Hadron spectroscopy: Lattice QCD

Christopher Thomas, University of Cambridge

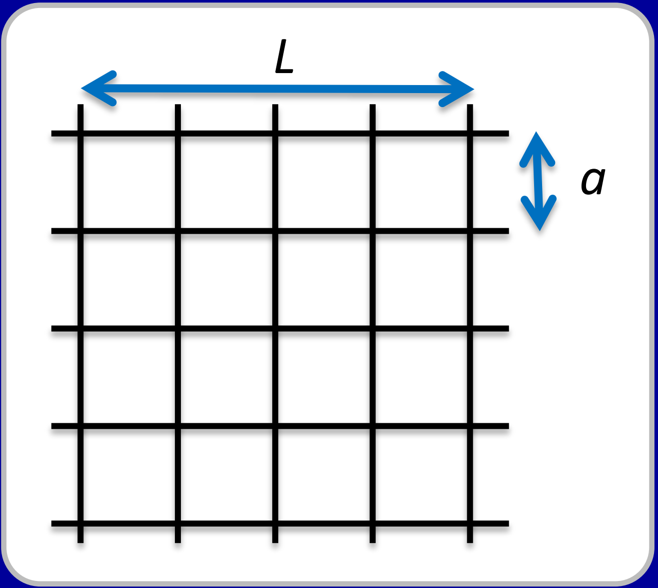
c.e.thomas@damtp.cam.ac.uk

PANIC 2017, Beijing, 1 – 5 September 2017

UNIVERSITY OF CAMBRIDGE

Lattice QCD Spectroscopy

Systematically-improvable first-principles calculations



- **Discretise** spacetime in a **finite volume**
- Compute correlation fns. numerically (Euclidean time, $t \rightarrow i\,t$)

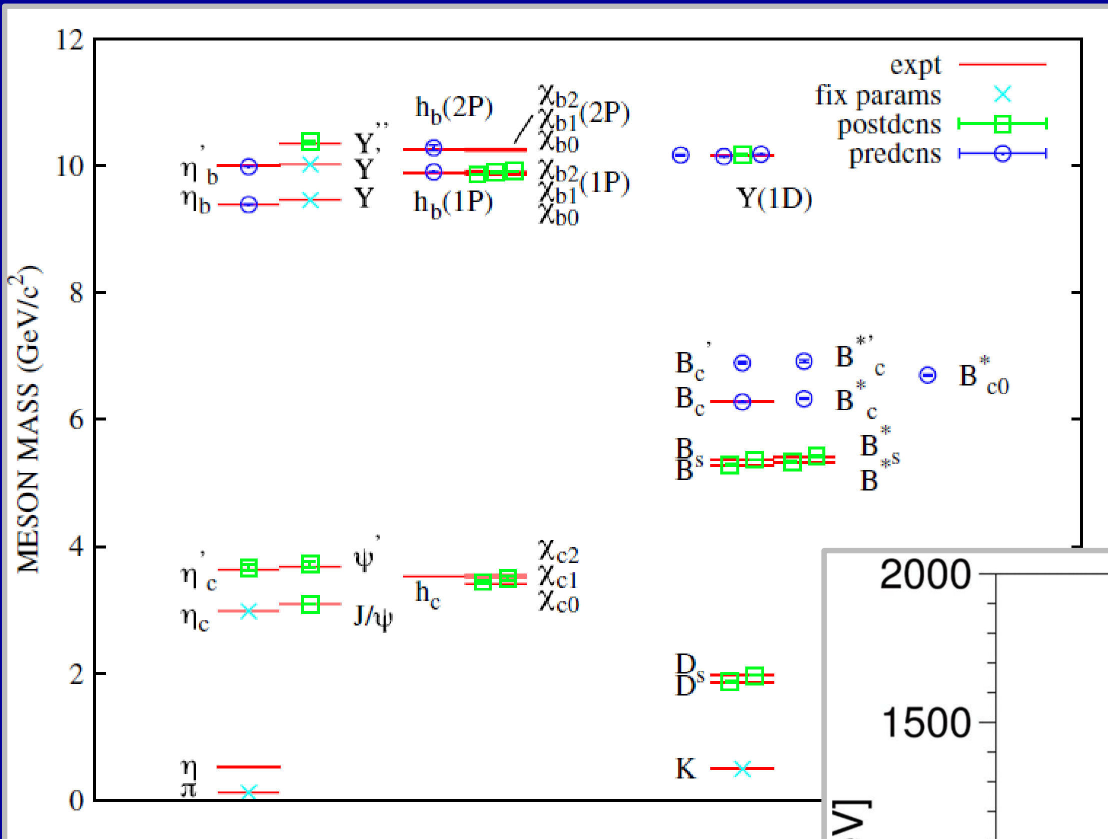
Note:

- Finite a and L
- Possibly heavy u, d quarks ($\rightarrow$ unphysical m_π)

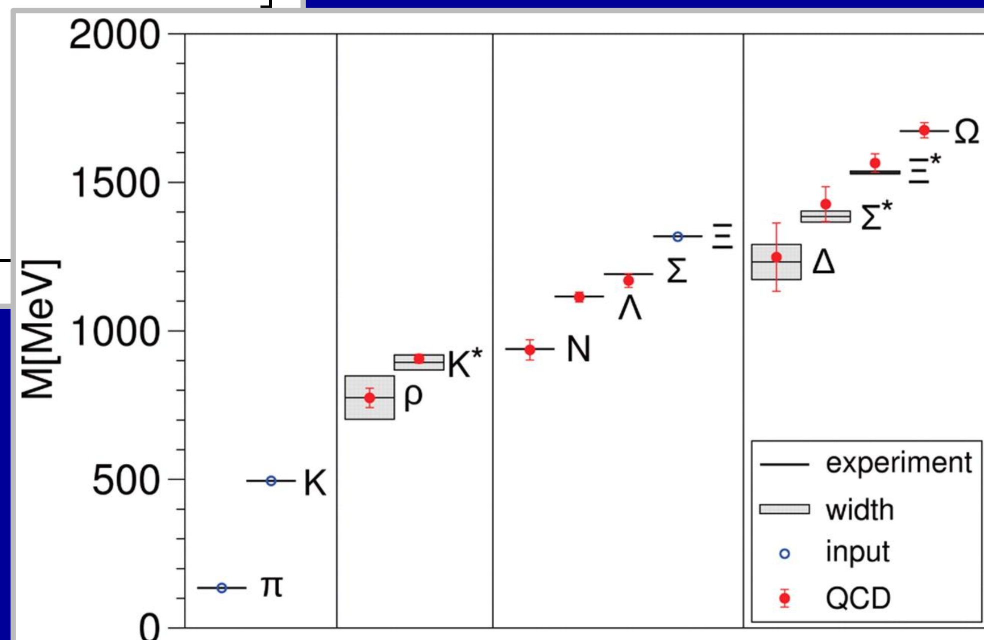
Finite-volume energy eigenstates from:

$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t) \mathcal{O}_j^\dagger(0) | 0 \rangle$$

Lower-lying mesons and baryons

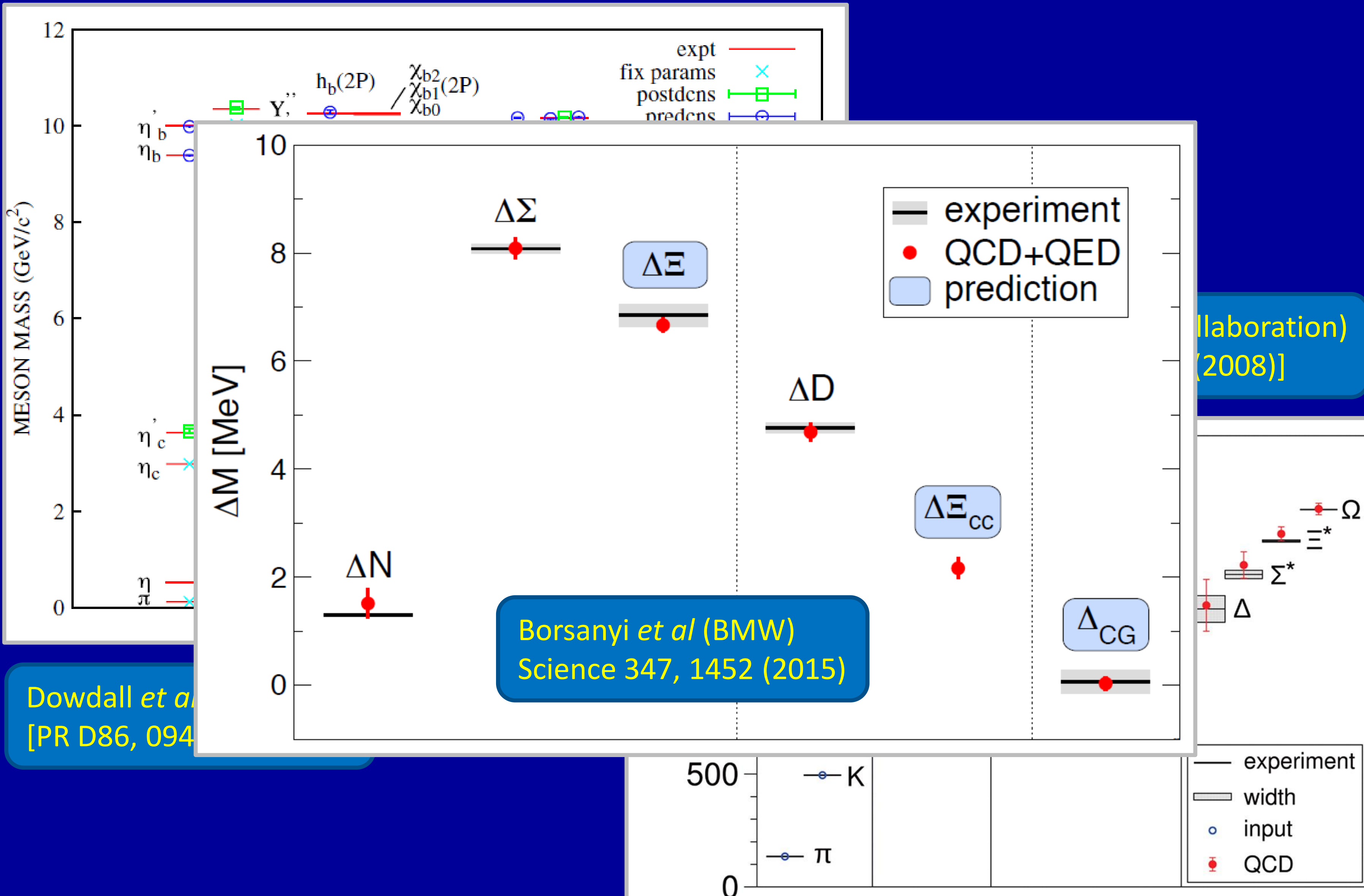


Durr *et al* (BMW Collaboration)
[Science 322, 1224 (2008)]



Dowdall *et al* (HPQCD)
[PR D86, 094510 (2012)]

Lower-lying mesons and baryons



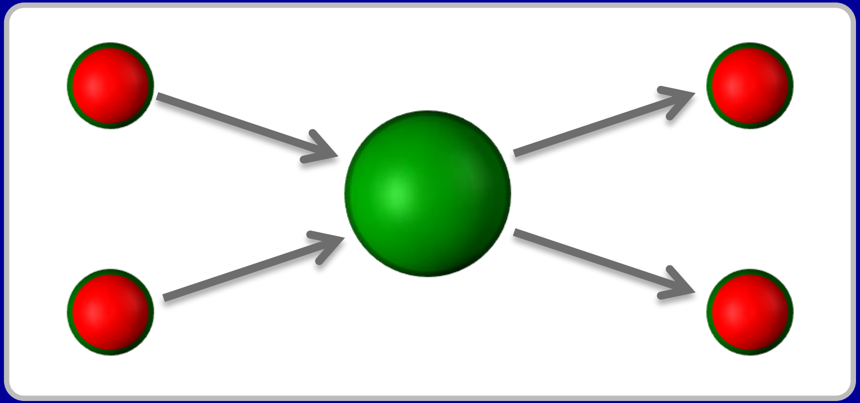
Dowdall *et al* [PR D86, 094...

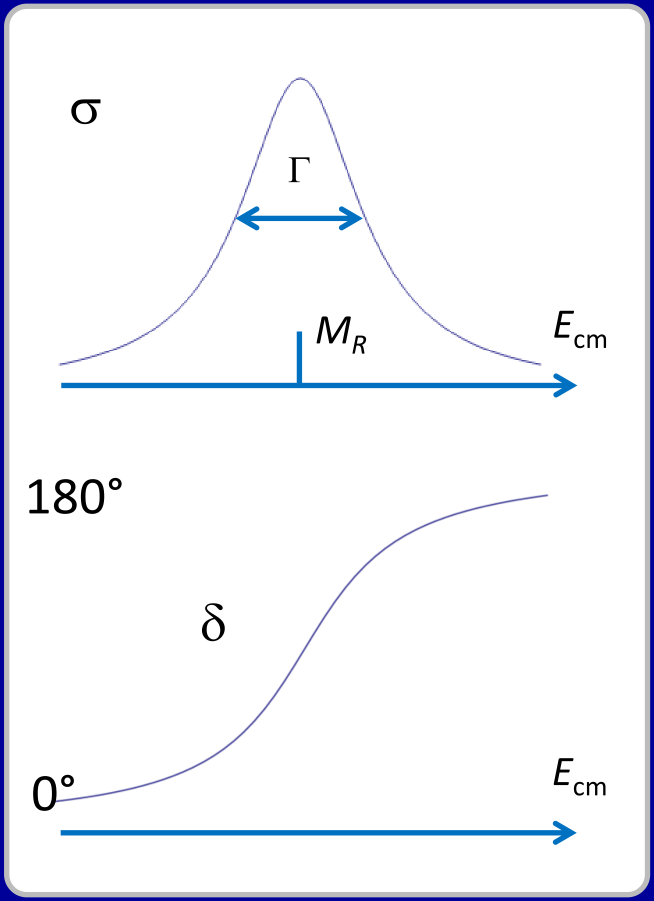
Borsanyi *et al* (BMW)
Science 347, 1452 (2015)

...llaboration)
...(2008)]

Scattering and resonances

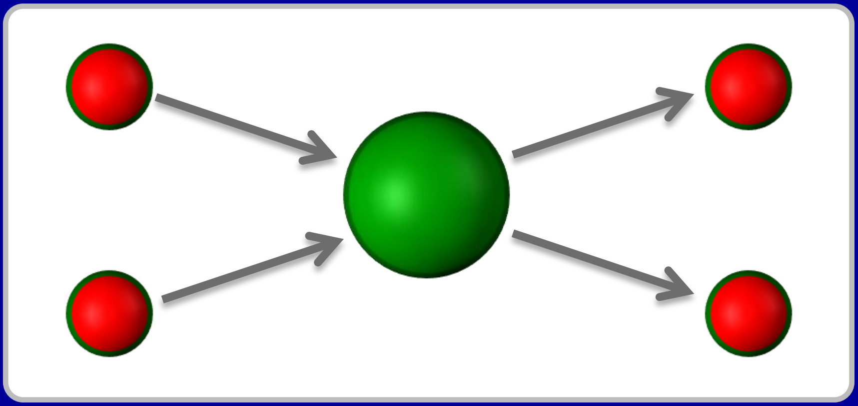
Most hadrons appear as resonances in scattering of lighter hadrons

