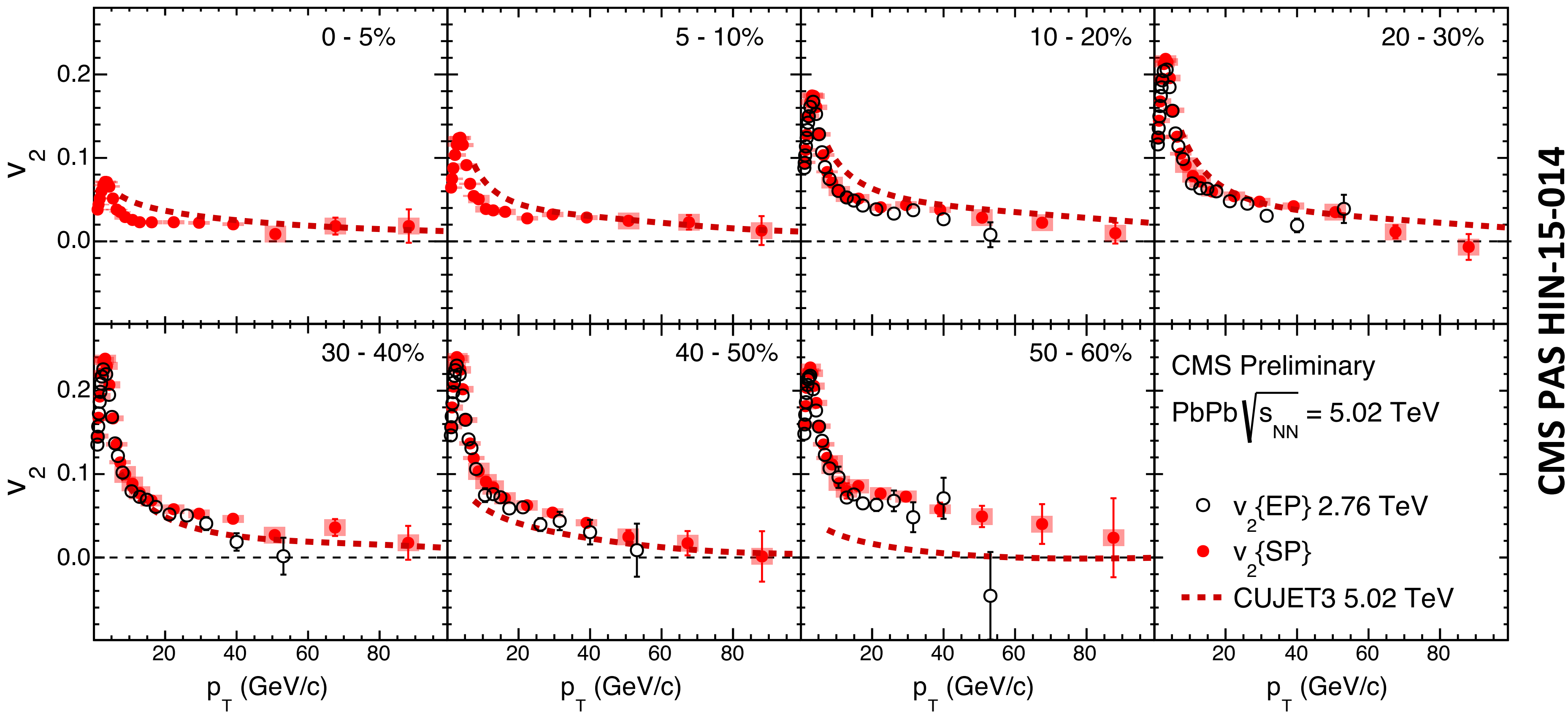




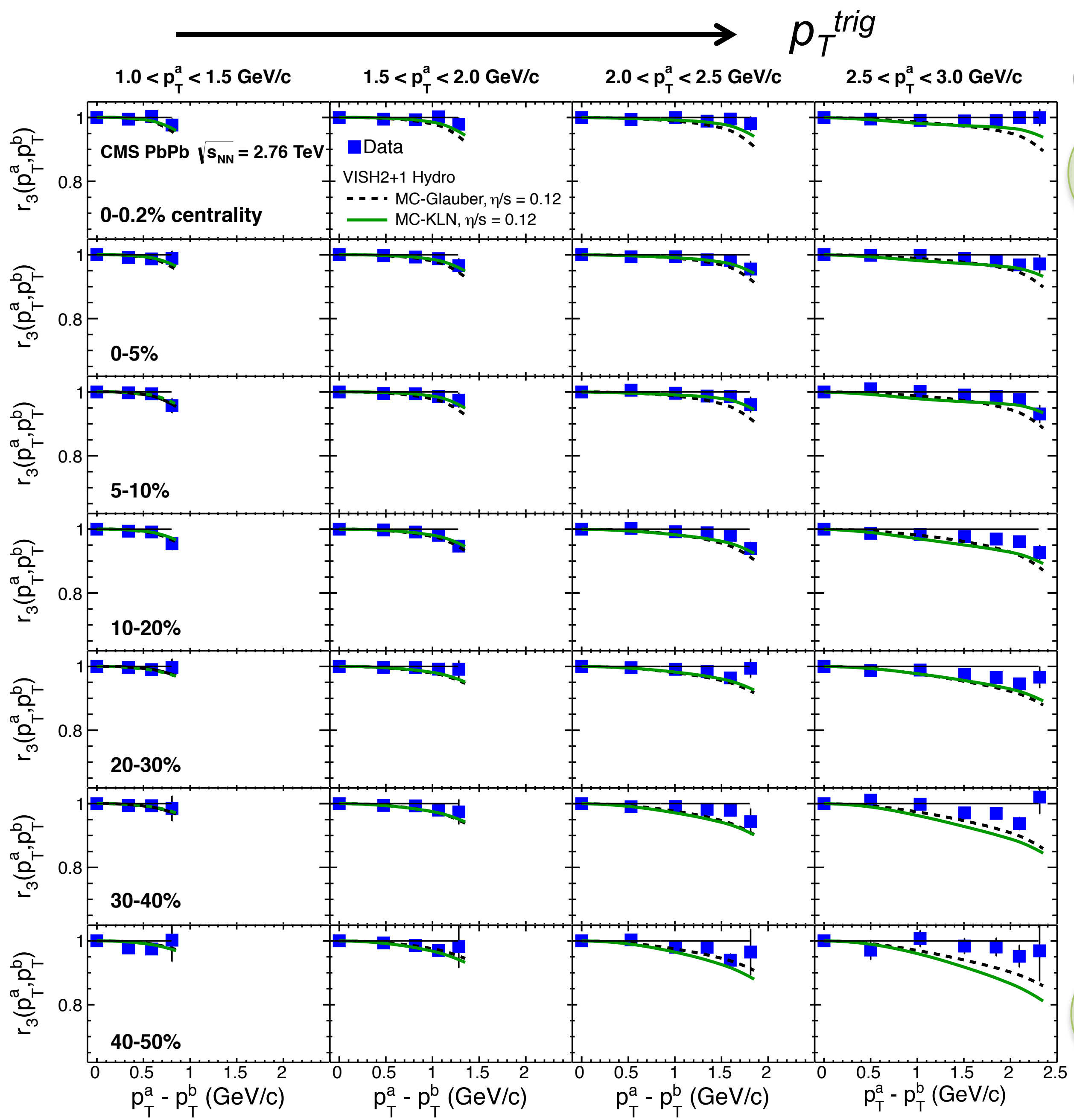


Backup slides

Comparison with lower energy and CUJET3



- A slight increase of v_2 wrt results (EP method) from 2.76 TeV collision energy
- CUJET3 predictions roughly compatible with the data at high-p_T (over 40 GeV/c)
