Study of the Lorentz structure of tau decays at Belle

The 21st Particle and Nuclei International Conference
@China National Convention Center

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- Belle experiment
- Study of Michel parameters at Belle
    - $\tau^- \rightarrow l^- \bar{\nu}_l \nu_\tau$
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    - $\tau^- \rightarrow l^- \bar{\nu}_l \nu_\tau l'^+ l'^-$
- Conclusion & Future Plan

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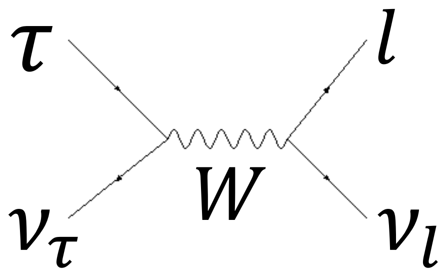
Introduction

Standard Model (SM)'s prediction: A Lorentz structure of charged weak current has a V-A structure.
We study the Lorentz structure through the measurement of Michel parameters.

$$\mathcal{L} \propto \sum_{\substack{i=S,V,T \\ \lambda,\rho=L,R}} \{g^i_{\lambda\rho}[\overline{l'_\lambda}\Gamma^i(\nu_{l'})_\xi][\overline{(\nu_l)_\kappa}\Gamma_i l_\rho]\}$$

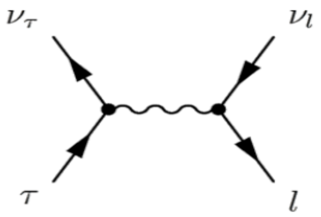
$$\Gamma^S = I, \Gamma^V = \gamma^\mu, \Gamma^T = \sigma^{\mu\nu}/\sqrt{2}$$



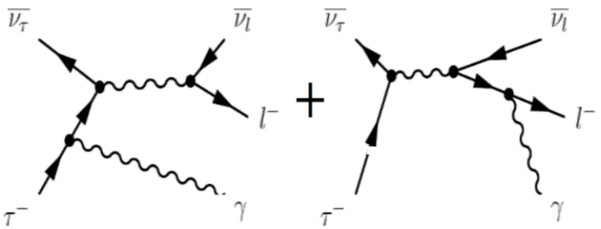
In the SM, g^V_{LL} = 1, and others are $g^i_{\lambda\rho} = 0$

Michel parameters are the bi-linear combination of coupling constants $g^i_{\lambda\rho}$.
The different type of Michel parameter appears in three modes shown below.

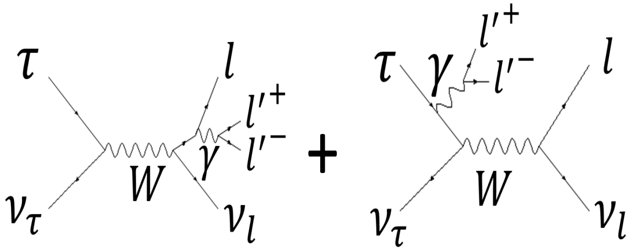
$\tau^- \to l^- \bar{\nu}_l \nu_\tau$



$\tau^- \to l^- \bar{\nu}_l \nu_\tau \gamma$



$\tau^- \to l^- \bar{\nu}_l \nu_\tau l'^+ l'^-$



Introduction

Measurable Michel parameters (MPs) for each tau's leptonic decay and what the coupling constants $g^{i}_{\lambda\rho}$ are contained, are shown below.

$\tau^- \to l^- \bar{\nu}_l \nu_\tau,$

$$\rho = \frac{3}{4} - \frac{3}{4}\left(|g^V_{LR}|^2 + |g^V_{RL}|^2 + 2|g^T_{LR}|^2 + 2|g^T_{RL}|^2 + \Re(g^S_{LR}g^{T*}_{LR} + g^S_{RL}g^{T*}_{RL})\right)$$

$$\eta = \frac{1}{2}\Re\left(6g^V_{RL}g^{T*}_{LR} + 6g^V_{LR}g^{T*}_{RL} + g^S_{RR}g^{V*}_{LL} + g^S_{RL}g^{V*}_{LR} + g^S_{LR}g^{V*}_{RL} + g^S_{LL}g^{V*}_{RR}\right)$$

$$\xi = 4\Re(g^S_{LR}g^{T*}_{LR}) - 4\Re(g^S_{RL}g^{T*}_{RL}) + |g^V_{LL}|^2 + 3|g^V_{LR}|^2 - 3|g^V_{RL}|^2 - |g^V_{RR}|^2 +$$
$$+5|g^T_{LR}|^2 - 5|g^T_{RL}|^2 + \frac{1}{4}|g^S_{LL}|^2 - \frac{1}{4}|g^S_{LR}|^2 + \frac{1}{4}|g^S_{RL}|^2 - \frac{1}{4}|g^S_{RR}|^2$$

$$\xi\delta = \frac{3}{16}|g^S_{LL}|^2 - \frac{3}{16}|g^S_{LR}|^2 + \frac{3}{16}|g^S_{RL}|^2 - \frac{3}{16}|g^S_{RR}|^2 - \frac{3}{4}|g^T_{LR}|^2 + \frac{3}{4}|g^T_{RL}|^2 +$$
$$+\frac{3}{4}|g^V_{LL}|^2 - \frac{3}{4}|g^V_{RR}|^2 + \frac{3}{4}\Re(g^S_{LR}g^{T*}_{LR}) - \frac{3}{4}\Re(g^S_{RL}g^{T*}_{RL})$$

$\tau^- \to l^- \bar{\nu}_l \nu_\tau \gamma$

$$\bar{\eta} = \left|g^V_{RL}\right|^2 + \left|g^V_{LR}\right|^2 + \frac{1}{8}\left(\left|g^S_{RL} + 2g^T_{RL}\right|^2 + \left|g^S_{LR} + 2g^T_{LR}\right|^2\right) + 2\left(\left|g^T_{RL}\right|^2 + \left|g^T_{LR}\right|^2\right)$$

$$\xi\kappa = \left|g^V_{RL}\right|^2 - \left|g^V_{LR}\right|^2 + \frac{1}{8}\left(\left|g^S_{RL} + 2g^T_{RL}\right|^2 - \left|g^S_{LR} + 2g^T_{LR}\right|^2\right) + 2\left(\left|g^T_{RL}\right|^2 - \left|g^T_{LR}\right|^2\right)$$

$\tau^- \to l^- \bar{\nu}_l \nu_\tau l'^+ l'^-$

$$Q_{LL} = \frac{1}{4}|g^S_{LL}|^2 + |g^V_{LL}|^2$$

$$Q_{RL} = \frac{1}{4}|g^S_{RL}|^2 + |g^V_{RL}|^2 + |g^T_{RL}|^2$$

$$Q_{RL} = \frac{1}{4}|g^S_{LR}|^2 + |g^V_{LR}|^2 + |g^T_{LR}|^2$$

$$Q_{RR} = \frac{1}{4}|g^S_{RR}|^2 + |g^V_{RR}|^2$$

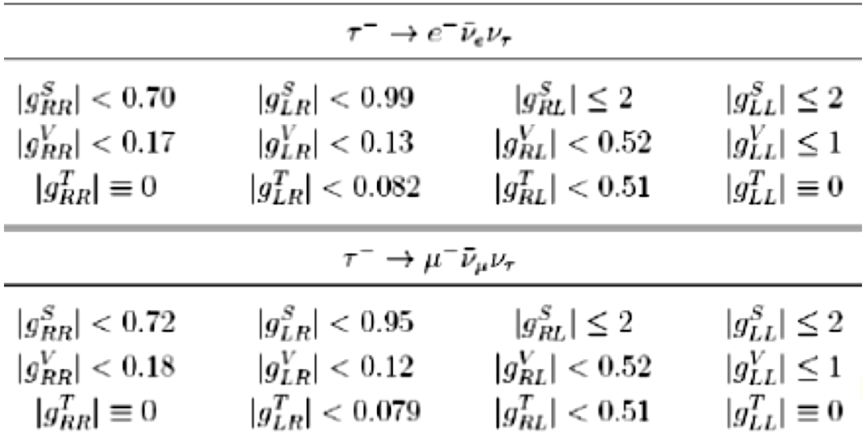
$$B_{RL} = \frac{1}{16}|g^S_{RL} + 6g^T_{RL}|^2 + |g^V_{RL}|^2$$

$$B_{LR} = \frac{1}{16}|g^S_{LR} + 6g^T_{LR}|^2 + |g^V_{LR}|^2$$

$$I_\alpha = \frac{1}{4}g^V_{LR}(g^S_{RL} + 6g^T_{RL})^* + \frac{1}{4}g^{V*}_{RL}(g^S_{LR} + 6g^T_{LR})$$

$$I_\beta = g^V_{LL}g^{S*}_{RR}/2 + g^{V*}_{RR}g^S_{LL}/2$$

Recent Constraints to $g^{i}_{\lambda\rho}$



$\tau^- \to e^- \bar{\nu}_e \nu_\tau$			
$\lvert g^S_{RR}\rvert < 0.70$	$\lvert g^S_{LR}\rvert < 0.99$	$\lvert g^S_{RL}\rvert \leq 2$	$\lvert g^S_{LL}\rvert \leq 2$
$\lvert g^V_{RR}\rvert < 0.17$	$\lvert g^V_{LR}\rvert < 0.13$	$\lvert g^V_{RL}\rvert < 0.52$	$\lvert g^V_{LL}\rvert \leq 1$
$\lvert g^T_{RR}\rvert \equiv 0$	$\lvert g^T_{LR}\rvert < 0.082$	$\lvert g^T_{RL}\rvert < 0.51$	$\lvert g^T_{LL}\rvert \equiv 0$
$\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau$			
$\lvert g^S_{RR}\rvert < 0.72$	$\lvert g^S_{LR}\rvert < 0.95$	$\lvert g^S_{RL}\rvert \leq 2$	$\lvert g^S_{LL}\rvert \leq 2$
$\lvert g^V_{RR}\rvert < 0.18$	$\lvert g^V_{LR}\rvert < 0.12$	$\lvert g^V_{RL}\rvert < 0.52$	$\lvert g^V_{LL}\rvert \leq 1$
$\lvert g^T_{RR}\rvert \equiv 0$	$\lvert g^T_{LR}\rvert < 0.079$	$\lvert g^T_{RL}\rvert < 0.51$	$\lvert g^T_{LL}\rvert \equiv 0$

Some coupling constants $g^{i}_{\lambda\rho}$ are not well known as shown in right-side table.

Measurement of MPs is the important clue for studying Lorentz structure.

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Belle Experiment (Tsukuba, Japan)

- Integrated Luminosity: 1000 fb^{-1}
- Energy of Collision e^- (8 GeV) / e^+ (3.5 GeV)
- $\sqrt{s} = 10.58\,\mathrm{GeV}$ (Same energy to generate BB-pair)
- Low background & Energy of center of mass frame is known
- Pair creation ($B\bar{B}$, $\tau^+\tau^-$, etc). One-side can be used for tagging.

$> 1\ \mathrm{ab}^{-1}$

On resonance:

$\Upsilon(5S)$: 121 fb^{-1}

$\Upsilon(4S)$: 711 fb^{-1}

$\Upsilon(3S)$: 3 fb^{-1}

$\Upsilon(2S)$: 25 fb^{-1}

$\Upsilon(1S)$: 6 fb^{-1}

Off reson./scan:

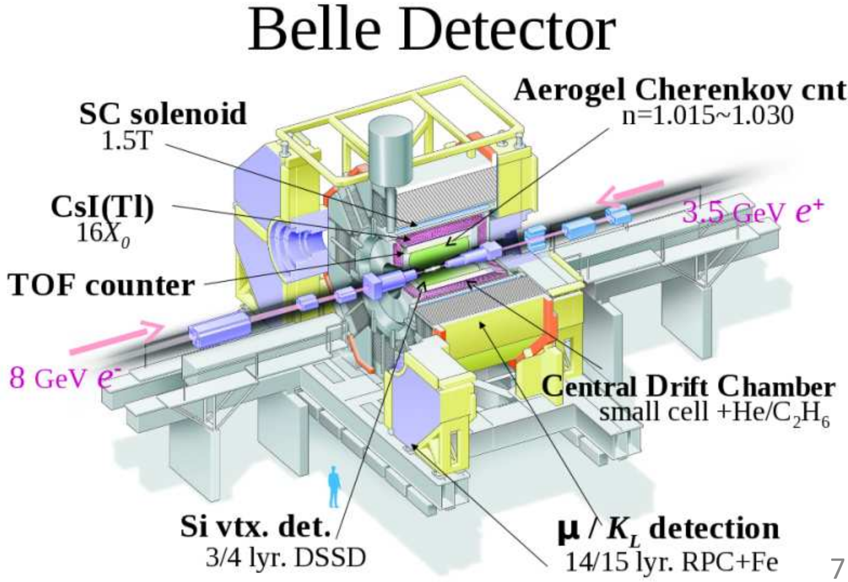
$\sim 100\ \mathrm{fb}^{-1}$

$e^+e^- \rightarrow B\bar{B}$ σ_{BB} ~ 1.05 nb (neutral+charged)

$e^+e^- \rightarrow \tau^+\tau^-$ $\sigma_{\tau\tau}$ = (0.919$\pm$0.003) nb

Belle is a B factory, and also a tau-factory

$N_{\tau\tau} \sim 9.0\times10^8$



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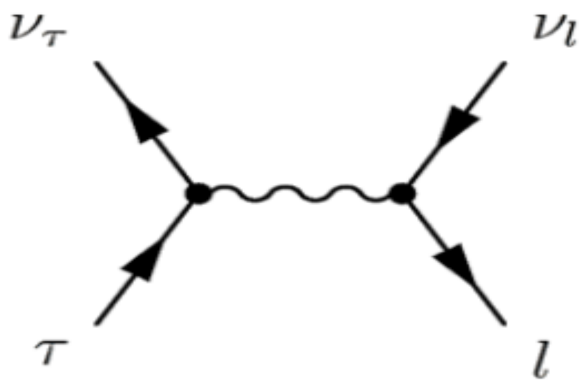
$$\tau^- \to l^- \bar{\nu}_l \nu_\tau$$



Michel Parameter

↓

Target MPs: ρ, η, ξ, δ

$$\rho = \frac{3}{4} - \frac{3}{4}\left(|g^V_{LR}|^2 + |g^V_{RL}|^2 + 2|g^T_{LR}|^2 + 2|g^T_{RL}|^2 + \Re(g^S_{LR}g^{T*}_{LR} + g^S_{RL}g^{T*}_{RL})\right)$$

$$\eta = \frac{1}{2}\Re\left(6g^V_{RL}g^{T*}_{LR} + 6g^V_{LR}g^{T*}_{RL} + g^S_{RR}g^{V*}_{LL} + g^S_{RL}g^{V*}_{LR} + g^S_{LR}g^{V*}_{RL} + g^S_{LL}g^{V*}_{RR}\right)$$

$$\xi = 4\Re(g^S_{LR}g^{T*}_{LR}) - 4\Re(g^S_{RL}g^{T*}_{RL}) + |g^V_{LL}|^2 + 3|g^V_{LR}|^2 - 3|g^V_{RL}|^2 - |g^V_{RR}|^2 +$$

$$+5|g^T_{LR}|^2 - 5|g^T_{RL}|^2 + \frac{1}{4}|g^S_{LL}|^2 - \frac{1}{4}|g^S_{LR}|^2 + \frac{1}{4}|g^S_{RL}|^2 - \frac{1}{4}|g^S_{RR}|^2$$

$$\xi\delta = \frac{3}{16}|g^S_{LL}|^2 - \frac{3}{16}|g^S_{LR}|^2 + \frac{3}{16}|g^S_{RL}|^2 - \frac{3}{16}|g^S_{RR}|^2 - \frac{3}{4}|g^T_{LR}|^2 + \frac{3}{4}|g^T_{RL}|^2 +$$

$$+\frac{3}{4}|g^V_{LL}|^2 - \frac{3}{4}|g^V_{RR}|^2 + \frac{3}{4}\Re(g^S_{LR}g^{T*}_{LR}) - \frac{3}{4}\Re(g^S_{RL}g^{T*}_{RL})$$

Formalism of $\tau^- \to l^- \bar{\nu}_l \nu_\tau$'s differential decay width

$$\frac{d\Gamma(\tau^\mp)}{d\Omega dx} = \frac{4G_F^2 M_\tau E_{\max}^4}{(2\pi)^4}\sqrt{x^2 - x_0^2}\Bigg(x(1-x) + \frac{2}{9}\rho(4x^2 - 3x - x_0^2) + \eta x_0(1-x)$$

$$\mp\frac{1}{3}P_\tau cos\theta_\ell\xi\sqrt{x^2 - x_0^2}\left[1 - x + \frac{2}{3}\delta\left(4x - 4 + \sqrt{1 - x_0^2}\right)\right]\Bigg),\ x = \frac{E_\ell}{E_{\max}},\ x_0 = \frac{m_\ell}{E_{\max}}$$