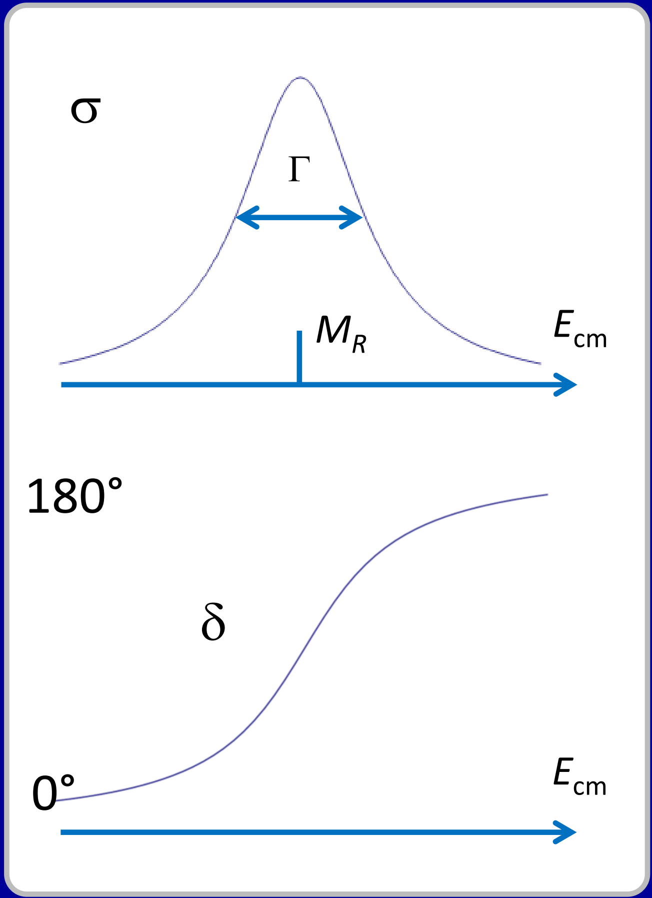


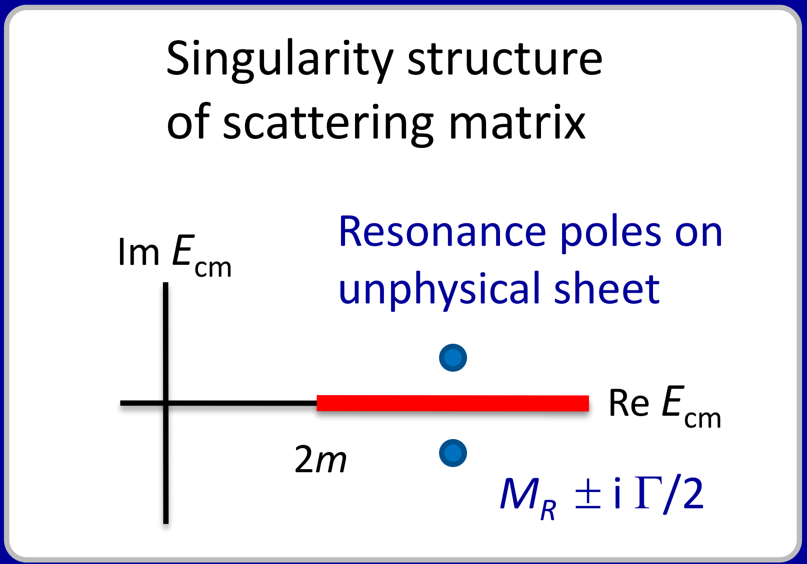


Scattering and resonances

Most hadrons appear as resonances in scattering of lighter hadrons

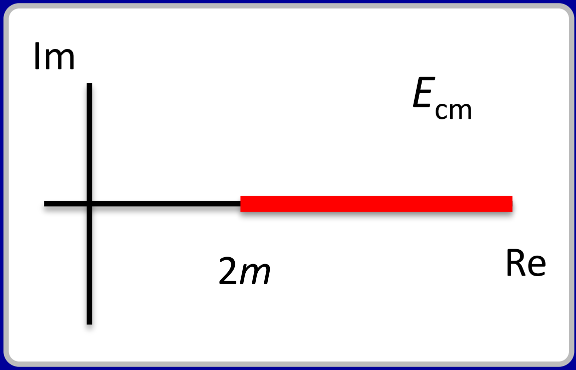






Scattering in Lattice QCD

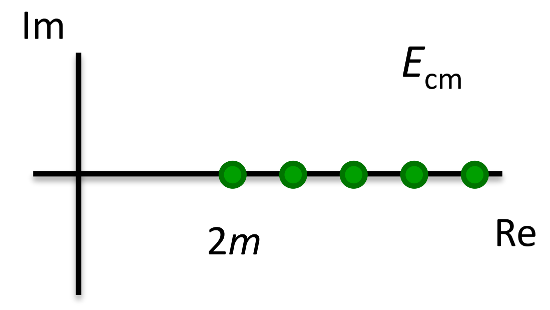
Infinite volume – contin. spectrum above thresh.

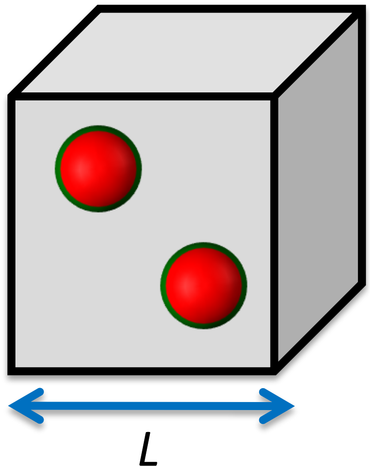


Scattering in Lattice QCD

Infinite volume – contin. spectrum above thresh.

Finite volume – discrete spectrum





Non-interacting: $\vec{k}_{A,B} = \frac{2\pi}{L}(n_x, n_y, n_z)$

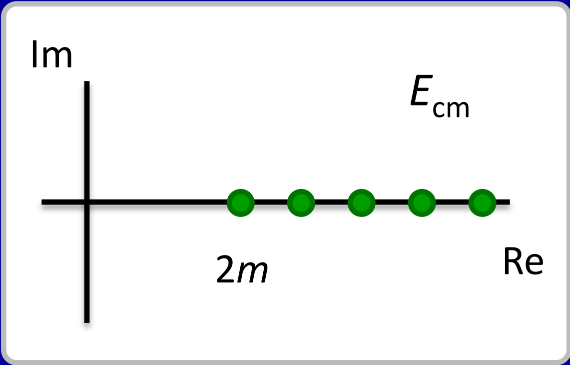
Interacting: $\vec{k}_{A,B} \neq \frac{2\pi}{L}(n_x, n_y, n_z)$

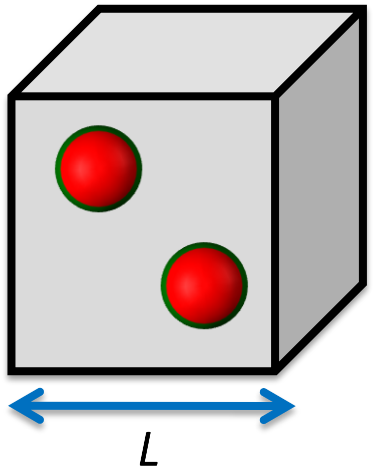
[periodic b.c.s]

Scattering in Lattice QCD

Infinite volume – contin. spectrum above thresh.

Finite volume – discrete spectrum





Non-interacting: $\vec{k}_{A,B} = \frac{2\pi}{L}(n_x, n_y, n_z)$

Interacting: $\vec{k}_{A,B} \neq \frac{2\pi}{L}(n_x, n_y, n_z)$

$$\mathrm{t}(E_{\mathrm{cm}}) = \begin{pmatrix} t_{\pi\pi\to\pi\pi}(E_{\mathrm{cm}}) & t_{\pi\pi\to K\bar{K}}(E_{\mathrm{cm}}) \\ t_{K\bar{K}\to\pi\pi}(E_{\mathrm{cm}}) & t_{K\bar{K}\to K\bar{K}}(E_{\mathrm{cm}}) \end{pmatrix}$$

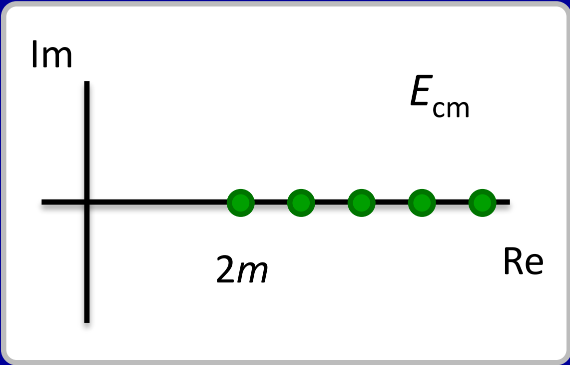
Lüscher method (and extensions): relate **finite-volume energy levels** to **infinite-volume scattering *t*-matrix**.

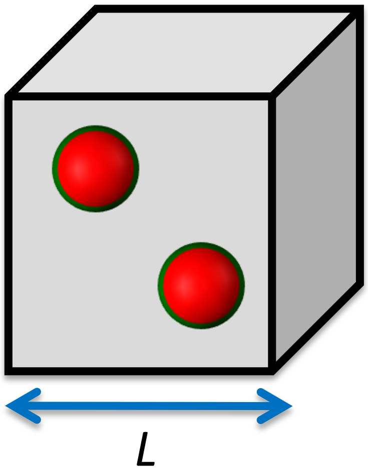
Elastic scattering: 1-to-1 correspondence (ignoring partial-wave mixing).
But **in general under-constrained problem** (determinant equ. at each E_{cm})
→ parameterize E_{cm} dependence of *t*-matrix and fit $\{E_{lat}\}$ to $\{E_{param}\}$
Consider many different parameterizations (e.g. *K*-matrix, eff. range, B.W.)

Scattering in Lattice QCD

Infinite volume – contin. spectrum above thresh.

Finite volume – discrete spectrum





Non-interacting: $\vec{k}_{A,B} = \frac{2\pi}{L}(n_x, n_y, n_z)$

Interacting: $\vec{k}_{A,B} \neq \frac{2\pi}{L}(n_x, n_y, n_z)$

$$\mathrm{t}(E_{\mathrm{cm}}) = \begin{pmatrix} t_{\pi\pi\to\pi\pi}(E_{\mathrm{cm}}) & t_{\pi\pi\to K\bar{K}}(E_{\mathrm{cm}}) \\ t_{K\bar{K}\to\pi\pi}(E_{\mathrm{cm}}) & t_{K\bar{K}\to K\bar{K}}(E_{\mathrm{cm}}) \end{pmatrix}$$

Lüscher method (and extensions): relate **finite-volume energy levels** to **infinite-volume scattering *t*-matrix**.

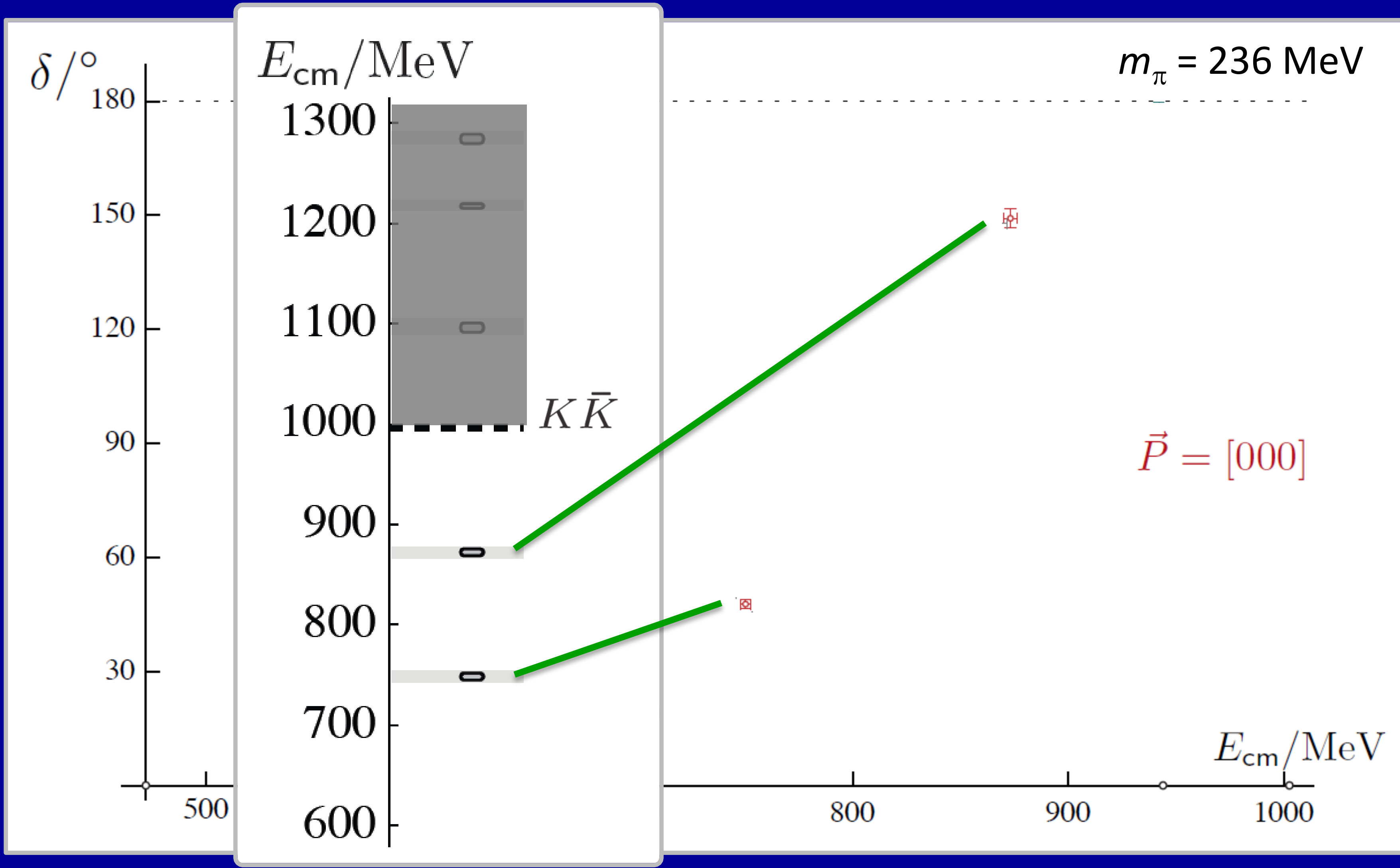
Elas .
But cm)
Con V.)

Currently limited to hadron-hadron scattering – progress being made on formalism for channels with > 2 hadrons.