- At lower p_T, CUJET3 overpredicts the experimental v_2



0-0.2%

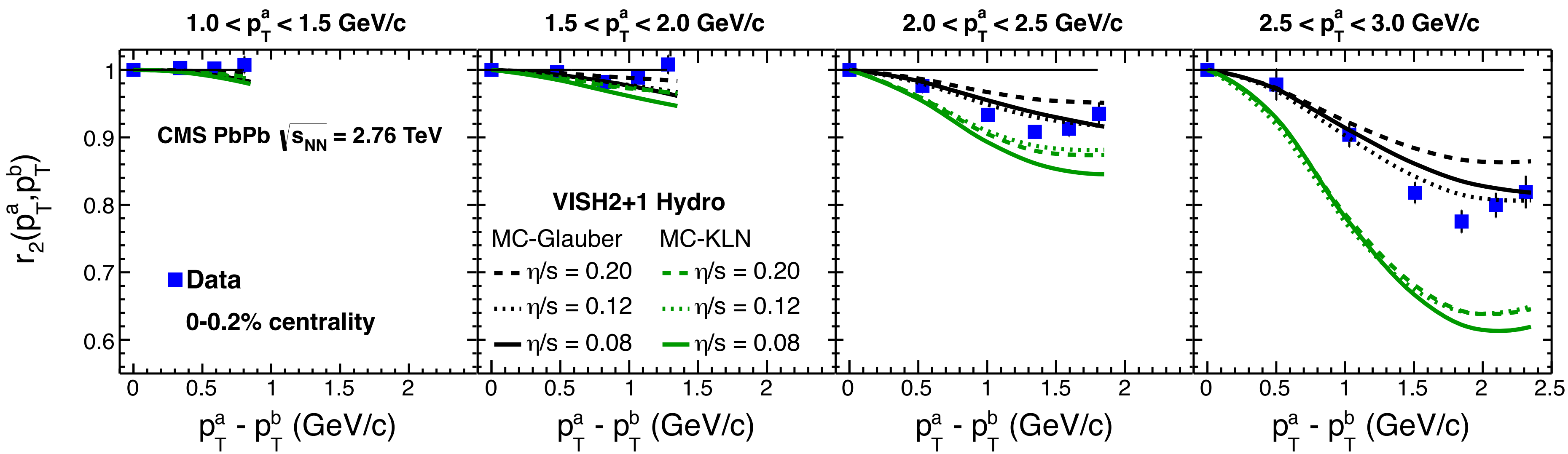
PbPb case

- Factorization holds better for V_3
- Breaking visible only for the highest p_T^{trig}-p_T^{assoc}
- Very weakly depends on centrality

