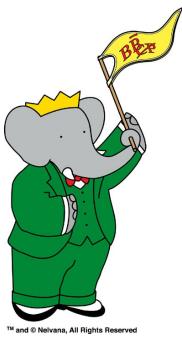


# Measurement of exclusive hadronic cross sections with the BaBar detector and implications on the $g-2$ of the muon

Fabrizio Bianchi  
INFN & University of Torino

On behalf of the BaBar collaboration

PANIC 2017, Beijing, September 1-5, 2017



# Outline

- $g-2$  of the muon
- ISR analysis at BaBar
- Recent results:
  - $\pi^+\pi^-\pi^0\pi^0$
  - $\pi^+\pi^-\eta$
  - $K_S K_L \pi^0, K_S K_L \eta, K_S K_L \pi^0\pi^0$
  - $K_S K^+\pi^-\pi^0, K_S K^+\pi^-\eta$
- Conclusions

# $a_\mu$ of the Muon (1)

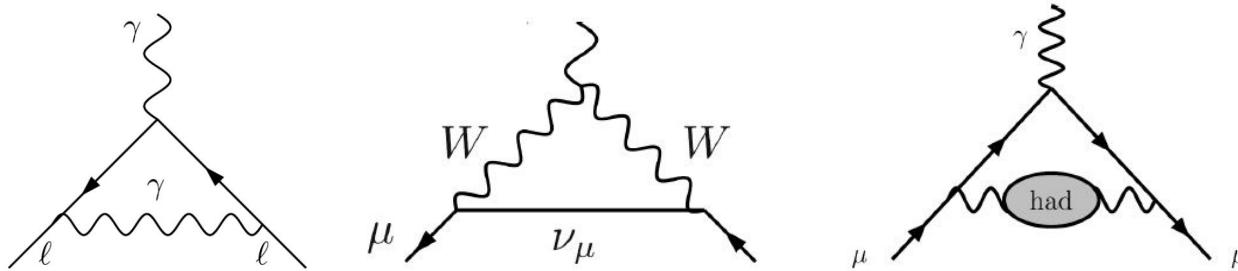
$$\vec{\mu} = g \frac{e}{2m} \vec{s}$$

Dirac equation predicts  $g=2$  for point like fermions

$$a_\mu = \frac{g_\mu - 2}{2}$$

Higher orders contributions lead to non zero  $a_\mu$

$$a_\mu^{SM} = a_\mu^{QED} + a_\mu^{weak} + a_\mu^{hadronic}$$



$a_\mu$  is sensitive to New Physics

$$a_\mu = a_\mu^{SM} + a_\mu^{NP}$$

# $a_\mu$ of the Muon (2)

T. Teubner, PhiPsi 2017

Experiments have achieved an amazing precision and are still improving

E821@BNL (1997-2001):

G.W. Bennett *et al.*,

Phys. Rev. D **77**, 072003 (2006)

$$a_\mu = (11\,659\,209.1 \pm 6.3) \times 10^{-10} \text{ (0.54 ppm)}$$

E989 @ FNAL (2017-...):

F. Gray *et al.*, arXiv: 1510.003

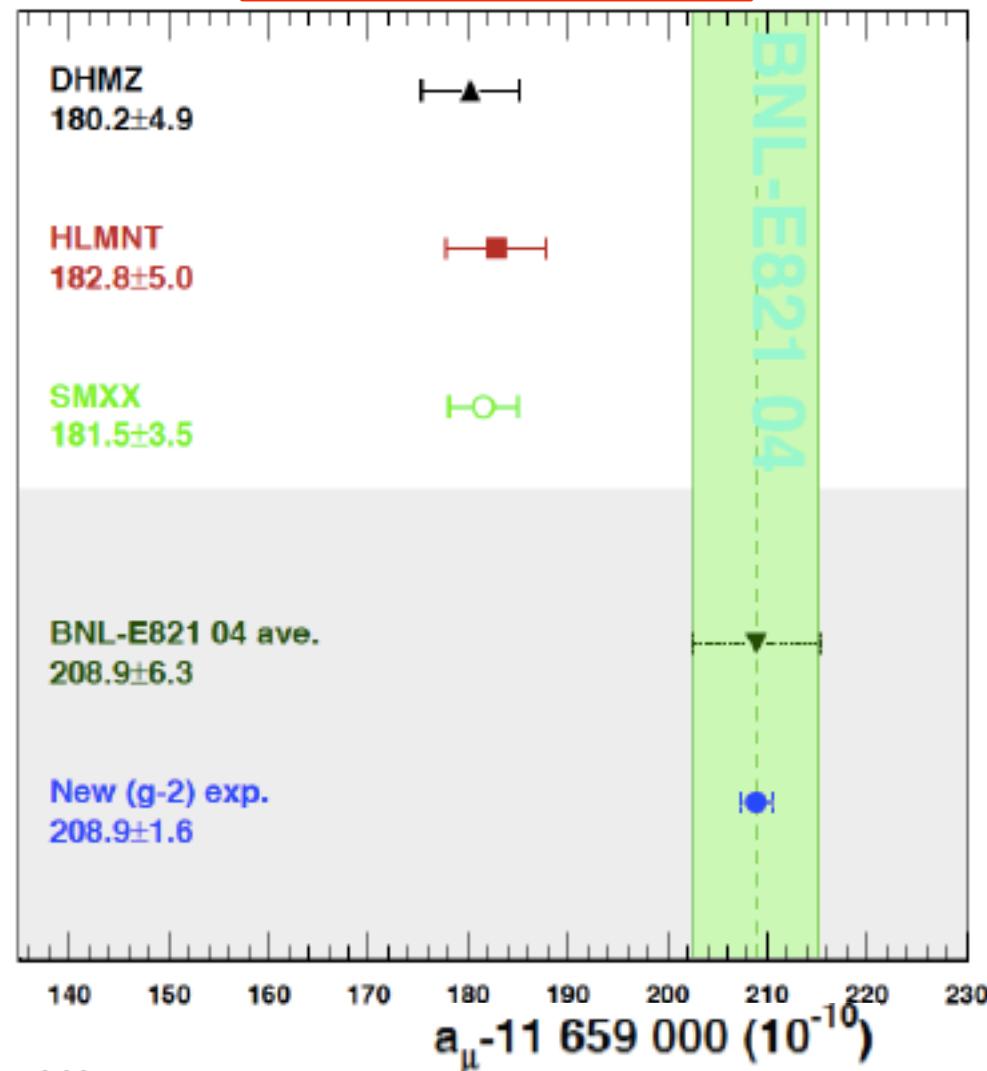
$$a_\mu = \dots \text{ (0.14 ppm)}$$

E34 @ J-PARC:

T. Mibe *et al.*,

Chin. Phys. C **34** (2010) 745

$$a_\mu = \dots \text{ (0.1 ppm)}$$



# $a_\mu$ of the Muon (3)

Comparison with measurement ( $\times 10^{-10}$ )

$a_\mu^{\text{total-SM}}$	$11659181.7 \pm 4.2$
$a_\mu^{\text{BNL-E821}}$	$11659209.1 \pm 6.3$
Data - SM	$27.4 \pm 7.6 (3.6\sigma)$



1. Need more precise measurements of  $a_\mu$
2. Need to improve accuracy of SM prediction

Individual SM contributions ( $\times 10^{-10}$ )

$a_\mu^{\text{QED}}$	$11658471.895 \pm 0.008$
$a_\mu^{\text{EW}}$	$15.4 \pm 0.1$
$a_\mu^{\text{had,LO-VP}}$	$692.6 \pm 3.3$
$a_\mu^{\text{had,HO-VP}}$	$-8.63 \pm 0.09$
$a_\mu^{\text{had,LbLs}}$	$10.5 \pm 2.6$

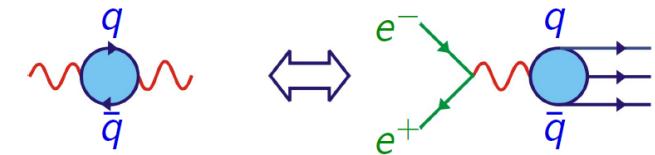


Uncertainties from hadronic contributions dominate

# $a_\mu$ of the Muon (4)

The hadronic leading order vacuum polarization contribution is calculated using dispersion relations from experimental data on the total cross section of the  $e^+e^-$  annihilation into hadrons