Recent review in Briceño, Dudek, Young [arXiv:1706.06223]

The ρ resonance: **elastic** ππ scattering

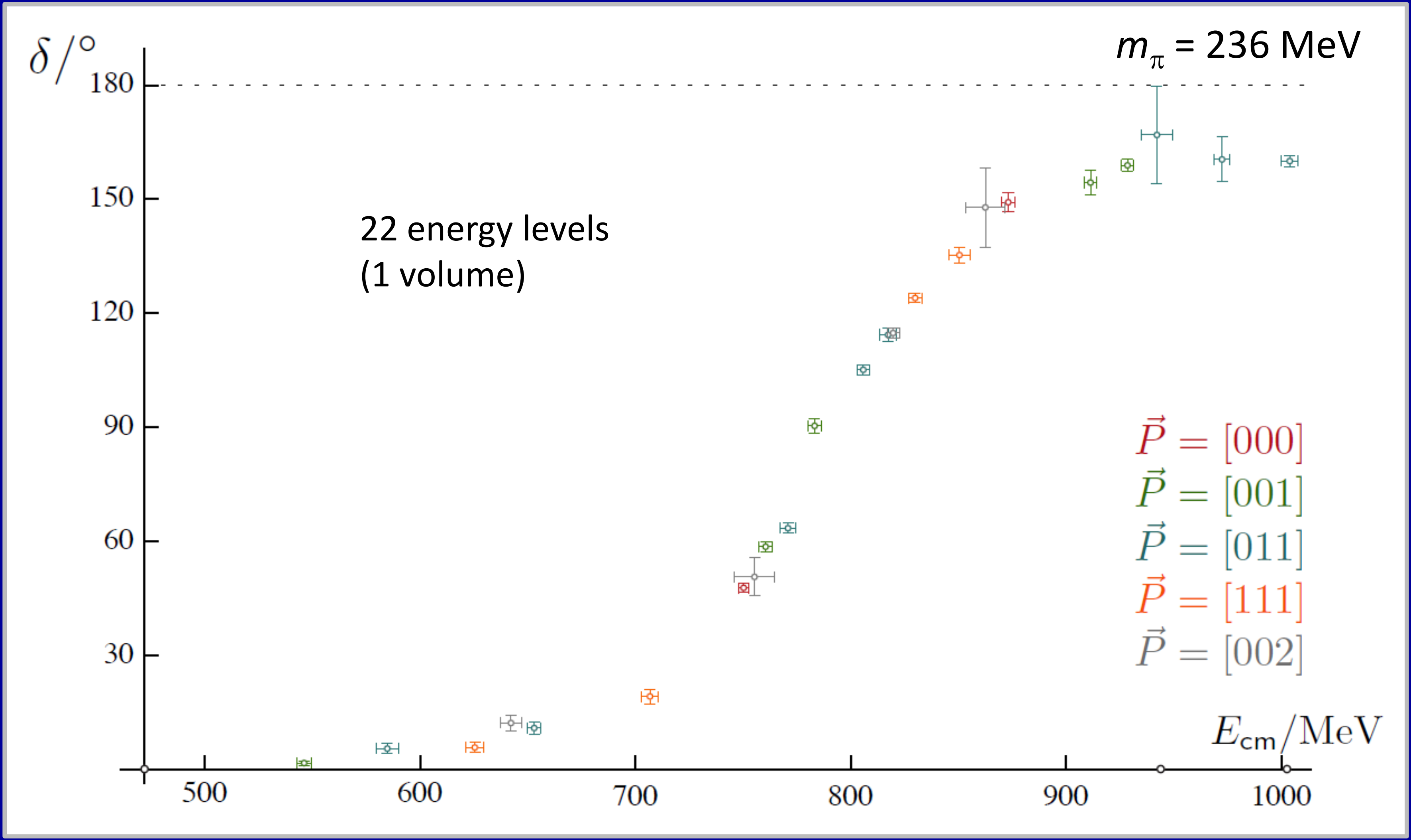
($J^{PC} = 1^{--}$, I = 1)



(HadSpec) [PR D87, 034505 (2013); PR D92, 094502 (2015)]

The ρ resonance: **elastic** ππ scattering

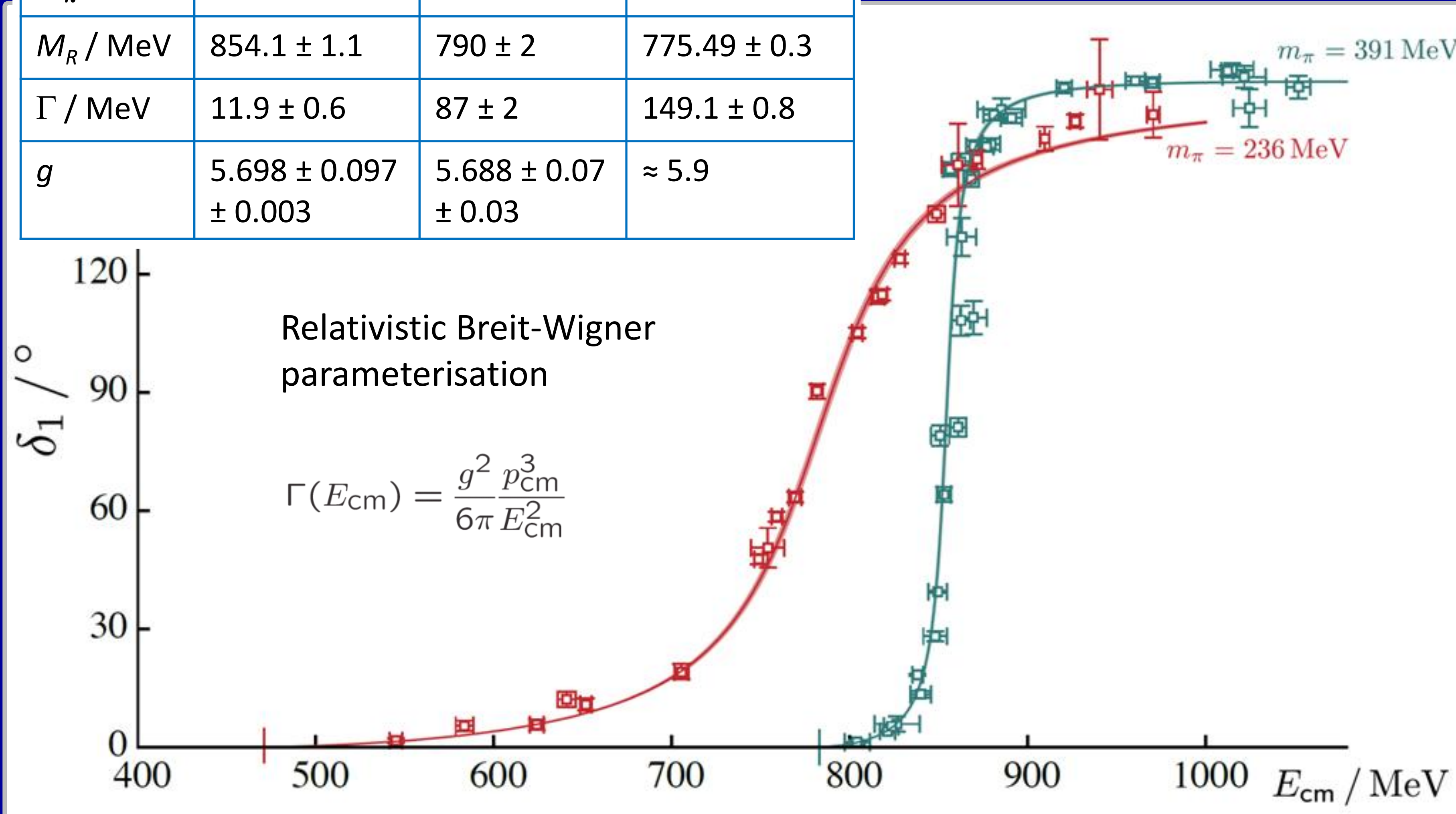
($J^{PC} = 1^{--}$, I = 1)



(HadSpec) [PR D87, 034505 (2013); PR D92, 094502 (2015)]

The ρ resonance: **elastic** ππ scattering

m_π / MeV	391	236	Experimental
M_R / MeV	854.1 ± 1.1	790 ± 2	775.49 ± 0.3
Γ / MeV	11.9 ± 0.6	87 ± 2	149.1 ± 0.8
g	5.698 ± 0.097 ± 0.003	5.688 ± 0.07 ± 0.03	≈ 5.9

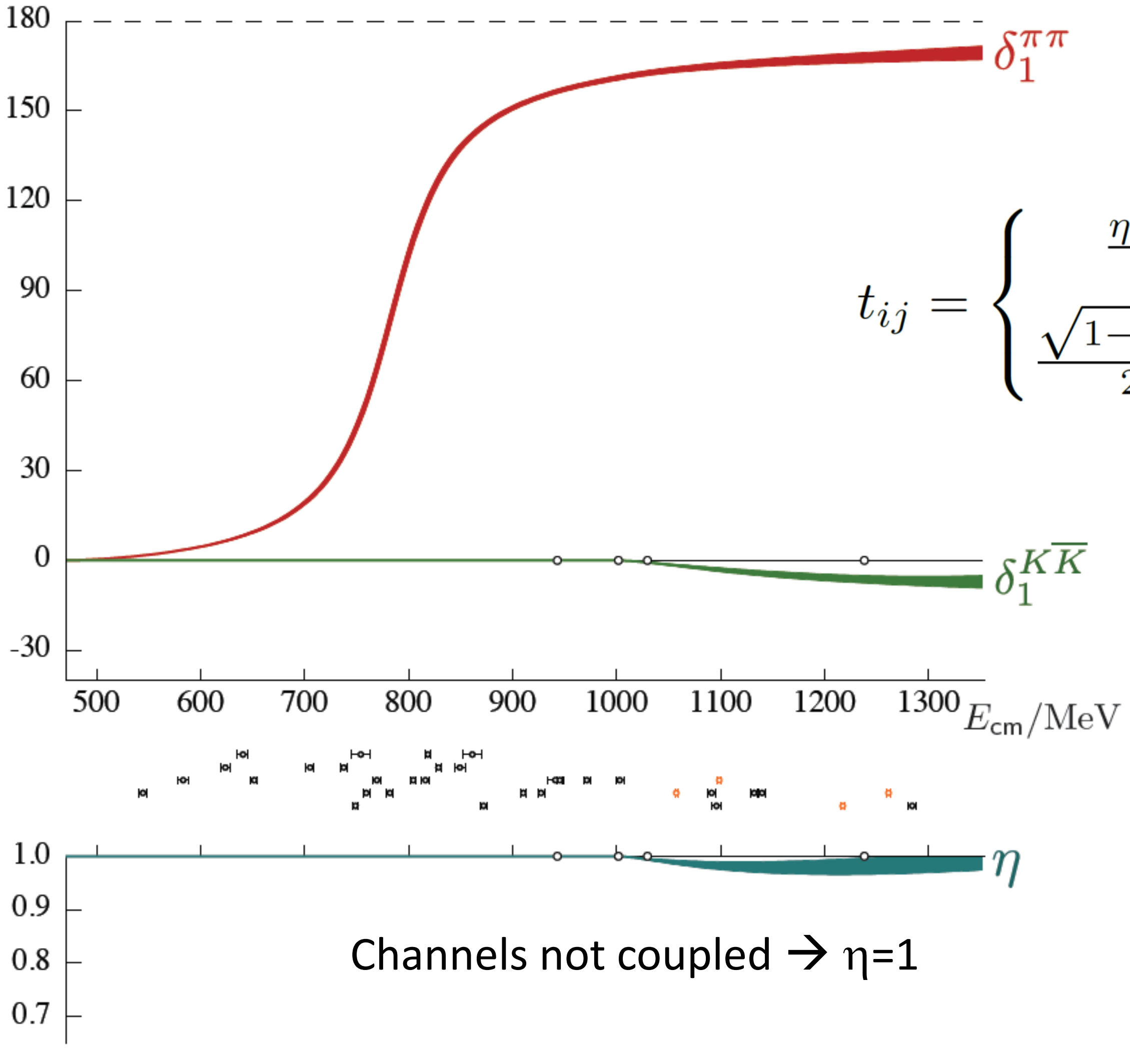


Relativistic Breit-Wigner parameterisation

$$\Gamma(E_{\mathsf{cm}}) = \frac{g^2}{6\pi} \frac{p_{\mathsf{cm}}^3}{E_{\mathsf{cm}}^2}$$

(HadSpec) [PR D87, 034505 (2013); PR D92, 094502 (2015)]

The ρ resonance: **coupled-channel** $\pi\pi, K\bar{K}$



$$t_{ij} = \begin{cases} \dfrac{\eta\, e^{2i\delta_i} - 1}{2i\,\rho_i} & (i = j) \\ \dfrac{\sqrt{1-\eta^2}\, e^{i(\delta_i+\delta_j)}}{2\sqrt{\rho_i\,\rho_j}} & (i \neq j) \end{cases}$$

Wilson *et al* (HadSpec) [PR D92, 094502 (2015)]

The ρ resonance: **elastic** $\pi\pi$ scattering: other calcs.

Some other recent lattice QCD calculations:

- Bali *et al* (RQCD) [PR D93, 054509 (2016)]: $m_\pi \approx 150$ MeV ($N_f = 2$)
- Bulava *et al* [NP B910, 842 (2016)]: $m_\pi \approx 240$ MeV
- Guo *et al* [PR D94, 034501 (2016)]: $m_\pi = 315, 226$ MeV ($N_f = 2$)
- Fu & Wang [PR D94, 034505 (2016)]: $m_\pi = 176 – 346$ MeV
- Alexandrou *et al* [arXiv:1704.05439]: $m_\pi = 317$ MeV

- Also chiral extrapolations/analyses including lattice data:
    - Bolton, Briceño, Wilson [PL B757, 50 (2016)]
    - Hu *et al* [PRL 117, 122001 (2016)]
    - Hu *et al* [PR D96, 034520 (2017)]

The ρ resonance: **elastic** ππ scattering: other calcs.

s:

6)]: $m_\pi \approx 150$ MeV ($N_f = 2$)

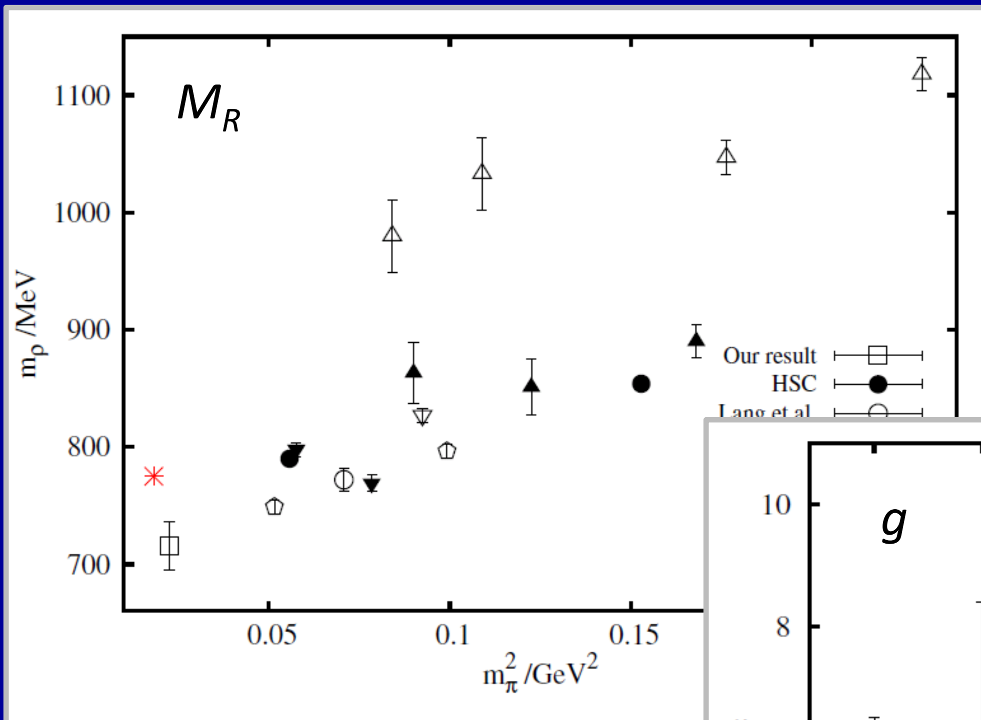
≈ 240 MeV

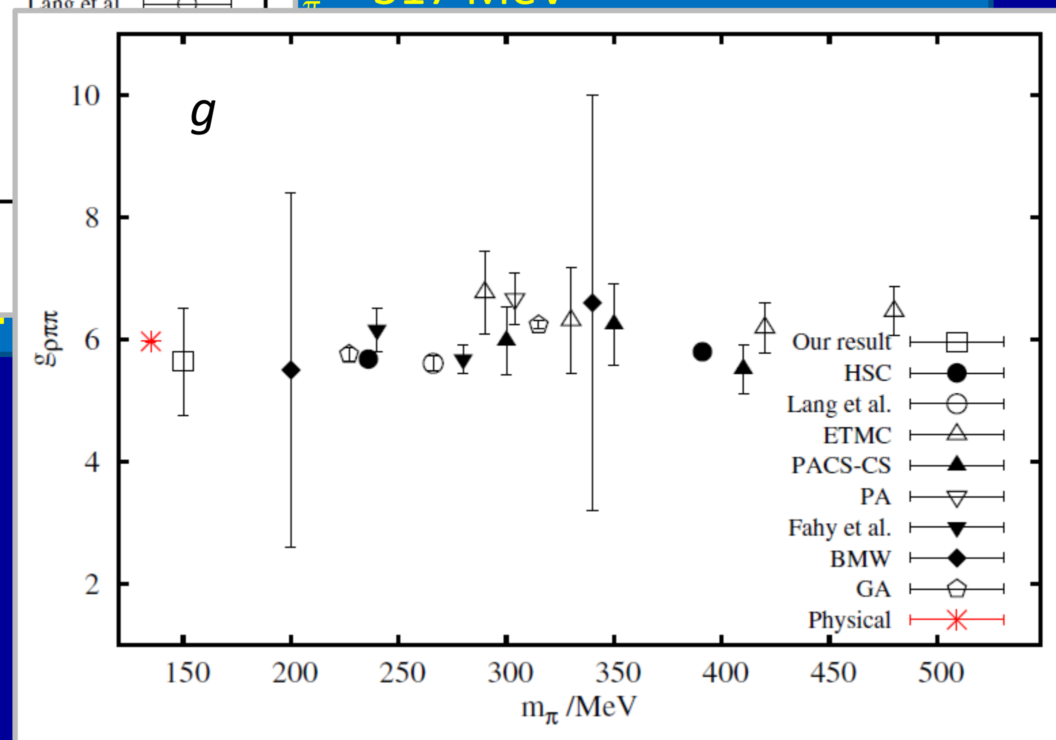
= 315, 226 MeV ($N_f = 2$)