40-50%

arXiv: 1503.01692
PRC 92 (2015) 034911

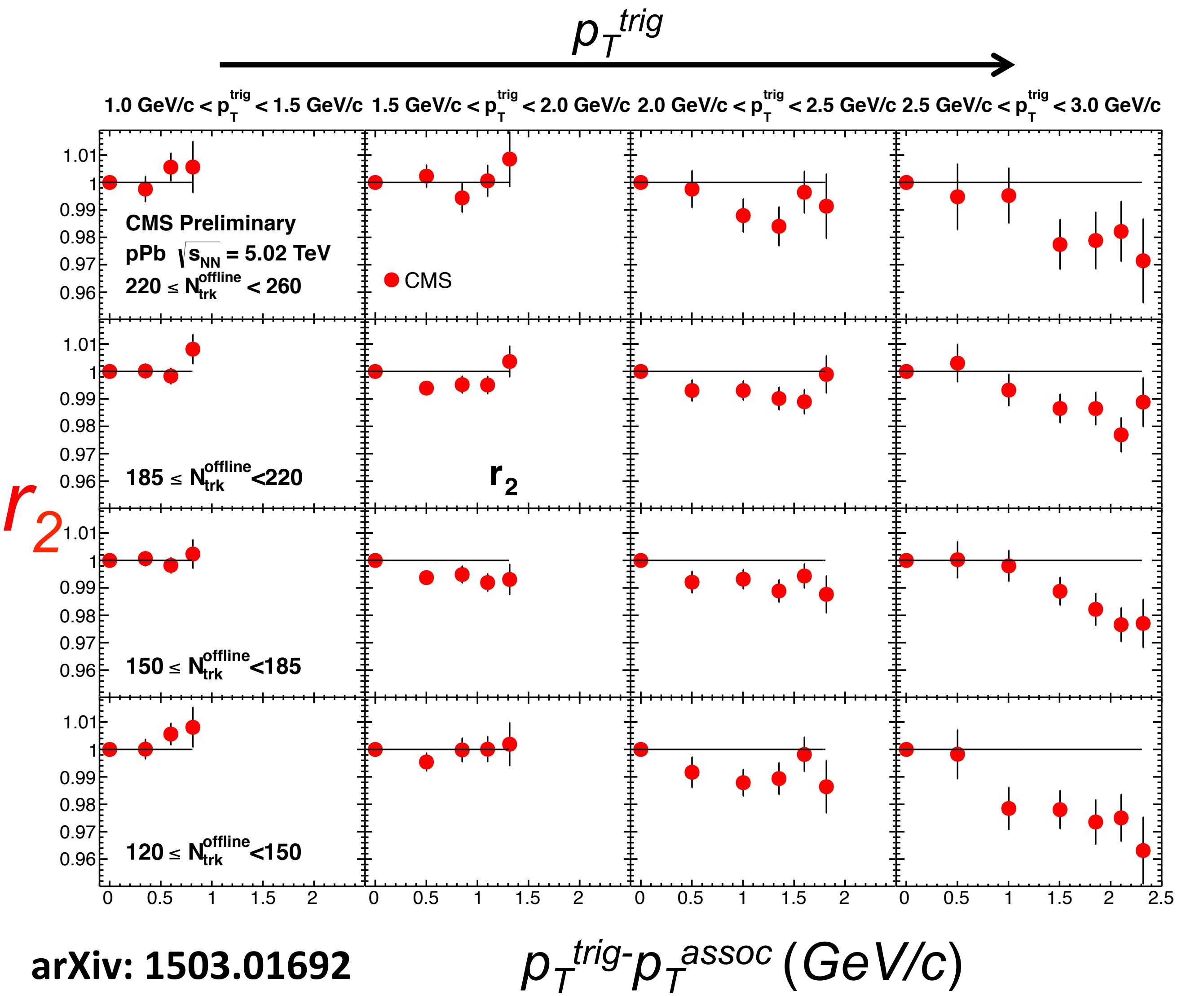
r_2 in ultra-central PbPb collisions and VISH2+1



arXiv: 1503.01692, PRC 92 (2015) 034911 **VISH2+1: PRC 87 (2013) 034913**

- The effect increases with rise of p_T^{trig} and p_T^{trig}-p_T^{assoc}
- The biggest effect seen in ultra-central collisions while for semi-central collisions, the effect achieves only a size of 2−3%
- The VISH2+1 model qualitatively gives a good description of CMS data for both MC-Glauber and MC-KLN initial conditions
- Large insensitivity to η/s ➔ an independent constraint to the initial-state

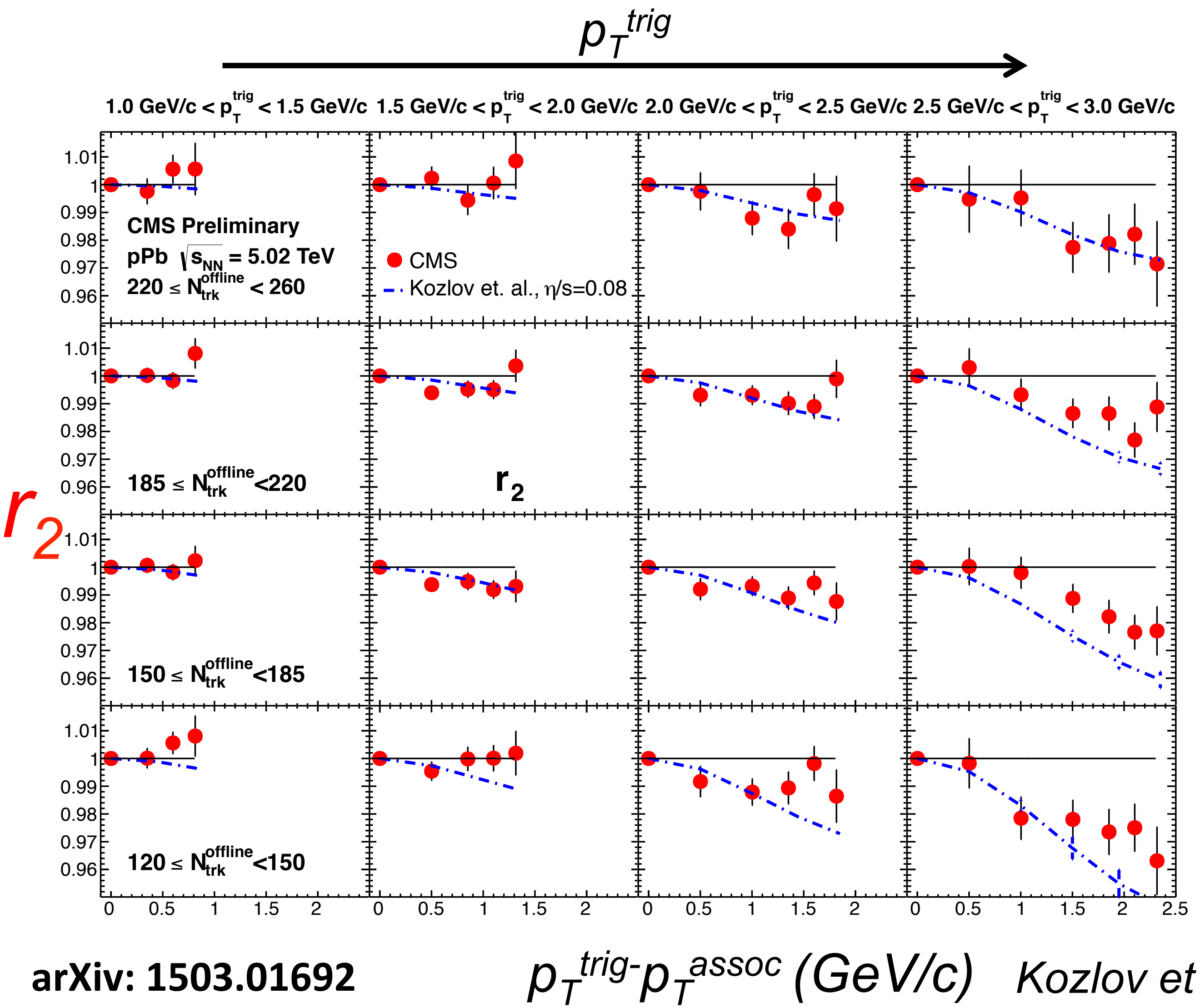
r_2 from high-multiplicity pPb collisions