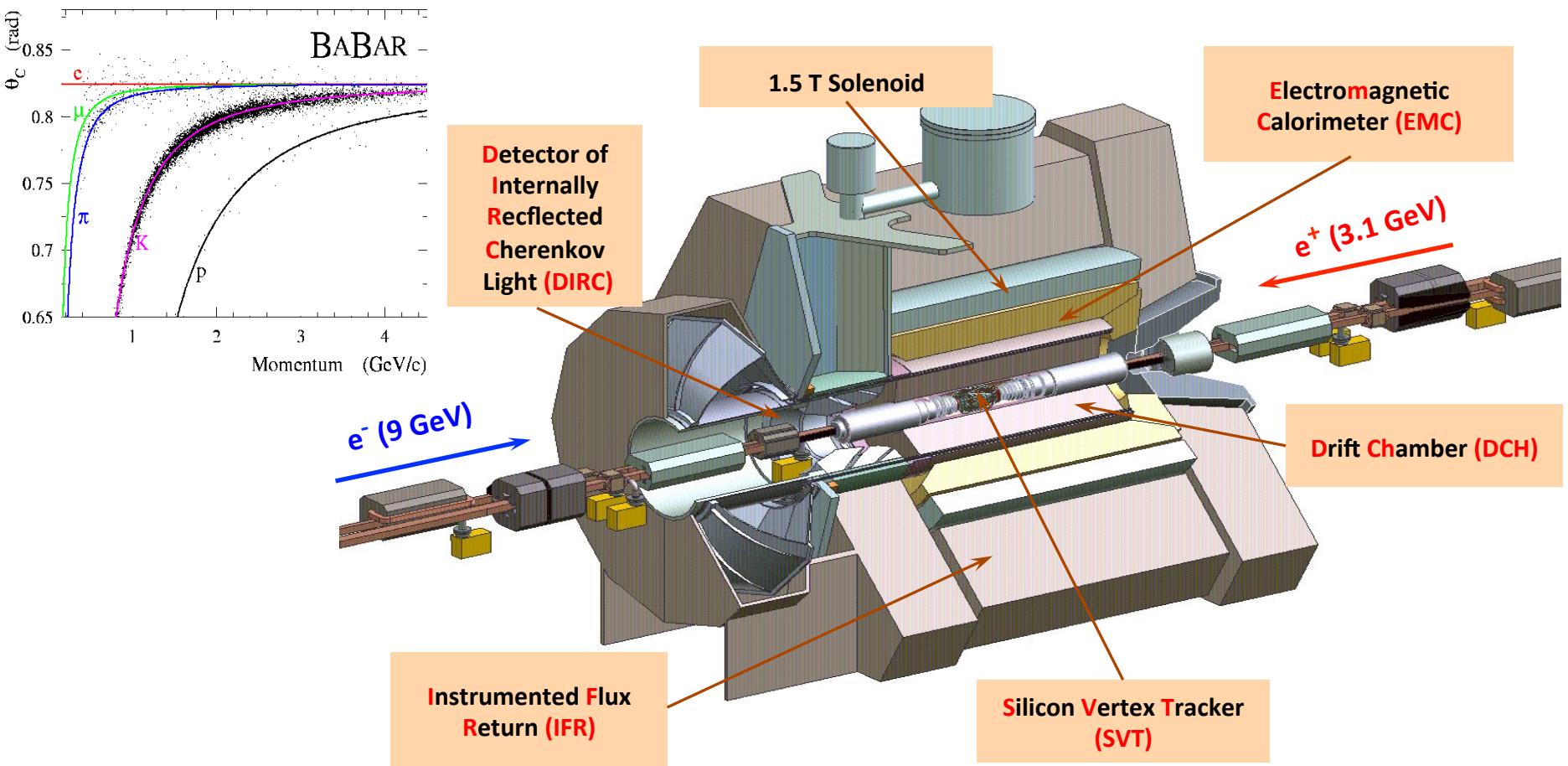
$$a_\mu^{\text{had,LO}} = \frac{m_\mu^2}{12\pi^3} \int_{s_{\text{th}}}^\infty ds \frac{1}{s} \hat{K}(s) \sigma_{\text{had}}(s)$$



Lower energies ( $E < 2$  GeV) give dominant contribution

- To get  $\sigma_{\text{had}}^0$  sum cross sections of measured exclusive channels and use isospin relations for missing channels
- Experimental options are:
  - perform a direct scan (require tunable beam energy)
  - take advantage of radiative return (ISR events)

# The BaBar Detector



BaBar collected data at the PEP-II asymmetric collider at SLAC between 1999 and 2008.

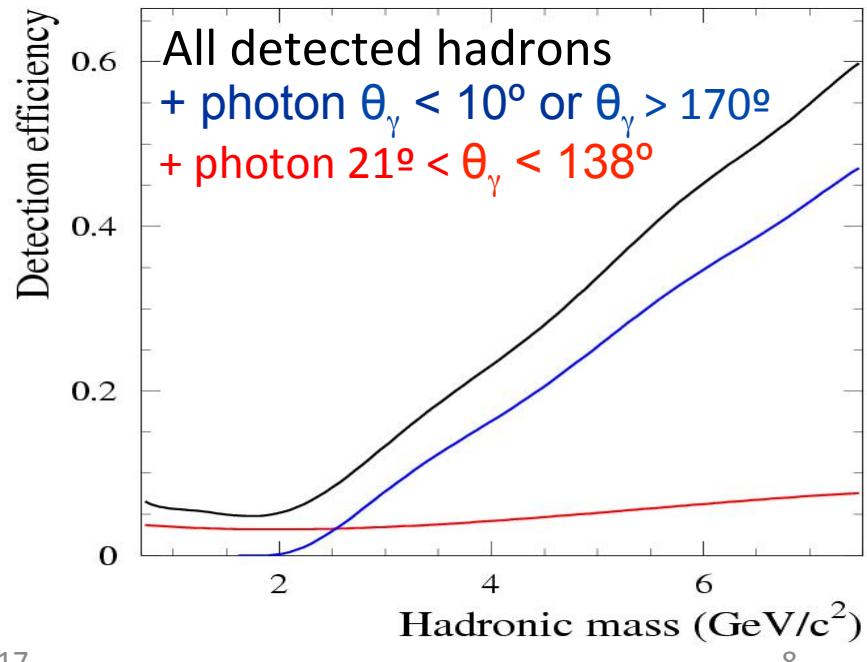
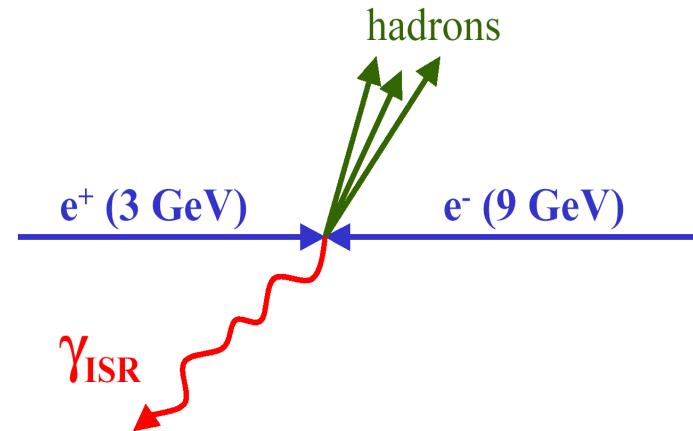
For ISR analysis, a data sample of  $469 \text{ fb}^{-1}$  collected at (or near) a c. m. energy of 10.58 GeV (at or near the Y(4S) resonance).

# ISR Method at BaBar

The mass spectrum of the hadronic system in the reaction  $e^+e^- \rightarrow f\gamma$  is related to the cross section of the reaction  $e^+e^- \rightarrow f$ .

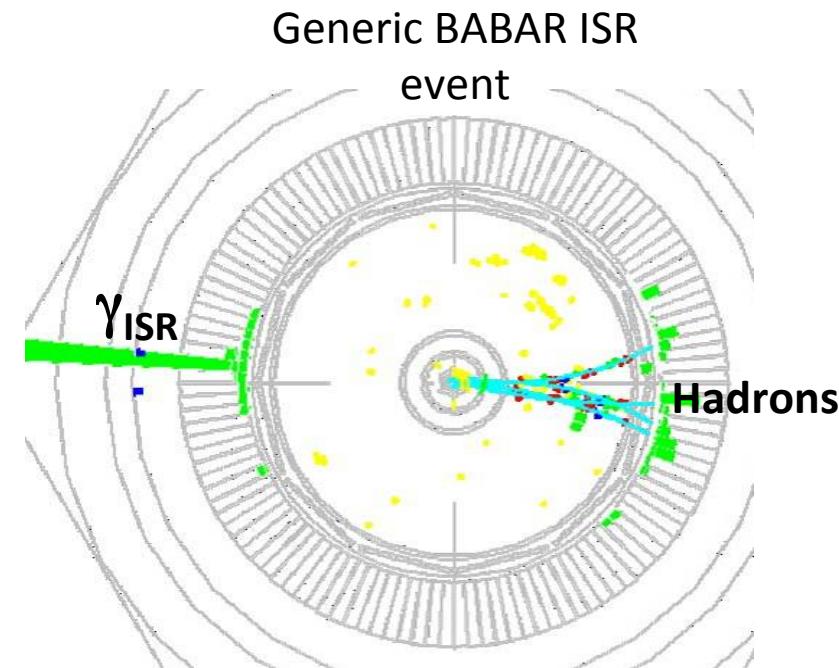
$$\frac{d\sigma(s, x)}{dx d(\cos \theta)} = W(s, x, \theta) \cdot \sigma_0(s(1-x)), \quad x = \frac{2E_\gamma}{\sqrt{s}}$$

The ISR photon is emitted predominantly along the beam axis. The produced hadronic system is boosted against the ISR photon. Due to limited detector acceptance the mass region below 2 GeV can be studied only with detected photon.