π = 176 – 346 MeV

π = 317 MeV

Hu *et al* [PR D93, 05

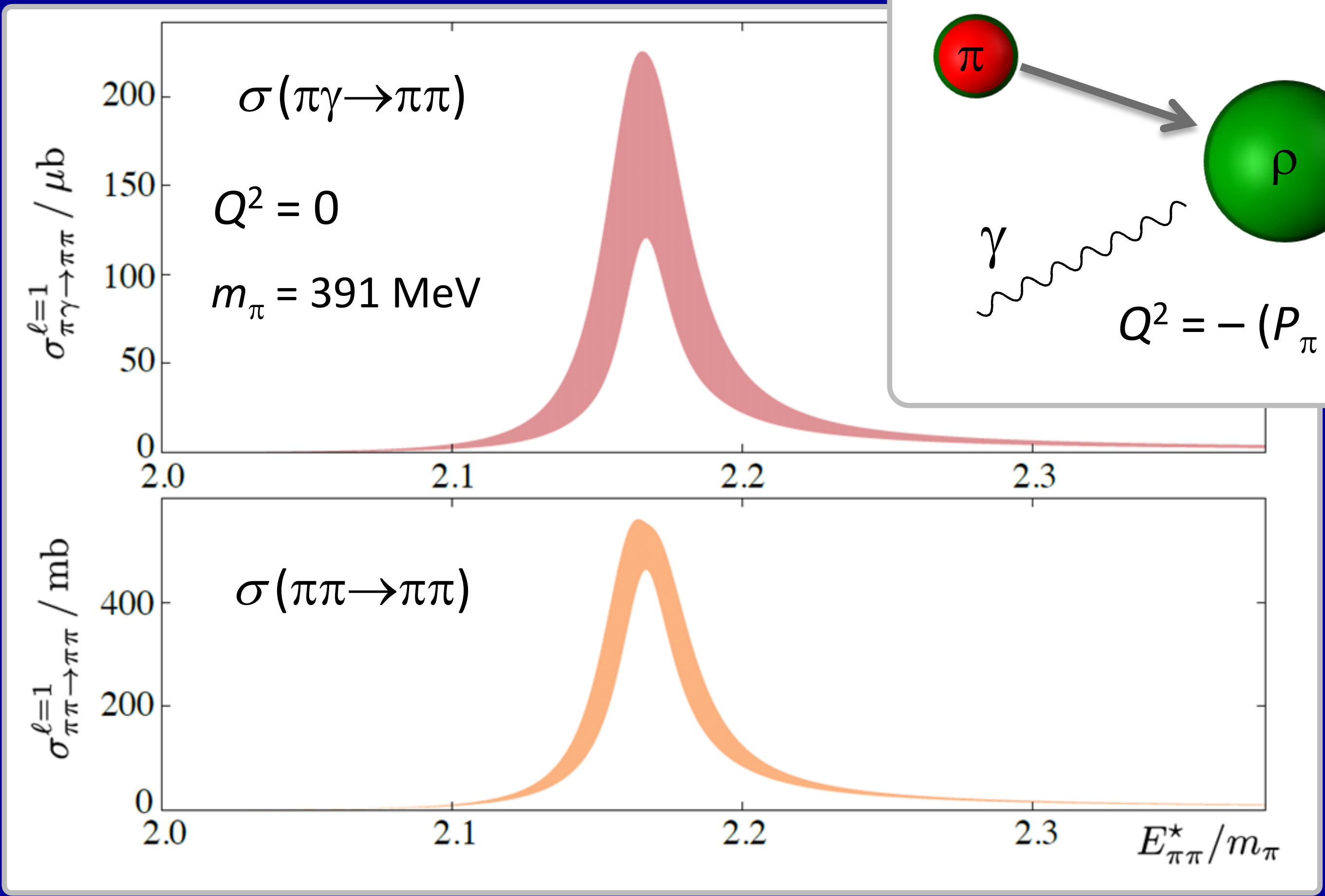


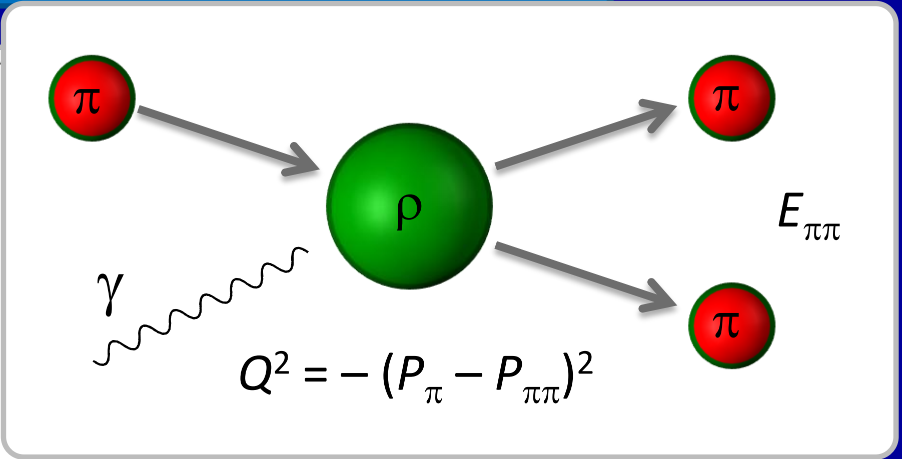


Bali *et al* (RQCD)
[PR D93, 054509 (2016)]

Resonant $\pi^+ \gamma \to \rho \to \pi^+ \pi^0$ amplitude

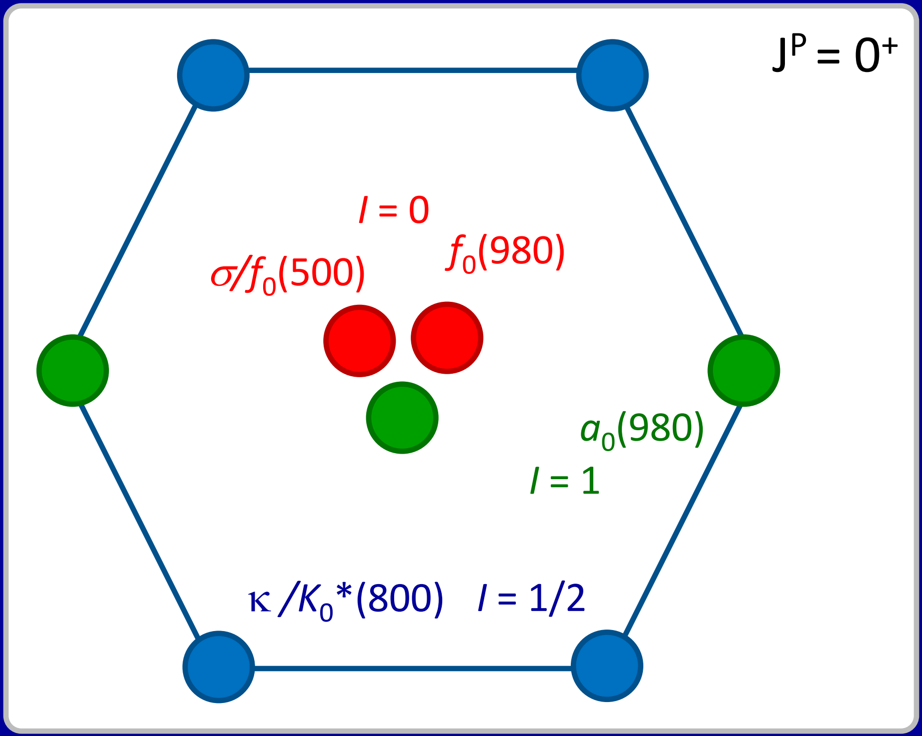
Need: $C_{ij}(t_f, t, t_i) = \langle 0|O_i(t_f)\ \bar{\psi}(t)\gamma^\mu\psi(t)\ O_j(t_i)|0\rangle$



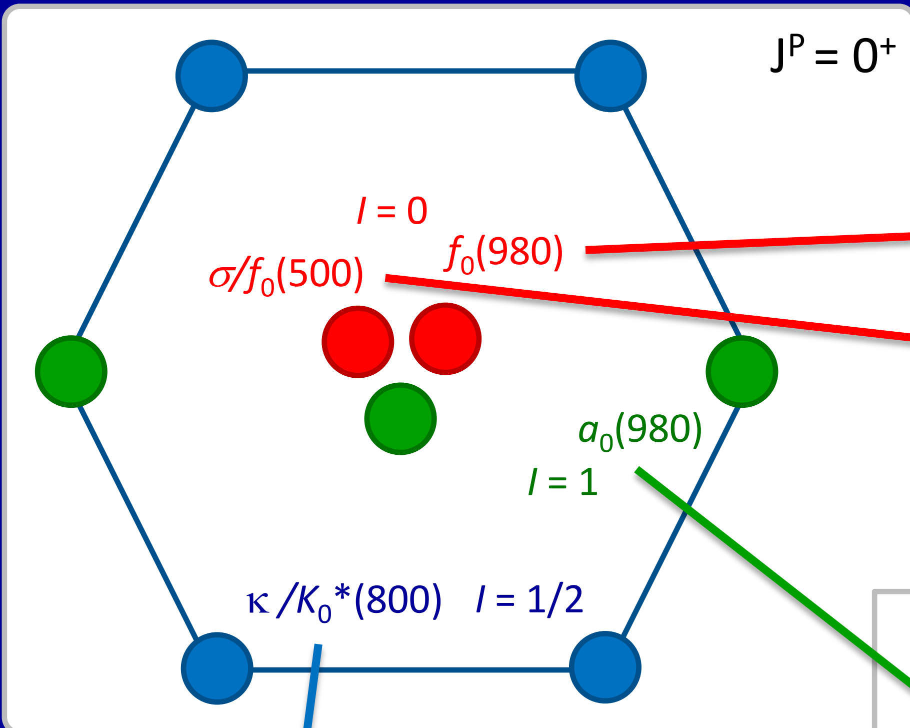


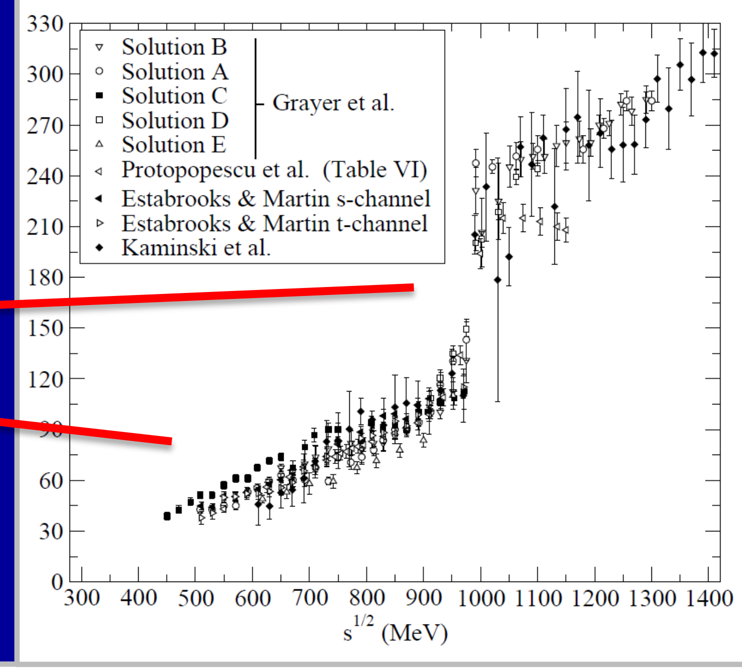
Briceño *et al* (HadSpec) [PRL 115, 242001 (2015); PRD 93, 114508 (2016)]

Light scalar mesons (< 1 GeV)

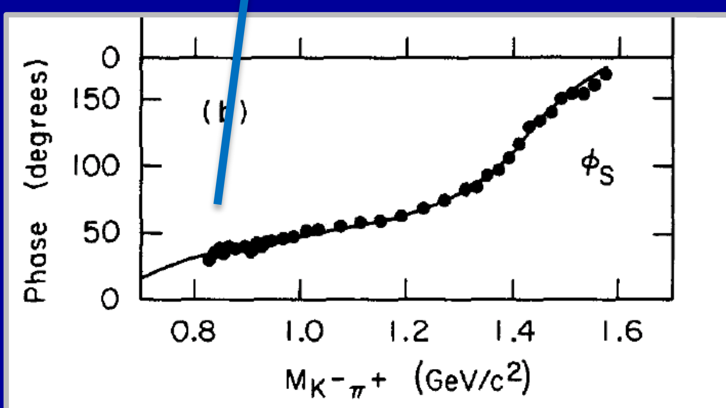


Light scalar mesons (< 1 GeV)

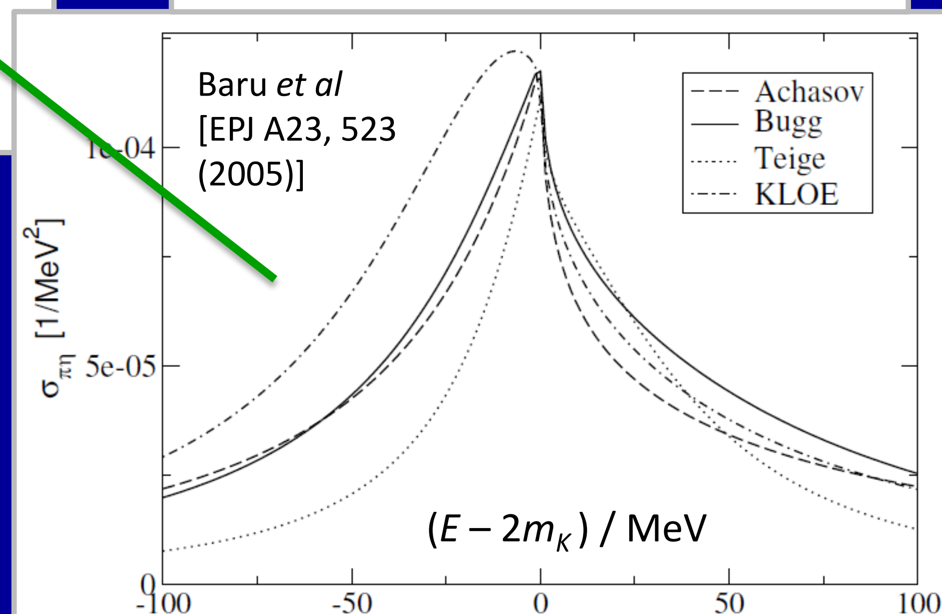




Pelaez [Phys. Rep. 658, 1 (2016)]