- The effect increases with p_T^{trig} and p_T^{trig}-p_T^{assoc}
- Maximum around 2-3%
- Nearly no dependence on multiplicity

arXiv: 1503.01692
PRC 92 (2015) 034911

pPb r_2: comparison to Kozlov et. al hydro model

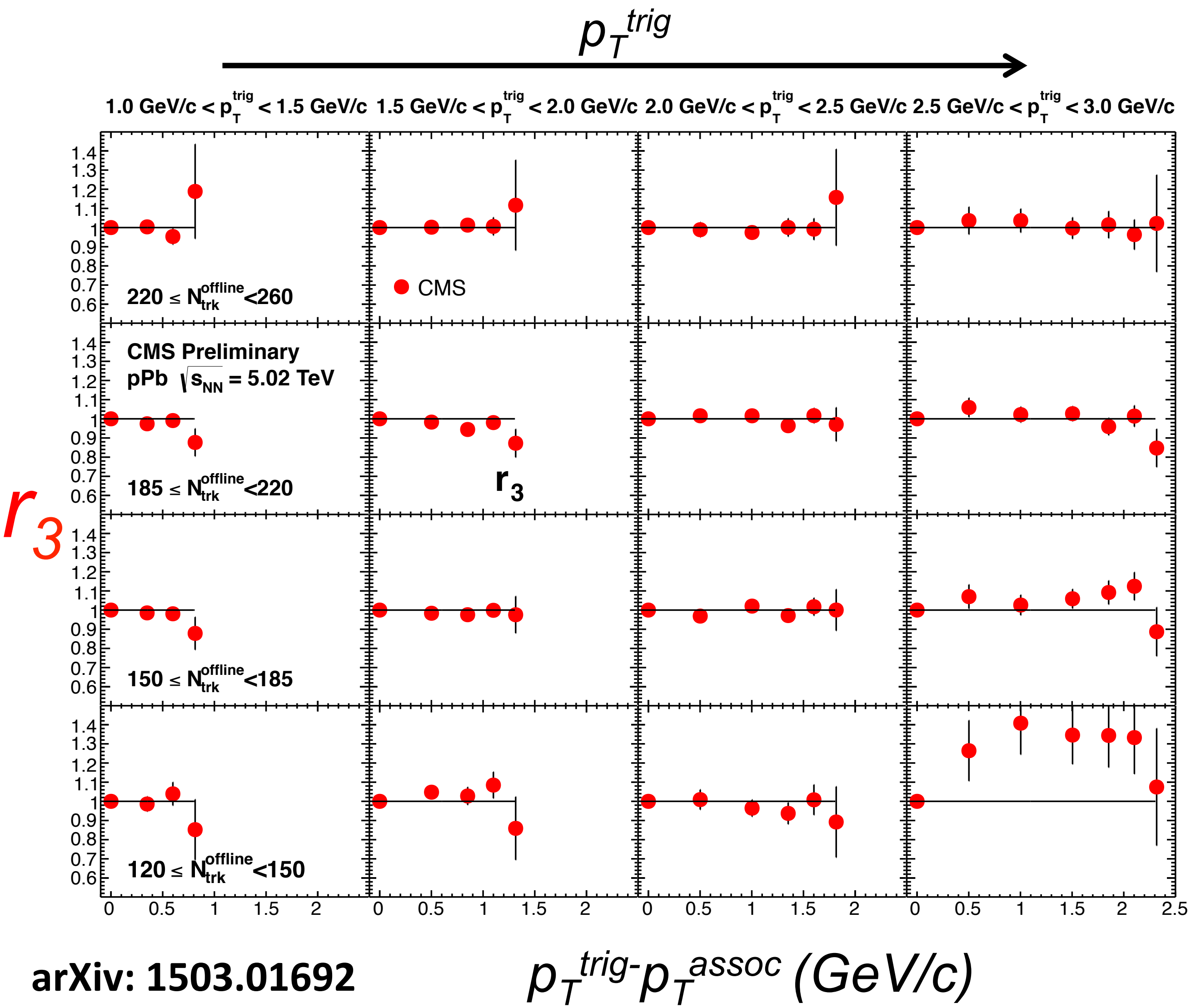


Kozlov et al. hydro model qualitatively describes data

arXiv: 1503.01692
PRC 92 (2015) 034911

Kozlov et al.: arXiv:1405.3976

r_3 from high-multiplicity pPb collisions



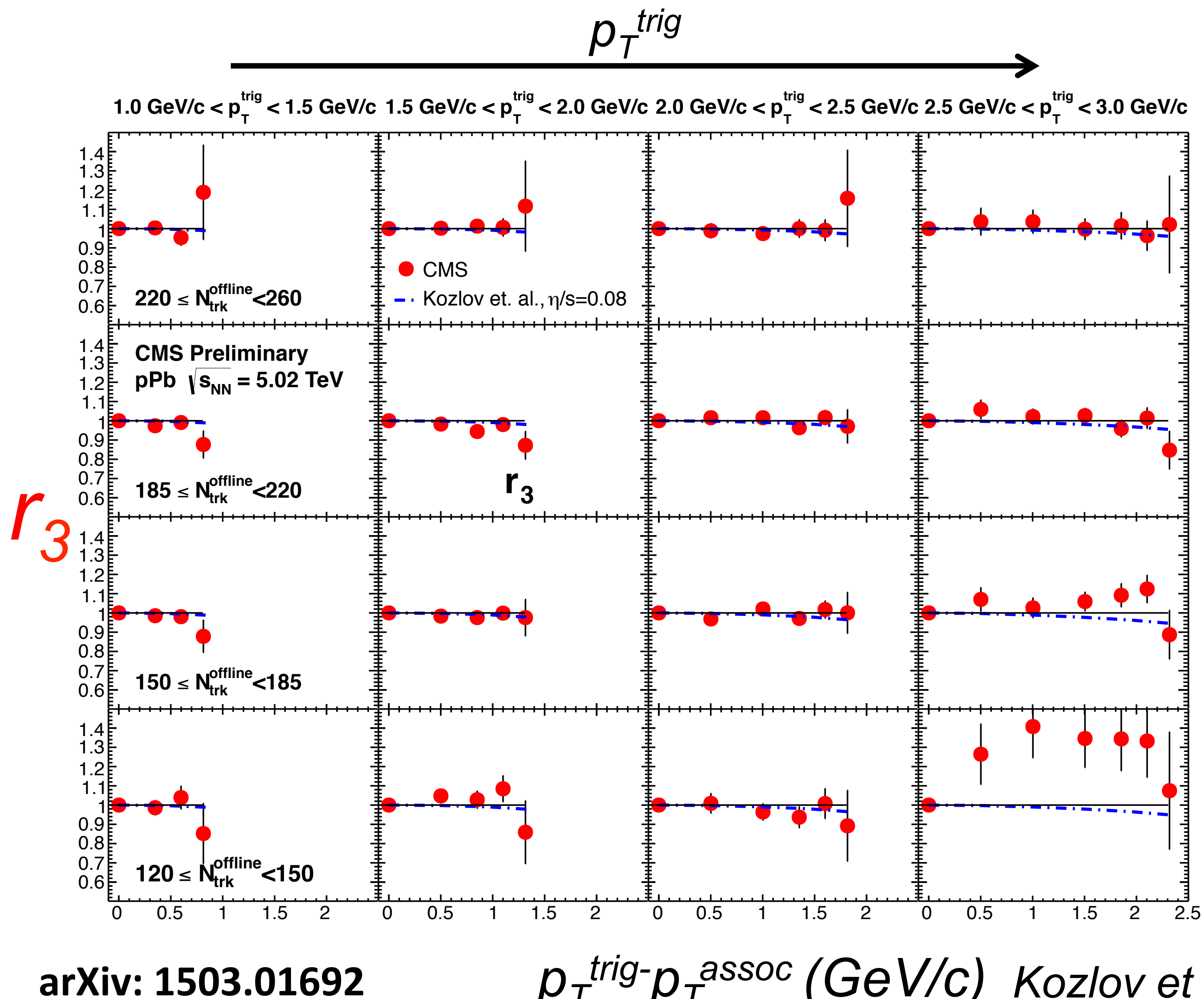
$220<N_{trk}^{offline}<260$

- V_3 factorize better than V_2
- A direct indication of non-flow effect seen in r_3 for the highest p_T^{trig} in lower multiplicity bins

