Role of the isospin 3/2 component in nd elastic scattering and breakup

Collaboration of:

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Outline:

1. Exact treatment of 3N reactions: Faddeev equations

2. Isospin structure of the transition amplitude

3. Charge independence breaking of NN interactions: main source for T=3/2 component contribution

4. Results for nucleon-deuteron reactions:
   - elastic nd scattering
   - nd breakup reaction
   - total cross sections

5. Summary and conclusions

3N continuum Faddeev equation (W.Gloeckle et al., Phys.Rept.274,107(1996)):

$$T|\phi\rangle = tP|\phi\rangle + (1+tG_0)V_{123}^{(1)}(1+P)|\phi\rangle + tPG_0T|\phi\rangle + (1+tG_0)V_{123}^{(1)}(1+P)G_0T|\phi\rangle$$

$|\phi\rangle$ -initial state composed of a deuteron and a momentum eigenstate of the projectile nucleon

$G_0 = \dfrac{1}{E-H_0}$ - the free propagator of three nucleons

P - a sum of a cyclical and anticyclical permutation of the three nucleons

t - the two-body t-operator $\quad t = V_{23} + V_{23}G_0^{2N}t$

$V_{123} = V_{123}^{(1)} + V_{123}^{(2)} + V_{123}^{(3)}$ - complete 3NF

$V_{123}^{(i)}$ - is that part of the 3NF which singles out the particle „i" and which is symmetrical under the exchange of the other two particles

Faddeev equation is numerically solved in momentum space using a partial wave decomposition

Elastic scattering amplitude

$$\langle\phi'|U|\phi\rangle = \langle\phi'|PG_0^{-1} + PT + V_{123}^{(1)}(1+P)(1+G_0T)|\phi\rangle$$

Breakup amplitude

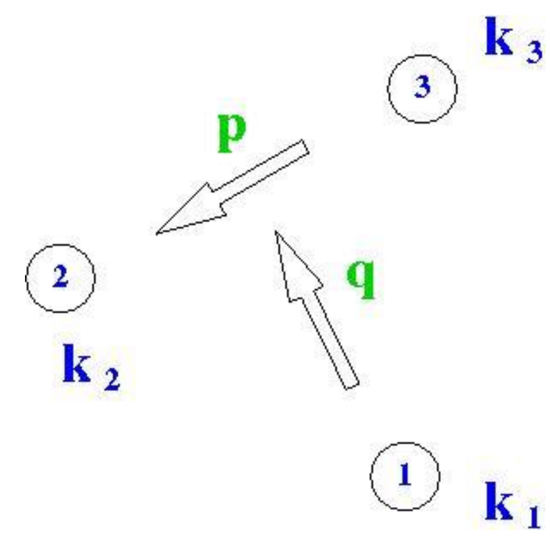
$$\langle\phi_0|U_0|\phi\rangle = \langle\phi_0|(1+P)T|\phi\rangle$$

Technical performance: W.Gloeckle et al., Phys. Rept.274,107(1996)):

$$T|\phi\rangle = tP|\phi\rangle + tPG_0T|\phi\rangle$$

$$|pq\alpha\rangle \equiv |pqa\rangle\left|(t\frac{1}{2})T\right\rangle = \left|pq(ls)j(\lambda\frac{1}{2})I(jI)J(t\frac{1}{2})T\right\rangle$$

$$\sum_\alpha \int_0^\infty p^2dp\int_0^\infty q^2dq\,|pq\alpha\rangle\langle pq\alpha| = \mathrm{I}$$



$$\langle pq\alpha|T|\phi\rangle = \langle pq\alpha|tP|\phi\rangle + \sum_{\alpha'}\int{}' \sum_{\alpha''}\int{}'' \langle pq\alpha|t|p'q'\alpha'\rangle$$

$$\langle p'q'\alpha'|P|"q"\alpha"\rangle\langle p"q"\alpha"|G_0T|\phi\rangle$$

- coupled set of integral equations in 2 continuous variables for amplitudes <pqα|T|ϕ>
- finite range of nuclear interaction finite number of channels α
- with j_{max}=5 about 150 coupled integral equations
- solved be generating Neuman series which is then summed up by Pade method

Let us divide the partial wave states into those with the total izospin T=1/2:

$$| pq\alpha \rangle \equiv | pq, \text{angular momenta, spins} \rangle \left| (t\frac{1}{2})T = \frac{1}{2} M_T \right\rangle, \text{ (t=0,1)}$$

and with T=3/2:

$$| pq\beta \rangle \equiv | pq, \text{angular momenta, spins} \rangle \left| (t\frac{1}{2})T = \frac{3}{2} M_T \right\rangle, \text{ (t=1)}$$

Assuming charge conservation and employing the notation where the neutron (proton) isospin projection is 1/2 (-1/2) the 2N t-operator in the three-particle isospin space can be decomposed for the nd system as:

$$\left\langle (t\frac{1}{2})TM_T = \frac{1}{2} \right| \mathrm{t} \left| (t'\frac{1}{2})T'M_{T'} = \frac{1}{2} \right\rangle = \delta_{tt'}\delta_{TT'}\delta_{T1/2}\left[\delta_{t0}\mathrm{t}_{\mathrm{np}}^{t=0} + \delta_{t1}(\frac{2}{3}\mathrm{t}_{\mathrm{nn}}^{t=1} + \frac{1}{3}\mathrm{t}_{\mathrm{np}}^{t=1}) \right]$$

$$+\delta_{tt'}\delta_{t1}(1-\delta_{TT'})\frac{\sqrt{2}}{3}(\mathrm{t}_{\mathrm{nn}}^{t=1} - \mathrm{t}_{\mathrm{np}}^{t=1})$$

$$+\delta_{tt'}\delta_{t1}\delta_{TT'}\delta_{T3/2}(\frac{1}{3}\mathrm{t}_{\mathrm{nn}}^{t=1} + \frac{2}{3}\mathrm{t}_{\mathrm{np}}^{t=1})$$

where t_{nn} and t_{np} are solutions of the Lippman-Schwinger equations driven by the v_{nn} and v_{np} potentials, respectively.

As a result one gets the amplitudes $\langle pq\alpha|T|\phi\rangle$ and $\langle pq\beta|T|\phi\rangle$ which fulfill the following set of coupled integral equations:

$$
\begin{aligned}
\left\langle pq\alpha\right|\mathrm{T}\left|\phi\right\rangle =& \sum_{\alpha'}\int_{p'q'}\left\langle pq\alpha\right|\mathrm{t}\left|p'q'\alpha'\right\rangle\left\langle p'q'\alpha'\right|\mathrm{P}\left|\phi\right\rangle \\
&+\sum_{\alpha'}\int_{p'q'}\left\langle pq\alpha\right|\mathrm{V}^{(1)}\left|p'q'\alpha'\right\rangle\left\langle p'q'\alpha'\right|(1+\mathrm{P})\left|\phi\right\rangle \\
&+\sum_{\alpha'}\int_{p'q'}\left\langle pq\alpha\right|\mathrm{t}\left|p'q'\alpha'\right\rangle\left\langle p'q'\alpha'\right|\mathrm{G}_0\mathrm{V}^{(1)}(1+\mathrm{P})\left|\phi\right\rangle \\
&+\sum_{\alpha'}\int_{p'q'}\left\langle pq\alpha\right|\mathrm{t}\left|p'q'\alpha'\right\rangle\left\langle p'q'\alpha'\right|\mathrm{P}\mathrm{G}_0\mathrm{T}\left|\phi\right\rangle \\
&+\sum_{\beta'}\int_{p'q'}\left\langle pq\alpha\right|\mathrm{t}\left|p'q'\beta'\right\rangle\left\langle p'q'\beta'\right|\mathrm{P}\mathrm{G}_0\mathrm{T}\left|\phi\right\rangle \\
&+\sum_{\alpha'}\int_{p'q'}\left\langle pq\alpha\right|\mathrm{V}^{(1)}\left|p'q'\alpha'\right\rangle\left\langle p'q'\alpha'\right|(1+\mathrm{P})\mathrm{G}_0\mathrm{T}\left|\phi\right\rangle \\
&+\sum_{\alpha'}\int_{p'q'}\sum_{\alpha''}\int_{p''q''}\left\langle pq\alpha\right|\mathrm{t}\left|p'q'\alpha'\right\rangle\left\langle p'q'\alpha'\right|\mathrm{G}_0\mathrm{V}^{(1)}\left|p''q''\alpha''\right\rangle \\
&\times\left\langle p''q''\alpha''\right|(1+\mathrm{P})\mathrm{G}_0\mathrm{T}\left|\phi\right\rangle \\
&+\sum_{\beta'}\int_{p'q'}\sum_{\beta''}\int_{p''q''}\left\langle pq\alpha\right|\mathrm{t}\left|p'q'\beta'\right\rangle\left\langle p'q'\beta'\right|\mathrm{G}_0\mathrm{V}^{(1)}\left|p''q''\beta''\right\rangle \\
&\times\left\langle p''q''\beta''\right|(1+\mathrm{P})\mathrm{G}_0\mathrm{T}\left|\phi\right\rangle
\end{aligned}
$$

$$
\begin{aligned}
\langle pq\beta|\mathrm{T}|\phi\rangle = & \sum_{\alpha'}\int_{p'q'}\langle pq\beta|\mathrm{t}|p'q'\alpha'\rangle\langle p'q'\alpha'|\mathrm{P}|\phi\rangle \\
& + \sum_{\alpha'}\int_{p'q'}\langle pq\beta|\mathrm{t}|p'q'\alpha'\rangle\langle p'q'\alpha'|\mathrm{G_0V^{(1)}(1+P)}|\phi\rangle \\
& + \sum_{\alpha'}\int_{p'q'}\langle pq\beta|\mathrm{t}|p'q'\alpha'\rangle\langle p'q'\alpha'|\mathrm{PG_0T}|\phi\rangle \\
& + \sum_{\beta'}\int_{p'q'}\langle pq\beta|\mathrm{t}|p'q'\beta'\rangle\langle p'q'\beta'|\mathrm{PG_0T}|\phi\rangle \\
& + \sum_{\beta'}\int_{p'q'}\langle pq\beta|\mathrm{V^{(1)}}|p'q'\beta'\rangle\langle p'q'\beta'|\mathrm{(1+P)G_0T}|\phi\rangle \\
& + \sum_{\alpha'}\int_{p'q'}\sum_{\alpha''}\int_{p''q''}\langle pq\beta|\mathrm{t}|p'q'\alpha'\rangle\langle p'q'\alpha'|\mathrm{G_0V^{(1)}}|p''q''\alpha''\rangle \\
& \times\langle p''q''\alpha''|\mathrm{(1+P)G_0T}|\phi\rangle \\
& + \sum_{\beta'}\int_{p'q'}\sum_{\beta''}\int_{p''q''}\langle pq\beta|\mathrm{t}|p'q'\beta'\rangle\langle p'q'\beta'|\mathrm{G_0V^{(1)}}|p''q''\beta''\rangle \\
& \times\langle p''q''\beta''|\mathrm{(1+P)G_0T}|\phi\rangle
\end{aligned}
$$

The form of the couplings follows from the fact that the incoming nd state $|\phi\rangle$ has isospin T=1/2, the permutation operator is diagonal in the total isospin, and the 3NF is assumed to conserve the total izospin.

The form of the couplings follows from the fact that the incoming nd state |ϕ> has izospin T=1/2, the permutation operator is diagonal in the total isospin, the 3NF is assumed to conserve the total izospin, and the 2N t-matrix has the follwing izospin structure:

$$\left\langle (t\frac{1}{2})TM_T=\frac{1}{2}\right| \mathrm{t} \left| (t'\frac{1}{2})T'M_{T'}=\frac{1}{2}\right\rangle = \delta_{tt'}\delta_{TT'}\delta_{T1/2}\left[\delta_{t0}\mathrm{t}_{\mathrm{np}}^{t=0}+\delta_{t1}(\frac{2}{3}\mathrm{t}_{\mathrm{nn}}^{t=1}+\frac{1}{3}\mathrm{t}_{\mathrm{np}}^{t=1})\right]$$
$$+\delta_{tt'}\delta_{t1}(1-\delta_{TT'})\frac{\sqrt{2}}{3}(\mathrm{t}_{\mathrm{nn}}^{t=1}\text{-}\mathrm{t}_{\mathrm{np}}^{t=1})$$
$$+\delta_{tt'}\delta_{t1}\delta_{TT'}\delta_{T3/2}(\frac{1}{3}\mathrm{t}_{\mathrm{nn}}^{t=1}+\frac{2}{3}\mathrm{t}_{\mathrm{np}}^{t=1})$$

It follows that a difference between neutron-proton and neutron-neutron (or proton-proton) interactions (charge independence breaking of the NN forces (CIB)) will cause transitions between 3N isospin T=1/2 and T=3/2 states.

Since the incoming nucleon-deuteron state |ϕ> has isospin T=1/2 and the free propagator G_0 as well as the permutation operator P are diagonal in isospin, it follows that the isospin T=3/2 amplitudes **<pqβ |T |ϕ>** are generated by CIB in t=1 nucleon-nucleon states. Their magnitude is determined by magnitude of CIB in NN interactions.

These **T=3/2 amplitudes will not contribute directly to the elastic scattering** (the outgoing state |ϕ'> has isospin T=1/2), but **will contribute directly to the nd breakup** transition amplitude where the outgoing state $|\phi_0\rangle = |$**pq**> has T=1/2 and T=3/2 contributions.

Largest CIB effects are in $^1S_0$ NN force, what is clearly seen in the values of the corresponding np and nn (pp) scattering lengths.

The nonzero values of T=3/2 <pqβ|T|ϕ> amplitudes will result only in those t=1 3N partial waves where 2N t-matrices t_{np} and t_{nn} differ. In such a case for each T=1/2 α-state the corresponding T=3/2 β-state will appear – that leads to increase of the total numer of 3N partial wave states, making calculations more complex and time consuming.

For 3N total angular momentum J=11/2 and higher the number of partial wave states (=number of coupled integral equations in two continuous variables p and q to be solved) amounts to **142** when all 2N subsystem angular momenta up to j=5 are included. When in all t=1 states CIB is taken into account that numer will increase to **207**. When CIB only in $^1S_0$ is considered that numer is **143**.

We solved 3N continuum Faddeev equations taking j_{max}=5 and J_{max}=25 with semi-localy regularized chiral N4LO NN potential (regulator R=0.9 fm) alone (E.Epelbaum et al.,Eur. Phys. J. A51 (2015)) or combined with the N2LO 3NF. The strengths of the one-pion exchange term in that 3NF was C_D=6.0 and of the 3N contact term C_E=-1.094. That combination of NN and 3NF reproduces 3H binding energy and provides quite a good descritpion of the nd elastic scattering cross section data at higher energies.

We checked that conclusions remain unchanged when taking AV18 potential alone or combined with the Urbana IX 3NF

How large is CIB in NN interactions ?