In the SM: $\rho = \frac{3}{4}$, $\eta = 0$, $\xi = 1$, $\delta = \frac{3}{4}$

Reference: D. Epifanov's slide@PhiPsi-2017

Formalism of $\tau^- \to l^-\bar{\nu}_l\nu_\tau$'s differential decay width

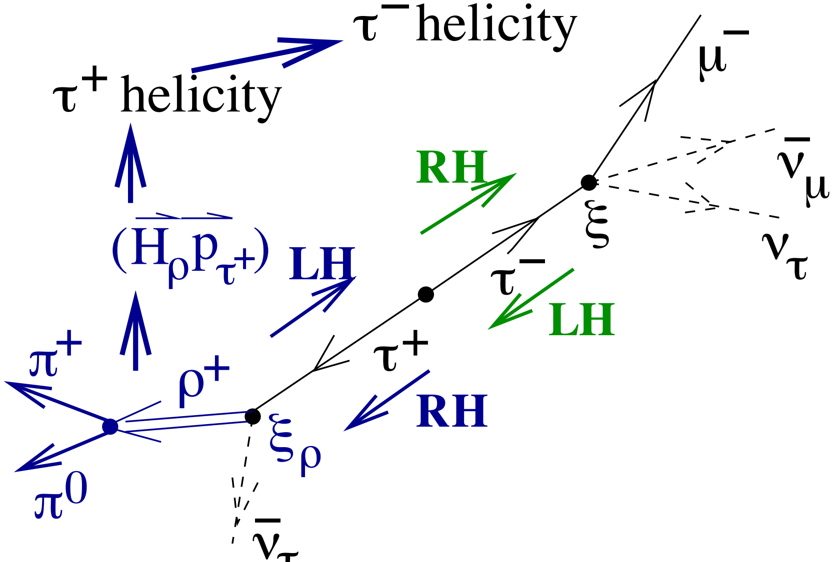
$$\frac{d\Gamma(\tau^{\mp})}{d\Omega dx} = \frac{4G_F^2 M_\tau E_{\max}^4}{(2\pi)^4}\sqrt{x^2-x_0^2}\Big(x(1-x)+\frac{2}{9}\rho(4x^2-3x-x_0^2)+\eta x_0(1-x)$$
$$\mp\frac{1}{3}P_\tau cos\theta_\ell\xi\sqrt{x^2-x_0^2}\Big[1-x+\frac{2}{3}\delta\big(4x-4+\sqrt{1-x_0^2}\big)\Big]\Big),\ x=\frac{E_\ell}{E_{\max}},\ x_0=\frac{m_\ell}{E_{\max}}$$

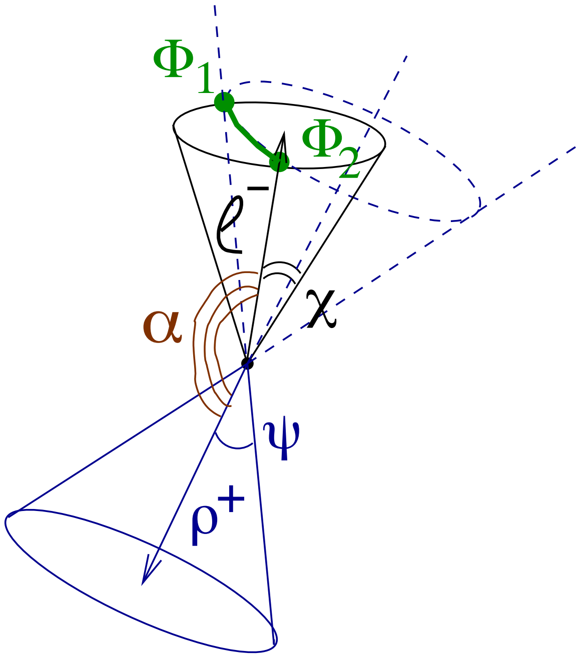
In the SM: $\rho=\frac{3}{4}$, $\eta=0$, $\xi=1$, $\delta=\frac{3}{4}$

Point of this analysis:

1. Information of τ −spin direction is needed.

2. To extract τ −spin direction, we use a mode $\tau^{\pm}\to\rho^{\pm}\nu$ for tag-side.





Reference: D. Epifanov's slide@PhiPsi-2017

Method, study of $(\ell\nu\nu;\ \rho\nu)$ and $(\rho\nu;\ \rho\nu)$ events

Reference: D. Epifanov's slide@PhiPsi-2017

Effect of τ spin-spin correlation is used to measure ξ and δ MP.
Events of the $(\tau^\mp \to \ell^\mp\nu\nu;\ \tau^\pm \to \rho^\pm\nu)$ topology are used to measure: ρ, η, $\xi_\rho\xi$ and $\xi_\rho\xi\delta$, while $(\tau^\mp \to \rho^\mp\nu;\ \tau^\pm \to \rho^\pm\nu)$ events are used to extract ξ_ρ^2.

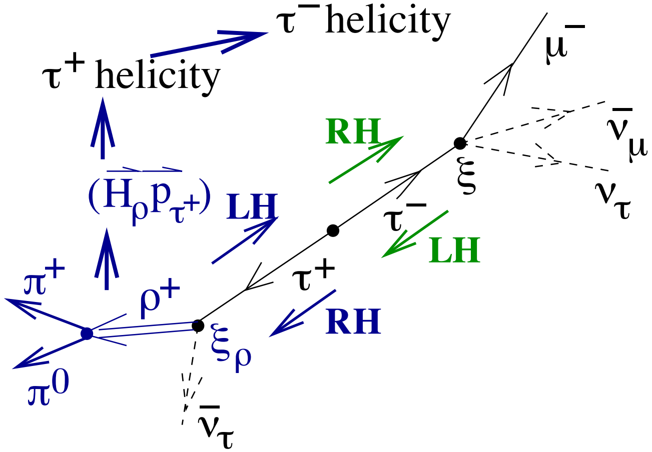
$$\frac{d\sigma(\ell^\mp\nu\nu,\rho^\pm\nu)}{dE^*_\ell\, d\Omega^*_\ell\, d\Omega^*_\rho\, dm^2_{\pi\pi}\, d\tilde{\Omega}_\pi\, d\Omega_\tau} = A_0 + \rho A_1 + \eta A_2 + \xi_\rho\xi A_3 + \xi_\rho\xi\delta A_4 = \sum_{i=0}^{4} A_i\Theta_i$$

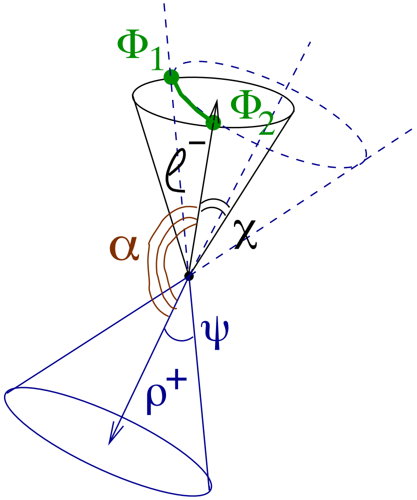
$$\mathcal{F}(\vec{z}) = \frac{d\sigma(\ell^\mp\nu\nu,\rho^\pm\nu)}{dp_\ell\, d\Omega_\ell\, dp_\rho\, d\Omega_\rho\, dm^2_{\pi\pi}\, d\tilde{\Omega}_\pi} = \int\limits_{\Phi_1}^{\Phi_2} \frac{d\sigma(\ell^\mp\nu\nu,\rho^\pm\nu)}{dE^*_\ell\, d\Omega^*_\ell\, d\Omega^*_\rho\, dm^2_{\pi\pi}\, d\tilde{\Omega}_\pi\, d\Omega_\tau}\left|\frac{\partial(E^*_\ell,\Omega^*_\ell,\Omega^*_\rho,\Omega_\tau)}{\partial(p_\ell,\Omega_\ell,p_\rho,\Omega_\rho,\Phi_\tau)}\right| d\Phi_\tau$$

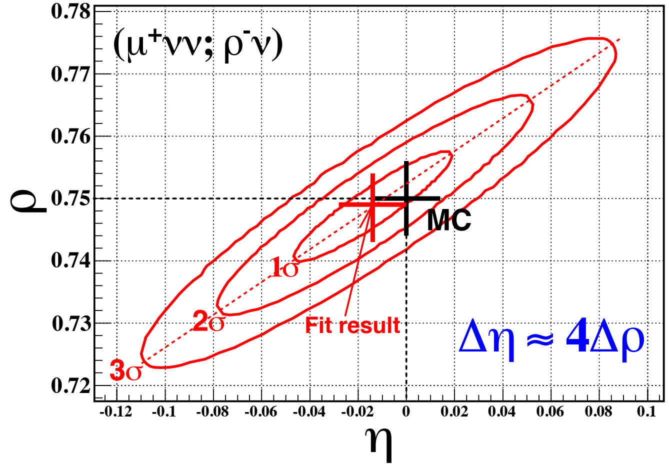
$$L = \prod_{k=1}^{N}\mathcal{P}^{(k)},\ \mathcal{P}^{(k)} = \mathcal{F}(\vec{z}^{(k)})/\mathcal{N}(\vec{\Theta}),\ \mathcal{N}(\vec{\Theta}) = \int \mathcal{F}(\vec{z})d\vec{z},\ \vec{\Theta} = (1,\rho,\eta,\xi_\rho\xi_\ell,\xi_\rho\xi_\ell\delta_\ell)$$

$$\mathcal{P}_{\mathrm{total}} = (1-\sum_{i=1}^{4}\lambda_i)\mathcal{P}^{\ell-\rho}_{signal} + \lambda_1\mathcal{P}^{\ell-3\pi}_{bg} + \lambda_2\mathcal{P}^{\pi-\rho}_{bg} + \lambda_3\mathcal{P}^{\rho-\rho}_{bg} + \lambda_4\mathcal{P}^{other}_{bg}\ (\mathrm{MC})$$

MP are extracted in the unbinned maximum likelihood fit of $(\ell\nu\nu;\ \rho\nu)$ events in the 9D phase space $\vec{z} = (p_\ell,\ \cos\theta_\ell,\ \phi_\ell,\ p_\rho,\ \cos\theta_\rho,\ \phi_\rho,\ m^2_{\pi\pi},\ \cos\tilde{\theta}_\pi,\ \tilde{\phi}_\pi)$ in CMS.

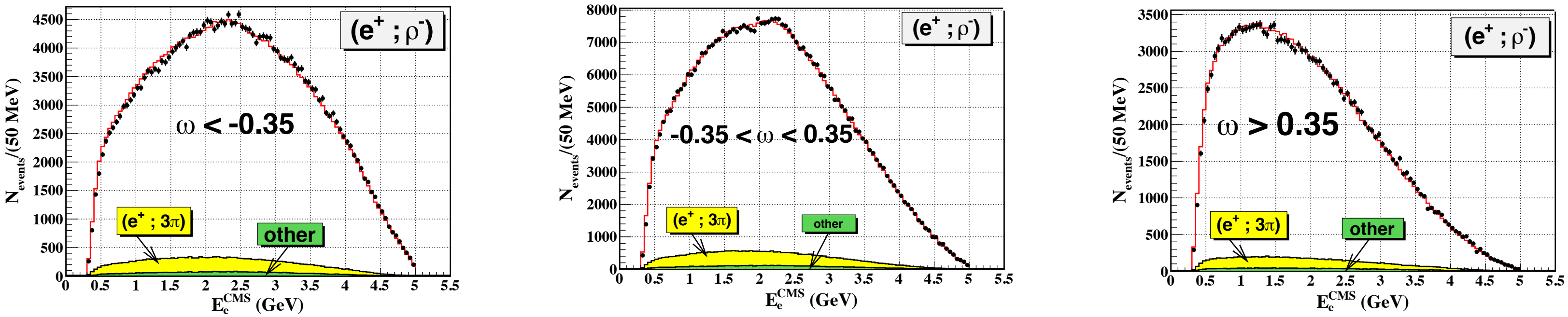






Data fits and systematic uncertainties

Helicity sensitive variable $\omega = \frac{1}{\Phi_2-\Phi_1}\int_{\Phi_1}^{\Phi_2}(\vec{H}_{\rho^\pm},\vec{n}_{\tau^\pm})d\Phi = <(\vec{H}_{\rho^\pm},\vec{n}_{\tau^\pm})>_{\Phi_\tau}$



Spin-spin correlation manifests itself through momentum-momentum correlations of final lepton and pions.

Source	$\Delta(\rho)$, %	$\Delta(\eta)$, %	$\Delta(\xi_\rho\xi)$, %	$\Delta(\xi_\rho\xi\delta)$, %
Physical corrections				
ISR+$\mathcal{O}(\alpha^3)$	0.10	0.30	0.20	0.15
$\tau \to \ell\nu\nu\gamma$	0.03	0.10	0.09	0.08
$\tau \to \rho\nu\gamma$	0.06	0.16	0.11	0.02
Background	0.20	0.60	0.20	0.20
Apparatus corrections				
Resolution $\oplus$ brems.	0.10	0.33	0.11	0.19
$\sigma(E_{\mathrm{beam}})$	0.07	0.25	0.03	0.15
Normalization				
$\Delta\mathcal{N}$	0.11	0.50	0.17	0.13
without Data/MC corr.	**0.29**	**0.95**	**0.38**	**0.38**
trigger eff. corr.	**$\sim$ 1**	**$\sim$ 2**	**$\sim$ 3**	**$\sim$ 3**

We are working on the Data/MC efficiency corrections (trigger, ℓID, track rec., π^0 rec.).

Reference: D. Epifanov's slide@PhiPsi-2017

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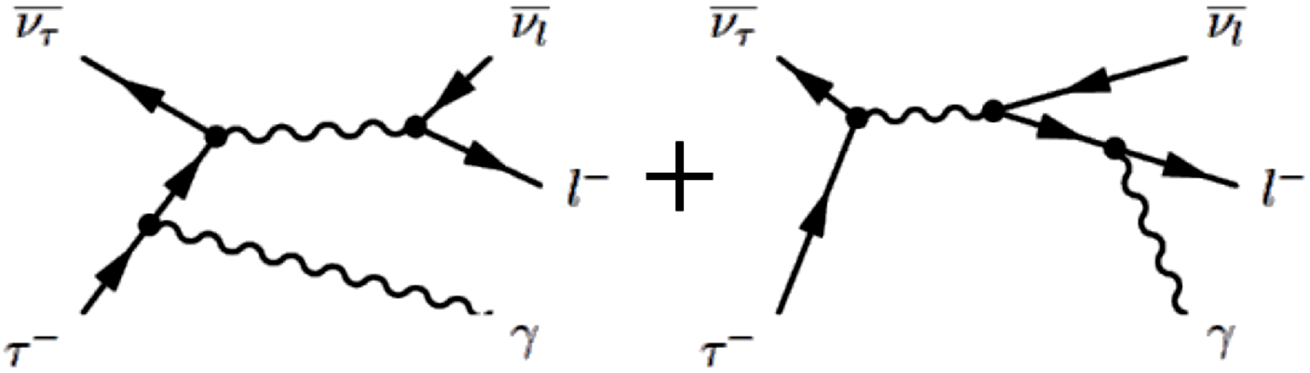


Target MPs: $\bar{\eta}$, $\xi\kappa$

$$\tau^- \to l^- \bar{\nu}_l \nu_\tau \gamma$$

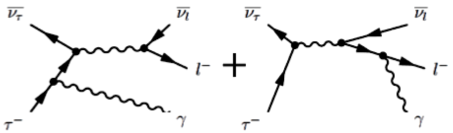
$$\begin{aligned} \bar{\eta} &= \left|g^V_{RL}\right|^2 + \left|g^V_{LR}\right|^2 + \frac{1}{8}\left(\left|g^S_{RL} + 2g^T_{RL}\right|^2 + \left|g^S_{LR} + 2g^T_{LR}\right|^2\right) + 2\left(\left|g^T_{RL}\right|^2 + \left|g^T_{LR}\right|^2\right) \\ \xi\kappa &= \left|g^V_{RL}\right|^2 - \left|g^V_{LR}\right|^2 + \frac{1}{8}\left(\left|g^S_{RL} + 2g^T_{RL}\right|^2 - \left|g^S_{LR} + 2g^T_{LR}\right|^2\right) + 2\left(\left|g^T_{RL}\right|^2 - \left|g^T_{LR}\right|^2\right) \end{aligned}$$

$\tau^- \to l^- \bar{\nu}_l \nu_\tau \gamma$ Target MPs: $\bar{\eta}, \ \xi\kappa$

New Michel parameters ($\bar{\eta}, \ \xi\kappa$) appear in a radiative decay of tau.

$$\bar{\eta} = |g^V_{RL}|^2 + |g^V_{LR}|^2 + \frac{1}{8}\left(|g^S_{RL} + 2g^T_{RL}|^2 + |g^S_{LR} + 2g^T_{LR}|^2\right) + 2\left(|g^T_{RL}|^2 + |g^T_{LR}|^2\right)$$

$$\xi\kappa = |g^V_{RL}|^2 - |g^V_{LR}|^2 + \frac{1}{8}\left(|g^S_{RL} + 2g^T_{RL}|^2 - |g^S_{LR} + 2g^T_{LR}|^2\right) + 2\left(|g^T_{RL}|^2 - |g^T_{LR}|^2\right)$$



The purpose of this study is to measure the values of $\bar{\eta}, \xi\kappa$

MP	ρ	η	$\xi\delta$	ξ_h	$\bar{\eta}$	$\xi\kappa$
SM	0.75	0	0.75	1	0	0
EX	0.747 ± 0.010	0.013 ± 0.020	0.746 ± 0.021	0.995 ± 0.007	not measured yet	

PDG Chin. Phys. C38, 090001 (2014).

- ☐ Observation of γ is equivalent to the measurement of polarization of the daughter lepton:
  - ● coupling of τ with the right handed daughter lepton $\propto 1 - \xi'$
  - ● $\xi' = -\xi - 4\xi\kappa + 8\xi\delta/3$

Final piece to reveal the $V - A$ structure!