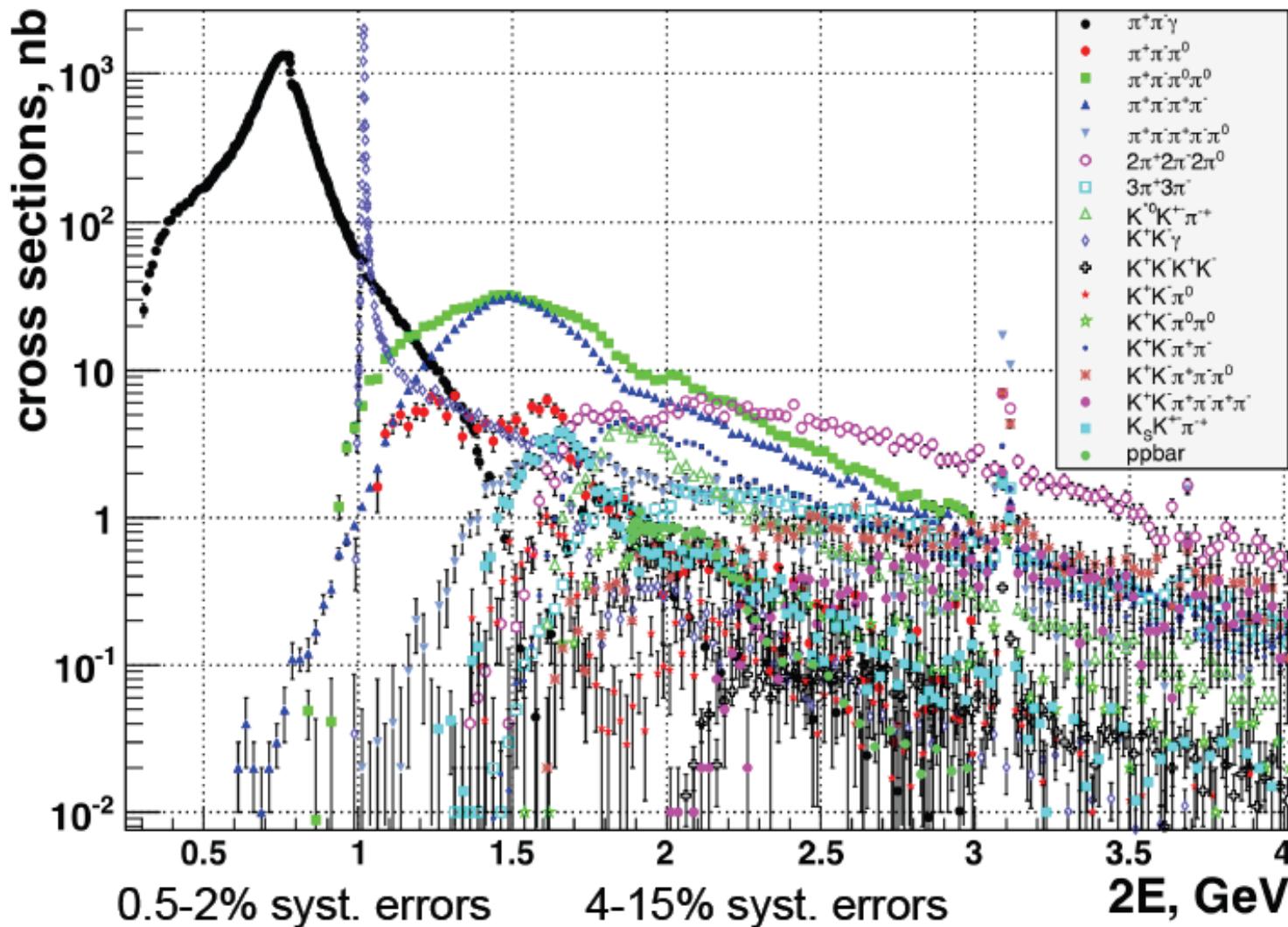
# Tagged ISR Analysis at BaBar

- Fully exclusive measurement
  - Photon with  $E_{CM} > 3 \text{ GeV}$ , which is assumed to be the ISR photon
  - All final hadrons are detected and identified
- Large-angle ISR forces the hadronic system into the detector fiducial region
  - A weak dependence of the detection efficiency on dynamics of the hadronic system (angular and momentum distributions in the hadron rest frame)  $\Rightarrow$  small model uncertainty
  - A weak dependence of the detection efficiency on hadron invariant mass  $\Rightarrow$  measurement near and above threshold with the same selection criteria.
- Kinematic fit with requirement of energy and momentum balance
  - excellent mass resolution
  - background suppression



Can access a wide range of energy in a single experiment: from threshold to  $\sim 5 \text{ GeV}$

# BaBar Tagged ISR Analysis



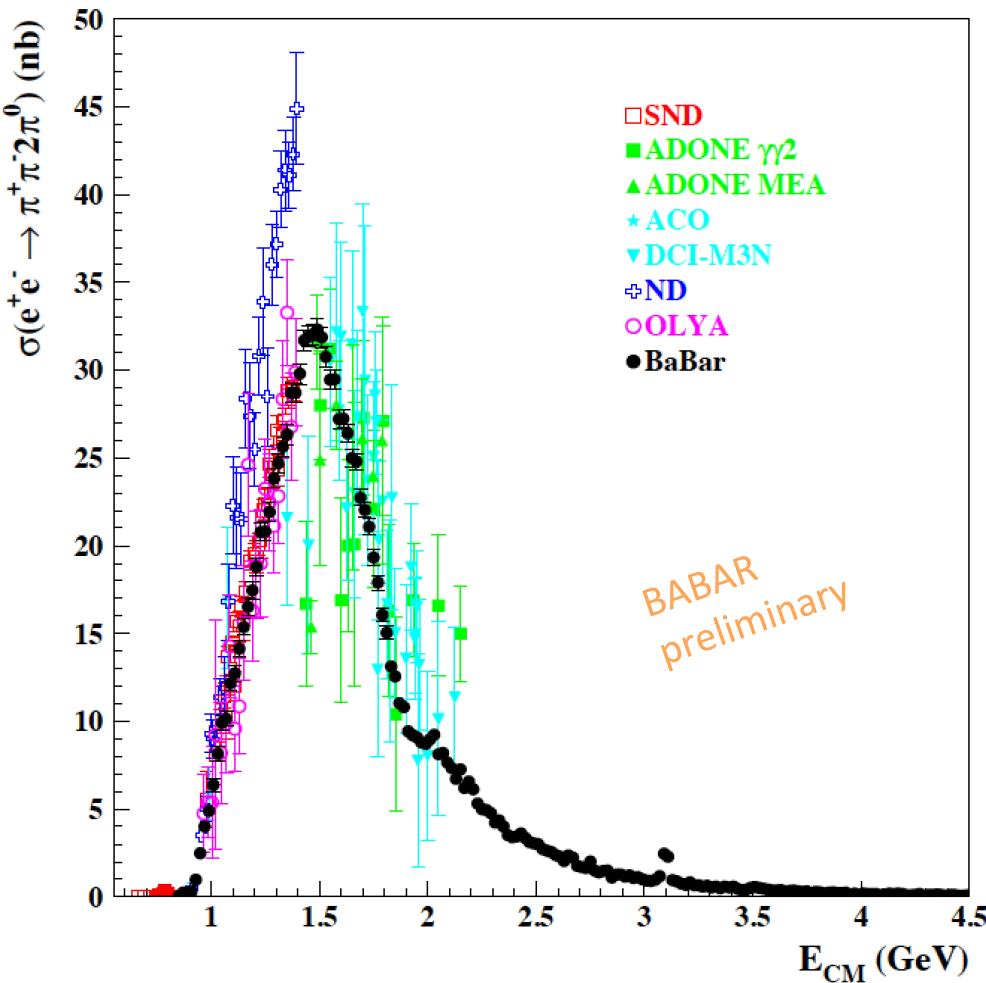
# Before the BaBar Measurements (Presented in This Talk)

BaBar measurements  
presented in this talk:

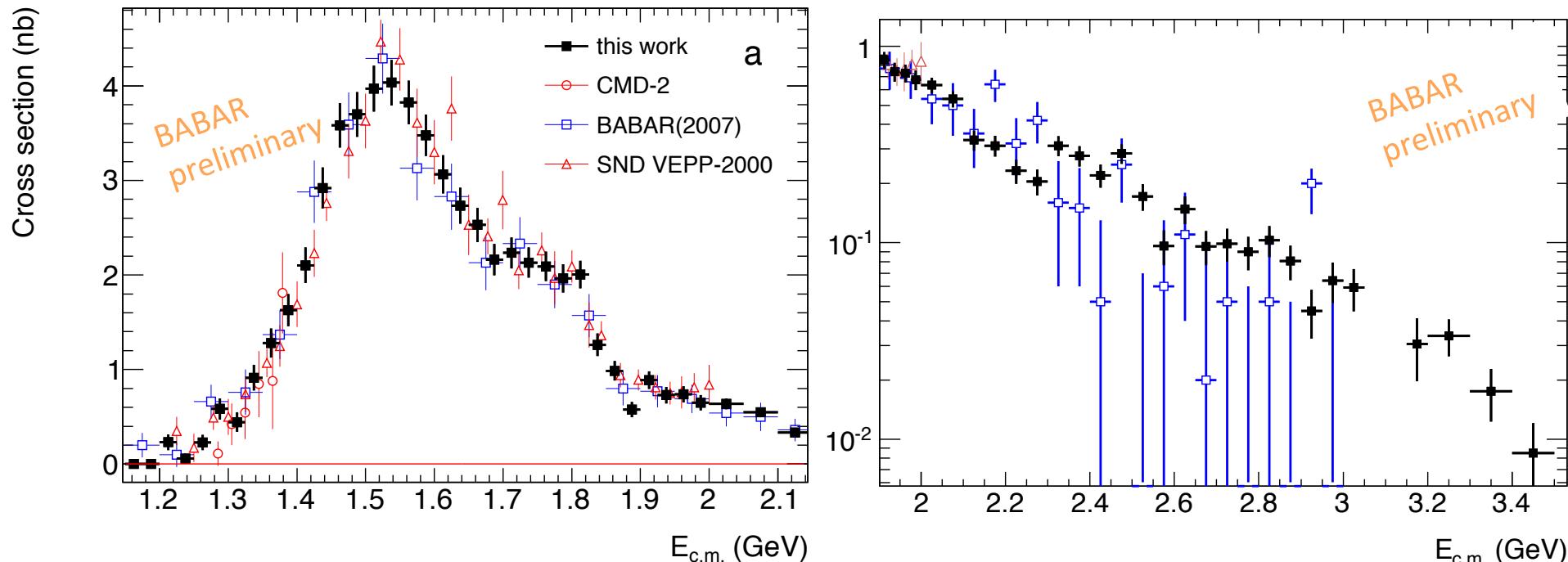
- $\pi^+\pi^-\pi^0\pi^0$
- $\pi^+\pi^-\eta$
- $K_S K_L \pi^0, K_S K_L \eta, K_S K_L \pi^0\pi^0$
- $K_S K^+ \pi^- \pi^0, K_S K^+ \pi^- \eta$

Channel	$a_\mu^{\text{had,LO}} [10^{-10}]$
$\pi^0\gamma$	$4.42 \pm 0.08 \pm 0.13 \pm 0.12$
$\eta\gamma$	$0.64 \pm 0.02 \pm 0.01 \pm 0.01$
$\pi^+\pi^-$	$507.80 \pm 1.22 \pm 2.50 \pm 0.56$
$\pi^+\pi^-\pi^0$	$46.00 \pm 0.42 \pm 1.03 \pm 0.98$
$2\pi^+2\pi^-$	$13.35 \pm 0.10 \pm 0.43 \pm 0.29$
$\pi^+\pi^-2\pi^0$	$18.01 \pm 0.14 \pm 1.17 \pm 0.40$
$2\pi^+2\pi^-\pi^0$ ( $\eta$ excl.)	$0.72 \pm 0.04 \pm 0.07 \pm 0.03$
$\pi^+\pi^-3\pi^0$ ( $\eta$ excl., from isospin)	$0.36 \pm 0.02 \pm 0.03 \pm 0.01$
$3\pi^+3\pi^-$	$0.12 \pm 0.01 \pm 0.01 \pm 0.00$
$2\pi^+2\pi^-2\pi^0$ ( $\eta$ excl.)	$0.70 \pm 0.05 \pm 0.04 \pm 0.09$
$\pi^+\pi^-4\pi^0$ ( $\eta$ excl., from isospin)	$0.11 \pm 0.01 \pm 0.11 \pm 0.00$
$\eta\pi^+\pi^-$	$1.15 \pm 0.06 \pm 0.08 \pm 0.03$
$\eta\omega$	$0.47 \pm 0.04 \pm 0.00 \pm 0.05$
$\eta 2\pi^+2\pi^-$	$0.02 \pm 0.01 \pm 0.00 \pm 0.00$
$\eta\pi^+\pi^-2\pi^0$ (estimated)	$0.02 \pm 0.01 \pm 0.01 \pm 0.00$
$\omega\pi^0$ ( $\omega \rightarrow \pi^0\gamma$ )	$0.89 \pm 0.02 \pm 0.06 \pm 0.02$
$\omega\pi^+\pi^-, \omega 2\pi^0$ ( $\omega \rightarrow \pi^0\gamma$ )	$0.08 \pm 0.00 \pm 0.01 \pm 0.00$
$\omega$ (non- $3\pi, \pi\gamma, \eta\gamma$ )	$0.36 \pm 0.00 \pm 0.01 \pm 0.00$
$K^+K^-$	$21.63 \pm 0.27 \pm 0.58 \pm 0.36$
$K_S^0 K_L^0$	$12.96 \pm 0.18 \pm 0.25 \pm 0.24$
$\phi$ (non- $K\bar{K}, 3\pi, \pi\gamma, \eta\gamma$ )	$0.05 \pm 0.00 \pm 0.00 \pm 0.00$
$KK\pi$ (partly from isospin)	$2.39 \pm 0.07 \pm 0.12 \pm 0.08$
$KK2\pi$ (partly from isospin)	$1.35 \pm 0.09 \pm 0.38 \pm 0.03$
$KK3\pi$ (partly from isospin)	$-0.03 \pm 0.01 \pm 0.02 \pm 0.00$
$\phi\eta$	$0.36 \pm 0.02 \pm 0.02 \pm 0.01$
$\omega K\bar{K}$ ( $\omega \rightarrow \pi^0\gamma$ )	$0.00 \pm 0.00 \pm 0.00 \pm 0.00$

$$e^+ e^- \rightarrow \pi^+ \pi^- \pi^0 \pi^0$$



- BABAR results are most precise and covers wider energy range
- Systematic uncertainty is 3.1% in the 1.2-2.7 GeV energy range. Dominated by background subtraction.
- Contribution to  $a_\mu$  for the range  $1.02 < E_{CM} < 1.8$  GeV is measured to be
  - $17.5 \pm 0.6$  (stat+syst)  $\times 10^{-10}$
  - 3.4% precision
- Previous result including the preliminary BaBar data from 2007 is
  - $18.0 \pm 1.2$  (stat+syst)  $\times 10^{-10}$
  - 6.7% precision



The BaBar results in the  $\eta \rightarrow \gamma\gamma$  mode agrees well with the previous measurements, but is more precise and covers wider energy range

Systematic uncertainty near the cross section maximum, 1.35-1.80 GeV, is 4.5%. Dominated by background subtraction.