$^1S_0$ NN partial wave state:

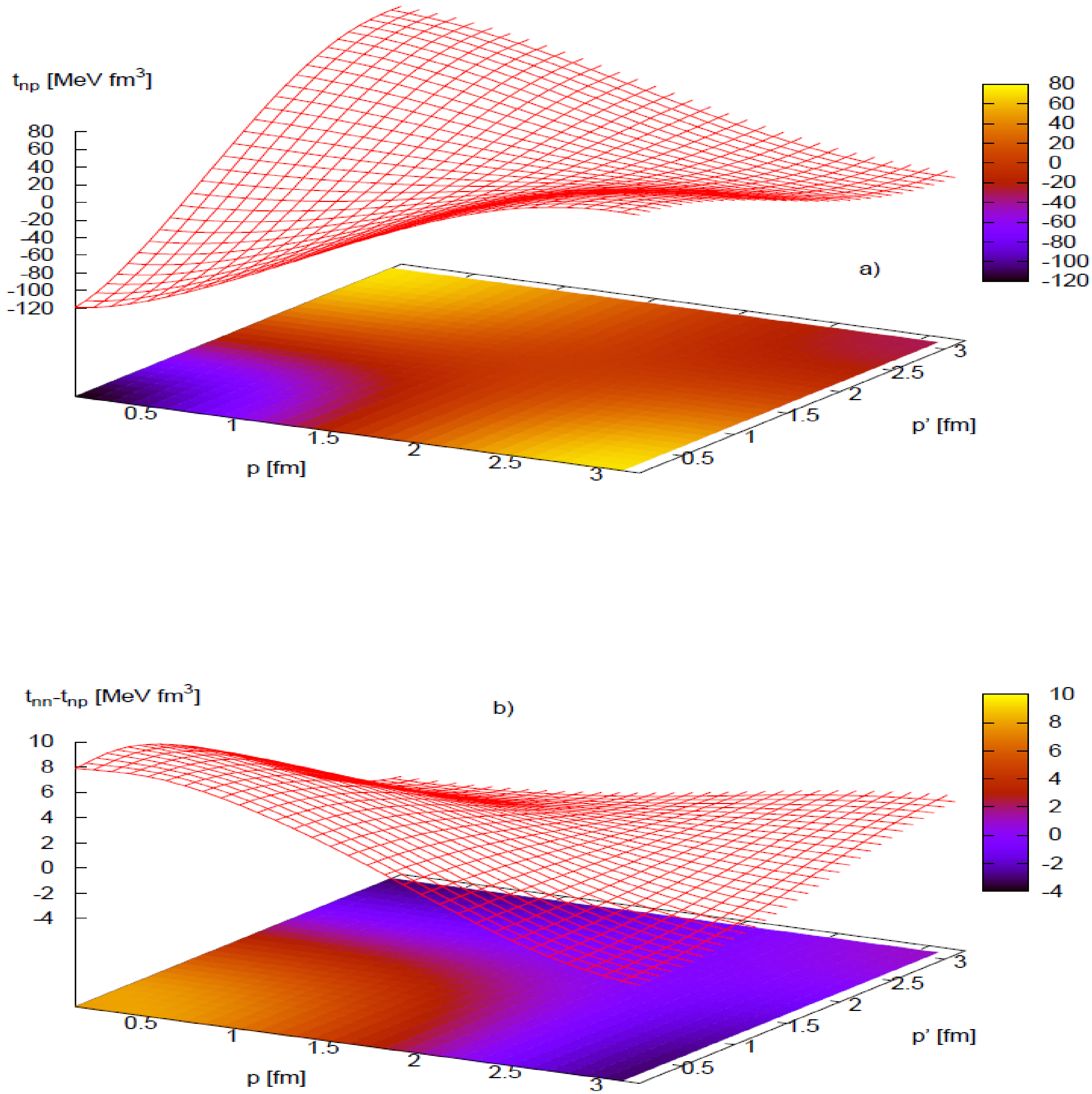
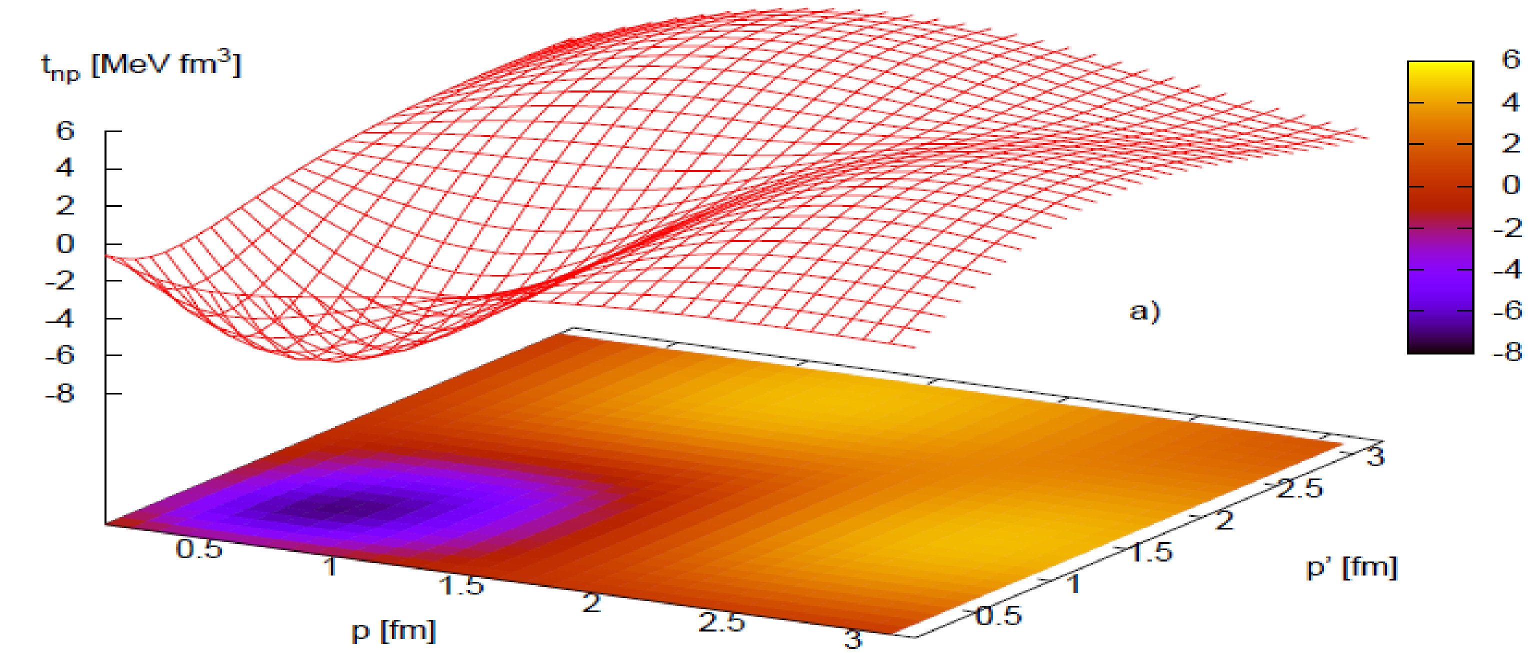


Fig. 1 (color online) The np t-matrix $t_{np}(p,p';E-\frac{3}{4}q^2)$ (a) and the difference of the nn and np t-matrices (b) at incoming neutron laboratory energy $E_{lab}=13$ MeV for the $^1S_0$ partial wave, as a function of the relative NN momenta p and p' at a particular value of the spectator nucleon momentum $q=0.528$ fm^{-1} at which the 2N subsystem energy is equal to the binding energy of the deuteron E_d: $E-\frac{3}{4}q^2=E_d$.