- ☐ $\bar{\eta}$ gives a constraint on each term

$$\bar{\eta} = |g^V_{RL}|^2 + |g^V_{LR}|^2 + \frac{1}{8}\left(|g^S_{RL} + 2g^T_{RL}|^2 + |g^S_{LR} + 2g^T_{LR}|^2\right) + 2\left(|g^T_{RL}|^2 + |g^T_{LR}|^2\right)$$

$\tau \to e\nu_e\nu_\tau$		
$\lvert g^S_{RR}\rvert < 0.70$	$\lvert g^V_{RR}\rvert < 0.17$	$\lvert g^T_{RR}\rvert \equiv 0$
$\lvert g^S_{LR}\rvert < 0.99$	$\lvert g^V_{LR}\rvert < 0.13$	$\lvert g^T_{LR}\rvert < 0.082$
$\lvert g^S_{RL}\rvert < 2.01$	$\lvert g^V_{RL}\rvert < 0.52$	$\lvert g^T_{RL}\rvert < 0.51$
$\lvert g^S_{LL}\rvert < 2.01$	$\lvert g^V_{LL}\rvert < 1.005$	$\lvert g^T_{LL}\rvert \equiv 0$
$\tau \to \mu\nu_\mu\nu_\tau$		
$\lvert g^S_{RR}\rvert < 0.72$	$\lvert g^V_{RR}\rvert < 0.18$	$\lvert g^T_{RR}\rvert \equiv 0$
$\lvert g^S_{LR}\rvert < 0.95$	$\lvert g^V_{LR}\rvert < 0.12$	$\lvert g^T_{LR}\rvert < 0.079$
$\lvert g^S_{RL}\rvert < 2.01$	$\lvert g^V_{RL}\rvert < 0.52$	$\lvert g^T_{RL}\rvert < 0.51$
$\lvert g^S_{LL}\rvert < 2.01$	$\lvert g^V_{LL}\rvert < 1.005$	$\lvert g^T_{LL}\rvert \equiv 0$

PDG Tau decay parameters

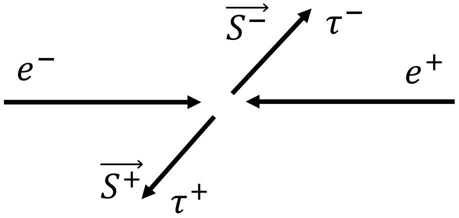
$|g^V_{RL}| < 0.52$
$|g^T_{RL}| < 0.51$
$|g^S_{RL}| < 2.01$
$|g^S_{LR}| < 0.95$

These values are not well known yet

Further constraint on NP models

Reference: N. Shimizu's slide@TAU2016

Method



☐ $d\Gamma(\tau^- \to l^-\bar{\nu}\nu\gamma) \propto \boldsymbol{A^-} - \overrightarrow{\boldsymbol{B^-}} \cdot \overrightarrow{S^-}$

$$\boxed{\begin{aligned} d\Gamma = & \underline{n_V + \left(1-\frac{4}{3}\rho\right)(2n_S + n_V - n_T) + \bar{\boldsymbol{\eta}}(2n_S - 2n_V + n_T)}\ \boldsymbol{A^-} \\ & - \overrightarrow{S^-} \cdot \underline{\boldsymbol{\xi} \textstyle\sum_{k=l,\gamma} \overrightarrow{e^k} \left\{n_V^k - \frac{1}{3}\left(1-\frac{4}{3}\delta\right)(2n_S^k + 5n_V^k - n_T^k) + \boldsymbol{\kappa}(2n_S^k - 2n_V^k + n_T^k)\right\}}\ \overrightarrow{\boldsymbol{B^-}} \end{aligned}}$$

where $\overrightarrow{e^k}$ is direction of $k = l, \gamma$ and n_*^* are known function, whose arguments are experimentally observable.

☐ Measurement of $\boldsymbol{\xi\kappa}$ requires information of τ's spin

☐ We use correlation of the $\tau\tau$ pairs

- The cross section $\frac{d\sigma}{d\Omega_\tau}$ under the definite direction of spin $(\overrightarrow{S^-}, \overrightarrow{S^+})$ is expressed as $\frac{d\sigma}{d\Omega_\tau} = D_0 + D_{ij}S_i^- S_j^+$: D_0 spin-independent, D_{ij} spin-dependent coefficients.

☐ We use $\tau^+ \to \rho^+(\to \pi^+\pi^0)\bar{\nu}$ decay as a spin analyzer.

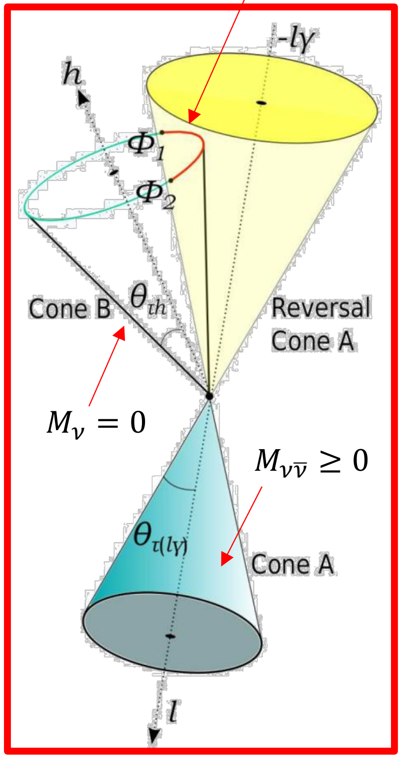
- $d\Gamma(\tau^+ \to \rho^+\nu) = \boldsymbol{A^+} - \overrightarrow{\boldsymbol{B^+}} \cdot \overrightarrow{S^+}$

$$\boldsymbol{d\sigma(\tau^-\tau^+ \to (l^-\nu\bar{\nu}\gamma)(\rho^+\bar{\nu}))}$$
$$\boldsymbol{\propto D_0 A^- A^+ + D_{ij} B_i^- B_j^+}$$

PDF of signal

$\bar{\boldsymbol{\eta}}$ $\boldsymbol{\xi\kappa}$

Φ is an angle along the arc



☐ The Michel Parameters are fitted using the likelihood function: $\mathcal{L}(\bar{\eta}, \xi\kappa) = \sum \log(PDF)$

- "bare" PDF= $\dfrac{d\sigma}{\underbrace{dE_l^* d\Omega_l^* dE_\gamma^* d\Omega_\gamma^*}_{\tau^- \text{ side}} \underbrace{d\Omega_\rho^* dm_\rho^2 d\tilde{\Omega}_\pi}_{\tau^+ \text{ side}} d\Omega_\tau}$

$$\boxed{J = \left|\frac{\partial(E_l^*, \Omega_l^*)}{\partial(P_l, \Omega_l)}\right| \cdot \left|\frac{\partial(E_\gamma^*, \Omega_\gamma^*)}{\partial(P_\gamma, \Omega_\gamma)}\right| \cdot \left|\frac{\partial(\Omega_\rho^*, \Omega_\tau)}{\partial(P_\rho, \Omega_\rho, \Phi)}\right|}$$

Convert to CMS

$$\frac{d\sigma}{dP_l d\Omega_l dP_\gamma d\Omega_\gamma dP_\rho d\Omega_\rho dm_\rho^2 d\tilde{\Omega}_\pi \boxed{d\Phi}}$$

integrate out

Method

PDF of signal

$$S(\vec{x}) = \frac{\mathrm{d}\sigma}{\mathrm{d}P_l \mathrm{d}\Omega_l \mathrm{d}P_\gamma \mathrm{d}\Omega_\gamma \mathrm{d}P_\rho \mathrm{d}\Omega_\rho \mathrm{d}m_\rho^2 \mathrm{d}\widetilde{\Omega}_\pi} = \int_{\Phi_1}^{\Phi_2} \mathrm{d}\Phi \frac{\mathrm{d}\sigma}{\mathrm{d}P_l \mathrm{d}\Omega_l \mathrm{d}P_\gamma \mathrm{d}\Omega_\gamma \mathrm{d}P_\rho \mathrm{d}\Omega_\rho \mathrm{d}m_\rho^2 \mathrm{d}\widetilde{\Omega}_\pi \mathrm{d}\Phi}$$

- ● $S(\vec{x})$: the visible PDF for observable $\vec{x}$ ($N_{\mathrm{dim.}} = 12$)
- ● $S(\vec{x})$ has a form: $\vec{x} = \{P_l, \Omega_l, P_\gamma, \Omega_\gamma, P_\rho, \Omega_\rho, m_{\pi\pi}^2, \tilde{\Omega}_\pi\}$

☐ Event selection & existence of BG → the visible PDF is formulated as sum of signal and BGs

- ● $P_{tot}(\vec{x}) = (1 - \sum \lambda_i) \cdot \frac{S(\vec{x})\varepsilon(\vec{x})}{\int \mathrm{d}\vec{x} S(\vec{x})\varepsilon(\vec{x})} + \sum \lambda_i \frac{B_i(\vec{x})\varepsilon(\vec{x})}{\int \mathrm{d}\vec{x} B_i(\vec{x})\varepsilon(\vec{x})}$

i: index of background

$S(\vec{x})$: PDF of signal

$B_i(\vec{x})$: PDF of i-th background

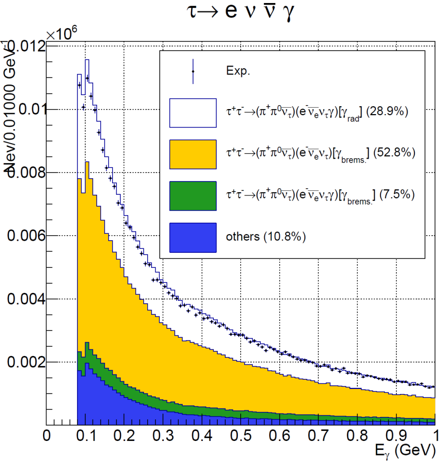
λ_i: fraction of i-th background component

$\varepsilon(\vec{x})$: selection efficiency

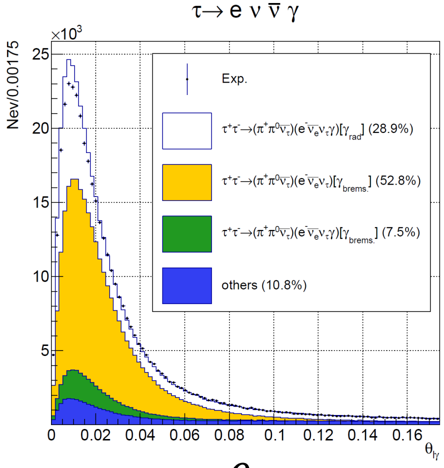
Then Michel parameters are fitted by likelihood function:

$$\mathcal{L}(\bar{\eta}, \xi\kappa) = \sum \log(PDF)$$

$\tau \rightarrow e\nu\bar{\nu}\gamma$ candidates

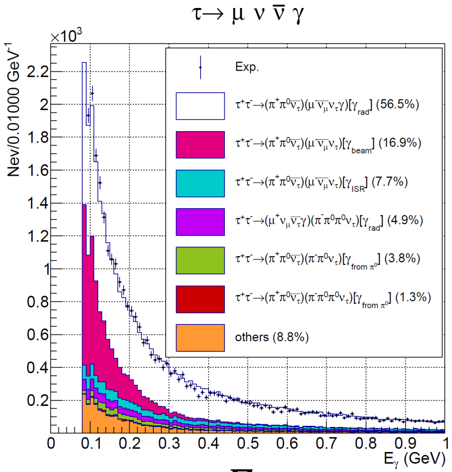


E_γ

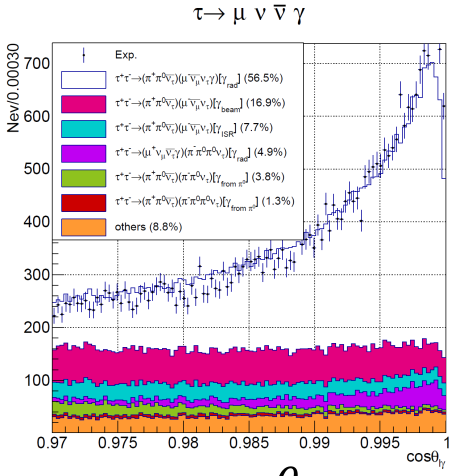


$\theta_{e\gamma}$

$\tau \rightarrow \mu\nu\bar{\nu}\gamma$ candidates



E_γ



$\cos\theta_{\mu\gamma}$

Result

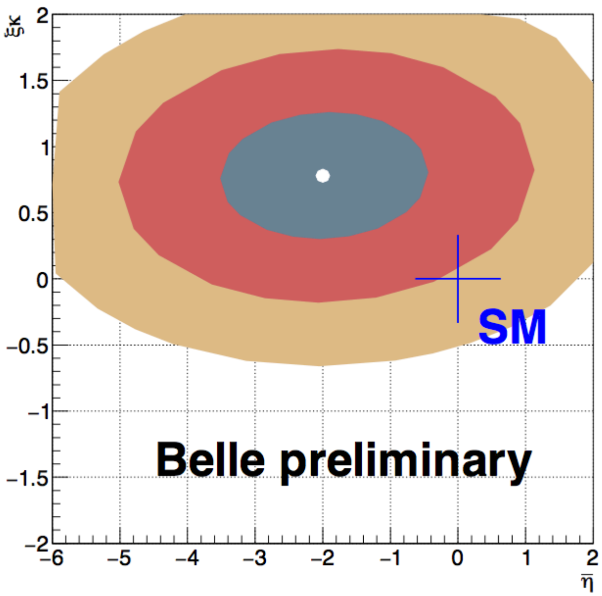
Source	$\sigma^e_{\bar{\eta}}$	$\sigma^e_{\xi\kappa}$	$\sigma^\mu_{\bar{\eta}}$	$\sigma^\mu_{\xi\kappa}$
Normalization	4.3	0.94	0.15	0.04
Background PDF	2.5	0.24	0.67	0.22
Branching ratios	3.8	0.05	0.25	0.01
Cluster merge in ECL	2.2	0.46	0.02	0.06
Detector resolution	0.74	0.20	0.22	0.02
Data/MC eff. corr.	1.9	0.14	0.04	0.04
Total	7.0	1.1	0.76	0.24

Belle preliminary

$$\bar{\eta} = -1.3 \pm 1.5 \pm 0.8$$

$$\xi\kappa = 0.5 \pm 0.4 \pm 0.2$$

Contour of likelihood



Reference: N. Shimizu's slide@TAU2016
Reference: D. Epifanov's slide@PhiPsi-2017

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- Introduction
- Belle experiment
- Study of Michel parameters at Belle
  - $\tau^- \rightarrow l^- \bar{\nu}_l \nu_\tau$
  - $\tau^- \rightarrow l^- \bar{\nu}_l \nu_\tau \gamma$
  - $\tau^- \rightarrow l^- \bar{\nu}_l \nu_\tau l'^+ l'^-$
- Conclusion & Future Plan

Target MPs:

$Q_{LL}, Q_{RL}, Q_{LR}, Q_{RR}, B_{RL}, B_{LR}, I_\alpha, I_\beta$

$$\tau^- \to l^- \bar{\nu}_l \nu_\tau l'^+ l'^-$$

$$Q_{LL} = \frac{1}{4}|g^S_{LL}|^2 + |g^V_{LL}|^2$$

$$Q_{RL} = \frac{1}{4}|g^S_{RL}|^2 + |g^V_{RL}|^2 + |g^T_{RL}|^2$$

$$Q_{RL} = \frac{1}{4}|g^S_{LR}|^2 + |g^V_{LR}|^2 + |g^T_{LR}|^2$$

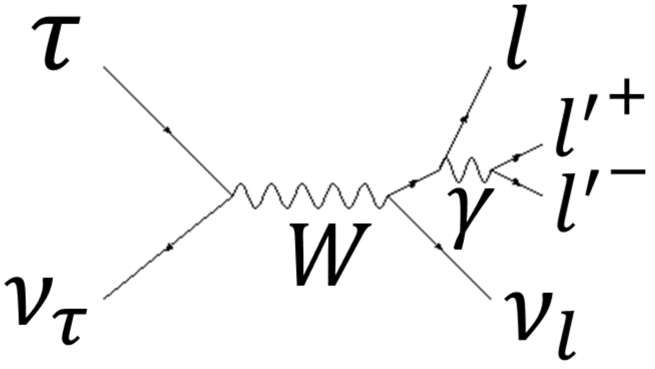
$$Q_{RR} = \frac{1}{4}|g^S_{RR}|^2 + |g^V_{RR}|^2$$

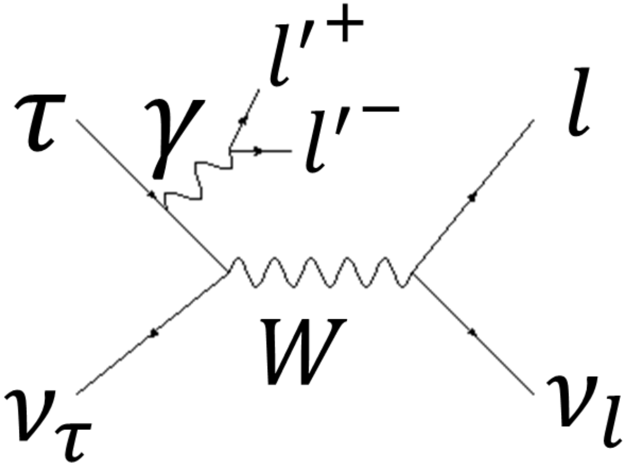
$$B_{RL} = \frac{1}{16}|g^S_{RL} + 6g^T_{RL}|^2 + |g^V_{RL}|^2$$

$$B_{LR} = \frac{1}{16}|g^S_{LR} + 6g^T_{LR}|^2 + |g^V_{LR}|^2$$

$$I_\alpha = \frac{1}{4}g^V_{LR}(g^S_{RL} + 6g^T_{RL})^* + \frac{1}{4}g^{V*}_{RL}(g^S_{LR} + 6g^T_{LR})$$

$$I_\beta = g^V_{LL}g^{S*}_{RR}/2 + g^{V*}_{RR}g^S_{LL}/2$$



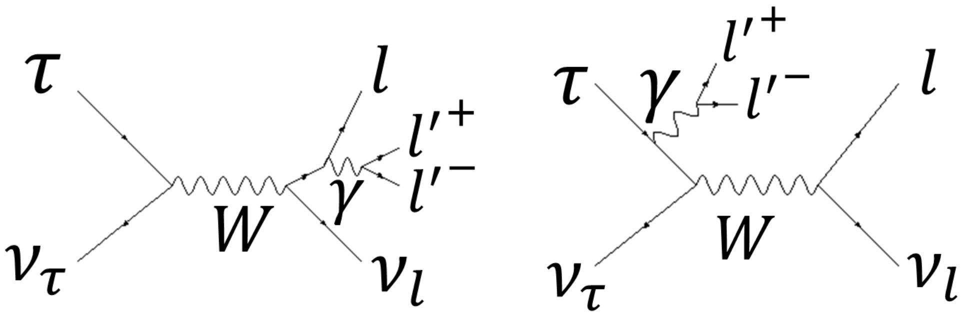


$\tau^\pm \to l^\pm l'^+ l'^- \nu_\tau \nu_l \; (l, l' = e, \mu)$

Target MPs:

$Q_{LL}, Q_{RL}, Q_{LR}, Q_{RR}, B_{RL}, B_{LR}, I_\alpha, I_\beta$

We measure the Michel-like parameters through the measurement of branching fraction of five-body leptonic decays of tau



$$Q_{LL} = \frac{1}{4}|g^S_{LL}|^2 + |g^V_{LL}|^2$$
$$Q_{RL} = \frac{1}{4}|g^S_{RL}|^2 + |g^V_{RL}|^2 + |g^T_{RL}|^2$$
$$Q_{RL} = \frac{1}{4}|g^S_{LR}|^2 + |g^V_{LR}|^2 + |g^T_{LR}|^2$$
$$Q_{RR} = \frac{1}{4}|g^S_{RR}|^2 + |g^V_{RR}|^2$$
$$B_{RL} = \frac{1}{16}|g^S_{RL} + 6g^T_{RL}|^2 + |g^V_{RL}|^2$$
$$B_{LR} = \frac{1}{16}|g^S_{LR} + 6g^T_{LR}|^2 + |g^V_{LR}|^2$$
$$I_\alpha = \frac{1}{4}g^V_{LR}(g^S_{RL} + 6g^T_{RL})^* + \frac{1}{4}g^{V*}_{RL}(g^S_{LR} + 6g^T_{LR})$$
$$I_\beta = g^V_{LL}g^{S*}_{RR}/2 + g^{V*}_{RR}g^S_{LL}/2$$

Reference:
W. Fetscher, H. J. Gerber and K. F. Johnson,
Phys. Lett. B 173, 102 (1986)

Prediction from theory

$\tau^\pm \to e^\pm e^+ e^- \nu_\tau \nu_e$
$\tau^\pm \to e^\pm \mu^+ \mu^- \nu_\tau \nu_e$
$\tau^\pm \to \mu^\pm e^+ e^- \nu_\tau \nu_\mu$
$\tau^\pm \to \mu^\pm \mu^+ \mu^- \nu_\tau \nu_\mu$

Channel	
$\mathrm{BR}(\tau^- \to e^- e^+ e^- \bar{\nu}_e \nu_\tau) \times 10^5$	4.21 ± 0.01
$\mathrm{BR}(\tau^- \to e^- \mu^+ \mu^- \bar{\nu}_e \nu_\tau) \times 10^7$	1.247 ± 0.001
$\mathrm{BR}(\tau^- \to \mu^- e^+ e^- \bar{\nu}_\mu \nu_\tau) \times 10^5$	1.984 ± 0.004
$\mathrm{BR}(\tau^- \to \mu^- \mu^+ \mu^- \bar{\nu}_\mu \nu_\tau) \times 10^7$	1.183 ± 0.001

Ref. [JHEP 1604, 185 (2016)]

$$\frac{d\Gamma_5}{dx_1 d\Omega_1 dx_2 d\Omega_2 dx_3 d\Omega_3} = \frac{M^2|\vec{\mathbf{p}}_1||\vec{\mathbf{p}}_2||\vec{\mathbf{p}}_3|}{3 \cdot 2^{21}\pi^{10}} \underline{\mathcal{T}^s_{\alpha\beta} I^{\alpha\beta}(P)}$$

$l^\pm$ $\quad x_1 = \frac{2E_1}{M_\tau}$

l'^+ $\quad x_2 = \frac{2E_2}{M_\tau}$

l'^- $\quad x_3 = \frac{2E_3}{M_\tau}$

$$\mathcal{T}^s_{\alpha\beta} I^{\alpha\beta}(P) = e^4|G_{\ell\ell'}|^2\Big[\Big(Q_{LL}T^Q_{LL} + Q_{RL}T^Q_{RL} + B_{RL}T^B_{RL} + L \leftrightarrow R\Big) + \Re e\Big(I_\alpha T^I_\alpha + I_\beta T^I_\beta\Big)\Big]$$

($T^Q_{LL}, T^Q_{RL}, T^Q_{LR}, T^Q_{RR}, T^B_{RL}, T^B_{LR}, T^I_\alpha$, and T^I_β are the function of kinematic variables)

Strategy of Measurement of MPs in $\boldsymbol{\tau^{\pm}\to l^{\pm}l'^{+}l'^{-}\nu_{\tau}\nu_{l}}$

- We do not use a kinematical fitting due to a lack of statistics
- We only do the measurement of branching fraction of $\boldsymbol{\tau^{\pm}\to l^{\pm}l'^{+}l'^{-}\nu_{\tau}\nu_{l}}$
- We use a discrepancy of branching fraction between MC and data

$$\frac{d\Gamma_5}{dx_1 d\Omega_1 dx_2 d\Omega_2 dx_3 d\Omega_3} = \frac{M^2|\vec{\mathbf{p}}_1||\vec{\mathbf{p}}_2||\vec{\mathbf{p}}_3|}{3\cdot 2^{21}\pi^{10}} \underline{\mathcal{T}^s_{\alpha\beta} I^{\alpha\beta}(P)}$$

$$\mathcal{T}^s_{\alpha\beta} I^{\alpha\beta}(P) = e^4|G_{\ell\ell'}|^2\Big[\Big(Q_{LL}T^Q_{LL} + Q_{RL}T^Q_{RL} + B_{RL}T^B_{RL} + L \leftrightarrow R\Big) + \Re e\Big(I_\alpha T^I_\alpha + I_\beta T^I_\beta\Big)\Big]$$

Strategy of Measurement of MPs in $\boldsymbol{\tau^{\pm}\to l^{\pm}l'^{+}l'^{-}\nu_{\tau}\nu_{l}}$

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$$\mathcal{T}^s_{\alpha\beta}I^{\alpha\beta}(P) = e^4|G_{\ell\ell'}|^2\Big[\Big(Q_{LL}T^Q_{LL} + Q_{RL}T^Q_{RL} + B_{RL}T^B_{RL} + L \leftrightarrow R\Big) + \Re e\Big(I_\alpha T^I_\alpha + I_\beta T^I_\beta\Big)\Big]$$

Integrate a differential decay width

$$BR_{\rm exp} = BR_{\rm SM}[Q_{LL} + bQ_{LR} + cB_{LR} + Q_{RR} + dQ_{RL} + eB_{RL} + \Re(fI_\alpha + gI_\beta)]$$

$\Delta b \sim g$ are the integrated value of the terms related to $T^Q_{RL}, T^Q_{LR}, T^Q_{RR}, T^B_{RL}, T^B_{LR}, T^I_\alpha$, and T^I_β

Strategy of Measurement of MPs in $\boldsymbol{\tau^\pm \to l^\pm l'^+ l'^- \nu_\tau \nu_l}$

- We do not use a kinematical fitting due to a lack of statistics
- We only do the measurement of branching fraction of $\boldsymbol{\tau^\pm \to l^\pm l'^+ l'^- \nu_\tau \nu_l}$
- We use a discrepancy of branching fraction between MC and data

$$\frac{d\Gamma_5}{dx_1 d\Omega_1 dx_2 d\Omega_2 dx_3 d\Omega_3} = \frac{M^2|\vec{\mathbf{p}}_1||\vec{\mathbf{p}}_2||\vec{\mathbf{p}}_3|}{3\cdot 2^{21}\pi^{10}} \underline{\mathcal{T}^s_{\alpha\beta} I^{\alpha\beta}(P)}$$

$$\mathcal{T}^s_{\alpha\beta} I^{\alpha\beta}(P) = e^4 |G_{\ell\ell'}|^2 \Big[\Big(Q_{LL} T^Q_{LL} + Q_{RL} T^Q_{RL} + B_{RL} T^B_{RL} + L \leftrightarrow R \Big) + \Re e \Big(I_\alpha T^I_\alpha + I_\beta T^I_\beta \Big) \Big]$$

Integrate a differential decay width

$$BR_{\text{exp}} = BR_{\text{SM}}[Q_{LL} + bQ_{LR} + cB_{LR} + Q_{RR} + dQ_{RL} + eB_{RL} + \Re(fI_\alpha + gI_\beta)]$$

$\Delta b \sim g$ are the integrated value of the terms related to $T^Q_{RL}, T^Q_{LR}, T^Q_{RR}, T^B_{RL}, T^B_{LR}, T^I_\alpha$, and T^I_β

Assume the discrepancy Δ $\quad BR^{\text{Measured}}_{\tau^\pm \to l^\pm l^+ l^- \nu_\tau \nu_l} = BR^{\text{SM predicted}}_{\tau^\pm \to l^\pm l^+ l^- \nu_\tau \nu_l} + \Delta$

We assume that, the discrepancy Δ is brought by only one term in BR_{exp}

We give constraints to Michel parameters by the discrepancy Δ:

$Q_{RL} < \Delta/(BR_{\text{SM}} b)$, $\quad B_{RL} < \Delta/(BR_{\text{SM}} c)$, $\quad Q_{RR} < \Delta/(BR_{\text{SM}})$, $\quad Q_{LR} < \Delta/(BR_{\text{SM}} d)$,

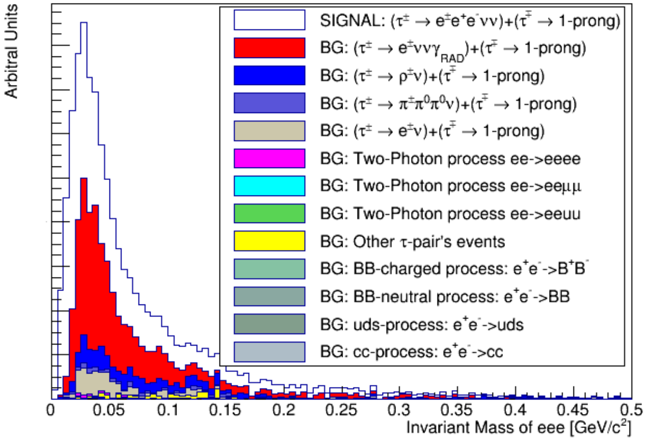
$B_{LR} < \Delta/(BR_{\text{SM}} e)$, $\quad I_\alpha < \Delta/(BR_{\text{SM}} f)$, $\quad I_\beta < \Delta/(BR_{\text{SM}} g)$,

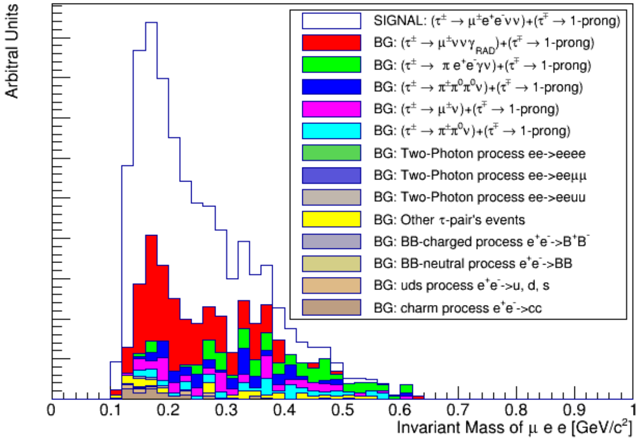
Result of MC study:

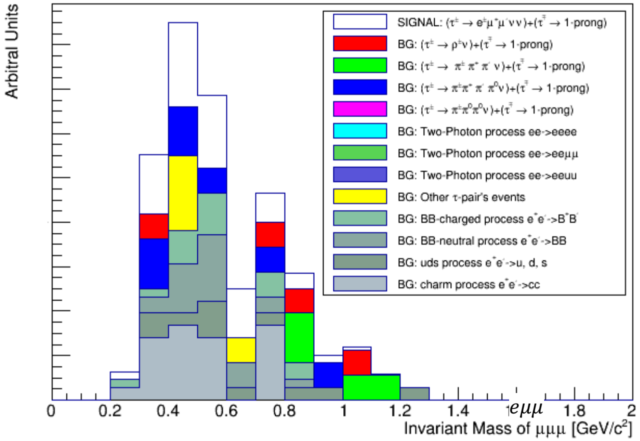
Monte Carlo (MC) sample of 4 million signal decays was used.

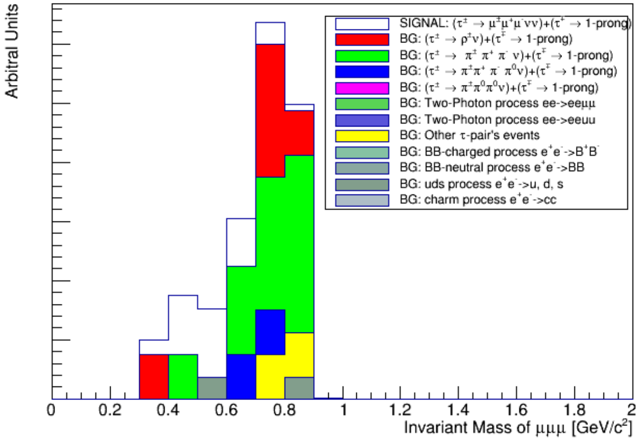
Histograms shown below are plotted with assuming the SM-predicted branching ratio.

τ^- decay mode	$e^-e^+e^-\bar{\nu}_e\nu_\tau$	$\mu^-e^+e^-\bar{\nu}_\mu\nu_\tau$	$e^-\mu^+\mu^-\bar{\nu}_e\nu_\tau$	$\mu^-\mu^+\mu^-\bar{\nu}_\mu\nu_\tau$
Detection efficiency	(1.769±0.004)%	(1.204±0.003)%	(3.561±0.006)%	(1.674±0.004)%
Main backgrounds	$e^-\bar{\nu}_e\nu_\tau\gamma$ $\to e^-\bar{\nu}_e\nu_\tau(e^+e^-)$ $\pi^-\pi^0\nu_\tau$ $\to\pi^-(\gamma\gamma)\nu_\tau$ $\to\pi^-((e^+e^-)\gamma)\nu_\tau$ (mis-ID π as e)	$\mu^-\bar{\nu}_\mu\nu_\tau\gamma$ $\to\mu^-\bar{\nu}_\mu\nu_\tau(e^+e^-)$ $\pi^-\pi^0\nu_\tau$ $\to\pi^-(e^+e^-\gamma)\nu_\tau$ $\pi^-\pi^0\pi^0\nu_\tau$ $\to\pi^-(\gamma\gamma)(\gamma\gamma)\nu_\tau$ $\to\pi^-((e^+e^-)\gamma)(\gamma\gamma)\nu_\tau$ (mis-ID π as μ)	$\pi^-\pi^0\nu_\tau$ $\to\pi^-(\gamma\gamma)\nu_\tau$ $\to\pi^-((e^+e^-)\gamma)\nu_\tau$ $\pi^-\pi^+\pi^-\nu_\tau$ (mis-ID π as μ, e) BB, cc, uds Processes	$\pi^-\pi^0\nu_\tau$ $\to\pi^-(\gamma\gamma)\nu_\tau$ $\to\pi^-((e^+e^-)\gamma)\nu_\tau$ $\pi^-\pi^+\pi^-\nu_\tau$ (mis-ID π as μ)
Expected number of signal events	1300	430	8	4
Fraction of the signal	47%	50%	22%	16%









$\boldsymbol{\tau^- \to l^- \bar{\nu}_l \nu_\tau l'^+ l'^-}$

Preliminary Estimation of systematic uncertainties (for branching fraction)

$\tau^\pm \to e^\pm e^+ e^- \nu_\tau \nu_e$ about 10%

Dominant errors:
- Statistical error
- Particle Identification's error

$\tau^\pm \to \mu^\pm e^+ e^- \nu_\tau \nu_\mu$ about 15%

Dominant errors:
- Statistical error
- Particle Identification's error

$\tau^\pm \to e^\pm \mu^+ \mu^- \nu_\tau \nu_e$ about 70%

Dominant errors:
- Statistical error

$\tau^\pm \to \mu^\pm \mu^+ \mu^- \nu_\tau \nu_\mu$ about 70%

Dominant errors:
- Statistical error

The data will be opened soon and the preliminary result of branching fractions will be finalized

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Conclusion & Future Plan

- To verify the SM's prediction that the Lorentz structure of the charged weak current has a V-A coupling, we study the Lorentz structure.
- For the study of Lorentz structure, a measurement of Michel parameters is ongoing for three modes below at Belle.
    - $\tau^- \to l^-\bar{\nu}_l\nu_\tau$ (Target MPs: $\rho,\ \eta,\ \xi,\ \delta$)
    - $\tau^- \to l^-\bar{\nu}_l\nu_\tau\gamma$ (Target MPs: $\bar{\eta},\ \xi\kappa$)
    - $\tau^- \to l^-\bar{\nu}_l\nu_\tau l'^+l'^-$ (Target MPs: $Q_{LL}, Q_{RL}, Q_{LR}, Q_{RR}, B_{RL}, B_{LR}, I_\alpha, I_\beta$)
- Belle has a good environment for the measurement of MPs of tau-lepton.
    - Low background & Energy of center of mass frame is known
    - Pair creation ($B\bar{B}$, $\tau^+\tau^-$, etc). One-side can be used for tagging.
    - Large number of tau-pairs ($N_{\tau\tau}\sim 9.0\times10^8$)
- For $\tau^- \to l^-\bar{\nu}_l\nu_\tau$ and $\tau^- \to l^-\bar{\nu}_l\nu_\tau\gamma$, the result of $\rho,\ \eta,\ \xi,\ \delta,\ \bar{\eta}, \xi\kappa$ will be published.
- For $\tau^- \to l^-\bar{\nu}_l\nu_\tau l'^+l'^-$, we try to give the constraints to MPs ($Q_{LL}, Q_{RL}, Q_{LR}, Q_{RR}, B_{RL}, B_{LR}, I_\alpha, I_\beta$) from branching fractions and the estimation of its systematic errors is ongoing and finalized soon then open data.