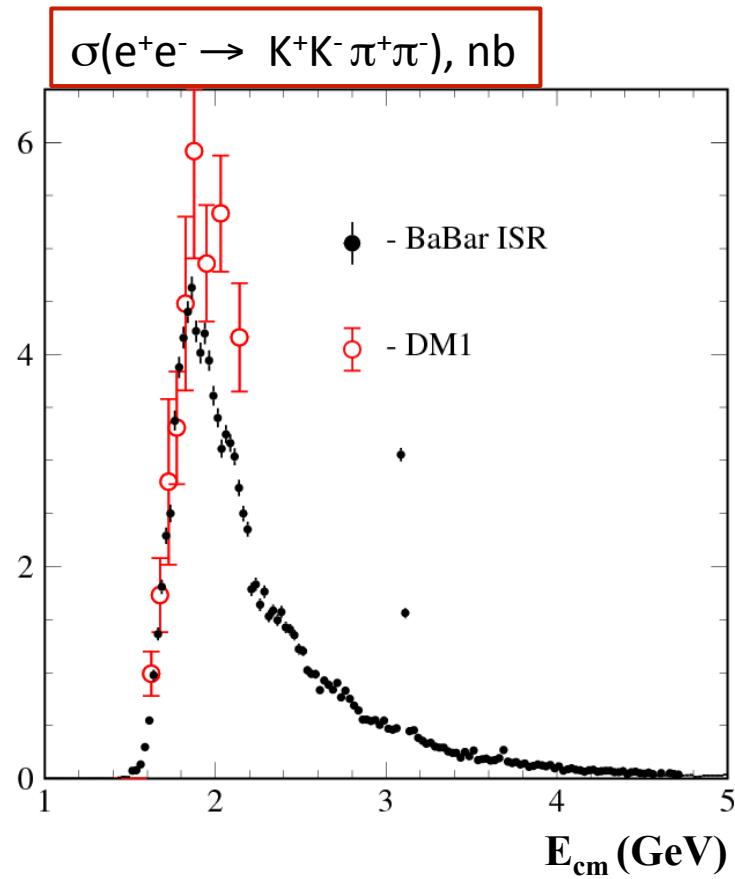
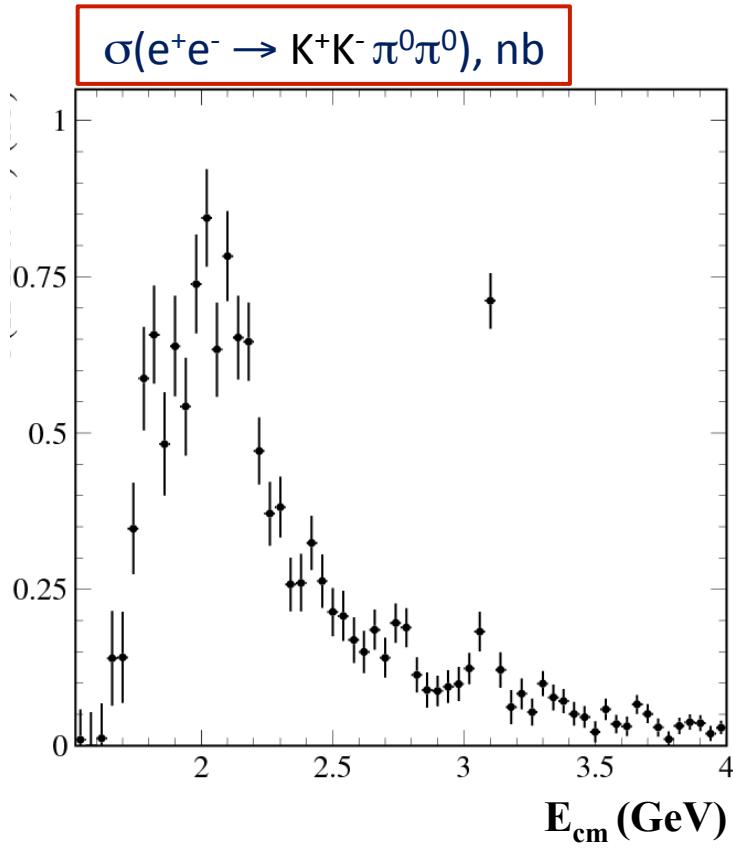
$$a_\mu^{\text{had LO}}(\sqrt{s} < 1.8 \text{ GeV}) = (1.18 \pm 0.06) \cdot 10^{-10} \text{ (this analysis)}$$

$$1.15 \pm 0.10 - \text{ (before this measurement)}$$

$$e^+ e^- \rightarrow K\bar{K}\pi\pi (1)$$

There are six combinations in the  $e^+e^- \rightarrow K\bar{K}\pi\pi$  processes .  
Four were measured previously

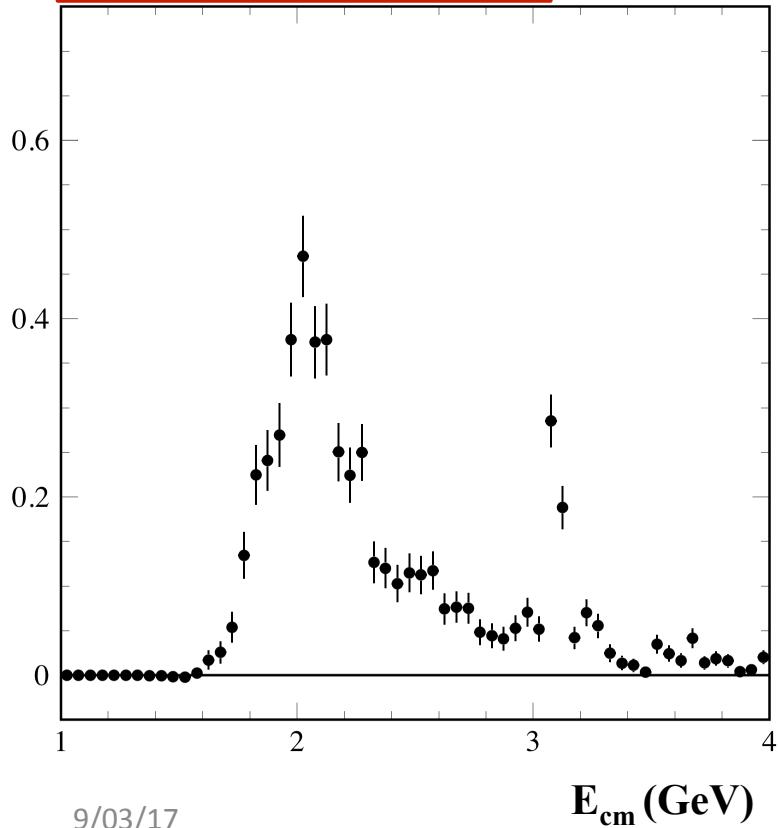
Phys. Rev. D 86, 012008 (2012)



$$e^+ e^- \rightarrow K\bar{K}\pi\pi (2)$$

Phys. Rev. D 89, 092002 (2014)

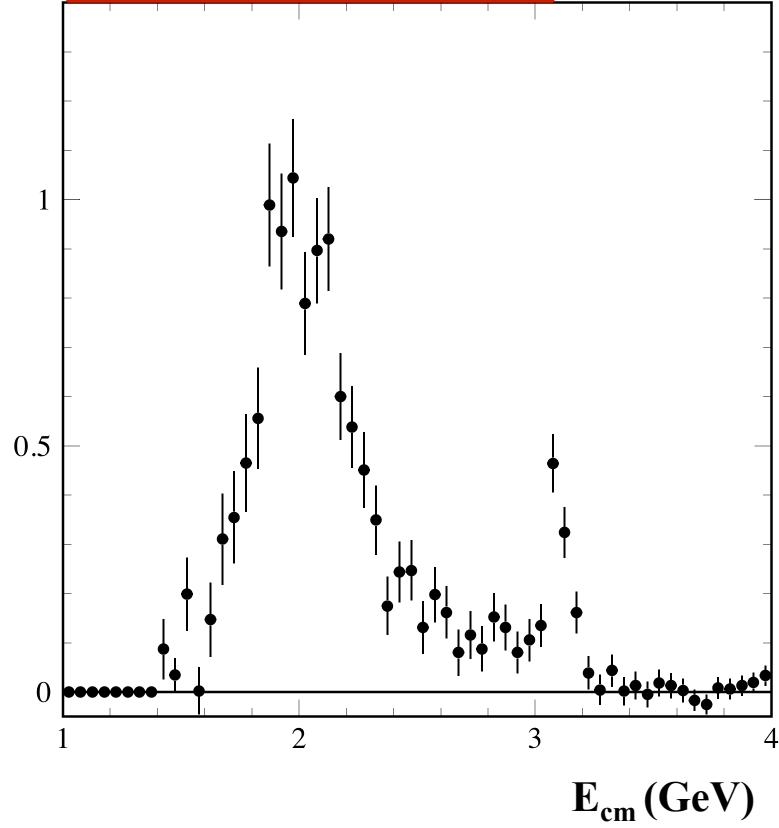
$\sigma(e^+e^- \rightarrow K_S K_S \pi^+\pi^-)$