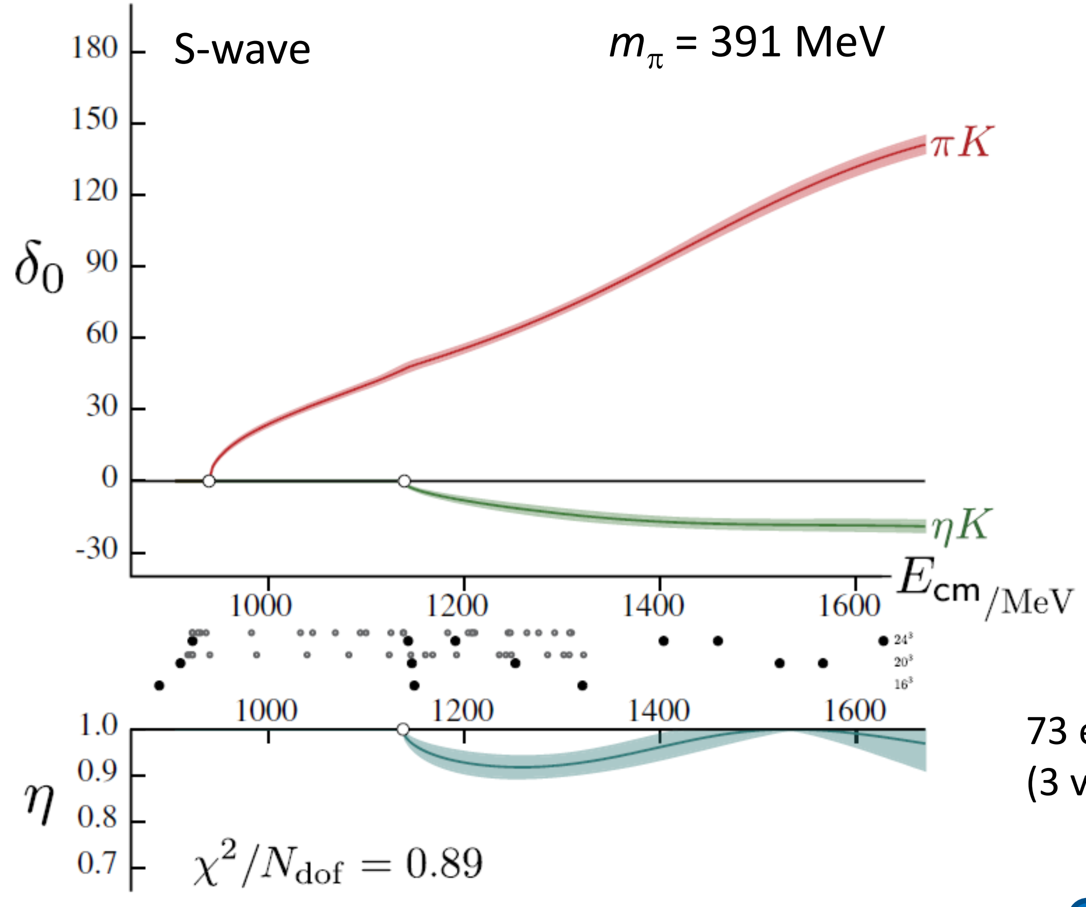
Aston *et al* (LASS) [NP B296, 493 (1988)]



Baru *et al* [EPJ A23, 523 (2005)]

κ in πK, ηK

$J^P = 0^+$, Isospin = ½, Strangeness = 1

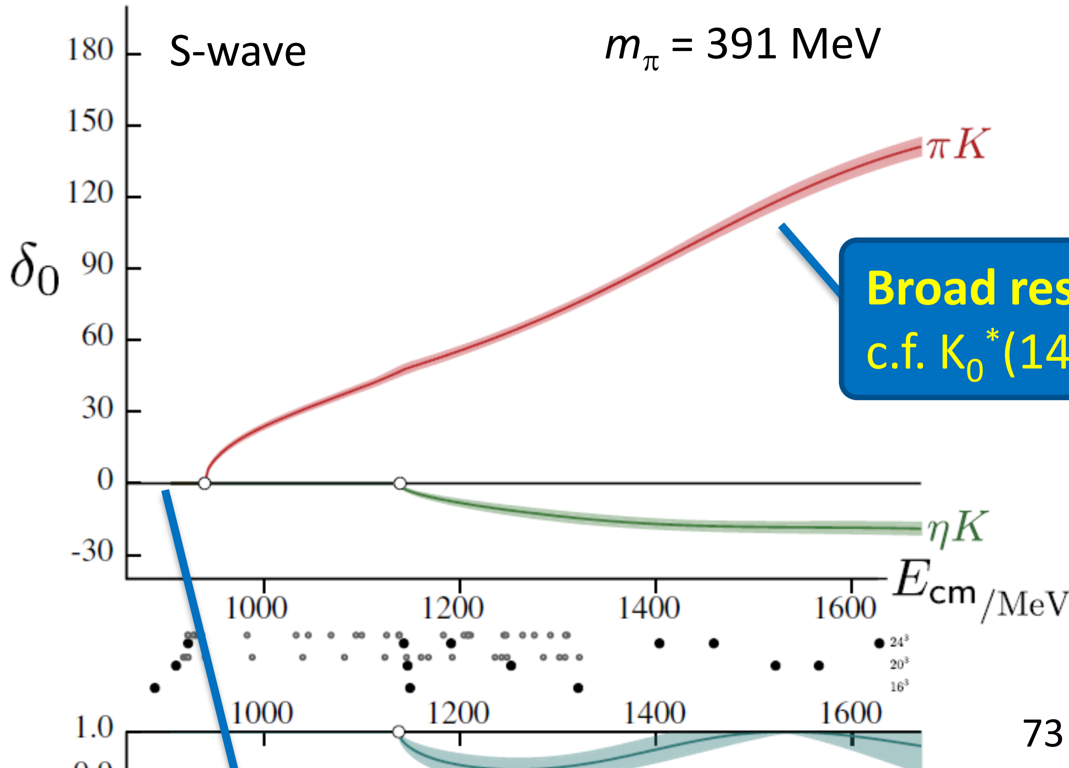


73 energy levels
(3 volumes)

Wilson, Dudek, Edwards, CT (HadSpec) [PRL 113, 182001 (2014); PR D91, 054008 (2015)]

κ in πK, ηK

$J^P = 0^+$, Isospin = ½, Strangeness = 1



Broad resonance
c.f. $K_0^*(1430)$

73 energy levels
(3 volumes)

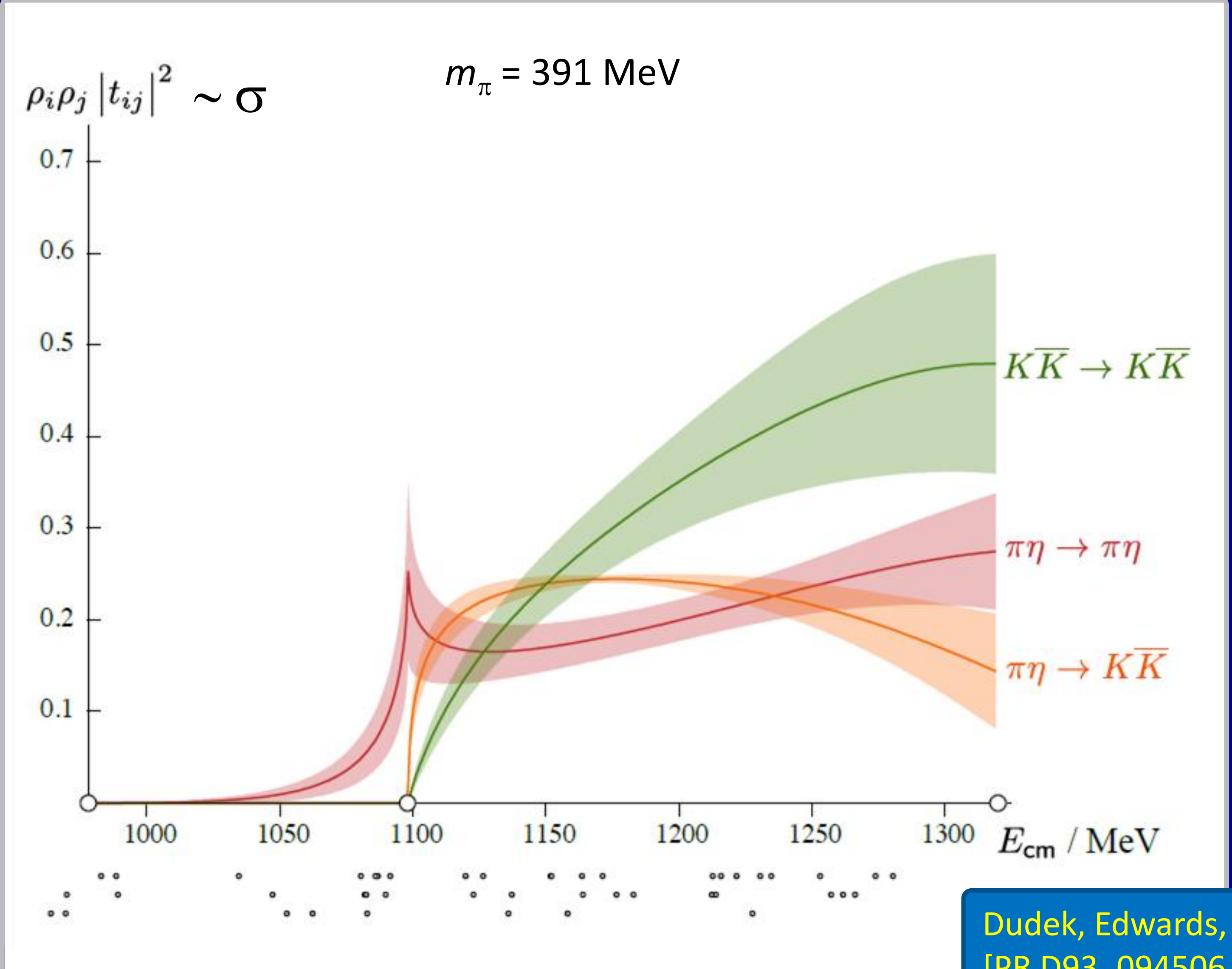
Virtual bound state [pole on real axis below threshold on unphysical sheet]

c.f. κ in unitarised χpt [Nebreda & Pelaez, PR D81, 054035 (2010)]

Wilson, Dudek, Edwards, CT (HadSpec) [PRL 113, 182001 (2014); PR D91, 054008 (2015)]

a_0 in $\pi\eta,\ K\bar{K}$

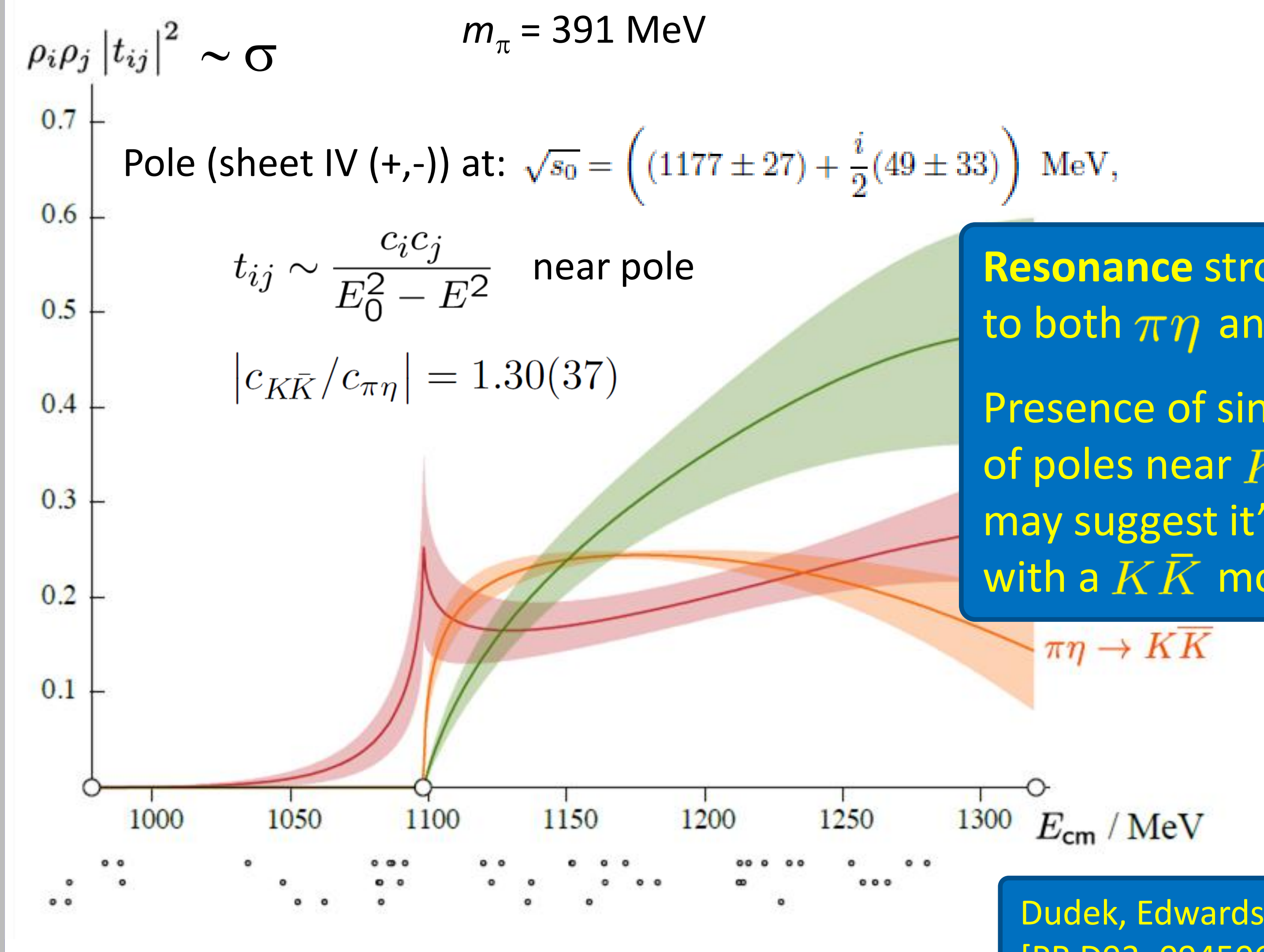
$J^P = 0^+,\ I = 1$



Dudek, Edwards, Wilson (HadSpec) [PR D93, 094506 (2016)]