Thank you!

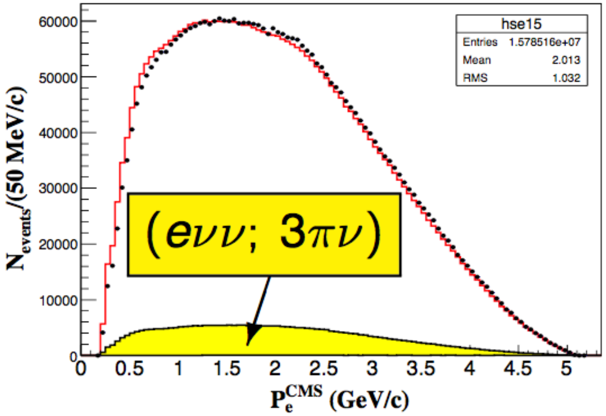
Backup

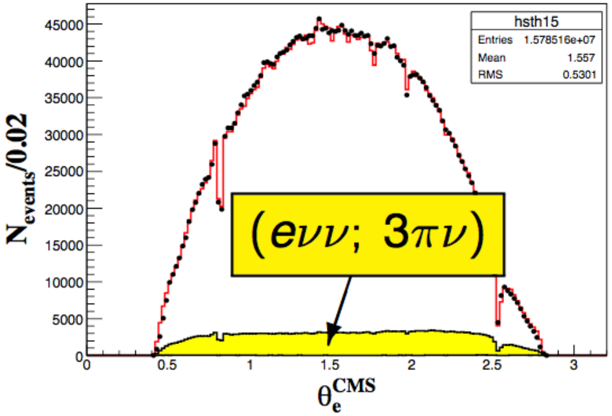
Backup ($\tau^- \rightarrow l^-\bar{\nu}_l\nu_\tau$)

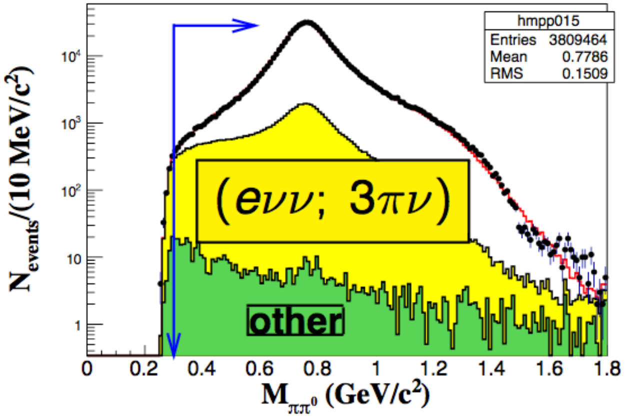
Selection criteria

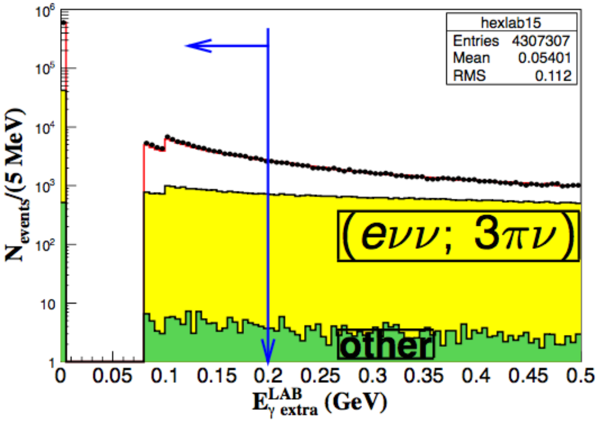
- After the standard preselections we take events with two oppositely charged tracks, one of them is identified as lepton (eID, $\mu ID > 0.9$) and the other one as pion ($PID(\pi/K) > 0.4$).
- π^0 candidate is reconstructed from the pair of gammas ($E_\gamma^{\rm LAB} > 80$ MeV) satisfying $115\ {\rm MeV}/c^2 < M_{\gamma\gamma} < 150\ {\rm MeV}/c^2$, $P_{\pi^0}^{\rm CMS} > 0.3\ {\rm GeV}/c$.
- $\cos(\vec{P}_{\rm lep}, \vec{P}_\pi) < 0$, $\cos(\vec{P}_{\rm lep}, \vec{P}_{\pi^0}) < 0$, $0.3\ {\rm GeV}/c^2 < M_{\pi\pi^0} < 1.8\ {\rm GeV}/c^2$.
- $E_{\rm rest\gamma}^{\rm LAB} < 0.2$ GeV

Detection efficiency $\varepsilon_{\rm det} \simeq 12\%$



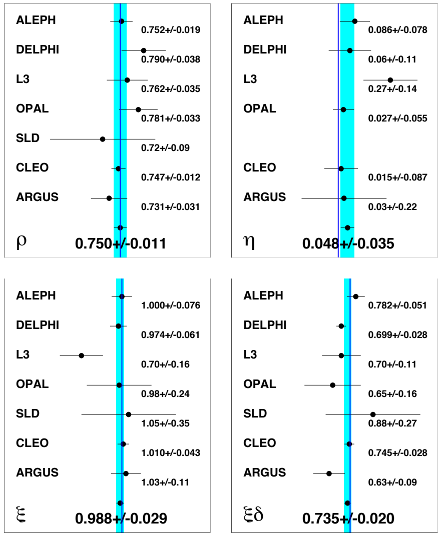






Status of Michel parameters in τ decays

Michel par.	Measured value	Experiment	SM value
ρ (e or μ)	$0.747 \pm 0.010 \pm 0.006$ 1.2%	CLEO-97	3/4
η (e or μ)	$0.012 \pm 0.026 \pm 0.004$ 2.6%	ALEPH-01	0
ξ (e or μ)	$1.007 \pm 0.040 \pm 0.015$ 4.3%	CLEO-97	1
$\xi\delta$ (e or μ)	$0.745 \pm 0.026 \pm 0.009$ 2.8%	CLEO-97	3/4
ξ_{h} (all hadr.)	$0.992 \pm 0.007 \pm 0.008$ 1.1%	ALEPH-01	1



With $\times 300$ Belle statistics we can improve MP uncertainties by one order of magnitude

In BSM models the couplings to τ are expected to be enhanced in comparison with μ.

Also contribution from New Physics in τ decays can be amplified by $(\frac{m_\tau}{m_\mu})^n$.

- **Type II 2HDM:** $\eta_\mu(\tau) = \frac{m_\mu M_\tau}{2}\left(\frac{tan^2\beta}{M^2_{H\pm}}\right)^2$; $\frac{\eta_\mu(\tau)}{\eta_e(\mu)} = \frac{M_\tau}{m_e} \approx 3500$
- **Tensor interaction:** $\mathcal{L} = \frac{g}{2\sqrt{2}} W^\mu \left\{\bar\nu\gamma_\mu(1-\gamma^5)\tau + \frac{\kappa^W_\tau}{2m_\tau}\partial^\nu\left(\bar\nu\sigma_{\mu\ nu}(1-\gamma^5)\tau\right)\right\}$,
  $-0.096 < \kappa^W_\tau < 0.037$: DELPHI Abreu EPJ C16 (2000) 229.
- **Unparticles:** Moyotl PRD 84 (2011) 073010, Choudhury PLB 658 (2008) 148.
- **Lorentz and CPTV:** Hollenberg PLB **701** (2011) 89
- **Heavy Majorana neutrino:** M. Doi *et al.*, Prog. Theor. Phys. **118** (2007) 1069.

Corrections, detector effects, background

Physical corrections:

- All $\mathcal{O}(\alpha^3)$ QED and electroweak higher order corrections to $e^+e^- \to \tau^+\tau^-(\gamma)$ are included
- Radiative leptonic decays $\tau^- \to \ell^-\bar{\nu}_\ell\nu_\tau\gamma$
- Radiative decay $\tau^- \to \pi^-\pi^0\nu_\tau\gamma$

Detector effects:

- Track momentum resolution
- γ energy and angular resolution
- Effect of external bremsstrahlung for $e-\rho$ events
- Beam energy spread
- EXP/MC efficiency corrections (trigger, track rec., π^0 rec., ℓID, πID)

Background:

The main background comes from $(\ell\nu\nu;\ \pi 2\pi^0\nu)$(~10%), $(\pi\nu;\ \pi\pi^0\nu)$(~1.5%) and $(\rho^+\nu;\ \rho^-\nu)$(~0.5%) events, it is included in PDF analytically. The remaining background(~2.0%) is taken into account using MC-based approach.

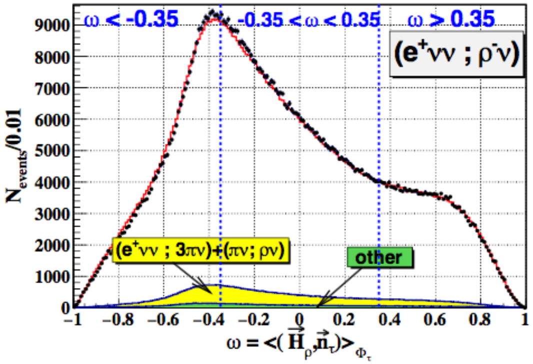
Background from the non-$\tau\tau$ events is $\lesssim 0.1\%$.

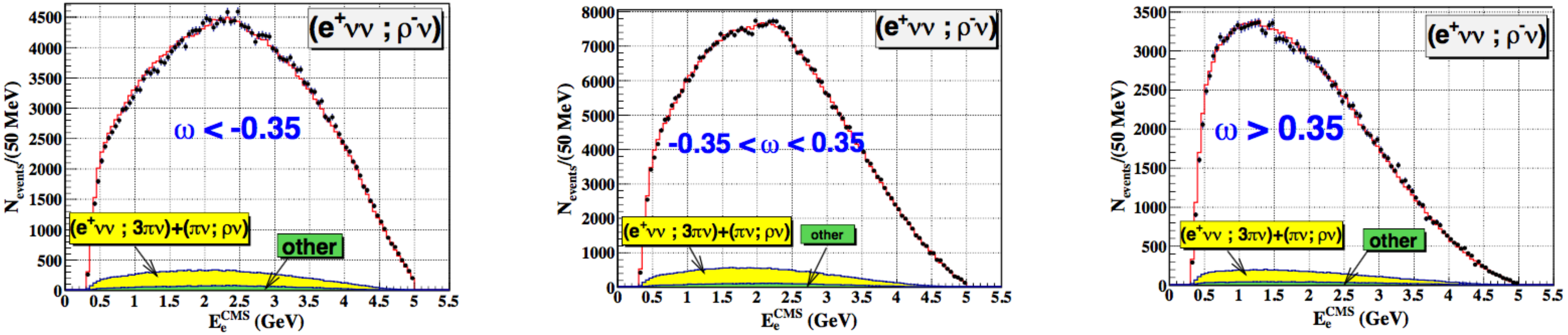
Helicity sensitive variable ω

M. Davier *et. al* Phys. Lett. B **306** (1993) 411.

Helicity sensitive variable ω is introduced as:

$$\omega = \frac{1}{\Phi_2-\Phi_1}\int_{\Phi_1}^{\Phi_2} (\vec{H}_{\rho^\pm}, \vec{n}_{\tau^\pm})\, d\Phi = < (\vec{H}_{\rho^\pm}, \vec{n}_{\tau^\pm}) >_{\Phi_\tau}$$





Spin-spin correlation manifests itself through momentum-momentum correlations of final lepton and pions.

Backup ($\tau^- \rightarrow l^- \bar{\nu}_l \nu_\tau \gamma$)

Method

Reference: N. Shimizu's slide@TAU2016

PDF of signal

$$S(\vec{x}) = \frac{\mathrm{d}\sigma}{\mathrm{d}P_l \mathrm{d}\Omega_l \mathrm{d}P_\gamma \mathrm{d}\Omega_\gamma \mathrm{d}P_\rho \mathrm{d}\Omega_\rho \mathrm{d}m_\rho^2 \mathrm{d}\widetilde{\Omega}_\pi} = \int_{\Phi_1}^{\Phi_2} \mathrm{d}\Phi \frac{\mathrm{d}\sigma}{\mathrm{d}P_l \mathrm{d}\Omega_l \mathrm{d}P_\gamma \mathrm{d}\Omega_\gamma \mathrm{d}P_\rho \mathrm{d}\Omega_\rho \mathrm{d}m_\rho^2 \mathrm{d}\widetilde{\Omega}_\pi \mathrm{d}\Phi}$$

- $S(\vec{x})$: the visible PDF for observable $\vec{x}$ ($N_{\mathrm{dim.}} = 12$)
- $S(\vec{x})$ has a form: $\vec{x} = \{P_l, \Omega_l, P_\gamma, \Omega_\gamma, P_\rho, \Omega_\rho, m^2_{\pi\pi}, \tilde{\Omega}_\pi\}$

☐ Event selection & existence of BG → the visible PDF is formulated as sum of signal and BGs

- $P_{tot}(\vec{x}) = (1 - \sum\lambda_i) \cdot \frac{S(\vec{x})\varepsilon(\vec{x})}{\int \mathrm{d}\vec{x} S(\vec{x})\varepsilon(\vec{x})} + \sum\lambda_i \frac{B_i(\vec{x})\varepsilon(\vec{x})}{\int \mathrm{d}\vec{x} B_i(\vec{x})\varepsilon(\vec{x})}$

i: index of background

$S(\vec{x})$: PDF of signal

$B_i(\vec{x})$: PDF of i-th background

λ_i: fraction of i-th background component

$\varepsilon(\vec{x})$: selection efficiency

☐ Normalization is evaluated by MC

$\sigma_0 = \int \mathrm{d}\vec{x} A_0(\vec{x})$

- $S(\vec{x}) = A_0(\vec{x}) + A_{\bar{\eta}}(\vec{x}) \cdot \bar{\boldsymbol{\eta}} + A_{\xi\kappa}(\vec{x}) \cdot \boldsymbol{\xi\kappa}$
- $\int \mathrm{d}\vec{x} S(\vec{x})\varepsilon(\vec{x}) = \int \mathrm{d}\vec{x} \varepsilon(\vec{x}) A_0(\vec{x}) \frac{A_0 + A_{\bar{\eta}}\bar{\boldsymbol{\eta}} + A_{\xi\kappa}\boldsymbol{\xi\kappa}}{A_0} = \frac{\sigma_0 \bar{\varepsilon}}{N_{\mathrm{sel}}} \sum_k \frac{A_0 + A_{\bar{\eta}}\bar{\boldsymbol{\eta}} + A_{\xi\kappa}\boldsymbol{\xi\kappa}}{A_0}$

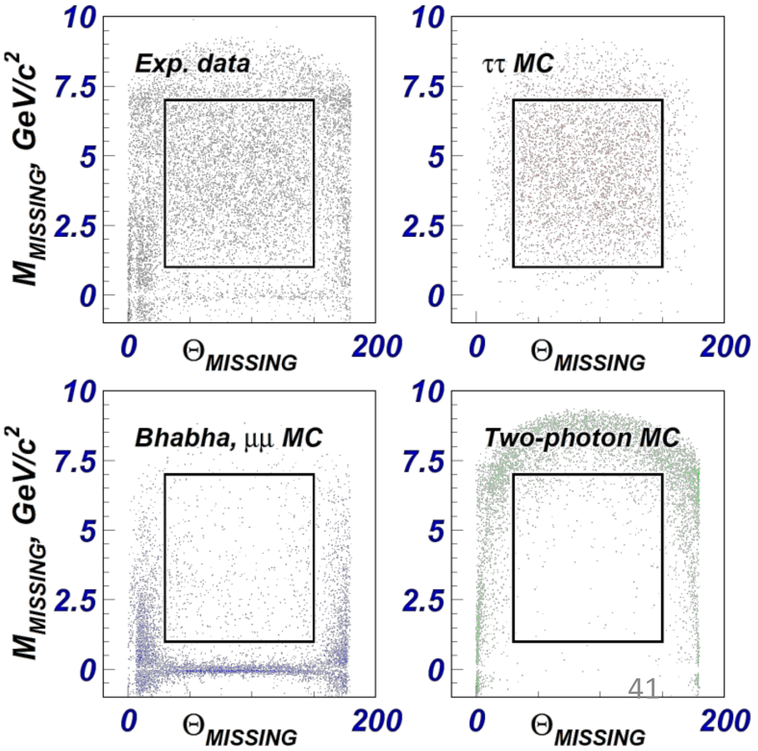
$$= \frac{\sigma_0 \bar{\varepsilon}}{N_{sel}} \left[1 + N_{\bar{\eta}} \cdot \bar{\boldsymbol{\eta}} + N_{\xi\kappa} \cdot \boldsymbol{\xi\kappa}\right]$$

σ_0: absolute normalization

$N_{\bar{\eta}}, N_{\xi\kappa}$: relative normalization

Event selection

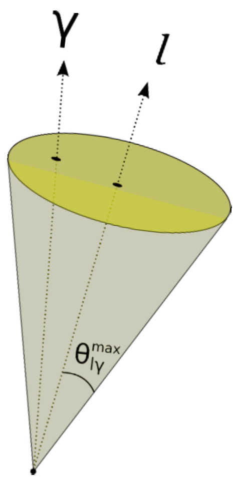
- We use all data taken with $\Upsilon(4S)$ energy: $703\ \mathrm{fb}^{-1}$
- Preselection of $\tau\tau$
  1. exactly two oppositely charged tracks
     - dr < 0.5 cm, $|\mathrm{dz}| < 2.5$ cm, one $P_t > 0.5\ \mathrm{GeV}/c$, the other $P_t > 0.1\ \mathrm{GeV}/c$
  2. ECL cluster energy < 9 GeV
  3. opening angle of two tracks $20° < \psi < 175°$
  4. N_γ<5 for E_γ>80 MeV
  5. $1\ \mathrm{GeV}/c^2 < M_{\mathrm{missing}} < 7\ \mathrm{GeV}/c^2$
  6. $30° < \theta_{\mathrm{missing}} < 175°$