$120<N_{trk}^{offline}<150$

arXiv: 1503.01692
PRC 92 (2015) 034911

pPb r_3: comparison to Kozlov et. al hydro model

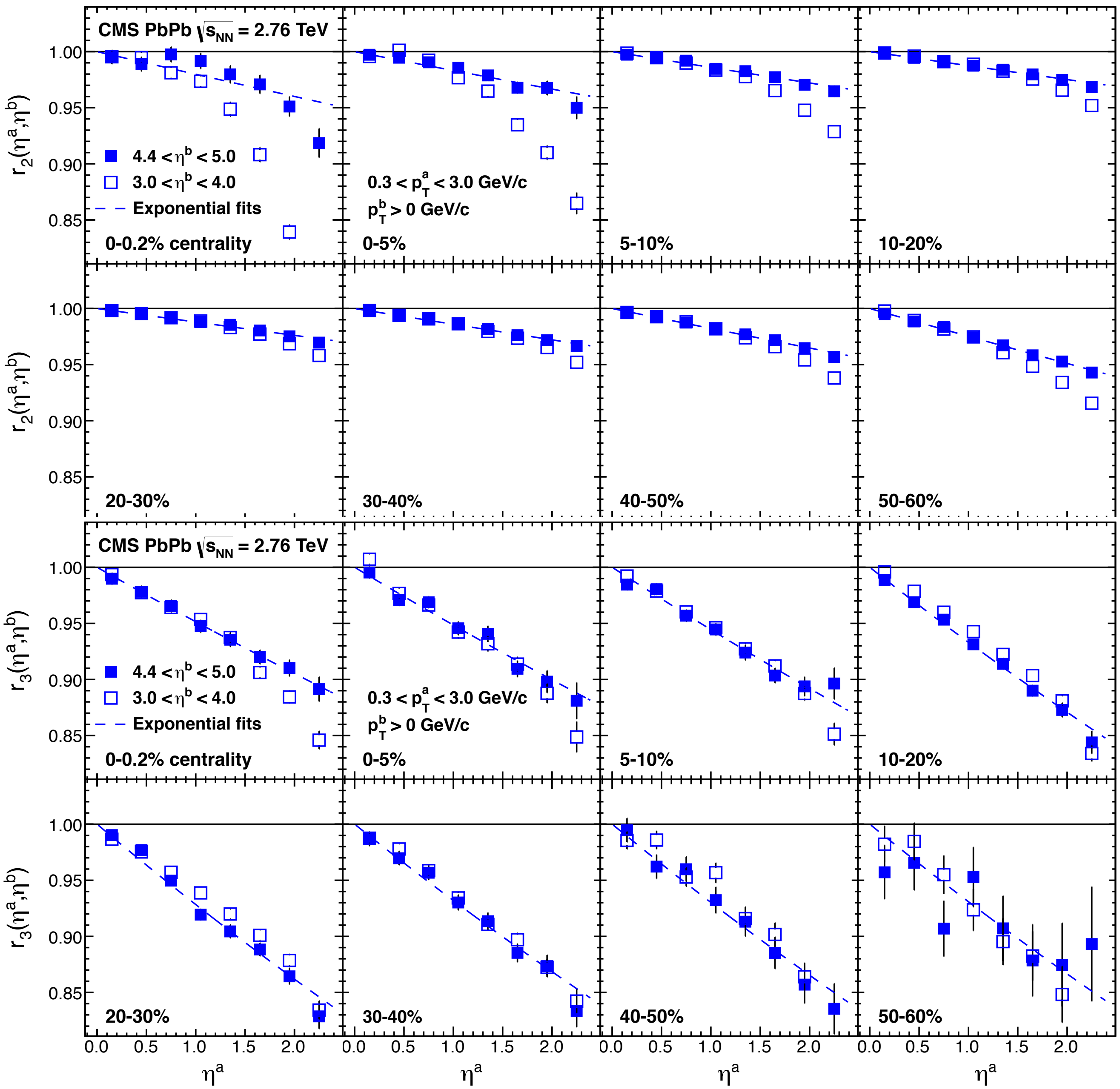


- Kozlov et al. hydro model qualitatively describes data except in lower multiplicity bins for the highest p_T^{trig}

Kozlov et al.: arXiv:1405.3976

arXiv: 1503.01692
PRC 92 (2015) 034911

η-dependent r_n in PbPb



- ❖ The r_2 factorization
- • breaking effect increases with increase of η^a
- ❖ Except for the most central collisions, the increase is approximately linear

arXiv: 1503.01692
PRC 92 (2015) 034911

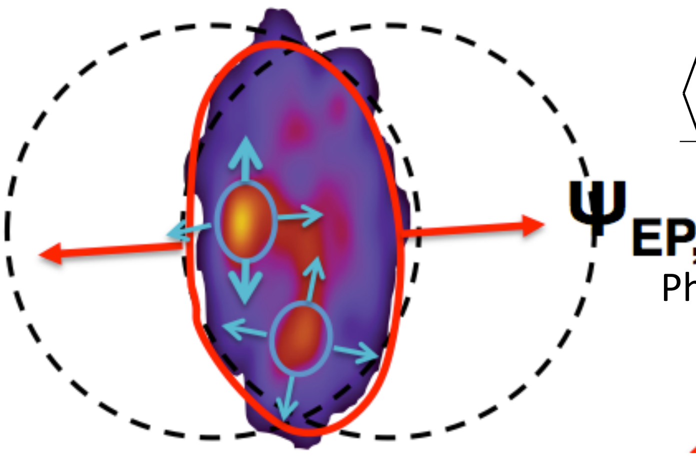
- ❖ The effect of factorization breaking is much stronger for higher-order harmonic r_3 – opposite to the p_T dependence