9/03/17

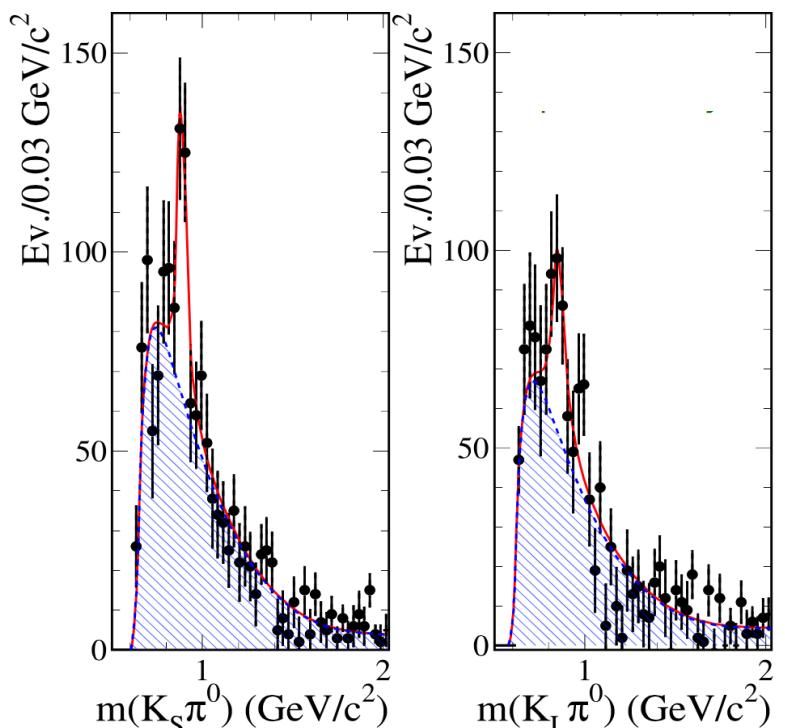
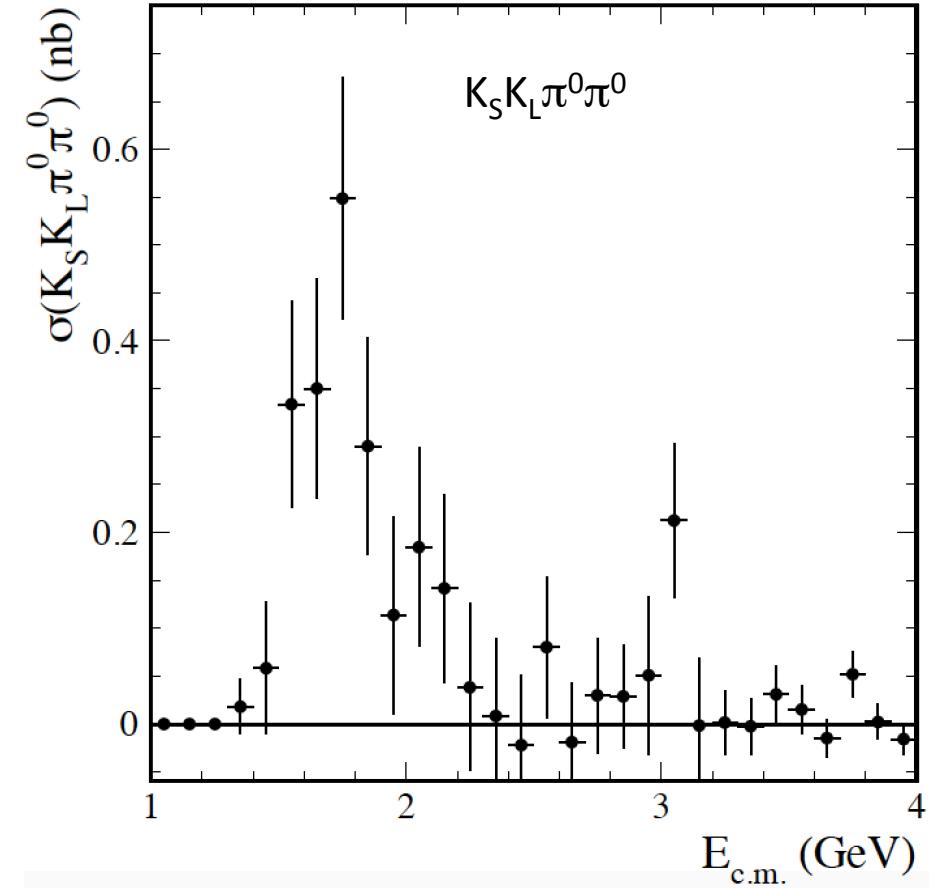
$E_{cm}$  (GeV)

$\sigma(e^+e^- \rightarrow K_S K_L \pi^+\pi^-), nb$

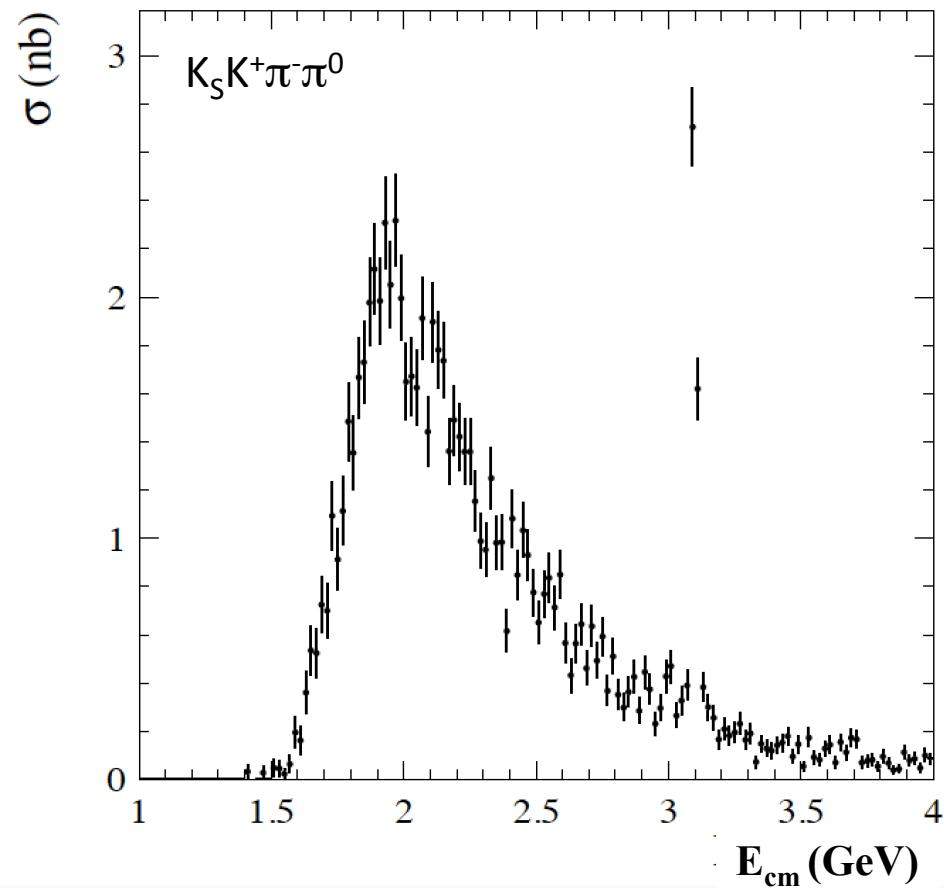


PANIC 2017

$E_{cm}$  (GeV)



- First measurement
- Systematic uncertainty is 25% at the peak, grows to 60% at 2 GeV.  
Dominated by background subtraction.
- Dominant  $K^*(892)K\pi$  intermediate state.