In particular, the last two requirements well discriminate other physics processes like

- Bhabha $e^+e^- \to e^+e^-$
- $e^+e^- \to \mu^+\mu^-$
- Two photon processes

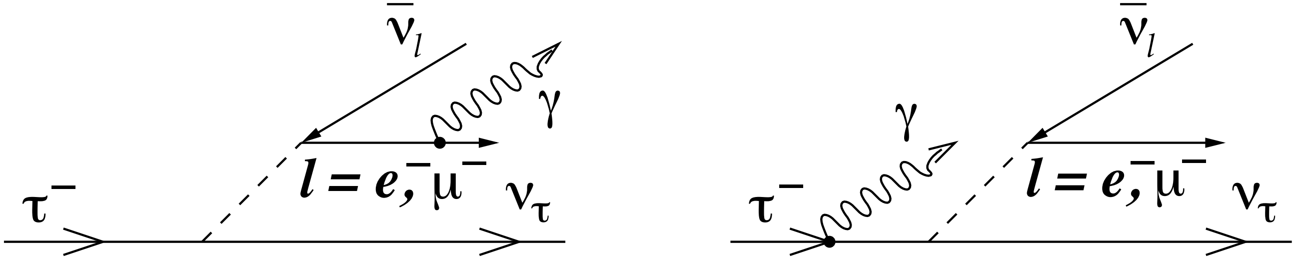
Event selection



- Final selection
  - Electron: $\frac{P_e}{P_e+P_x} > 0.9$, Muon: $\frac{P_\mu}{P_\mu+P_\pi+P_K} > 0.9$
  - Pion: $\frac{P_\pi}{P_\pi+P_K} > 0.4$
  - π^0 candidate: $115 \text{ MeV}/c^2 < m_{\gamma\gamma} < 150 \text{ MeV}/c^2$ for $E_\gamma > 80$ MeV
  - ρ candidate: $0.5 \text{ GeV}/c^2 < m_{\pi\pi} < 1.5 \text{ GeV}/c^2$
  - $\theta_{\rho(l\gamma)} > 90°$
  - $\cos\theta_{e\gamma} > 0.9848$, $\cos\theta_{\mu\gamma} > 0.9700$
  - Energy of photons which are not associated with any tracks $E_{\text{extra}\gamma} < 0.2$ GeV for $\tau \to e\nu\bar{\nu}\gamma$ and $E_{\text{extra}\gamma} < 0.3$ GeV for $\tau \to \mu\nu\bar{\nu}\gamma$

Michel parameters in $\tau \to \ell\nu\nu\gamma$, ($\ell = e, \mu$) (I)

C. Fronsdal and H. Uberall, Phys. Rev. **113** (1959) 654. ($m_\ell = 0$)
A. B. Arbuzov and T. V. Kopylova, JHEP **1609** (2016) 109. ($m_\ell \neq 0$)



Photon carries information about spin state of outgoing lepton, as a result two additional parameters, $\boldsymbol{\bar{\eta}}$ and $\boldsymbol{\xi\kappa}$, can be extracted.
These parameters were measured in τ decays at Belle for the first time.

$$\frac{d\Gamma(\tau^\mp \to \ell^\mp \nu_\ell \nu_\tau \gamma)}{dx\, dy\, d\Omega_\ell\, d\Omega_\gamma} = \Gamma_0 \frac{\alpha}{64\pi^3} \frac{\beta_\ell}{y} \Big[F(x, y, d) \pm P_\tau \left(\beta_\ell \cos\theta_\ell G(x, y, d) + \cos\theta_\gamma H(x, y, d) \right) \Big],$$

$$\Gamma_0 = G_{\mathrm{F}}^2 m_\tau^5 / 192\pi^3, \;\; \beta_\ell = \sqrt{1 - m_\ell^2 / E_\ell^2}, \;\; x = 2E_\ell / m_\tau, \;\; y = 2E_\gamma / m_\tau, \;\; d = 1 - \beta_\ell \cos\theta_{\ell\gamma}$$

$$F = F_0 + \boldsymbol{\bar{\eta}} F_1, \;\; G = G_0 + \boldsymbol{\xi\kappa} G_1, \;\; H = H_0 + \boldsymbol{\xi\kappa} H_1, \;\; \frac{d\sigma(\ell^\mp \nu\nu\gamma, \rho^\pm \nu)}{dE_\ell^*\, d\Omega_\ell^*\, dE_\gamma^*\, d\Omega_\gamma^*\, d\Omega_\rho^*\, dm_{\pi\pi}^2\, d\tilde{\Omega}_\pi\, d\Omega_\tau} = A_0 + \boldsymbol{\bar{\eta}} A_1 + \boldsymbol{\xi\kappa} A_2$$

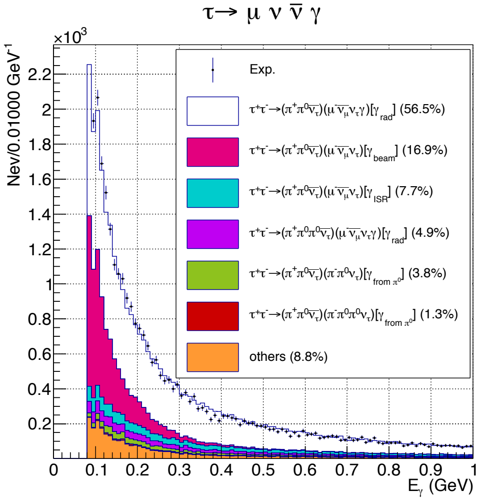
$$\mathcal{F}(\vec{z}) = \frac{d\sigma(\ell^\mp \nu\nu\gamma, \rho^\pm \nu)}{dp_\ell\, d\Omega_\ell\, dp_\gamma\, d\Omega_\gamma\, dp_\rho\, d\Omega_\rho\, dm_{\pi\pi}^2\, d\tilde{\Omega}_\pi} = \int\limits_{\Phi_1}^{\Phi_2} \frac{d\sigma(\ell^\mp \nu\nu\gamma, \rho^\pm \nu)}{dE_\ell^*\, d\Omega_\ell^*\, dE_\gamma^*\, d\Omega_\gamma^*\, d\Omega_\rho^*\, dm_{\pi\pi}^2\, d\tilde{\Omega}_\pi\, d\Omega_\tau} |\mathrm{JACOBIAN}|\, d\Phi_\tau$$

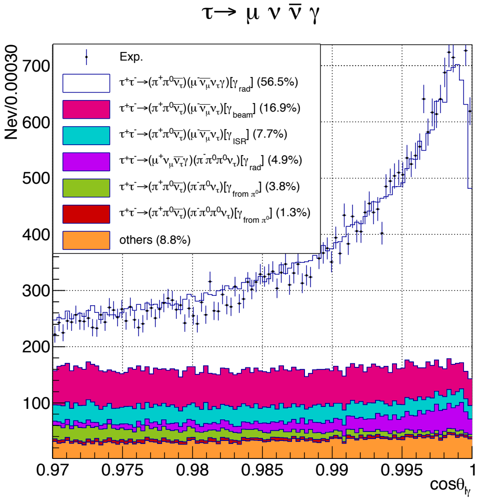
$$L = \prod_{k=1}^{N} \mathcal{P}^{(k)}, \;\; \mathcal{P}^{(k)} = \frac{\mathcal{F}(\vec{z}^{(k)})}{\mathcal{N}(\vec{\Theta})} = \frac{\mathcal{F}_0 + \mathcal{F}_1 \boldsymbol{\bar{\eta}} + \mathcal{F}_2 \boldsymbol{\xi\delta}}{\mathcal{N}_0 + \mathcal{N}_1 \boldsymbol{\bar{\eta}} + \mathcal{N}_2 \boldsymbol{\xi\delta}}, \;\; \mathcal{N}_k = \int \mathcal{F}_k(\vec{z}) d\vec{z}, \;\; (k = 0, 1, 2)$$

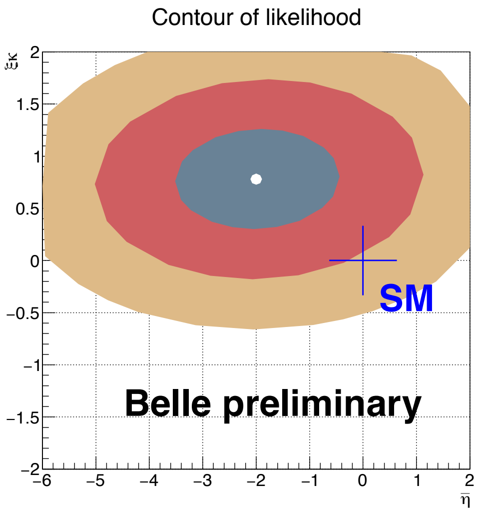
$\boldsymbol{\bar{\eta}}$ and $\boldsymbol{\xi\delta}$ are extracted in the unbinned maximum likelihood fit of ($\ell\nu\nu\gamma;\ \rho\nu$) events in the 12D phase space in CMS.

Michel parameters in $\tau \to \ell\nu\nu\gamma$, ($\ell = e, \mu$) (II)

$N_{\tau\tau} = 646 \times 10^6$, **selected: 71171** ($\mu\nu\nu\gamma;\ \rho\nu$) **and 776834** ($e\nu\nu\gamma;\ \rho\nu$) **events**







Source	$\sigma^e_{\bar\eta}$	$\sigma^e_{\xi\kappa}$	$\sigma^\mu_{\bar\eta}$	$\sigma^\mu_{\xi\kappa}$
Normalization	4.3	0.94	0.15	0.04
Background PDF	2.5	0.24	0.67	0.22
Branching ratios	3.8	0.05	0.25	0.01
Cluster merge in ECL	2.2	0.46	0.02	0.06
Detector resolution	0.74	0.20	0.22	0.02
Data/MC eff. corr.	1.9	0.14	0.04	0.04
Total	7.0	1.1	0.76	0.24

Belle preliminary

$$\bar\eta = -1.3 \pm 1.5 \pm 0.8$$

$$\xi\kappa = 0.5 \pm 0.4 \pm 0.2$$

Backup ($\boldsymbol{\tau^{\pm}\rightarrow l^{\pm}l'^{+}l'^{-}\nu_{\tau}\nu_{l}}$)

Preliminary Results

$$\tau^{\pm} \rightarrow l^{\pm} l'^{+} l'^{-} \nu_{\tau} \nu_{l} \; (l, l' = e, \mu)$$

Preliminary : Estimation of systematic uncertainties

$\tau^{\pm} \rightarrow e^{\pm} e^{+} e^{-} \nu_{\tau} \nu_{e}$

contents	syst. error (SVD1)	syst. error (SVD2)
PID correction	7.3%	5.0%
Tracking efficiency	1.1%	1.1%
Trigger correction	0.1%	0.1%
Tag-side	0.35%	0.35%
Luminosity	1.4%	1.4%
Background	5.5%	2.8%
Selection Cut	–	–
Total	9.3%	6.0%

~ 10%

$\tau^{\pm} \rightarrow \mu^{\pm} e^{+} e^{-} \nu_{\tau} \nu_{\mu}$

contents	syst. error (SVD1)	syst. error (SVD2)
PID correction	6.9%	6.4%
Tracking efficiency	1.1%	1.1%
Trigger correction	0.1%	0.1%
Tag-side	0.35%	0.35%
Luminosity	1.4%	1.4%
Background	9.6%	4.8%
Selection Cut	–	–
Total	12.0%	8.2%

~ 15%

$\tau^{\pm} \rightarrow e^{\pm} \mu^{+} \mu^{-} \nu_{\tau} \nu_{e}$

contents	syst. error (SVD1)	syst. error (SVD2)
PID correction	8.7%	7.4%
Tracking efficiency	1.1%	1.1%
Trigger correction	0.1%	0.1%
Tag-side	0.35%	0.35%
Luminosity	1.4%	1.4%
Background	71%	35%
Selection Cut	–	–
Total	72%	36%

~ 70%

$\tau^{\pm} \rightarrow \mu^{\pm} \mu^{+} \mu^{-} \nu_{\tau} \nu_{\mu}$

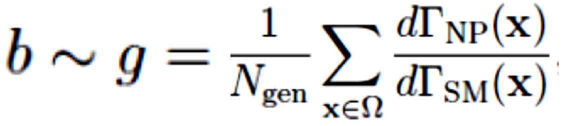
contents	syst. error (SVD1)	syst. error (SVD2)
PID correction	6.2%	8.4%
Tracking efficiency	1.1%	1.1%
Trigger correction	0.1%	0.1%
Tag-side	0.35%	0.35%
Luminosity	1.4%	1.4%
Background	71%	35%
Selection Cut	–	–
Total	72%	36%

~ 70%

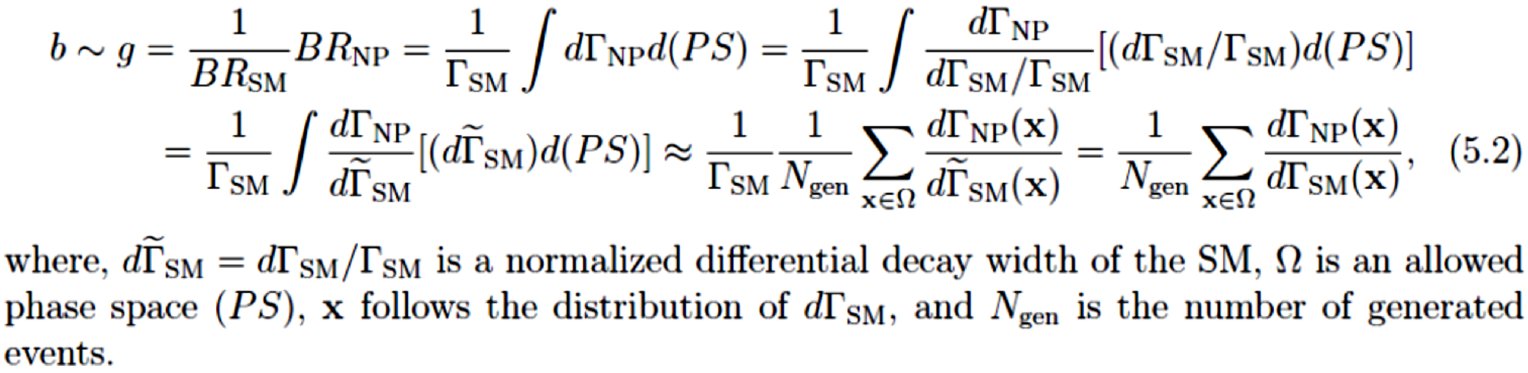
We are working to finalize the result and the estimation of systematic errors will be finished soon.

Theoretical Formula of BRs depends on Michel parameters

- Method of Monte Carlo integral is used
- Take the ratio of BR_{SM} to avoid considering the complicated common factor appears in the theoretical formula



$$b \sim g = \frac{1}{N_{\mathrm{gen}}} \sum_{\mathbf{x} \in \Omega} \frac{d\Gamma_{\mathrm{NP}}(\mathbf{x})}{d\Gamma_{\mathrm{SM}}(\mathbf{x})}$$



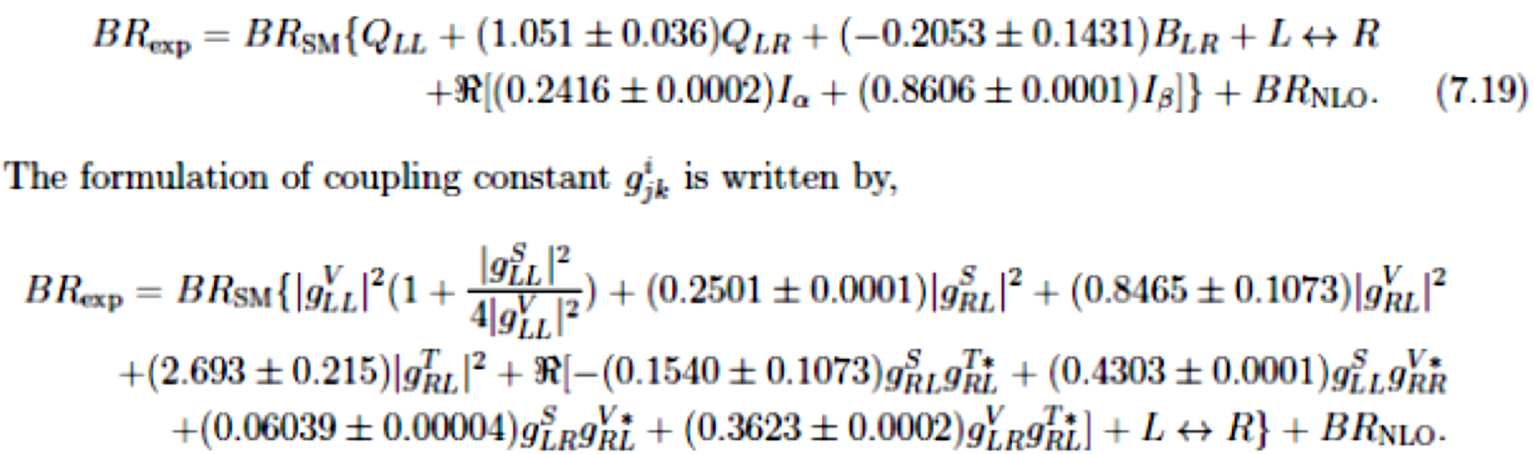
$$\begin{aligned} b \sim g = \frac{1}{BR_{\mathrm{SM}}} BR_{\mathrm{NP}} &= \frac{1}{\Gamma_{\mathrm{SM}}} \int d\Gamma_{\mathrm{NP}} d(PS) = \frac{1}{\Gamma_{\mathrm{SM}}} \int \frac{d\Gamma_{\mathrm{NP}}}{d\Gamma_{\mathrm{SM}}/\Gamma_{\mathrm{SM}}} [(d\Gamma_{\mathrm{SM}}/\Gamma_{\mathrm{SM}}) d(PS)] \\ &= \frac{1}{\Gamma_{\mathrm{SM}}} \int \frac{d\Gamma_{\mathrm{NP}}}{d\widetilde{\Gamma}_{\mathrm{SM}}} [(d\widetilde{\Gamma}_{\mathrm{SM}}) d(PS)] \approx \frac{1}{\Gamma_{\mathrm{SM}}} \frac{1}{N_{\mathrm{gen}}} \sum_{\mathbf{x} \in \Omega} \frac{d\Gamma_{\mathrm{NP}}(\mathbf{x})}{d\widetilde{\Gamma}_{\mathrm{SM}}(\mathbf{x})} = \frac{1}{N_{\mathrm{gen}}} \sum_{\mathbf{x} \in \Omega} \frac{d\Gamma_{\mathrm{NP}}(\mathbf{x})}{d\Gamma_{\mathrm{SM}}(\mathbf{x})}, \quad (5.2) \end{aligned}$$

where, $d\widetilde{\Gamma}_{\mathrm{SM}} = d\Gamma_{\mathrm{SM}}/\Gamma_{\mathrm{SM}}$ is a normalized differential decay width of the SM, Ω is an allowed phase space (PS), $\mathbf{x}$ follows the distribution of $d\Gamma_{\mathrm{SM}}$, and N_{gen} is the number of generated events.