- ❖ Almost linear increase of the effect size
- ❖ Parameterization:

$$r_n\left(\eta^a,\eta^b\right) \approx e^{-2F_n^{\eta}\eta^a}$$

Factorization breaking - connection to the PCA

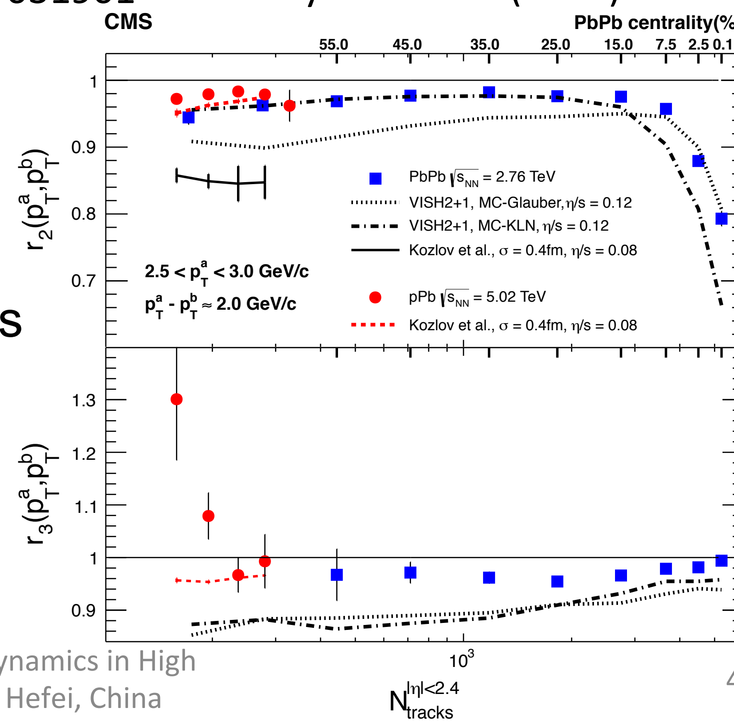
- Initial-state fluctuations → the EP (Ψ_n) depends on p_T *and on* η → factorization is broken. New observable introduced:

$$r_n = \frac{V_{n\Delta}(p_{T1},p_{T2})}{\sqrt{V_{n\Delta}(p_{T1},p_{T1})}\sqrt{V_{n\Delta}(p_{T2},p_{T2})}} = \frac{\left\langle v_n(p_{T1})v_n(p_{T2})\cos\left[n(\Psi_n(p_{T1})-\Psi_n(p_{T2}))\right]\right\rangle}{\sqrt{v_n^2(p_{T1})v_n^2(p_{T2})}} = \begin{cases} 1 & \text{holds} \\ <1 & \text{brakes} \\ >1 & \text{non-flow} \end{cases}$$