$^3P_0$ NN partial wave state:



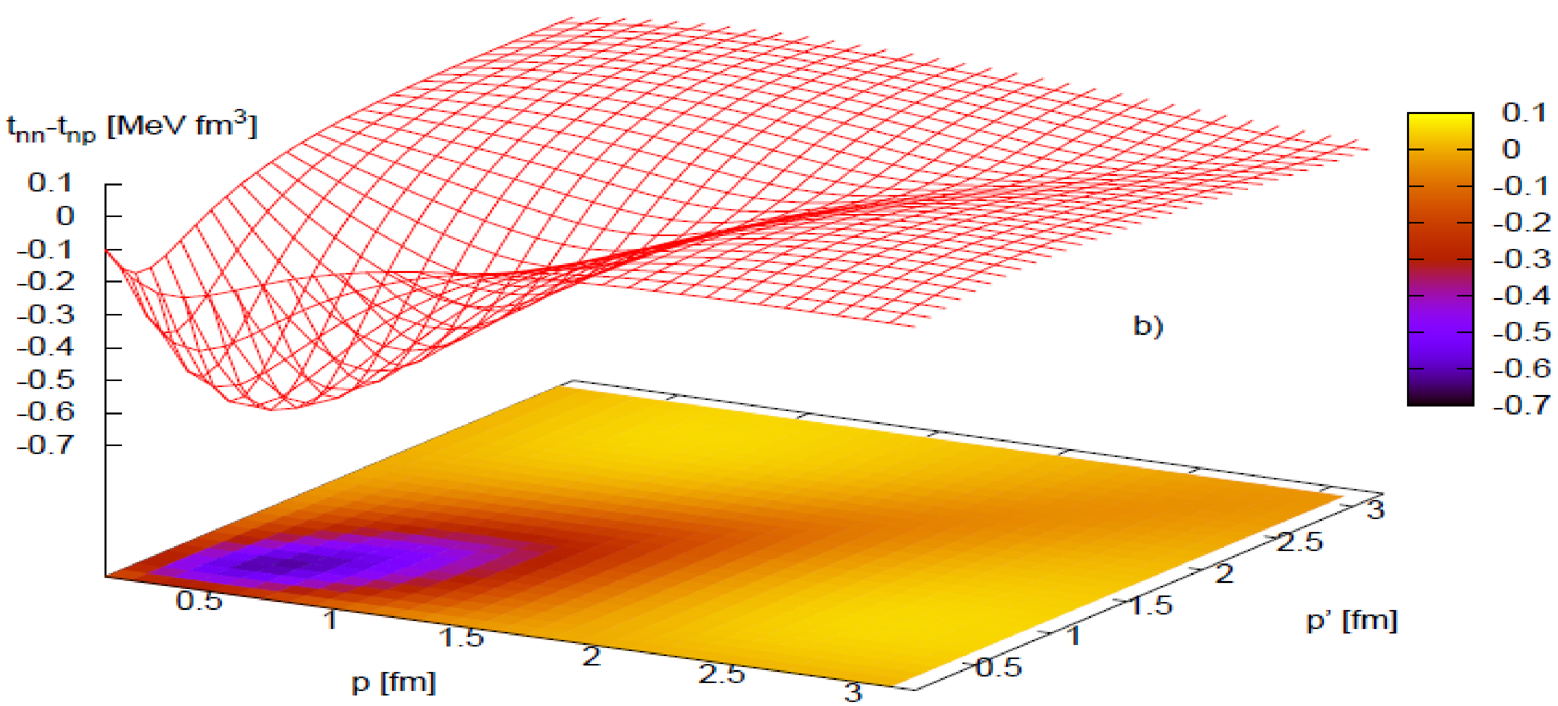


Fig. 2 (color online) The same as in Fig. 1 but for the $^3P_0$ partial wave.

In order to find out the importance of T=3/2 $\langle pq\beta|T|\phi\rangle$ - amplitudes and to check, what a minimal treatment of CIB is required to reproduce exact result, we have done a number of calculations, with and without 3N force, namely:

1. Full treatment including CIB (T=3/2 states) in all t=1 3N partial waves – it is an exact result to which others (next points) are only approximations !

2. CIB only in $^1S_0$ state.

3. No CIB (no T=3/2 states) but effective t-matrix $t_{eff}=2/3t_{nn}+1/3t_{np}$ in all t=1 states (”2/3-1/3 rule”).

4. No CIB and only np force in $^1S_0$.

5. No CIB and only nn force in $^1S_0$.

1	2	3	4	5	6	7	8	9
E_{lab} [MeV]	no CIB $V_{123}=0$ $^1S_0$ np	no CIB $V_{123}=0$ $^1S_0$ nn	no CIB $V_{123}=0$ $t_{\rm eff}$	no CIB V_{123} $t_{\rm eff}$	$^1S_0$ CIB $V_{123}=0$	$^1S_0$ CIB V_{123}	CIB $V_{123}=0$	CIB V_{123}
13.0	867.0	858.8	861.5	874.6	861.6	874.8	861.6	874.8
65.0	163.4	159.4	160.7	169.3	160.7	169.4	160.7	169.3
135.0	77.21	75.91	76.34	80.92	76.34	80.93	76.34	80.91
250.0	53.35	53.41	53.39	55.48	53.39	55.49	53.39	55.48