a_0 in $\pi\eta$, $K\bar{K}$

$J^P = 0^+$, $I = 1$



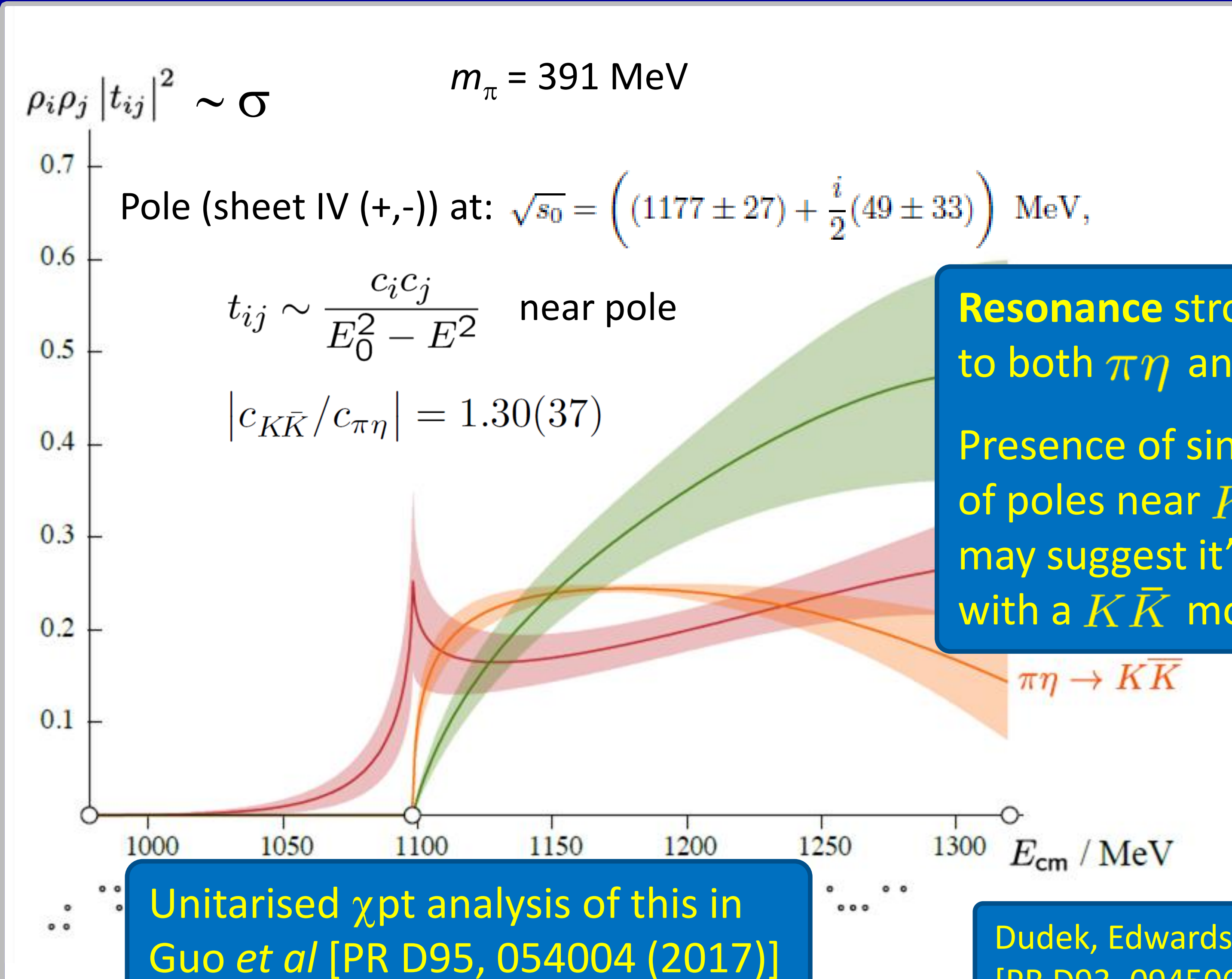
Resonance strongly coupled to both $\pi\eta$ and $K\bar{K}$

Presence of single conj. pair of poles near $K\bar{K}$ threshold may suggest it's associated with a $K\bar{K}$ molecular config.

Dudek, Edwards, Wilson (HadSpec) [PR D93, 094506 (2016)]

a_0 in $\pi\eta,\ K\bar{K}$

$J^P = 0^+, I = 1$



Resonance strongly coupled to both $\pi\eta$ and $K\bar{K}$

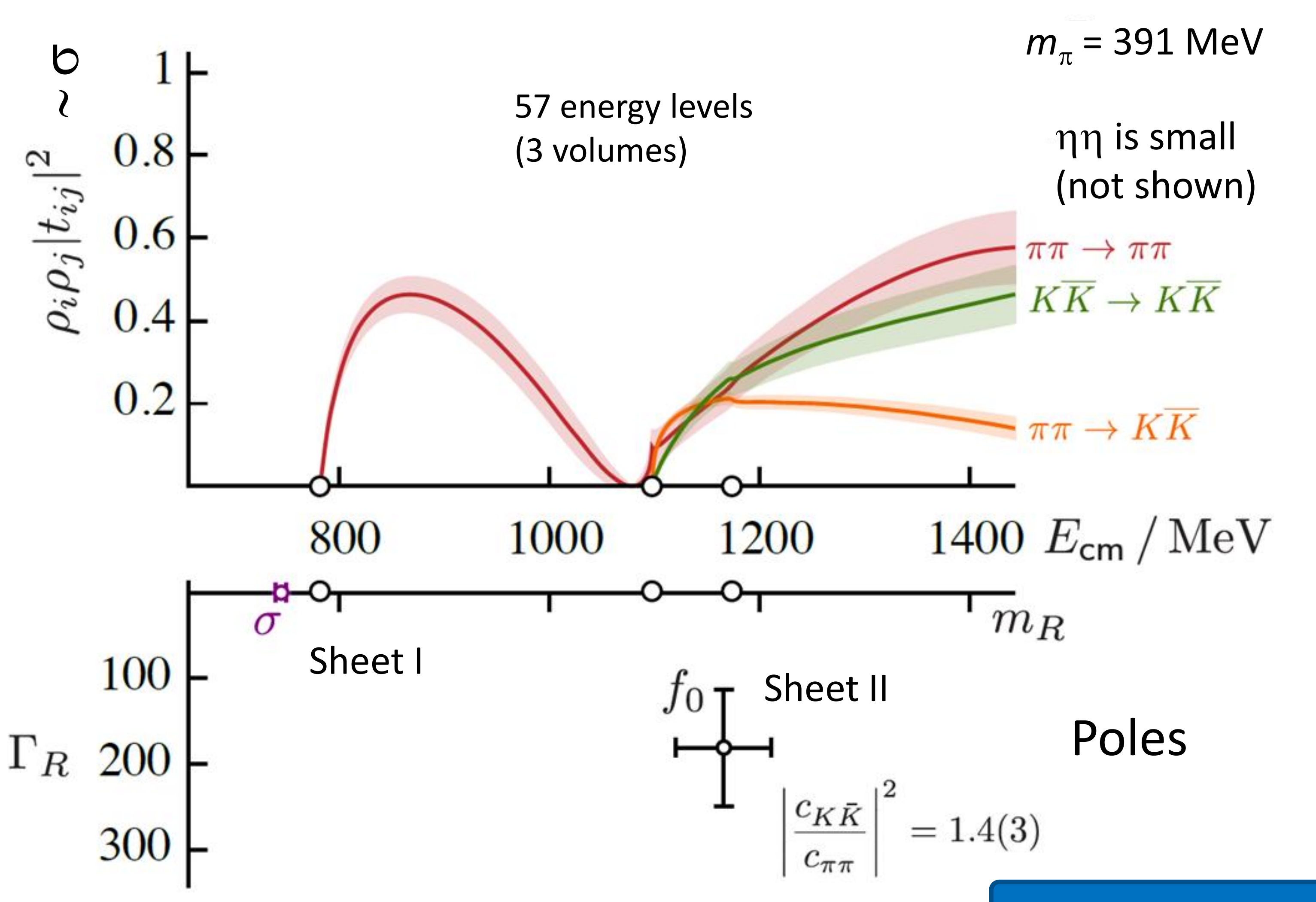
Presence of single conj. pair of poles near $K\bar{K}$ threshold may suggest it's associated with a $K\bar{K}$ molecular config.

Unitarised χpt analysis of this in Guo *et al* [PR D95, 054004 (2017)]

Dudek, Edwards, Wilson (HadSpec) [PR D93, 094506 (2016)]

σ, f_0(980) in $\pi\pi$, $K\bar{K}$, ηη

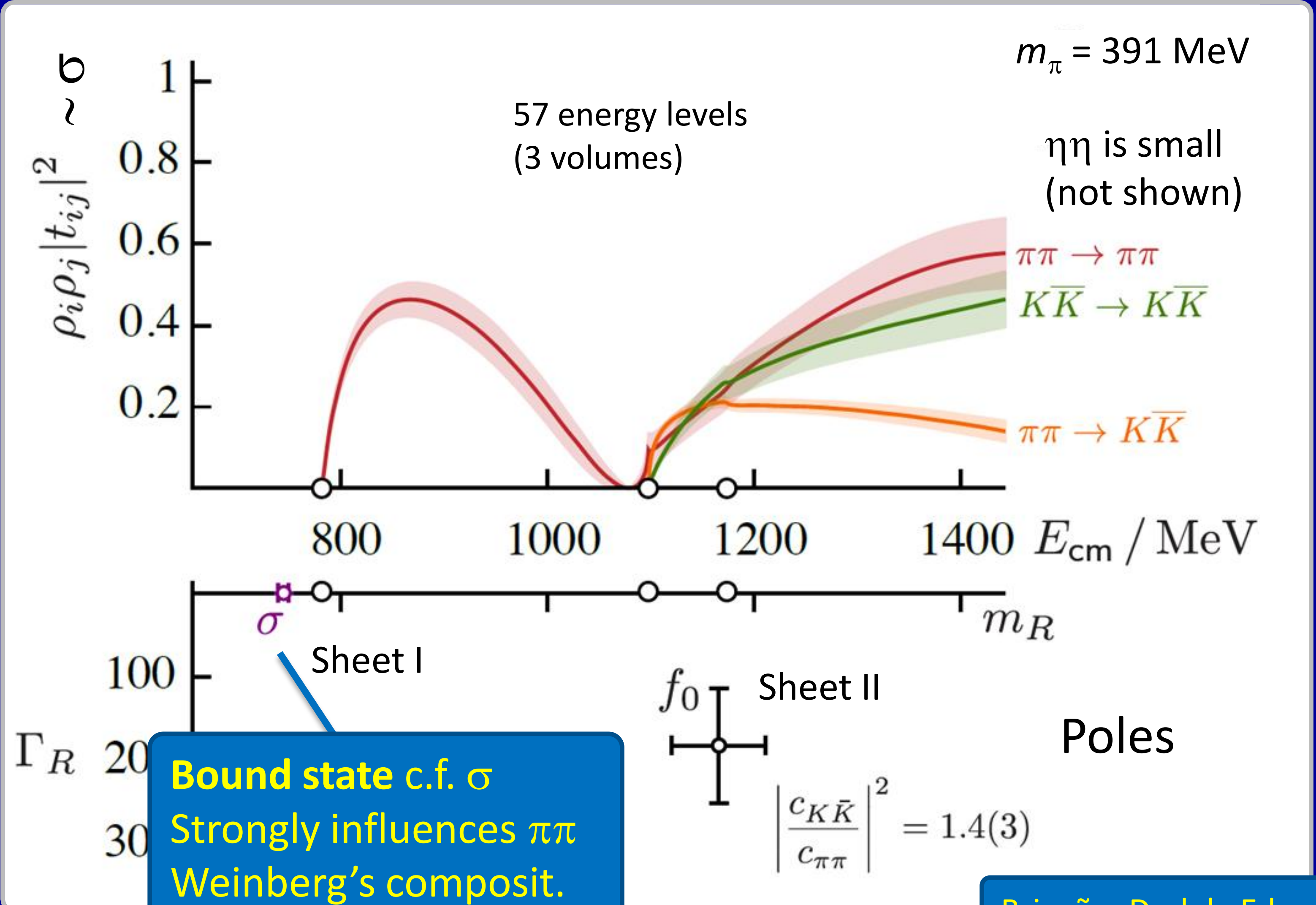
$J^P = 0^+$, I = 0



Briceño, Dudek, Edwards, Wilson (HadSpec) [arXiv:1708.06667]

σ, f_0(980) in $\pi\pi, K\bar{K}$, ηη

$J^P = 0^+, I = 0$



Bound state c.f. σ
Strongly influences ππ
Weinberg's composit. criterion: $Z \sim 0.3(1)$

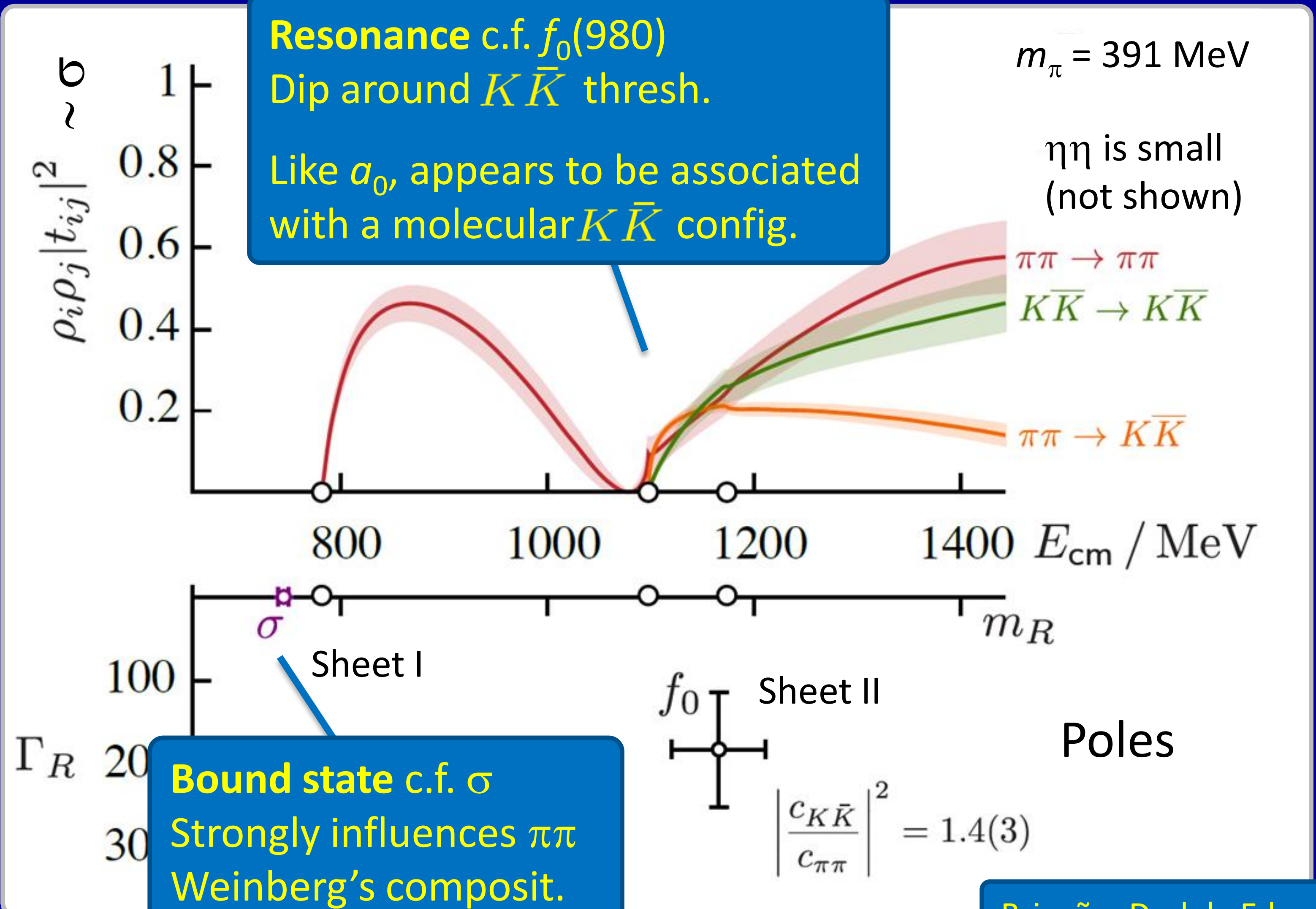
Briceño, Dudek, Edwards, Wilson (HadSpec) [arXiv:1708.06667]

σ, f_0(980) in ππ, $K\bar{K}$, ηη

$J^P = 0^+$, I = 0

Resonance c.f. f_0(980)
Dip around $K\bar{K}$ thresh.

Like a_0, appears to be associated with a molecular $K\bar{K}$ config.