Intermediate state

$$K^{*0} K_S^0 \pi^0$$

$$K^{*0} K^\pm \pi^\mp$$

$$K_2^*(1430)^0 K_S^0 \pi^0$$

$$K_2^*(1430)^0 K^\pm \pi^\mp$$

$$K^*(892)^\pm K_S^0 \pi^\mp$$

$$K^*(892)^\pm K^\mp \pi^0$$

$$K_2^*(1430)^\pm K_S^0 \pi^\mp$$

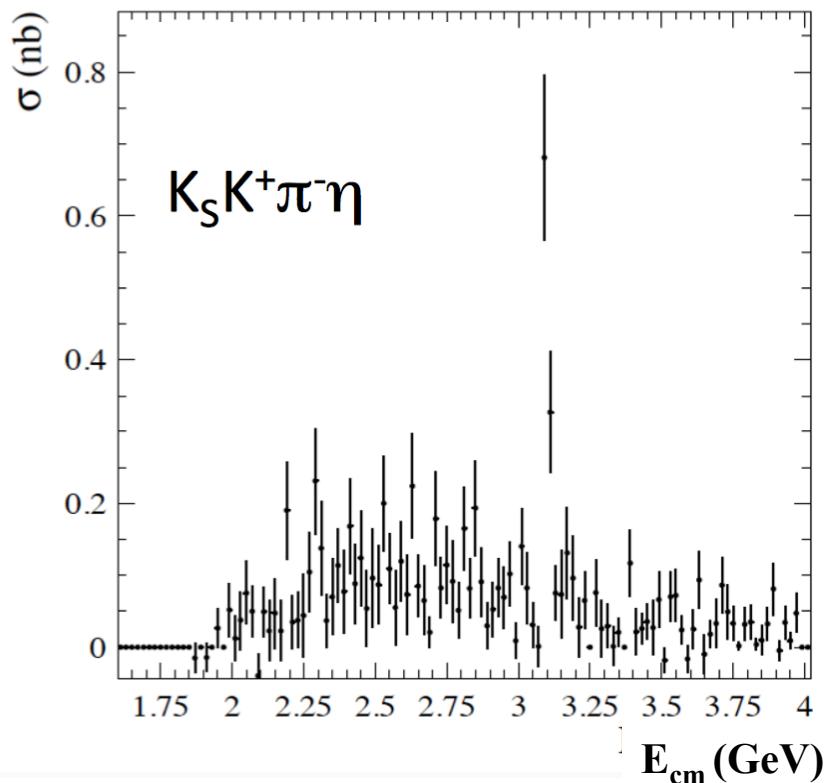
$$K_2^*(1430)^\pm K^\mp \pi^0$$

$$K^{*0} \bar{K}^{*0}$$

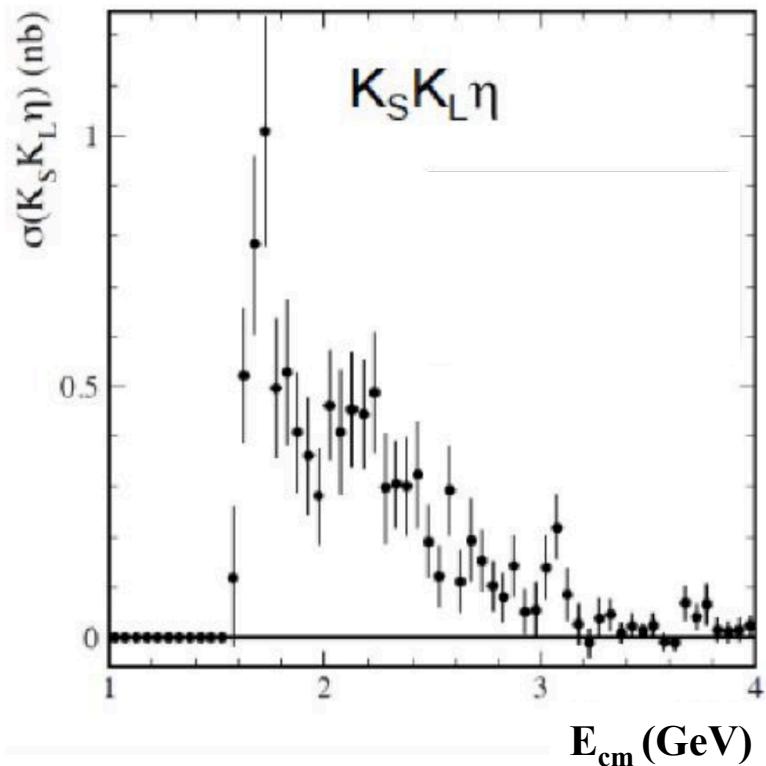
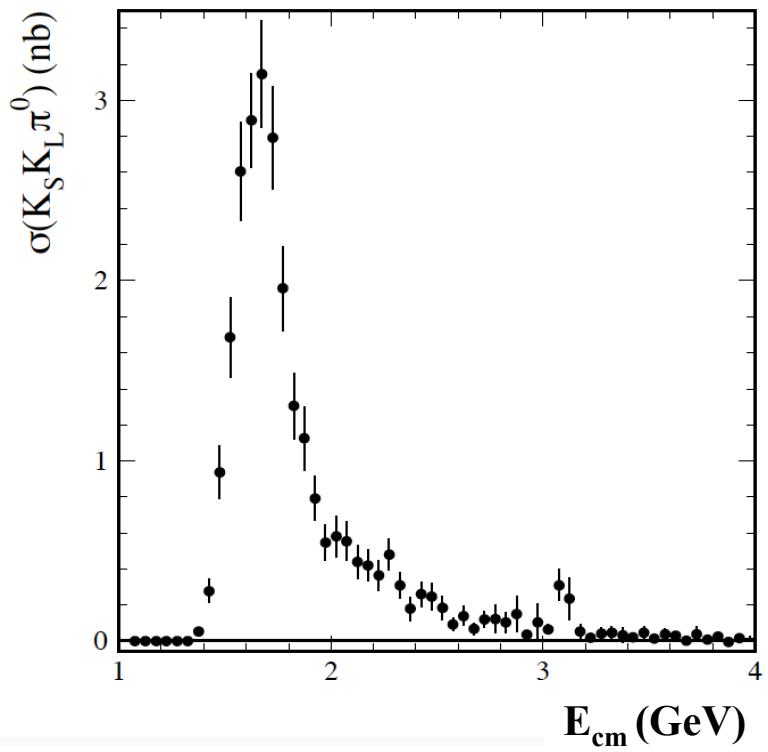
$$K^*(892)^+ K^*(892)^-$$

$$K_S^0 K^\pm \rho(770)^\mp$$

- First measurement
- Systematic uncertainty is 6-7% below 3 GeV. Dominated by background subtraction.
- 11 intermediate states - dominant are  $K^*(892)K\pi$ ,  $K_S K^+ \rho^-(770)$



- First measurement
- Systematic uncertainty is 12-19% below 3 GeV.  
Dominated by background subtraction
- Dominant  $K^*(892)K\eta$  intermediate state.

$$e^+ e^- \rightarrow K_S K_L \pi^0, K_S K_L \eta$$


- First measurement
- Systematic uncertainty is 10% near the peak, grows to 30% at 3.0 GeV. Dominated by background subtraction.
- Dominant  $K^*(892)K\pi$  intermediate state

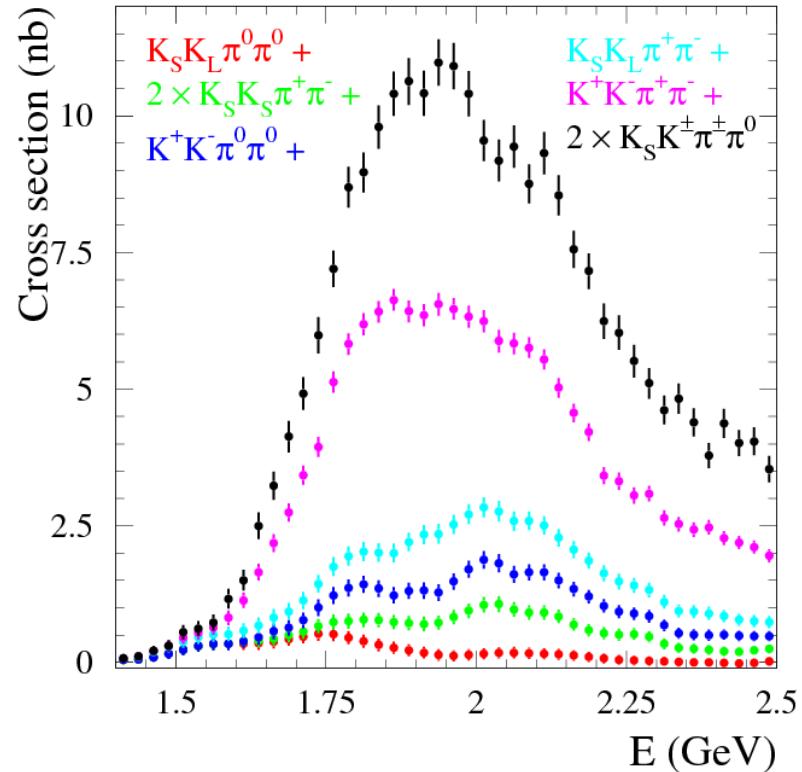
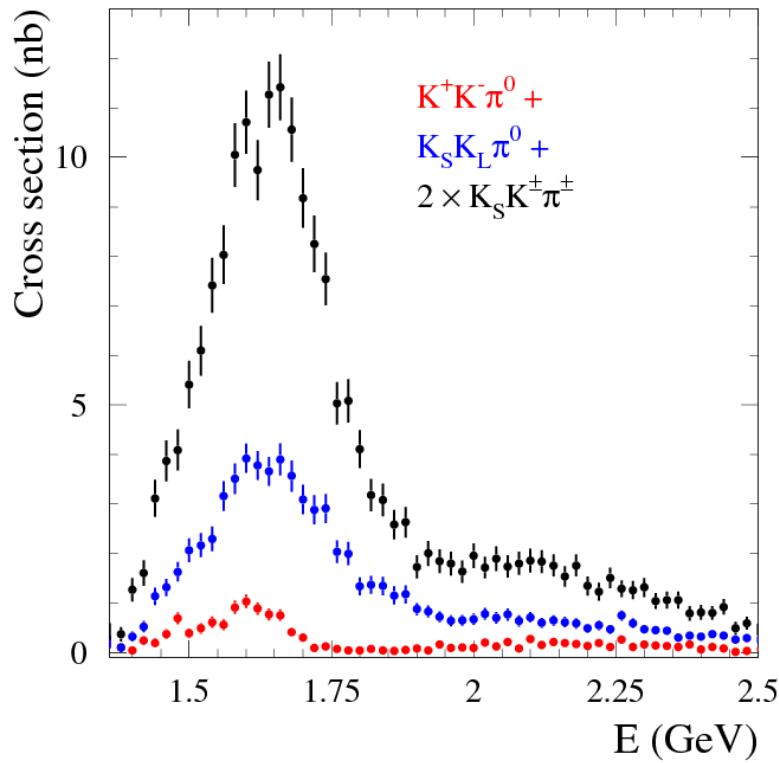
9/09/17

PANIC 2017

- First measurement
- Systematic uncertainty is 25% at the peak, grows to 60% at 2 GeV. Dominated by background subtraction

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# $e^+ e^- \rightarrow KK\pi, KK\pi\pi$ : all modes