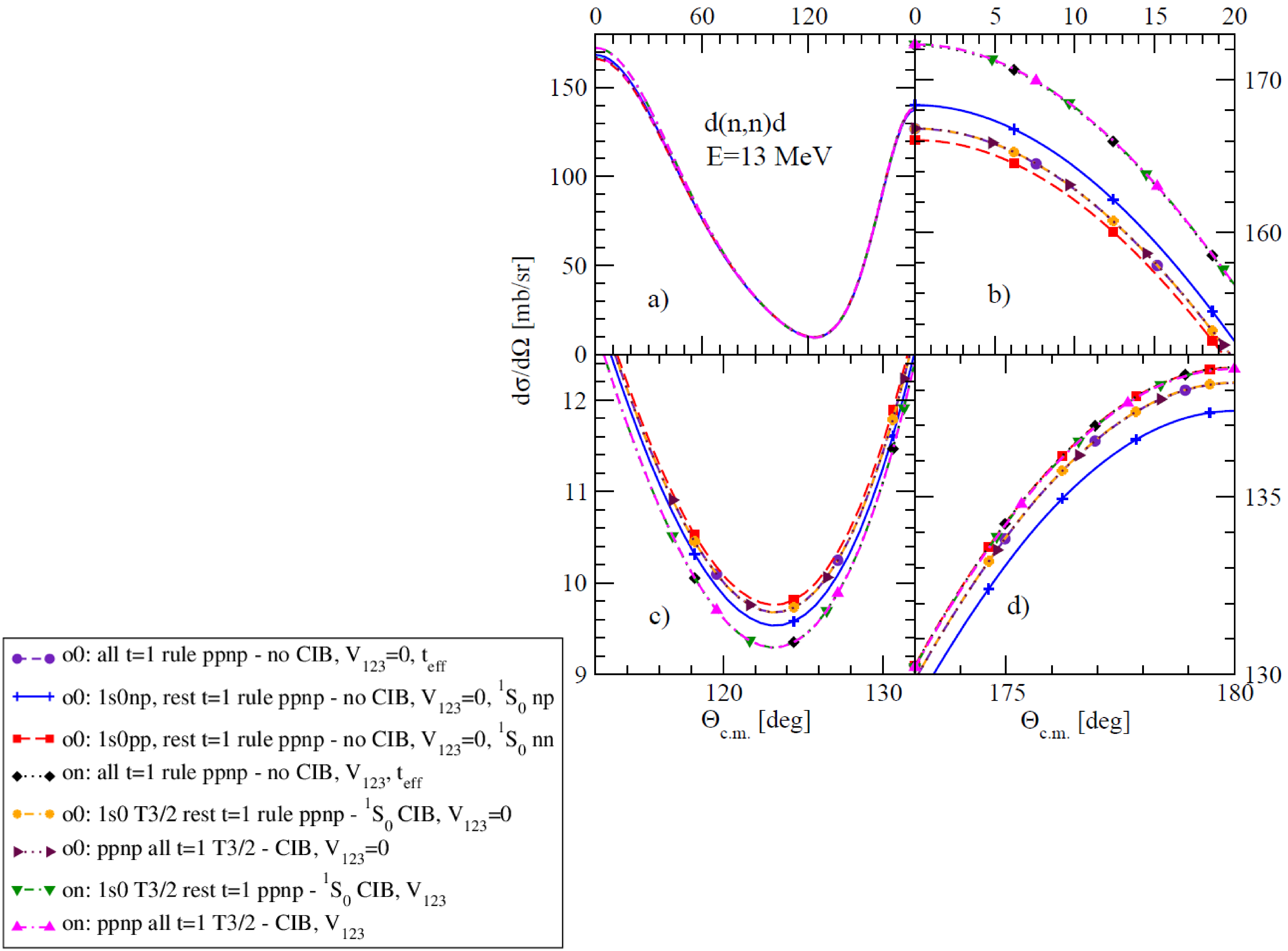
Table 1 The nd total cross section (in [mb]) at energies given in the first column. Dynamical models related to the particular columns are: in 2nd, 3rd, 4th, and 5th no CIB in any $t=1$ states was assumed. In the $^1S_0$ state the t-matrix has been taken as t_{np} and t_{nn} for the 2nd and 3rd, and as $t_{\rm eff}=(2/3)t_{nn}+(1/3)t_{np}$ for the 4th and 5th. In all other $t=1$ states $t_{\rm eff}$ was used. Descriptions $V_{123}=0$ and V_{123} means that the NN interaction was taken alone and combined with the 3NF, respectively. In the 6th and 7th columns CIB was exactly taken into account by using in $^1S_0$ state both t_{np} and t_{nn} t-matrices and state with isospin $T=3/2$ was taken into account. In all other $t=1$ states effective t-matrix $t_{\rm eff}$ was used. In the 8th and 9th columns for all $t=1$ states CIB was treated exactly by taking in addition to $T=1/2$ also $T=3/2$ states and the corresponding t_{np} and t_{nn} t-matrices.

1	2	3	4	5	6	7	8	9
E_{lab} [MeV]	no CIB $V_{123}=0$ $^1S_0$ np	no CIB $V_{123}=0$ $^1S_0$ nn	no CIB $V_{123}=0$ $t_{\rm eff}$	no CIB V_{123} $t_{\rm eff}$	$^1S_0$ CIB $V_{123}=0$	$^1S_0$ CIB V_{123}	CIB $V_{123}=0$	CIB V_{123}
13.0	699.8	699.3	699.4	711.8	699.5	712.0	699.5	712.0
65.0	71.43	69.53	70.15	75.68	70.15	75.70	70.15	75.67
135.0	20.80	20.31	20.46	22.63	20.46	22.64	20.46	22.62
250.0	8.769	8.774	8.769	9.472	8.769	9.475	8.769	9.471

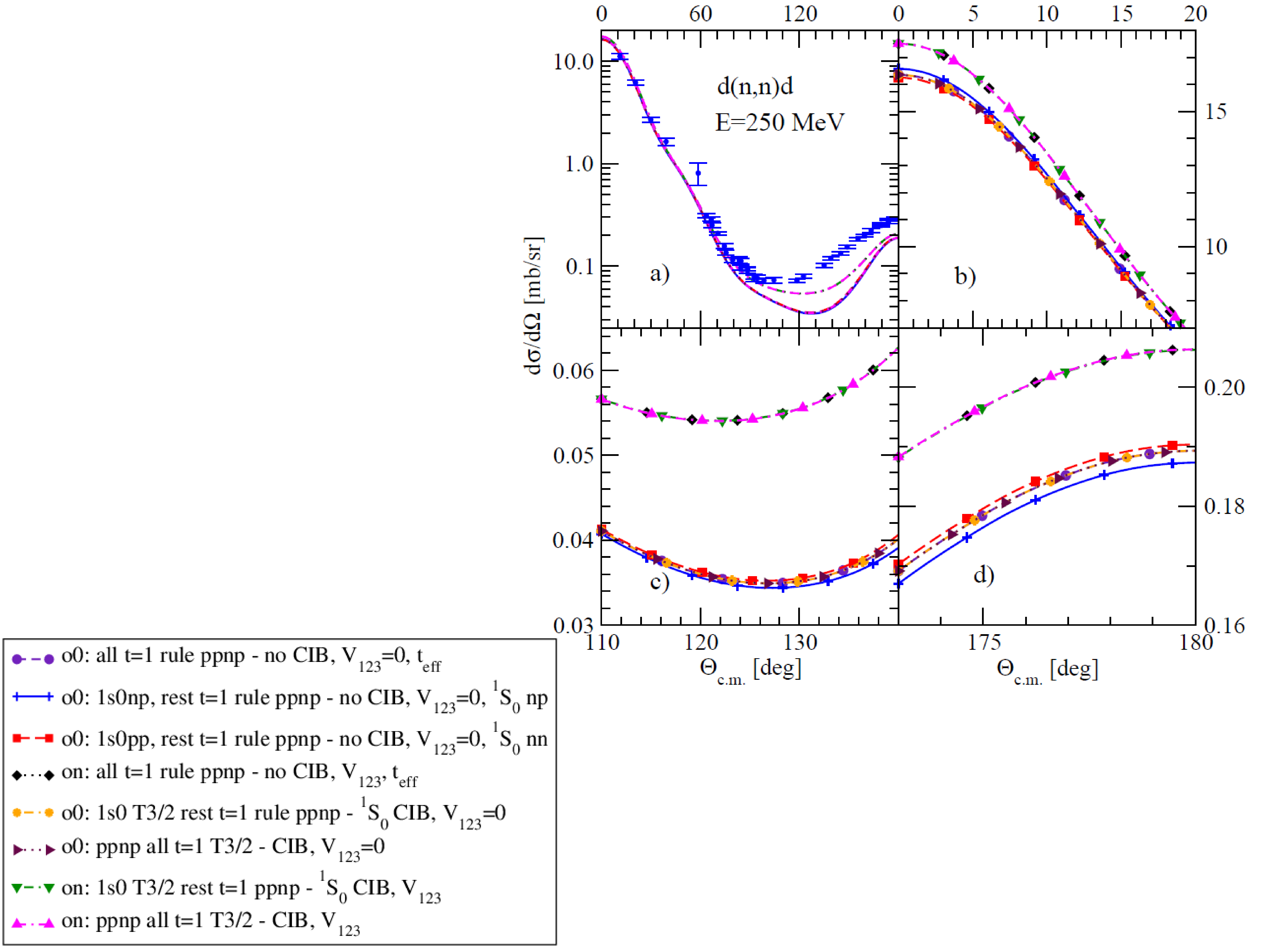
Table 2 The nd elastic scattering total cross section (in [mb]). For the description of underlying dynamics see Table 1.