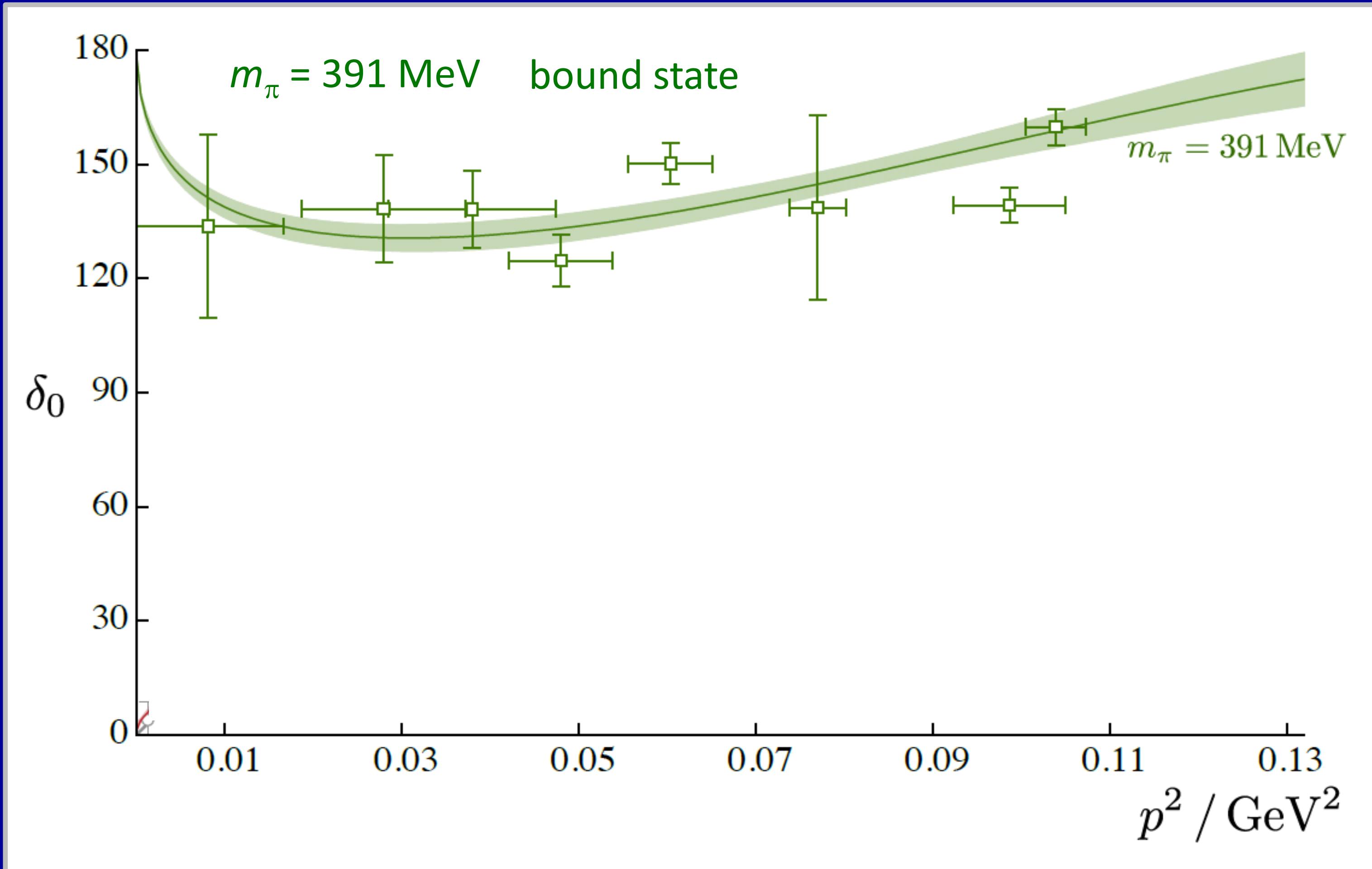


Bound state c.f. σ
Strongly influences ππ
Weinberg's composit. criterion: $Z \sim 0.3(1)$

Briceño, Dudek, Edwards, Wilson (HadSpec) [arXiv:1708.06667]

$f_0(500)/\sigma$ in elastic $\pi\pi$ scattering

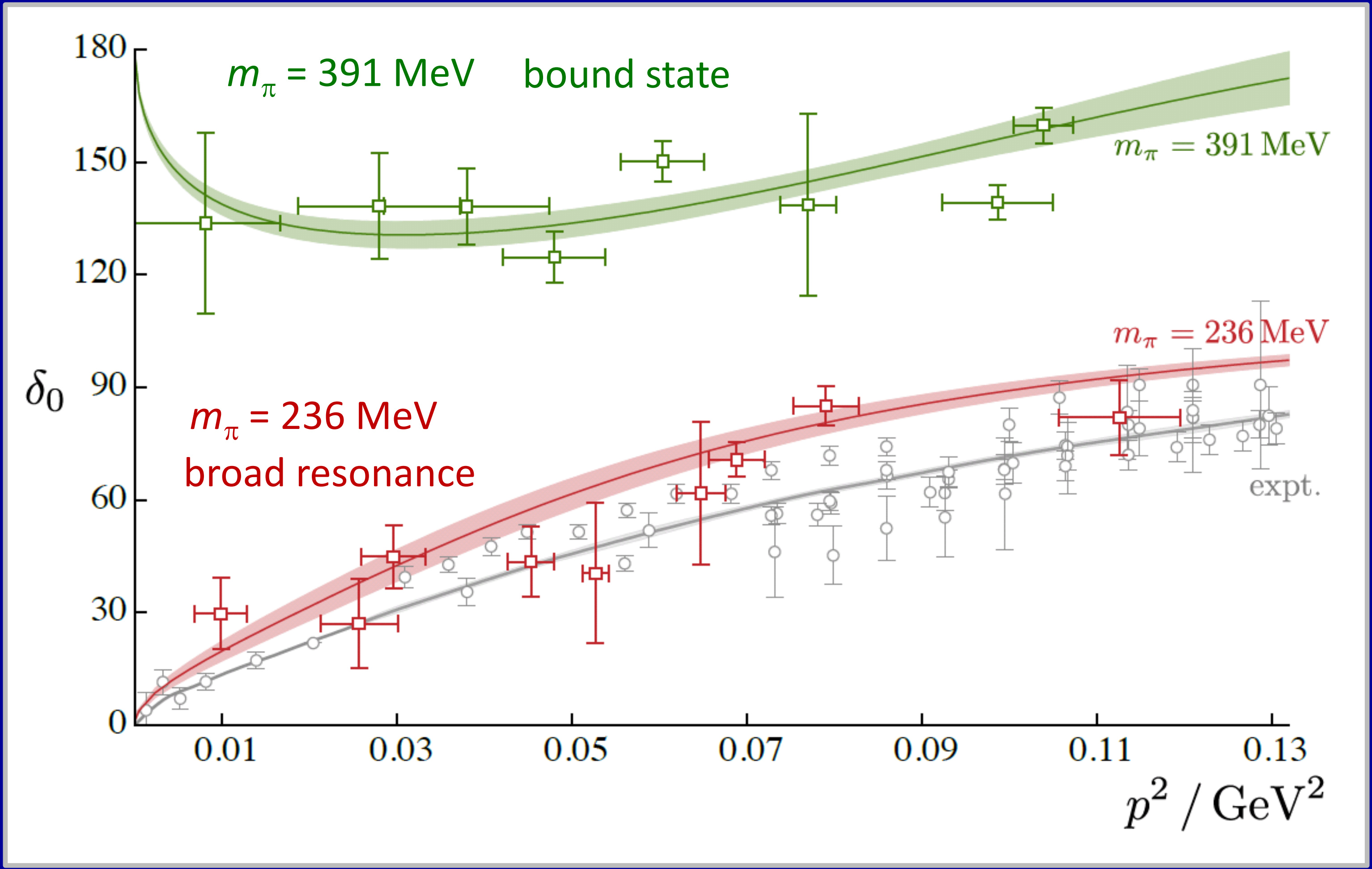
$J^P = 0^+, I = 0$



Briceño, Dudek, Edwards, Wilson (HadSpec) [PRL 118, 022002 (2017)]

$f_0(500)/\sigma$ in elastic $\pi\pi$ scattering

$J^P = 0^+$, $I = 0$

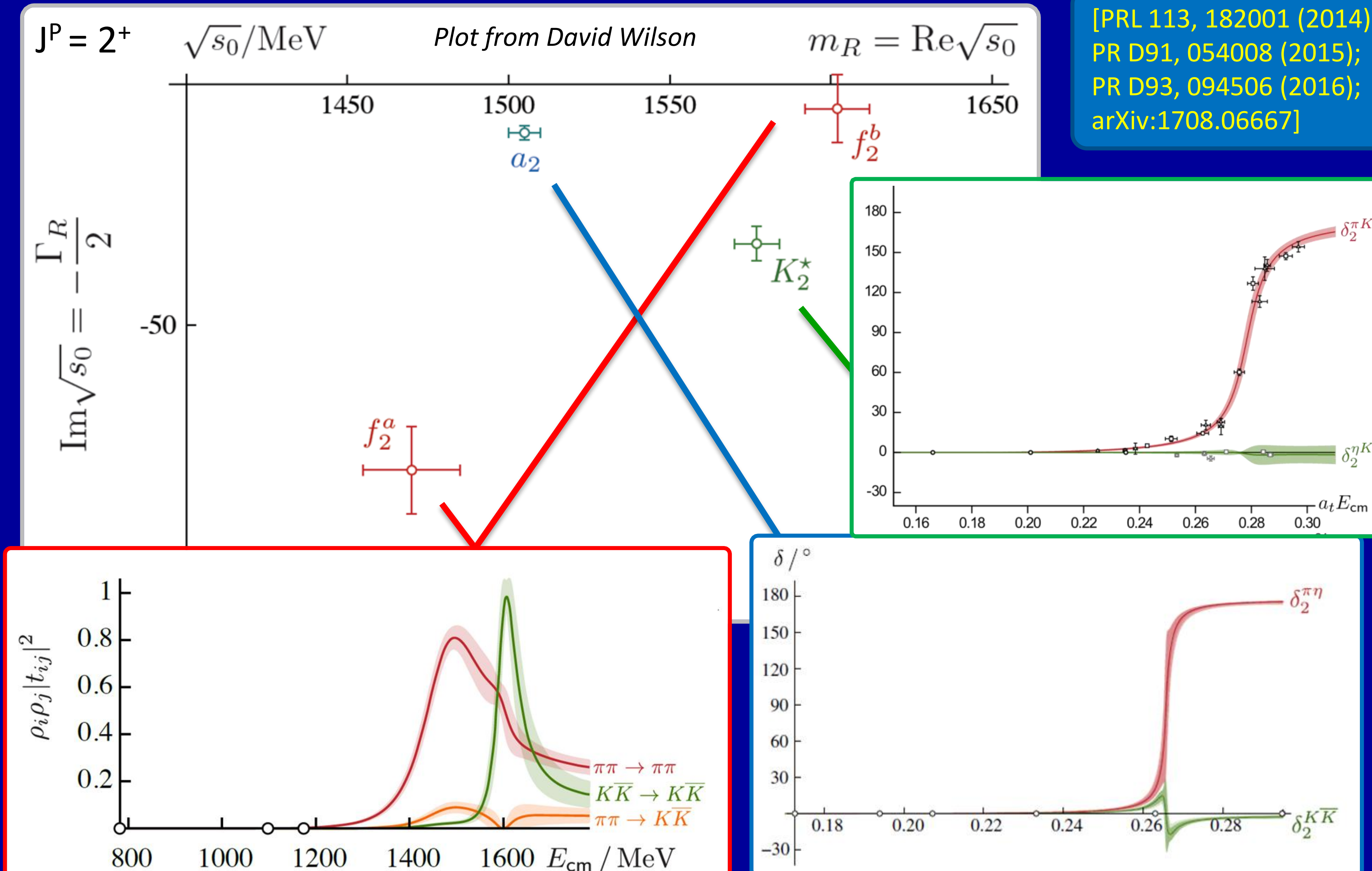


C.f. unitarised χpt in Hanart, Pelaez, Rios [PRL 100, 152001 (2008)]

Briceño, Dudek, Edwards, Wilson (HadSpec) [PRL 118, 022002 (2017)]

c.f. light tensor mesons with m_π = 391 MeV

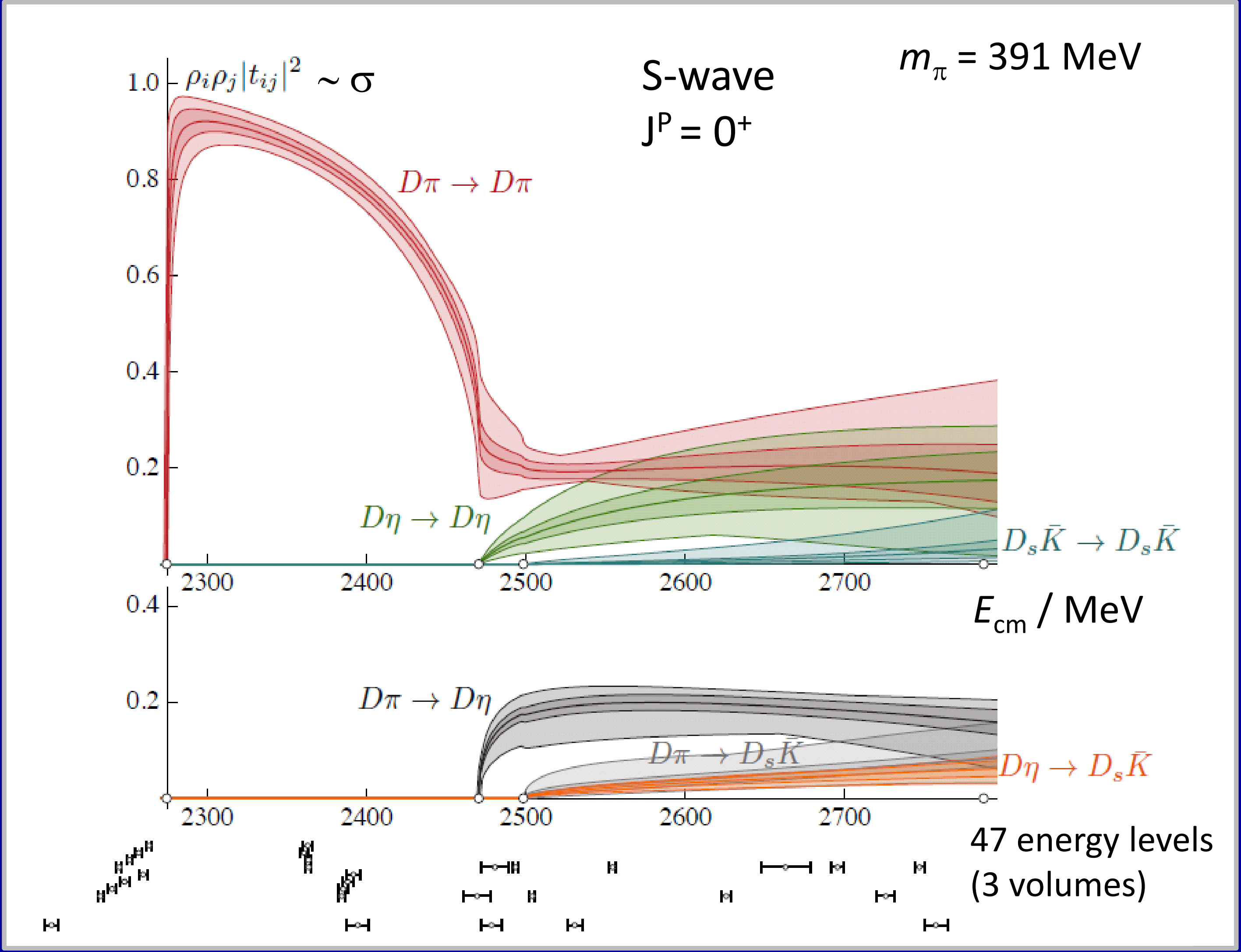
[PRL 113, 182001 (2014);
PR D91, 054008 (2015);
PR D93, 094506 (2016);
arXiv:1708.06667]