From next page, we show the result of calculated coefficients for our four target modes.

↓ from my note

$\tau^{\pm}\rightarrow e^{\pm}e^{+}e^{-}\nu_{\tau}\nu_{e}$

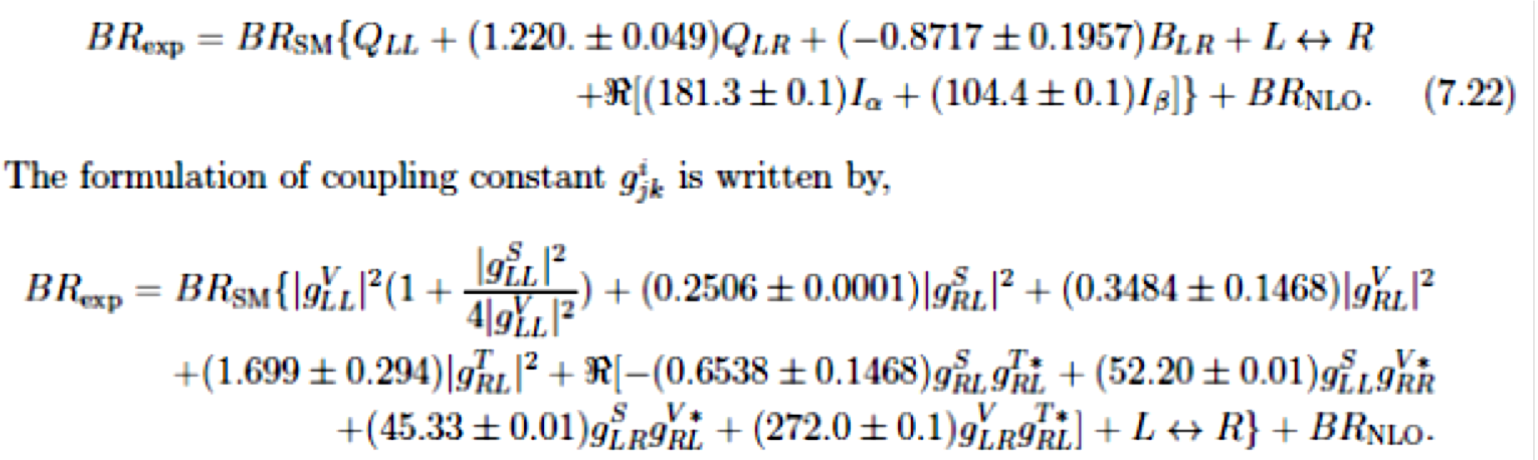


$$BR_{\mathrm{exp}} = BR_{\mathrm{SM}}\{Q_{LL} + (1.051 \pm 0.036)Q_{LR} + (-0.2053 \pm 0.1431)B_{LR} + L \leftrightarrow R$$
$$+\Re[(0.2416 \pm 0.0002)I_{\alpha} + (0.8606 \pm 0.0001)I_{\beta}]\} + BR_{\mathrm{NLO}}. \quad (7.19)$$

The formulation of coupling constant g^{i}_{jk} is written by,

$$BR_{\mathrm{exp}} = BR_{\mathrm{SM}}\{|g^{V}_{LL}|^2(1 + \frac{|g^{S}_{LL}|^2}{4|g^{V}_{LL}|^2}) + (0.2501 \pm 0.0001)|g^{S}_{RL}|^2 + (0.8465 \pm 0.1073)|g^{V}_{RL}|^2$$
$$+(2.693 \pm 0.215)|g^{T}_{RL}|^2 + \Re[-(0.1540 \pm 0.1073)g^{S}_{RL}g^{T*}_{RL} + (0.4303 \pm 0.0001)g^{S}_{LL}g^{V*}_{RR}$$
$$+(0.06039 \pm 0.00004)g^{S}_{LR}g^{V*}_{RL} + (0.3623 \pm 0.0002)g^{V}_{LR}g^{T*}_{RL}] + L \leftrightarrow R\} + BR_{\mathrm{NLO}}.$$

$\tau^{\pm}\rightarrow \mu^{\pm}e^{+}e^{-}\nu_{\tau}\nu_{\mu}$



$$BR_{\mathrm{exp}} = BR_{\mathrm{SM}}\{Q_{LL} + (1.220. \pm 0.049)Q_{LR} + (-0.8717 \pm 0.1957)B_{LR} + L \leftrightarrow R$$
$$+\Re[(181.3 \pm 0.1)I_{\alpha} + (104.4 \pm 0.1)I_{\beta}]\} + BR_{\mathrm{NLO}}. \quad (7.22)$$

The formulation of coupling constant g^{i}_{jk} is written by,

$$BR_{\mathrm{exp}} = BR_{\mathrm{SM}}\{|g^{V}_{LL}|^2(1 + \frac{|g^{S}_{LL}|^2}{4|g^{V}_{LL}|^2}) + (0.2506 \pm 0.0001)|g^{S}_{RL}|^2 + (0.3484 \pm 0.1468)|g^{V}_{RL}|^2$$
$$+(1.699 \pm 0.294)|g^{T}_{RL}|^2 + \Re[-(0.6538 \pm 0.1468)g^{S}_{RL}g^{T*}_{RL} + (52.20 \pm 0.01)g^{S}_{LL}g^{V*}_{RR}$$
$$+(45.33 \pm 0.01)g^{S}_{LR}g^{V*}_{RL} + (272.0 \pm 0.1)g^{V}_{LR}g^{T*}_{RL}] + L \leftrightarrow R\} + BR_{\mathrm{NLO}}.$$

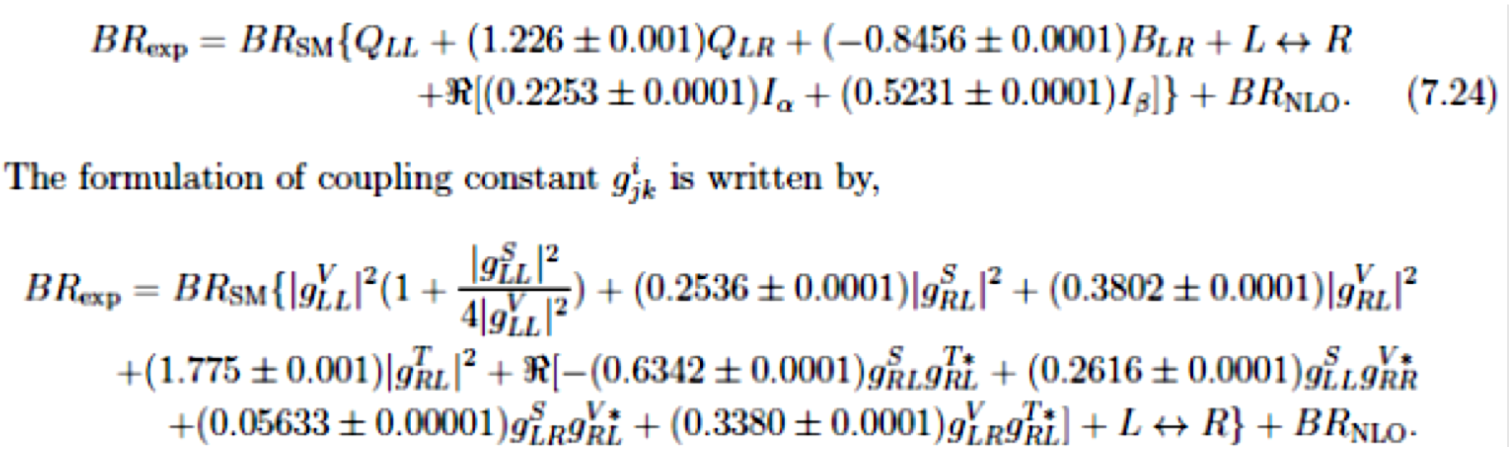
Because of pseudo peculiarity which appears in some terms, some result include large error. This pseudo peculiarity is caused mainly by the factor of virtual gamma conversion ($\gamma \rightarrow ee$)

$$|1/{q_{ee}}^2|^2 \sim |1/O(1\mathrm{MeV})^2|^2$$

in the matrix element.

$\tau^{\pm} \rightarrow e^{\pm}\mu^{+}\mu^{-}\nu_{\tau}\nu_{e}$

↓ from my note

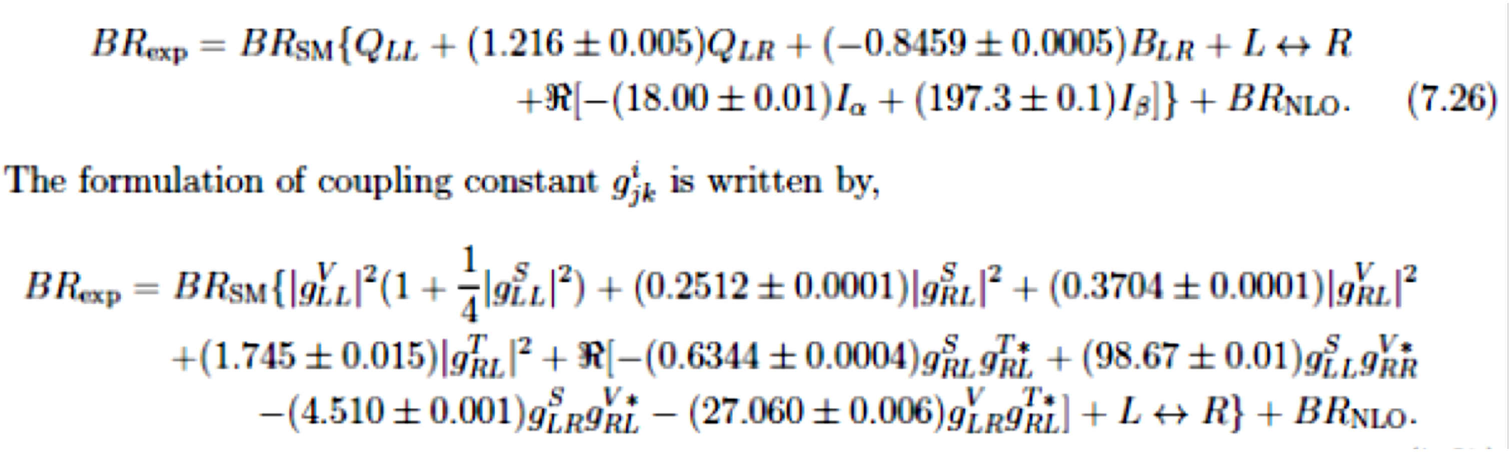


$$BR_{\mathrm{exp}} = BR_{\mathrm{SM}}\{Q_{LL} + (1.226 \pm 0.001)Q_{LR} + (-0.8456 \pm 0.0001)B_{LR} + L \leftrightarrow R \\ +\Re[(0.2253 \pm 0.0001)I_{\alpha} + (0.5231 \pm 0.0001)I_{\beta}]\} + BR_{\mathrm{NLO}}. \qquad (7.24)$$

The formulation of coupling constant g^{i}_{jk} is written by,

$$BR_{\mathrm{exp}} = BR_{\mathrm{SM}}\{|g^{V}_{LL}|^2(1 + \frac{|g^{S}_{LL}|^2}{4|g^{V}_{LL}|^2}) + (0.2536 \pm 0.0001)|g^{S}_{RL}|^2 + (0.3802 \pm 0.0001)|g^{V}_{RL}|^2 \\ +(1.775 \pm 0.001)|g^{T}_{RL}|^2 + \Re[-(0.6342 \pm 0.0001)g^{S}_{RL}g^{T*}_{RL} + (0.2616 \pm 0.0001)g^{S}_{LL}g^{V*}_{RR} \\ +(0.05633 \pm 0.00001)g^{S}_{LR}g^{V*}_{RL} + (0.3380 \pm 0.0001)g^{V}_{LR}g^{T*}_{RL}] + L \leftrightarrow R\} + BR_{\mathrm{NLO}}.$$

$\tau^{\pm} \rightarrow \mu^{\pm}\mu^{+}\mu^{-}\nu_{\tau}\nu_{\mu}$



$$BR_{\mathrm{exp}} = BR_{\mathrm{SM}}\{Q_{LL} + (1.216 \pm 0.005)Q_{LR} + (-0.8459 \pm 0.0005)B_{LR} + L \leftrightarrow R \\ +\Re[-(18.00 \pm 0.01)I_{\alpha} + (197.3 \pm 0.1)I_{\beta}]\} + BR_{\mathrm{NLO}}. \qquad (7.26)$$

The formulation of coupling constant g^{i}_{jk} is written by,

$$BR_{\mathrm{exp}} = BR_{\mathrm{SM}}\{|g^{V}_{LL}|^2(1 + \frac{1}{4}|g^{S}_{LL}|^2) + (0.2512 \pm 0.0001)|g^{S}_{RL}|^2 + (0.3704 \pm 0.0001)|g^{V}_{RL}|^2 \\ +(1.745 \pm 0.015)|g^{T}_{RL}|^2 + \Re[-(0.6344 \pm 0.0004)g^{S}_{RL}g^{T*}_{RL} + (98.67 \pm 0.01)g^{S}_{LL}g^{V*}_{RR} \\ -(4.510 \pm 0.001)g^{S}_{LR}g^{V*}_{RL} - (27.060 \pm 0.006)g^{V}_{LR}g^{T*}_{RL}] + L \leftrightarrow R\} + BR_{\mathrm{NLO}}.$$

POSSIBLE MEASUREMENT

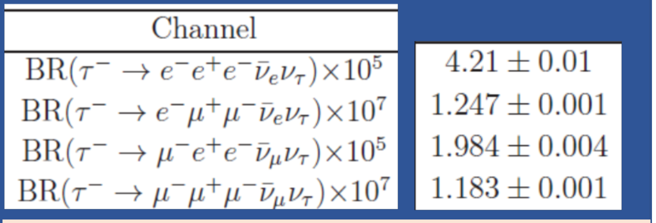
Because of expected statistics, we concentrate on the measurement through two modes:

$\tau^{\pm} \rightarrow e^{\pm} e^{+} e^{-} \nu_{\tau} \nu_{e}$, $\tau^{\pm} \rightarrow \mu^{\pm} e^{+} e^{-} \nu_{\tau} \nu_{\mu}$

The statistics of other two modes ($\tau^{\pm} \rightarrow e^{\pm} \mu^{+} \mu^{-} \nu_{\tau} \nu_{e}$, $\tau^{\pm} \rightarrow \mu^{\pm} \mu^{+} \mu^{-} \nu_{\tau} \nu_{\mu}$) expected to be small because of its expected branching fraction is small. And the measurement of Michel parameters is difficult from these two modes.