1	2	3	4	5	6	7	8	9
E_{lab} [MeV]	no CIB $V_{123}=0$ $^1S_0$ np	no CIB $V_{123}=0$ $^1S_0$ nn	no CIB $V_{123}=0$ $t_{\rm eff}$	no CIB V_{123} $t_{\rm eff}$	$^1S_0$ CIB $V_{123}=0$	$^1S_0$ CIB V_{123}	CIB $V_{123}=0$	CIB V_{123}
13.0	167.2	159.8	162.1	162.9	162.1	162.8	162.1	162.9
65.0	91.92	89.89	90.58	93.69	90.58	93.69	90.58	93.61
135.0	56.42	55.60	55.88	58.29	55.88	58.29	55.88	58.28
250.0	44.58	44.64	44.62	46.01	44.62	46.01	44.62	46.01

Table 3 The nd breakup total cross section (in [mb]). For the description of underlying dynamics see Table 1.



d(n,n)d
E=13 MeV
a)
b)
c)
d)
dσ/dΩ [mb/sr]
Θc.m. [deg]
Θc.m. [deg]
o0: all t=1 rule ppnp - no CIB, V123=0, teff
o0: 1s0np, rest t=1 rule ppnp - no CIB, V123=0, 1S0 np
o0: 1s0pp, rest t=1 rule ppnp - no CIB, V123=0, 1S0 nn
on: all t=1 rule ppnp - no CIB, V123, teff
o0: 1s0 T3/2 rest t=1 rule ppnp - 1S0 CIB, V123=0
o0: ppnp all t=1 T3/2 - CIB, V123=0
on: 1s0 T3/2 rest t=1 ppnp - 1S0 CIB, V123
on: ppnp all t=1 T3/2 - CIB, V123


d(n,n)d
E=250 MeV
a)
b)
c)
d)
dσ/dΩ [mb/sr]
Θc.m. [deg]
Θc.m. [deg]
o0: all t=1 rule ppnp - no CIB, V123=0, teff
o0: 1s0np, rest t=1 rule ppnp - no CIB, V123=0, 1S0 np
o0: 1s0pp, rest t=1 rule ppnp - no CIB, V123=0, 1S0 nn
on: all t=1 rule ppnp - no CIB, V123, teff
o0: 1s0 T3/2 rest t=1 rule ppnp - 1S0 CIB, V123=0
o0: ppnp all t=1 T3/2 - CIB, V123=0
on: 1s0 T3/2 rest t=1 ppnp - 1S0 CIB, V123
on: ppnp all t=1 T3/2 - CIB, V123

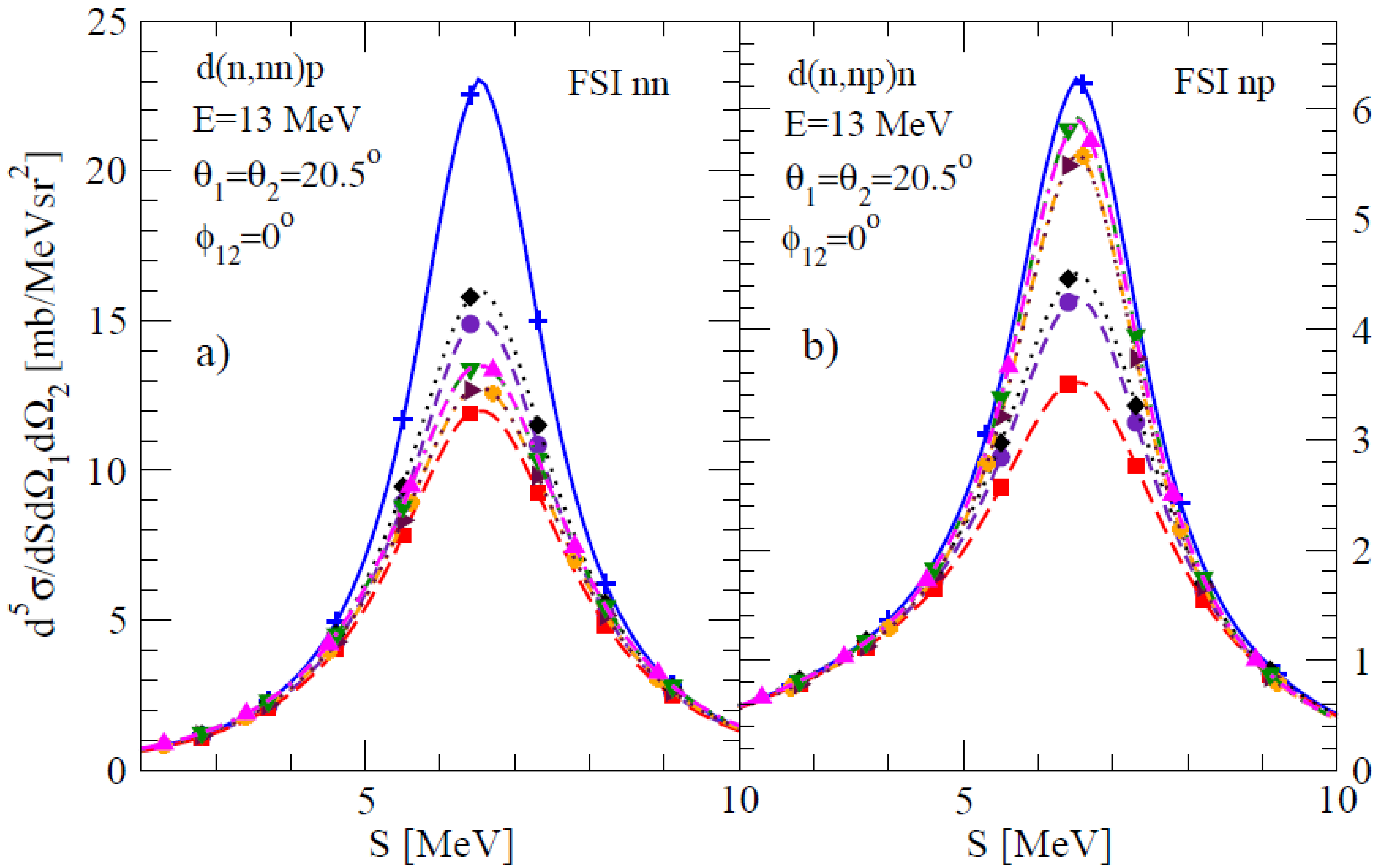
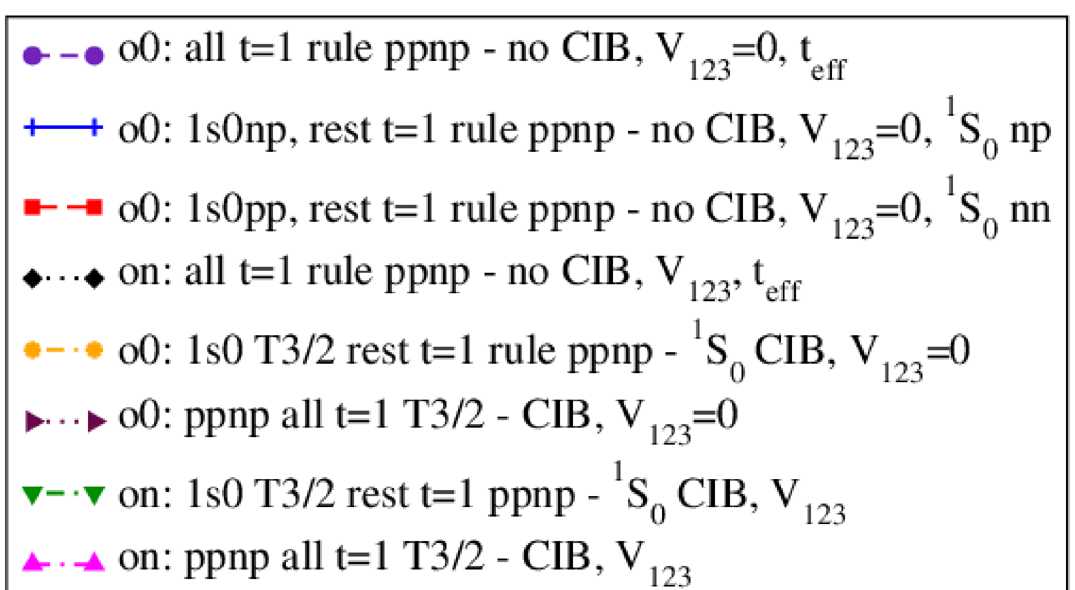