Charm-light (I=½): $D\pi$, $D\eta$, $D_s\bar{K}$

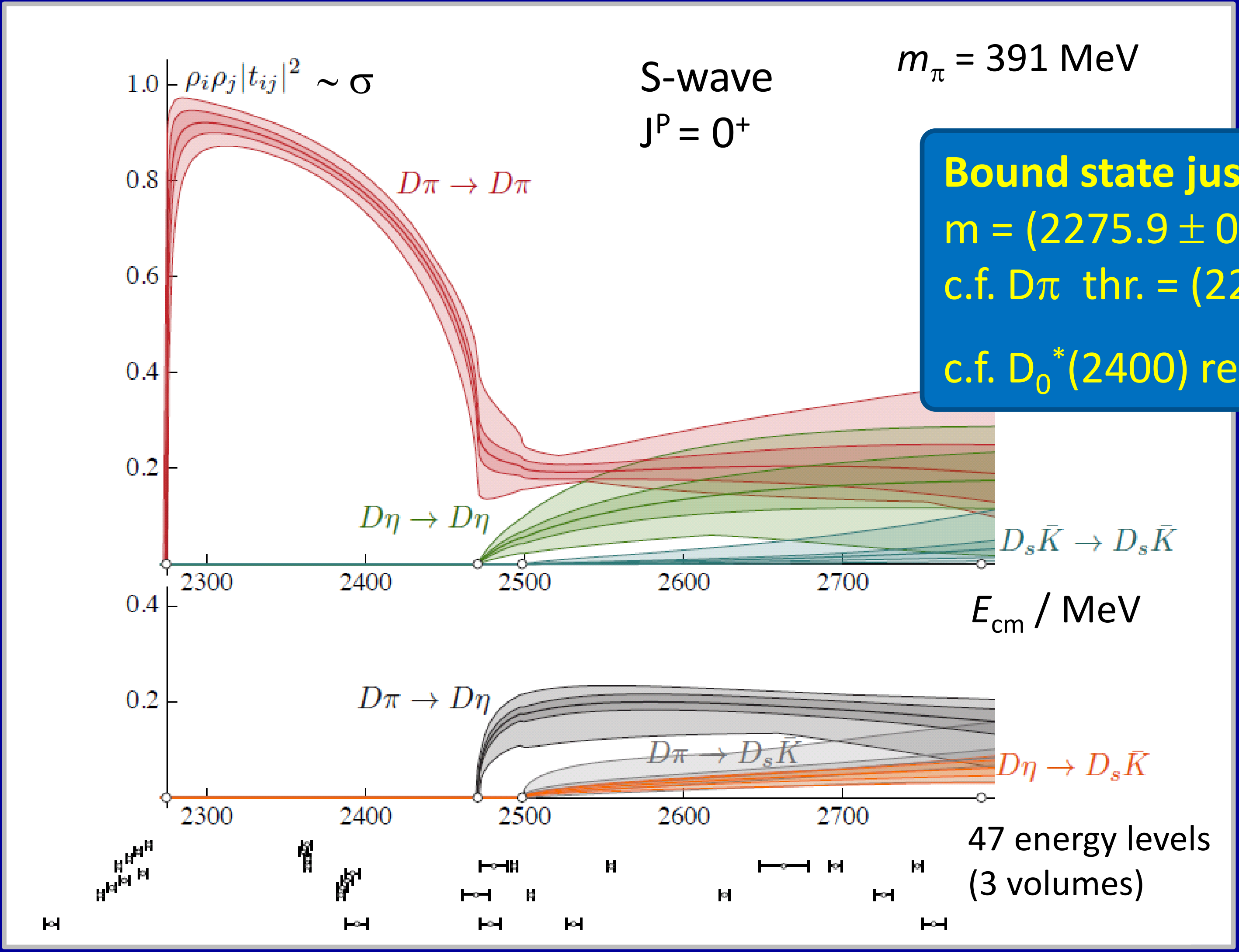
Charm-light (I=½): Dπ, Dη, $D_s\bar{K}$

Moir, Peardon, Ryan, CT, Wilson (HadSpec) [JHEP 1610, 011 (2016)]



Charm-light (I=½): Dπ, Dη, $D_s\bar{K}$

Moir, Peardon, Ryan, CT, Wilson (HadSpec) [JHEP 1610, 011 (2016)]



Bound state just below thresh.

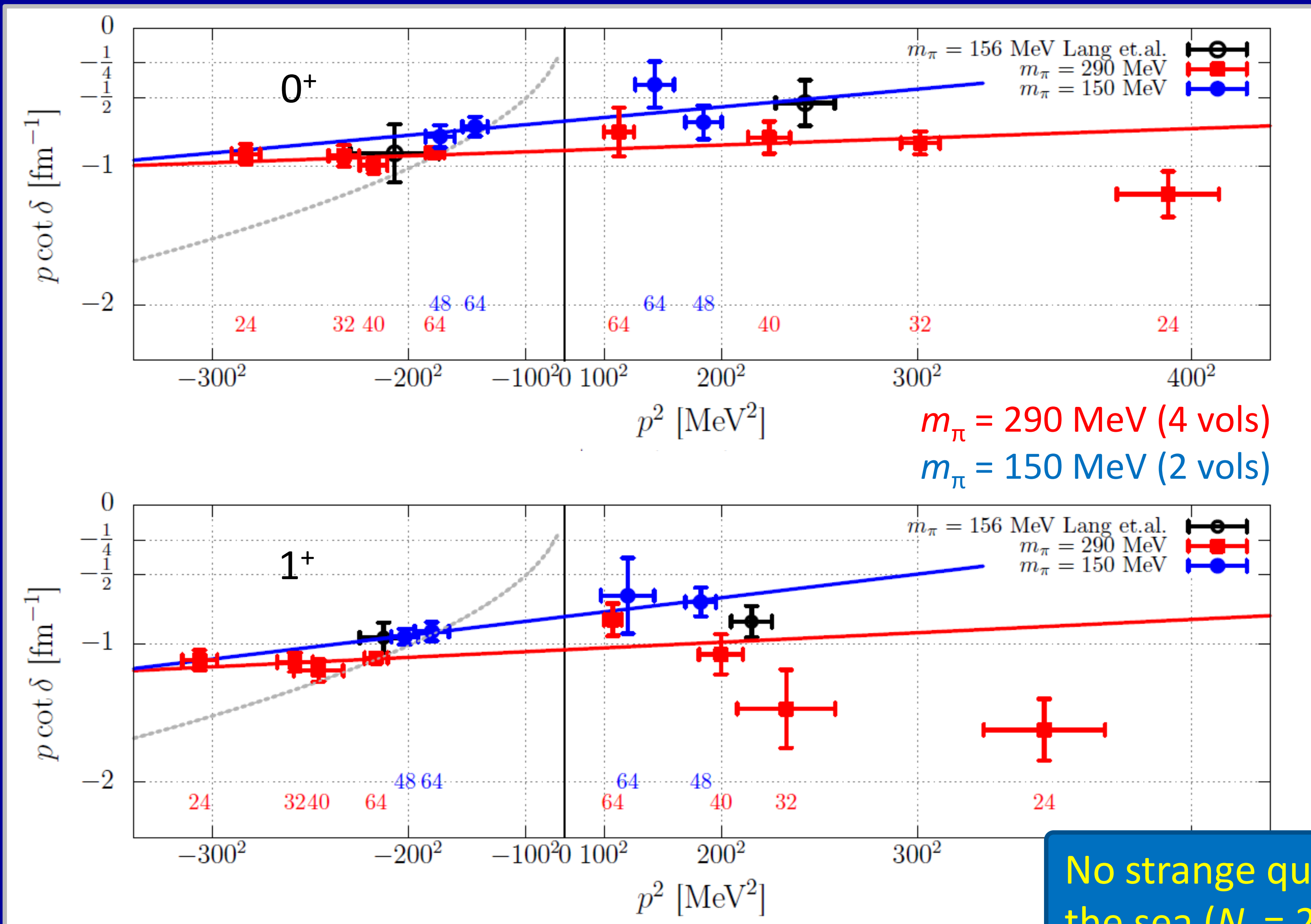
m = (2275.9 ± 0.9) MeV

c.f. Dπ thr. = (2276.4 ± 0.9) MeV

c.f. D_0^*(2400) resonance

Charm-strange (I=0): DK (0^+) and D*K (1^+)

Bali *et al* (RQCD) [arXiv:1706.01247]



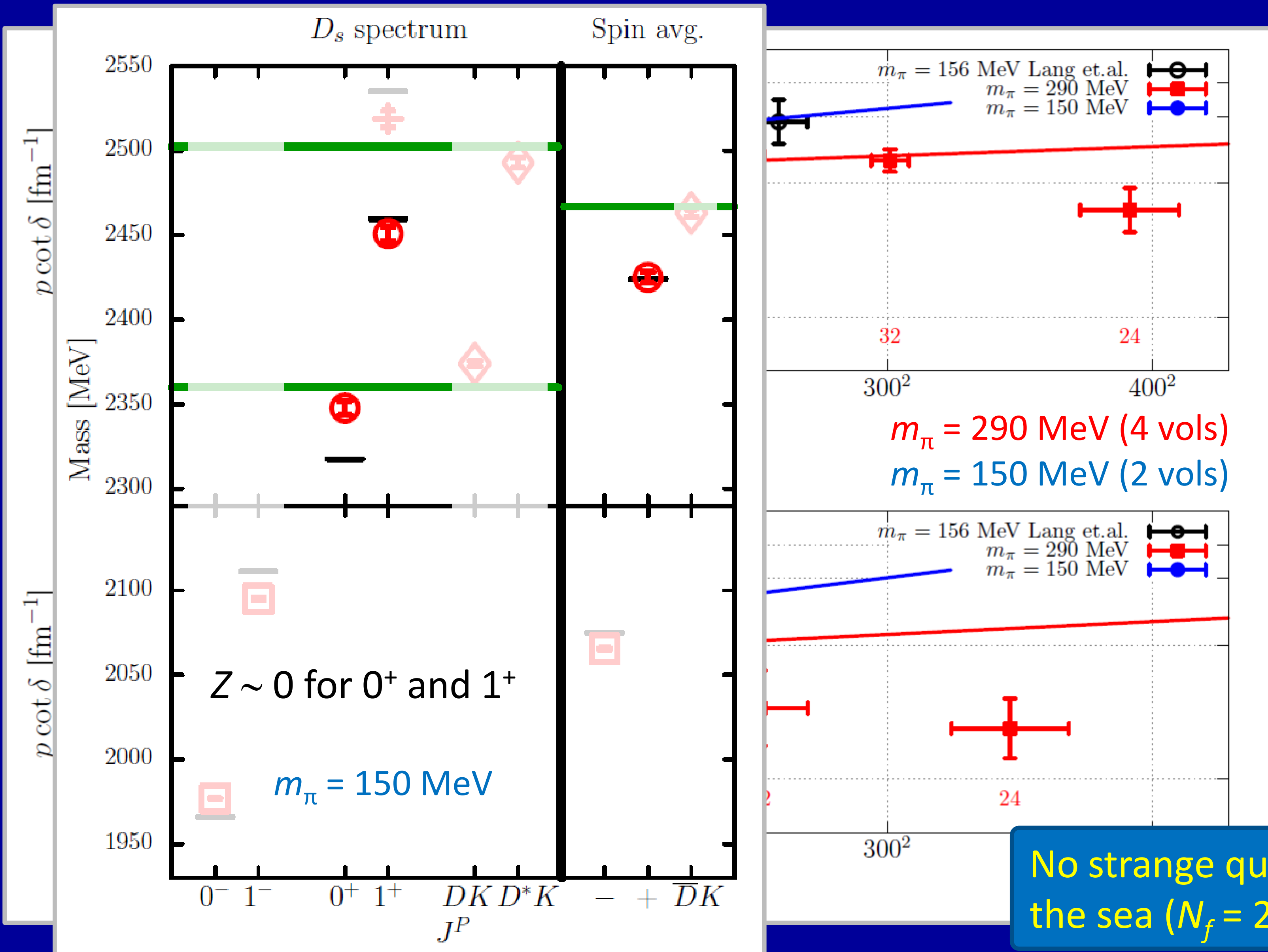
m_π = 290 MeV (4 vols)

m_π = 150 MeV (2 vols)

No strange quarks in the sea ($N_f = 2$)

Charm-strange (I=0): DK (0⁺) and D*K (1⁺)

Bali *et al* (RQCD)
[arXiv:1706.01247]



No strange quarks in the sea (N_f = 2)

Some recent work on charmonium(-like) mesons:

- Ozaki, Sasaki [PR D87, 014506 (2013)] – no sign of *Y*(4140) in J/ψ φ
- Prelovsek & Leskovec [PRL 111, 192001 (2013)] – 1^{++} I=0 near $D\bar{D}^*$ – *X*(3872)?
- Prelovsek *et al* [PL B727, 172; PR D91, 014504 (2015)] – no sign of Z^+(3900) in 1^{+-}
- Chen *et al* (CLQCD) [PR D89, 094506 (2014)] – 1^{++} I=1 $D\bar{D}^*$ weakly repulsive
- Padmanath *et al* [PR D92, 034501 (2015)] – 1^{++} I=0 [X(3872)?]; no I=1 or *Y*(4140)
- Lang *et al* [JHEP 1509, 089 (2015)] – I=0 $D\bar{D}$: 1^{--} ψ(3770) and 0^{++}
- Chen *et al* (CLQCD) [PR D92, 054507 (2015)] – 1^{+-} I=1 $D^*\bar{D}^*$ weakly repulsive?
- Chen *et al* (CLQCD) [PR D93, 114501 (2016)] – 0^{--}, 1^{+-} I=1 $D^*\bar{D}_1$ some attraction?
- Ikeda *et al* (HAL QCD) [PRL 117, 242001 (2016); arXiv:1706.07300] – π J/ψ, ρ η_c, $D\bar{D}^*$ using HAL QCD method – suggest Z^+(3900) is a threshold cusp
- Albaladejo *et al* [EPJ C76, 573 (2016)] – different scenarios for PR D91, 014504

Bottom mesons:

- Lang *et al* [PL B750, 17 (2015)] – *BK* (0^+) and *B*K* (1^+) I=0 bound states
- Lang *et al* [PR D94, 074509 (2016)] – $B_s\pi$, *BK* (I=1) $J^P = 0^+$ no sign of *X*(5568)

Heavy-flavour tetraquarks:

- Bicudo *et al* [PR D92, 014507 (2015); PR D93, 034501 (2016); PR D95, 034502 (2017)] – potential between static b antiquarks: $ud\bar{b}\bar{b}$ I=0 1^+ tetraquark
- Francis *et al* [PRL 118, 142001 (2017)] – $ud\bar{b}\bar{b}$ I=0 and $ls\bar{b}\bar{b}$ I=1/2 1^+ tetraquarks
- (Peters *et al* [arXiv:1609.00181 PoS Lattice2016])

Baryon resonances

Roper (excited nucleon, $J^P = \frac{1}{2}^+$, P-wave N π relevant) – situation not yet clear in lattice QCD calculations. See e.g. Lang, Leskovec, Padmanath, Prelovsek [PR D95, 014510 (2017)], Wu *et al* [PR D95, 114507 (2017)] and references therein

Summary

- **Significant progress** in LQCD calculations of **resonances**, near-threshold states, etc – **map out scattering amps**.
- Some examples of recent work:
  - ρ resonance
  - Light scalars (κ, $a_0(980)$, σ, $f_0(980)$)
  - Heavy mesons
- Also transitions, e.g. ρ resonance ($\pi\pi$) → $\pi\,\gamma$
- Tools to learn about structure (e.g. m_π dependence)
- Ongoing work on formalism (e.g. 3-hadron scattering)
- Connections with analysis of experimental data