Phys.Rev. C87 (2013) 031901

Phys.Rev. C87 (2013) 034913



Phys. Rev. C **92** (2015) 034911 *(arXiv:1503.01692)*

- If there is only one principal component for each harmonic *n* → $V_{n\Delta}(p_i,p_j)$ factorizes
- r_n i.e. $V_{n\Delta}(p_i,p_j)$ results are partially integrated, while mutually orthogonal eigenmodes contain all information

Factorization breaking - connection to the PCA

- The given harmonic order *n* has also higher (α>2) eigenmodes ordered from largest to smallest, while in r_n they are not clearly distinguished

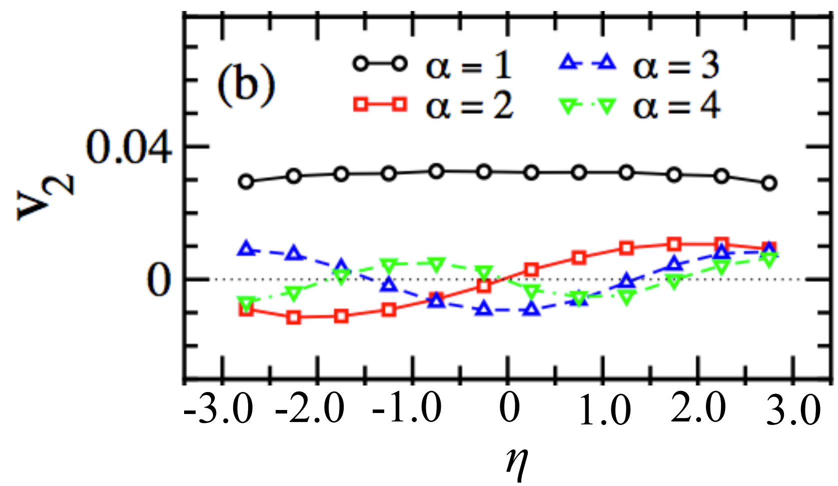
- The PCA can approximately reconstruct two-particle $V_{n\Delta}(p_i,p_j)$ coefficients

$$V_{n\Delta}(p_i,p_j) \approx \sum_{\alpha=1}^{k\leq N_b} V_n^{(\alpha)*}(p_i) V_n^{(\alpha)*}(p_j) \qquad \text{where } N_b{=}7$$

which can be used to calculate the factorization breaking ratio r_n

- So, the PCA is a good tool for analysis in hydrodynamics with fluctuations in the initial state

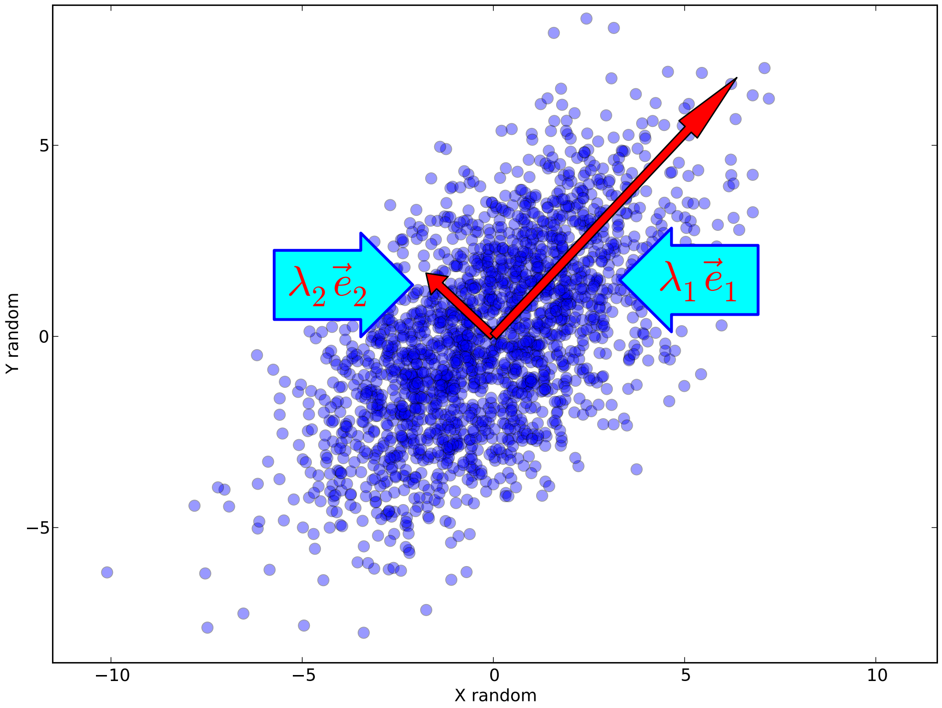
- Note that the PCA uses the whole p_T range simultaneously to extract the information on both leading and sub-leading flow modes



Phys.Rev.Lett. 114 (2015) 152301

Principal Component Analysis (PCA) method

A simple 2D example



- Random data generated by 2D multivariate Gauss distribution

$$\vec{X}_n = (x_1, x_2, \ldots, x_n)$$
$$\vec{Y}_n = (y_1, y_2, \ldots, y_n)$$

- a matrix

$$\Sigma = \begin{bmatrix} \mathrm{var}(X) & \mathrm{cov}(X,Y) \\ \mathrm{cov}(X,Y) & \mathrm{var}(Y) \end{bmatrix}$$