Fig. 5 (color online) The nd complete breakup d(n,nn)p cross section $d^5\sigma/d\Omega_1 d\Omega_2 dS$ for the nn (a) and np (b) FSI configuration at 13 MeV of the incoming neutron laboratory energy and laboratory angles of detected outgoing nucleons $\theta_1 = \theta_2 = 20.5^o$ and $\phi_{12} = 0^o$, as a function of the arc-length of the S-curve. For the description of lines and symbols see Fig. 3.



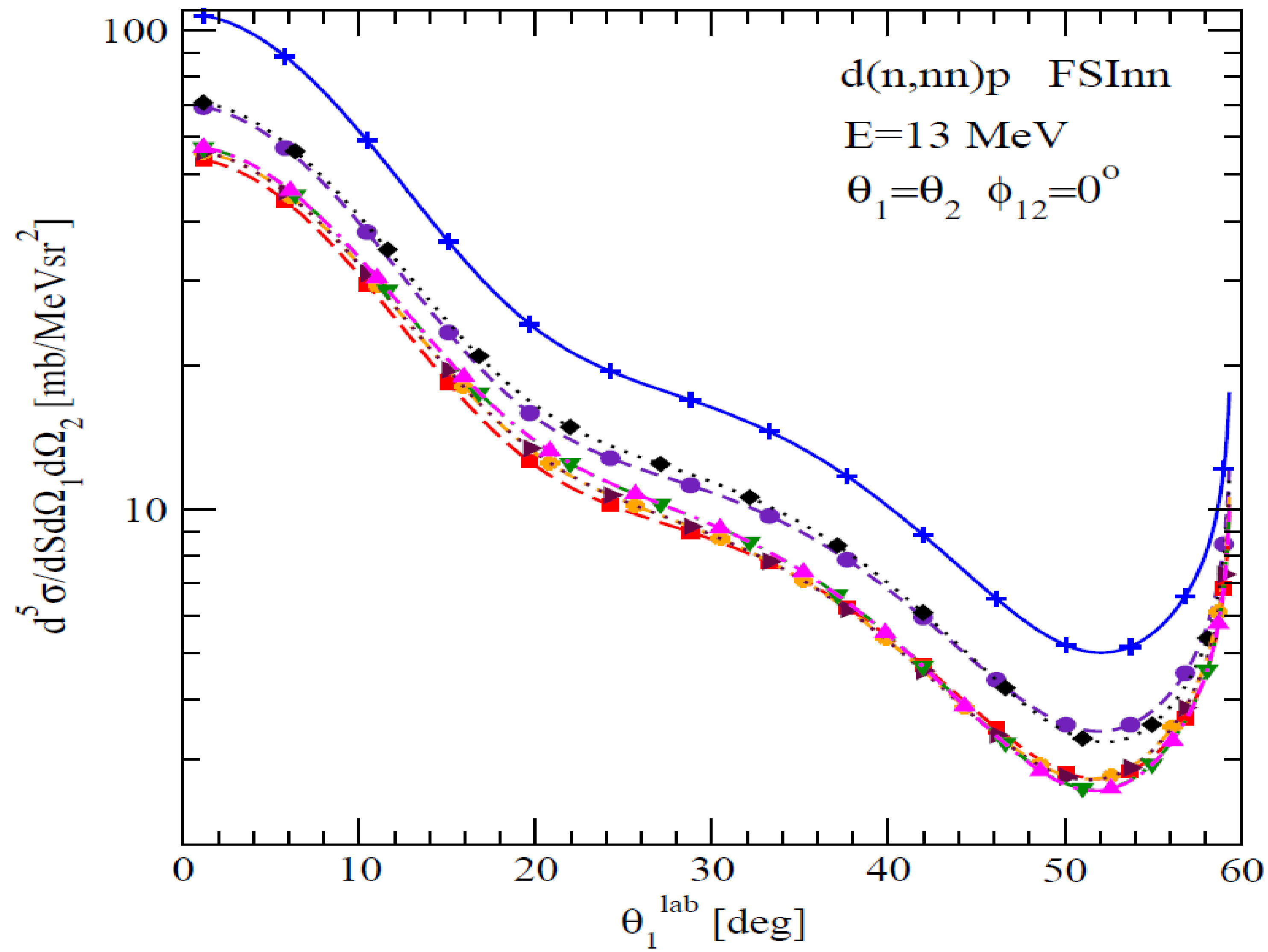
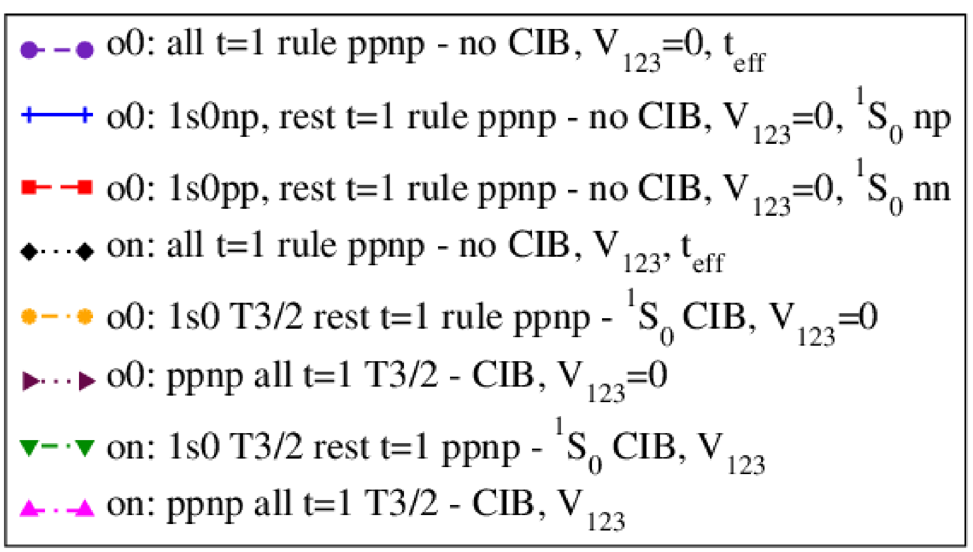


Fig. 6 (color online) The nd complete breakup d(n,nn)p cross section $d^5\sigma/d\Omega_1 d\Omega_2 dS$ calculated exactly at the neutron-neutron final state interaction condition (maximum of the cross section along the S-curve) at 13 MeV of the incoming neutron laboratory energy as a function of the laboratory production angle of the outgoing final-state-interacting neutrons $\theta_1^{lab} = \theta_2^{lab}$ and $\phi_{12} = 0^o$. For the description of lines and symbols see Fig. 3.