Expected BRs from the Standard Model

Prediction from theory



	Channel	
$\tau^{\pm} \rightarrow e^{\pm} e^{+} e^{-} \nu_{\tau} \nu_{e}$	$\mathrm{BR}(\tau^- \rightarrow e^- e^+ e^- \bar{\nu}_e \nu_\tau) \times 10^5$	4.21 ± 0.01
$\tau^{\pm} \rightarrow e^{\pm} \mu^{+} \mu^{-} \nu_{\tau} \nu_{e}$	$\mathrm{BR}(\tau^- \rightarrow e^- \mu^+ \mu^- \bar{\nu}_e \nu_\tau) \times 10^7$	1.247 ± 0.001
$\tau^{\pm} \rightarrow \mu^{\pm} e^{+} e^{-} \nu_{\tau} \nu_{\mu}$	$\mathrm{BR}(\tau^- \rightarrow \mu^- e^+ e^- \bar{\nu}_\mu \nu_\tau) \times 10^5$	1.984 ± 0.004
$\tau^{\pm} \rightarrow \mu^{\pm} \mu^{+} \mu^{-} \nu_{\tau} \nu_{\mu}$	$\mathrm{BR}(\tau^- \rightarrow \mu^- \mu^+ \mu^- \bar{\nu}_\mu \nu_\tau) \times 10^7$	1.183 ± 0.001

Ref. [JHEP 1604, 185 (2016)]

Result of Monte Carlo simulation

	$e^{\pm} e^{+} e^{-} \nu_{\tau} \nu_{e}$	$\mu^{\pm} e^{+} e^{-} \nu_{\tau} \nu_{\mu}$	$e^{\pm} \mu^{+} \mu^{-} \nu_{\tau} \nu_{e}$	$\mu^{\pm} \mu^{+} \mu^{-} \nu_{\tau} \nu_{\mu}$
Detection Efficiency	1.76 %	1.20%	3.56%	1.67%
Main Background(s)	$e \nu_{\tau} \nu_{e} \gamma, \pi \pi^{0} \nu_{\tau}$	$\mu \nu_{\tau} \nu_{\mu} \gamma, \pi \pi^{0} \pi^{0} \nu_{\tau}$ $, \pi \pi^{0} (\rightarrow e^{+} e^{-} \gamma) \nu_{\tau}$	$\pi \pi^{0} \nu_{\tau}$	$\pi \pi^{+} \pi^{-} \nu_{\tau}$
Expected number of signals at Belle	1300	430	8	4
Purity of signal	47%	50%	37%	16%

Previous Experiment

CLEOII measured the branching fraction of $\tau^{\pm}\rightarrow(e/\mu^{\pm})e^{+}e^{-}\nu_{\tau}\nu_{e/\mu}$

Result of CLEOII

$$Br(\tau\rightarrow ee^{+}e^{-}\nu_{\tau}\nu_{e}) = (2.7^{+1.5+0.4+0.1}_{-1.1-0.4-0.3})\times10^{-5}$$

$$Br(\tau\rightarrow\mu e^{+}e^{-}\nu_{\tau}\nu_{\mu}) < 3.2\times10^{-5}$$ (at 90% C.L.)

CLEO-II experiment

Decay mode	Number of events
$\tau^{\pm}\rightarrow e^{\pm}e^{+}e^{-}\nu_{\tau}\nu_{e}$	5
$\tau^{\pm}\rightarrow\mu^{\pm}e^{+}e^{-}\nu_{\tau}\nu_{\mu}$	1

Integrated luminosity $3.6fb^{-1}$

$N_{\tau\tau} = (3.28 \pm 0.05)\times10^{6}$

Detection efficiency:

$\tau\rightarrow ee^{+}e^{-}\nu_{\tau}\nu_{e}$ (2.7 ± 0.1)%

$\tau\rightarrow\mu e^{+}e^{-}\nu_{\tau}\nu_{\mu}$ (1.9 ± 0.1)%

- Main source of systematic error
  - Uncertainties of lepton identification efficiency
  - Uncertainties of reconstruction efficiency of slow tracks

Reference: Phys. Rev. Lett. 76, 2637 (1996)

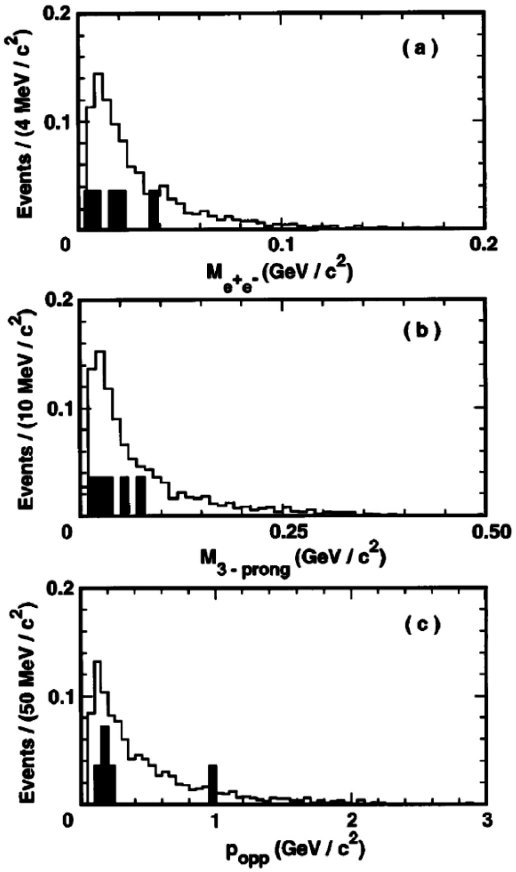


FIG. 3. Comparison of the kinematical distributions of the $\tau \rightarrow ee^{+}e^{-}\nu_{\tau}\nu_{e}$ Monte Carlo (solid line) and the data (shaded histogram) for events passing all selection requirements: (a) the $e^{+}e^{-}$ invariant mass averaged over two possible combinations, $M_{e^{+}e^{-}}$, (b) the 3-prong invariant mass, $M_{3\text{-prong}}$, and (c) the charge momentum of the electron on the 3-prong side with the charge opposite to that of the parent tau, p_{opp}. The normalization of the plots is arbitrary.

Selection

Pre-selection of tau-pair and **thrust selection** are applied at the first stage

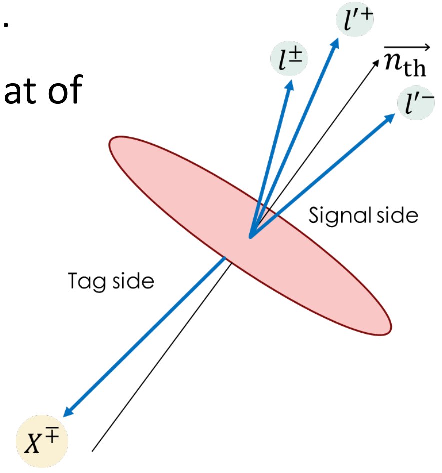
Thrust selection is applied as following method (in the CM-frame).

- Define thrust vector by $\overrightarrow{n_{\mathrm{th}}} = \max\{\frac{\sum_{\mathrm{sig,tag}} |\overrightarrow{n_{\mathrm{th}}} \cdot \overrightarrow{p_i}|}{\sum_{\mathrm{sig,tag}} |\overrightarrow{p_i}|}\}$
- Separate signal and tag side by:

  $[\{\overrightarrow{n_{\mathrm{th}}} \cdot \overrightarrow{p_{i_{\mathrm{sig}}}} < 0\}\&\&\{\overrightarrow{n_{\mathrm{th}}} \cdot \overrightarrow{p_{\mathrm{tag}}} > 0\}]$ or $[\{\overrightarrow{n_{\mathrm{th}}} \cdot \overrightarrow{p_{i_{\mathrm{sig}}}} > 0\}\&\&\{\overrightarrow{n_{\mathrm{th}}} \cdot \overrightarrow{p_{\mathrm{tag}}} < 0\}]$.
- Require the number of charged tracks in signal side to be three and that of tag side to be one
- Require $\sum_{\mathrm{sig}} Q = \pm 1$ and $\sum_{\mathrm{tag}} Q = \mp 1$

$\overrightarrow{p_{i_{\mathrm{sig}}}}$: momentum vector of electrons in signal-side

$\overrightarrow{p_{\mathrm{tag}}}$: momentum vector of charged particle in tag-side



※Pre-selection of tau-pair and second stage selection are written in a backup

We use 1-prong decay of tau as tag

Pre-Selection of Tau-pair

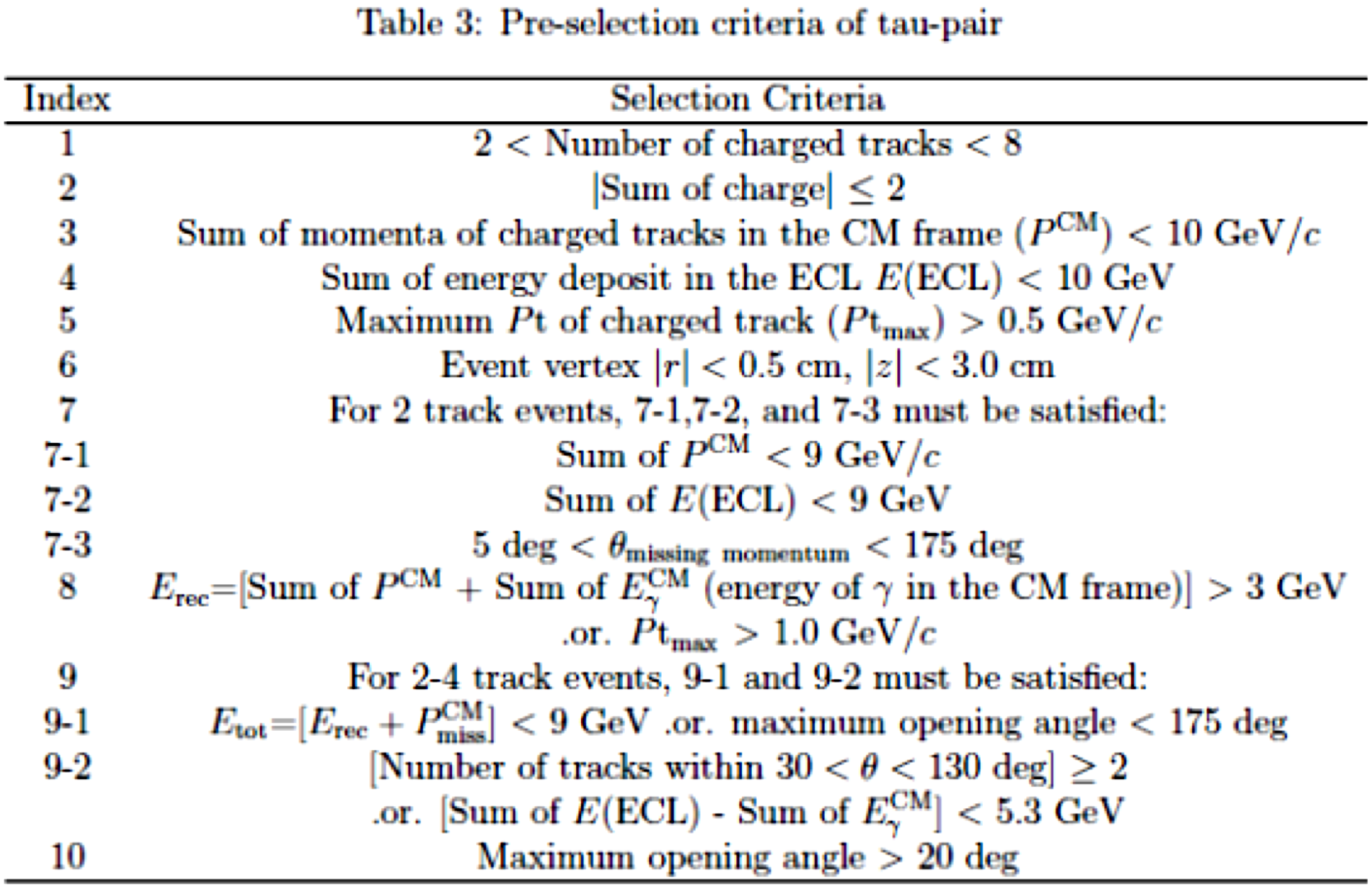


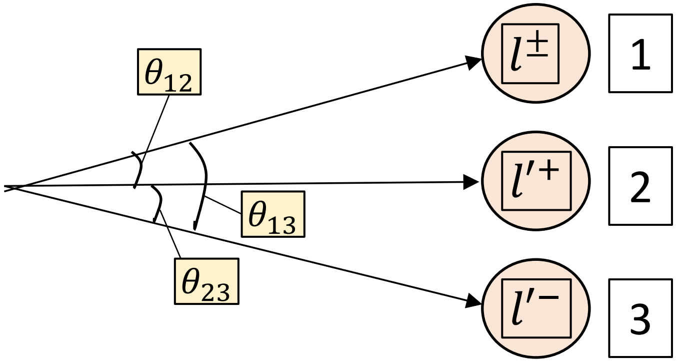
Table 3: Pre-selection criteria of tau-pair

Index	Selection Criteria
1	$2 <$ Number of charged tracks < 8
2	$\lvert$Sum of charge$\rvert \leq 2$
3	Sum of momenta of charged tracks in the CM frame (P^{CM}) < 10 GeV/c
4	Sum of energy deposit in the ECL E(ECL) < 10 GeV
5	Maximum Pt of charged track (Pt_{max}) > 0.5 GeV/c
6	Event vertex $\lvert r \rvert < 0.5$ cm, $\lvert z \rvert < 3.0$ cm
7	For 2 track events, 7-1,7-2, and 7-3 must be satisfied:
7-1	Sum of $P^{\mathrm{CM}} < 9$ GeV/c
7-2	Sum of E(ECL) < 9 GeV
7-3	5 deg $< \theta_{\mathrm{missing\ momentum}} <$ 175 deg
8	E_{rec}=[Sum of P^{CM} + Sum of E_{γ}^{CM} (energy of γ in the CM frame)] > 3 GeV .or. $Pt_{\mathrm{max}} > 1.0$ GeV/c
9	For 2-4 track events, 9-1 and 9-2 must be satisfied:
9-1	E_{tot}=[$E_{\mathrm{rec}} + P_{\mathrm{miss}}^{\mathrm{CM}}$] < 9 GeV .or. maximum opening angle < 175 deg
9-2	[Number of tracks within $30 < \theta < 130$ deg] ≥ 2 .or. [Sum of E(ECL) - Sum of E_{γ}^{CM}] < 5.3 GeV
10	Maximum opening angle > 20 deg

Second Stage Selection

Explanation of defined word

Sum of $\cos\theta_{ij}$: $\left(\sum_{i<j}\cos\theta_{ij}\right)$



Second Stage Selection

$$\tau^{\pm}\rightarrow e^{\pm}e^{+}e^{-}\nu_{\tau}\nu_{e}$$

1. Number of charged track = 4
2. Total charge (sum of Q_{sig} + sum of Q_{tag}) = 0
3. Number of photons (with $E(\gamma)_{CM}$> 0.06 GeV) <= 8
4. Total ECL energy deposition < 9 GeV
5. 1.5 GeV/c^2 < M_{missing} < 7 GeV/ c^2
6. Number of tracks in signal side = 3 && Number of track in tag side = 1
7. Max transverse momentum of electron in signal side $\left|\overrightarrow{p_{t\_i}}\right|$ > 0.15 GeV / c (CM-frame)

$$\tau^{\pm}\rightarrow e^{\pm}e^{+}e^{-}\nu_{\tau}\nu_{e}$$

8. Reconstructed vertex position of γ($\rightarrow e^+e^-$) should be $r(xy-\text{plane})$ < 1.5cm
9. Reconstructed vertex position of γ($\rightarrow e^+e^-$) should be $r(xyz-\text{space})$ < 3.0cm
10. The number of γ in signal-side <= 1 && The sum of energy of γ in signal-side < 0.5 GeV
11. Sum of $\cos\theta_{ij}$: $(\sum_{i<j}\cos\theta_{ij})$ >2.90
12. $|dz_i|$ of electrons in signal-side should be $|dz_i|$< 1cm
13. The momentum of virtual gamma in lab frame < 3 GeV/c
14. Polar angle of the missing momentum 30 deg < θ < 150 deg

Second Stage Selection

$$\tau^{\pm} \rightarrow \mu^{\pm} e^{+} e^{-} \nu_{\tau} \nu_{\mu}$$

1. Number of charged track = 4
2. Total charge (sum of Q_{sig} + sum of Q_{tag}) = 0
3. Number of photons (with $E(\gamma)_{CM}$> 0.06 GeV) <= 8
4. Total number of gamma in sig & tag –side <= 4
5. Total ECL energy deposition < 9 GeV
6. 1.5 GeV/c^2 < M_{missing} < 7 GeV/ c^2
7. Number of charged tracks in signal side = 3 && Number of charged track in tag side = 1
8. Reconstructed vertex position of $\gamma(\rightarrow e^{+}e^{-})$ should be $r(xy - \mathrm{plane})$ < 1.5cm
9. The sum of energy of γ in signal-side < 0.5 GeV
10. Sum of $\cos\theta_{ij}$: $(\sum_{i<j} \cos\theta_{ij})$ >2.93
11. Polar angle of the missing momentum 30 deg < θ < 150 deg

Second Stage Selection

$$\tau^{\pm} \rightarrow e^{\pm}\mu^{+}\mu^{-}\nu_{\tau}\nu_{e}$$

1. Number of charged track = 4
2. Total charge (sum of Q_{sig} + sum of Q_{tag}) = 0
3. Number of photons (with $E(\gamma)_{CM}$> 0.06 GeV) <= 8
4. Total ECL energy deposition < 9 GeV
5. 1.5 GeV/c^2 < M_{missing} < 7 GeV/ c^2
6. Number of tracks in signal side = 3 && Number of track in tag side = 1
8. The number of γ in signal-side <= 1 && The sum of energy of γ in signal-side < 0.3 GeV
9. Sum of $\cos\theta_{ij}$: $(\sum_{i<j}\cos\theta_{ij})$ >2.70
10. $(E_c/p)_{\mu}$ of muons in signal-side should be < 0.5
11. Polar angle of the missing momentum 30 deg < θ < 150 deg
12. Invariant mass of $\mu^{+}\mu^{-}$ should be < 0.4 GeV/c^2

Second Stage Selection

$$\tau^{\pm} \rightarrow \mu^{\pm}\mu^{+}\mu^{-}\nu_{\tau}\nu_{\mu}$$

1. Number of charged track = 4
2. Total charge (sum of Q_{sig} + sum of Q_{tag}) = 0
3. Number of photons (with $E(\gamma)_{CM}$> 0.06 GeV) <= 8
4. Total ECL energy deposition < 9 GeV
5. 1.5 GeV/c^2 < M_{missing} < 7 GeV/ c^2
6. Number of tracks in signal side = 3 && Number of track in tag side = 1

8. The number of γ in signal-side = 0
9. Sum of $\cos\theta_{ij}$: $(\sum_{i<j}\cos\theta_{ij})$ >2.85
10. $(E_c)_{\mu}$ of muons in signal-side should be < 0.4 GeV/c^2
11. Polar angle of the missing momentum 30 deg < θ < 150 deg
12. $|dz_i|$ of muons in signal-side should be $|dz_i|$< 0.1cm