- eigenvectors e_i and eigenvalues λ_i by diagonalization Σ

$$[e]^T \Sigma [e] = diag(\lambda_1, \lambda_2)$$

- First Principal Component: eigenvector e_1 points to maximum variance of data cloud. Its magnitude is $\sqrt{\lambda_1} e_1$
- Second Principal Component: eigenvector e_2 points to the next maximum variance of data cloud. Its magnitude is $\sqrt{\lambda_2} e_2$

PCA method in hydrodynamic flow - prescription

Two very recent theoretical papers: R.S.Bhalerao, J-Y. Ollitrault, S.Pal and D.Teaney, Phys.Rev.Lett. 114 (2015) 152301 and A.Mazeliauskas and D.Teaney, Phys.Rev. C91 (2015) 044902 introduced the PCA as a new method to study hydrodynamics flows

❖ *"The simplest correlations are pairs. The **principal component analysis** is a method which extracts all the information from pair correlations in a way which facilitates comparison between theory and experiment." J.-Y. Ollitrault*

In this analysis:

PhysRev.C **92** (2015) 034911
arXiv:1503.01692
and other CMS analyses

❖ **Input**: two-particle Fourier coefficients measured as

$$V_{n\Delta} = \langle\langle \cos(n\Delta\phi) \rangle\rangle_S - \langle\langle \cos(n\Delta\phi) \rangle\rangle_B$$ where

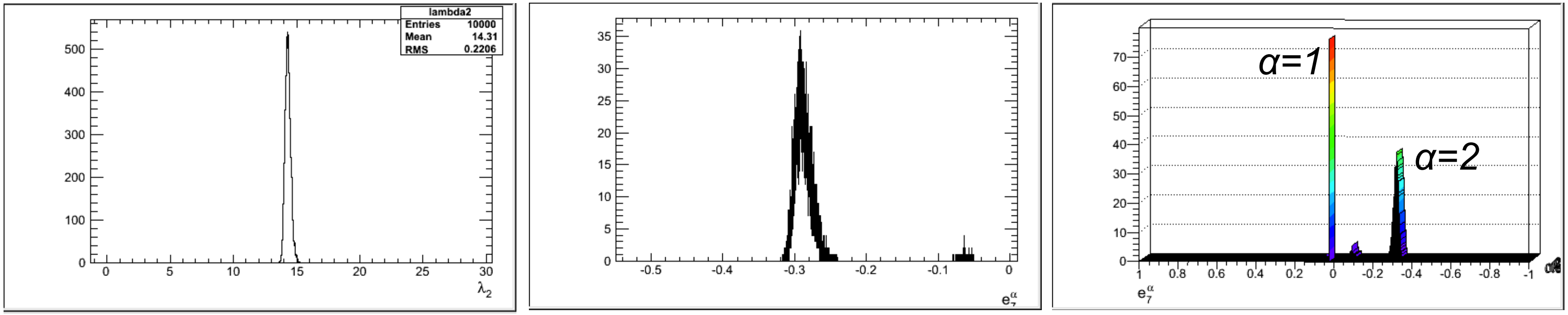
$\langle\langle \cos(n\Delta\phi) \rangle\rangle_S$ and $\langle\langle \cos(n\Delta\phi) \rangle\rangle_B$ are calculated for pairs with $|\Delta\eta|>2$

❖ 7 p_T bins ($0.3<p_T<3.0$ GeV/c); the eigenvalue problem of a matrix $[V_{n\Delta}(p_i,p_j)]$

$$\begin{pmatrix} e^{(1)} & e^{(2)} & . & . & . & . & e^{(7)} \end{pmatrix} \begin{bmatrix} V_{n\Delta}(p_1,p_1) & V_{n\Delta}(p_2,p_1) & V_{n\Delta}(p_3,p_1) & . & . & . & . \\ V_{n\Delta}(p_1,p_2) & V_{n\Delta}(p_2,p_2) & V_{n\Delta}(p_3,p_2) & . & . & . & . \\ V_{n\Delta}(p_1,p_3) & V_{n\Delta}(p_2,p_3) & V_{n\Delta}(p_3,p_3) & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & V_{n\Delta}(p_7,p_7) \end{bmatrix} \begin{pmatrix} e^{(1)} \\ e^{(2)} \\ . \\ . \\ . \\ . \\ e^{(7)} \end{pmatrix} = diag\begin{pmatrix} \lambda^{(1)} & \lambda^{(2)} & . & . & . & . & \lambda^{(7)} \end{pmatrix}$$

PCA method in hydrodynamic flow - prescription

λ distribution, α=2 *e distribution, α=2* *α=2 signal 200 times smaller wrt α=1*



CMS Preliminary

2.5<p_T<3.0 GeV/c

- The new introduced p_T dependent variable, **flow mode**, is defined as

$$V_n^{(\alpha)}(p_i) = \sqrt{\lambda^{(\alpha)}}e^{(\alpha)}(p_i)$$ where α=1,…,7

- corresponding single-particle flow mode $v_n^{(\alpha)}(p) = \frac{V_n^{(\alpha)}(p)}{\langle M(p)\rangle}$
- experimental data → $V_{n\Delta}(p_i,p_j)$ → it has its own statistical error $\Delta V_{n\Delta}(p_i,p_j)$
- The error propagation through $V_n^{(\alpha)}$ up to $v_n^{(\alpha)}$
- $\Delta\lambda^\alpha$ and $\Delta \mathbf{e}^\alpha$ as RMS of the distributions like ones shown above. Matrix elements $V_{n\Delta}$ were perturbed (10k times) within its $\Delta V_{n\Delta}$ → matrix $[V_{n\Delta}]$ **nonlinearly** perturbed