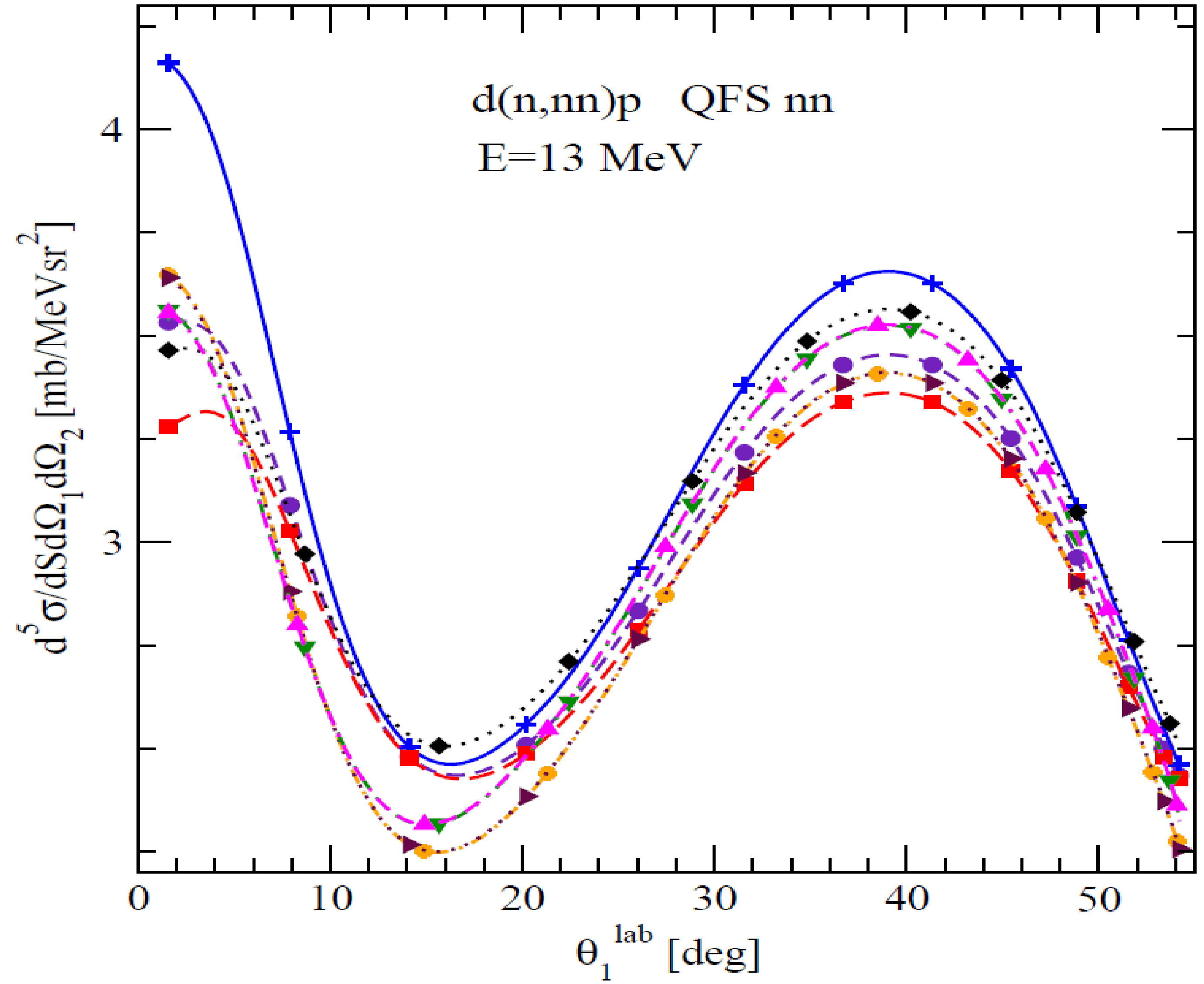
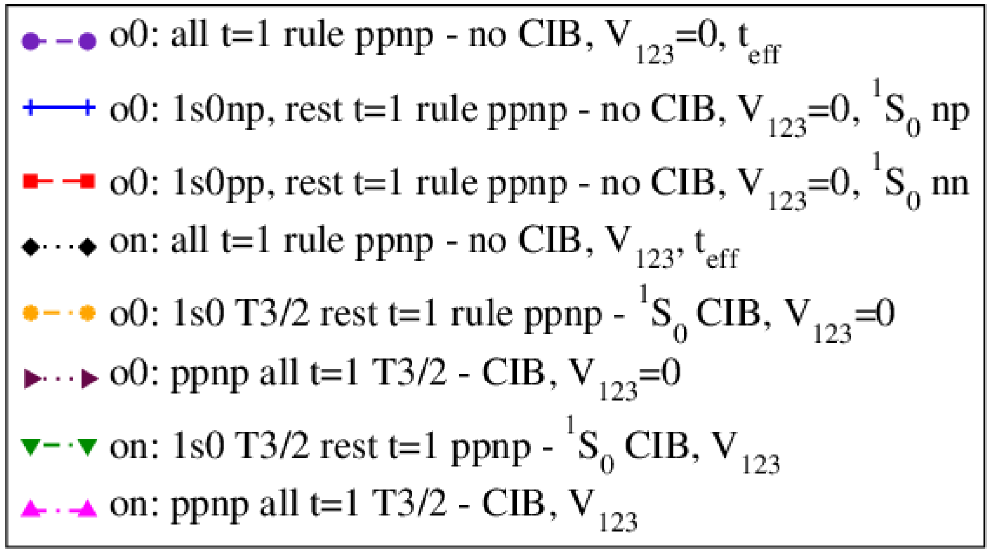


Fig. 7 (color online) The nd complete breakup d(n,nn)p cross section $d^5\sigma/d\Omega_1 d\Omega_2 dS$ calculated exactly at the neutron-neutron quasi-free-scattering condition (maximum of the cross section along the S-curve at $E_3^{lab} = 0$ and $\phi_{12} = 180^o$) at 13 MeV of the incoming neutron laboratory energy as a function of the laboratory angle of the outgoing neutron 1. For the description of lines and symbols see Fig. 3.



Summary and conclusions:

- We investigated the importance of the scattering amplitude components with the total 3N isospin T=3/2 in 3N reactions.
- The inclusion of these components is required to account for CIB effects of the NN interaction. The difference between np and nn (pp) forces leads to a situation in which also the matrix elements of the 3NF between T=3/2 states contribute to the considered 3N reactions.
- The modern NN interactions, which describe existing pp and np data with high precision, provide pp and np t-matrices in t=1 NN states, which differ up to ~10 %. Such a magnitude of CIB requires, that the isospin T=3/2 components are included in the calculation of the breakup reaction, especially for the regions of the breakup phase-space close to the FSI condition.
- However, in order to account practically for all CIB effects it is sufficient to restrict the inclusion of T=3/2 to the $^1S_0$ partial wave state only instead of doing it in all t=1 states.

- For elastic scattering we found that the T=3/2 components can be neglected completely and all CIB effects are accounted for by restricting to the total 3N isospin T=1/2 partial waves only and using the effective t-matrix generated with the ``2/3-1/3" rule $t_{eff}=(2/3)t_{nn}+(1/3)t_{np}$.

- These results allow one to reduce significantly the number of partial waves in time-consuming 3N calculations.

- The presented results show that in 3N reactions the T=3/2 components are overshadowed by the dominant T=1/2 contributions.

- It will be interesting to investigate reactions with three nucleons in which only T=3/2 components contribute in the final state such as e.g. $^3H + \pi^- \rightarrow n + n + n$. That will allow one to study the properties and the importance of 3NFs in the T=3/2 states.