- All modes have now been measured by BABAR
- $KK\pi$  is about 12% of the total cross section for  $E_{cm} = 1.65$  GeV
- $KK\pi\pi$  is about 25% of the total cross section for  $E_{cm} = 2.0$  GeV
- Precision on  $(g-2)/2$  improved (no reliance on isospin)

$$a_\mu(KK\pi) = (2.45 \pm 0.15) 10^{-10}$$

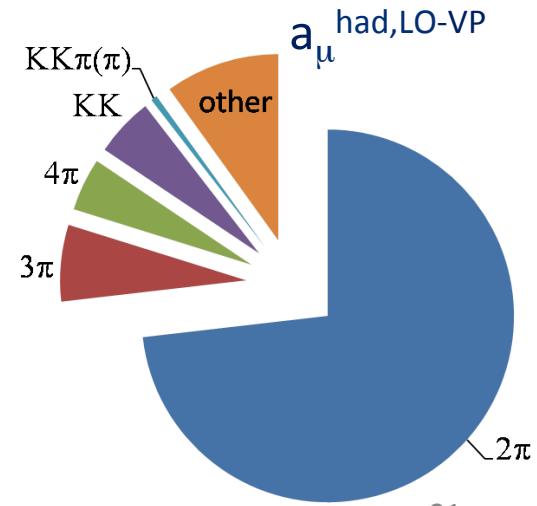
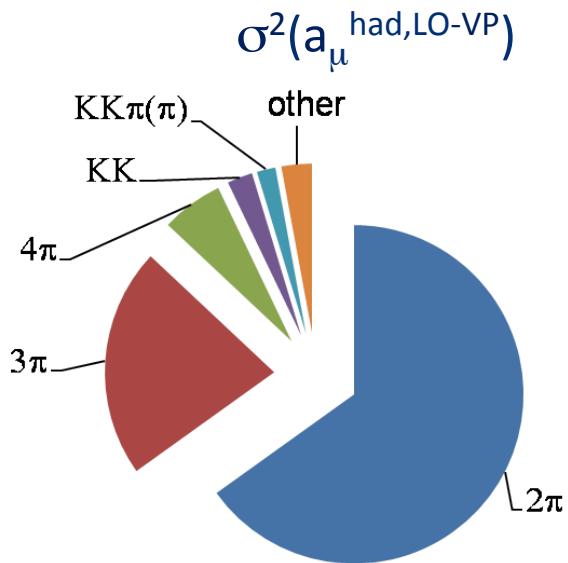
$$2.39 \pm 0.16$$

$$a_\mu(KK\pi\pi) = (0.85 \pm 0.05) 10^{-10}$$

$$1.35 \pm 0.39$$

# Summary

- Precise low-energy  $e^+e^-$  hadronic cross section data are needed to obtain an accurate SM prediction for  $a_\mu^{\text{had,LO-VP}}$
- Recent results on the  $e^+e^- \rightarrow \pi^+\pi^-\pi^0\pi^0$ ,  $KK\pi$ ,  $KK\pi\pi$  cross sections from BaBar reduce the uncertainty on  $a_\mu^{\text{had,LO-VP}}$
- However the exclusive data are incomplete in the region  $1.6 < E < 2.0$  GeV.
  - There is still no experimental information on final states like  $\pi^+\pi^-$ ,  $\pi^0\eta$ ,  $\pi^+\pi^-\eta\eta$ ,  $\pi^+\pi^-\pi^0\pi^0\pi^0$ ,  $\pi^+\pi^-\pi^0\pi^0\eta$ ,  $7\pi$ ...
- New results are expected from BaBar, as well as from BES III, SND, CMD-3
  - On a longer term, Belle II can have a significant impact



# Backup

# BaBar ISR References

$e^+e^- \rightarrow \pi^+\pi^-$

$e^+e^- \rightarrow K^+K^-$

$e^+e^- \rightarrow K_S K_L, K_S K_L \pi^+\pi^-, K_S K_S \pi^+\pi^-, K_S K_S K^+K^-$

$e^+e^- \rightarrow p \text{ anti-}p$

$e^+e^- \rightarrow \Lambda \text{ anti-}\Lambda, \Sigma^0 \text{ anti-}\Sigma^0, \Lambda \text{ anti-}\Sigma^0$

$e^+e^- \rightarrow \pi^+\pi^-\pi^0$

$e^+e^- \rightarrow K^+K^-\eta, K_S K^+\pi^- K^+K^-\pi^0$

$e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-$

$e^+e^- \rightarrow K^+K^-\pi^+\pi^-, K^+K^-\pi^0\pi^0, K^+K^-\pi^+\pi^-$

$e^+e^- \rightarrow 2(\pi^+\pi^-)\pi^0, 2(\pi^+\pi^-)\eta, K^+K^-\pi^+\pi^-\pi^0, K^+K^-\pi^+\pi^-\eta$

$e^+e^- \rightarrow 3(\pi^+\pi^-), 2(\pi^+\pi^-\pi^0), K^+K^-2(\pi^+\pi^-)$

Phys. Rev. Lett. 103 231801 (2009)

Phys. Rev. D 86, 032013 (2012)

Phys. Rev. D 88, 032013 (2013)

Phys. Rev. D 92, 072008 (2015)

Phys. Rev. D 89, 092002 (2014)

Phys. Rev. D 73, 012005 (2006)

Phys. Rev. D 87, 092005 (2013)

Phys. Rev. D 88, 072009 (2013)

Phys. Rev. D 76, 092006 (2007)

Phys. Rev. D 70, 072004 (2004)

Phys. Rev. D 77, 092002 (2008)

Phys. Rev. D 71, 052001 (2005)

Phys. Rev. D 85, 112009 (2012)

Phys. Rev. D 74, 091103 (2006)

Phys. Rev. D 76, 012008 (2007)

Phys. Rev. D 86, 012008 (2012)

Phys. Rev. D 76, 092005 (2007)

Phys. Rev. D 73, 052003 (2006)

# $a_\mu$ of the Muon (2)

T. Teubner, TAU 2016

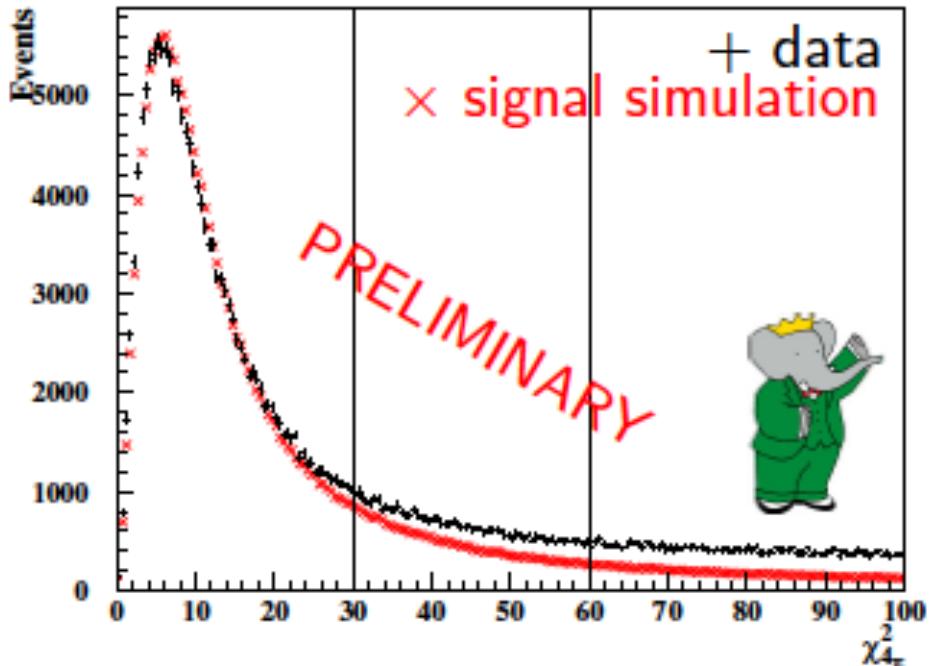
<b>QED</b> contribution	$11\ 658\ 471.808\ (0.015) \times 10^{-10}$	Kinoshita & Nio, Aoyama et al
<b>EW</b> contribution	$15.4\ (0.2) \times 10^{-10}$	Czarnecki et al
<b>Hadronic</b> contribution		
<b>LO</b> hadronic	$694.9\ (4.3) \times 10^{-10}$	HLMNT11
<b>NLO</b> hadronic	$-9.8\ (0.1) \times 10^{-10}$	HLMNT11
<b>light-by-light</b>	$10.5\ (2.6) \times 10^{-10}$	Prades, de Rafael & Vainshtein
<b>Theory TOTAL</b>	$11\ 659\ 182.8\ (4.9) \times 10^{-10}$	
<b>Experiment</b>	$11\ 659\ 208.9\ (6.3) \times 10^{-10}$	world avg
<b>Exp – Theory</b>	$26.1\ (8.0) \times 10^{-10}$	$3.3\ \sigma$ discrepancy

(Numbers taken from HLMNT11, arXiv:1105.3149)

$$e^+ e^- \rightarrow \pi^+ \pi^- \pi^0 \pi^0$$

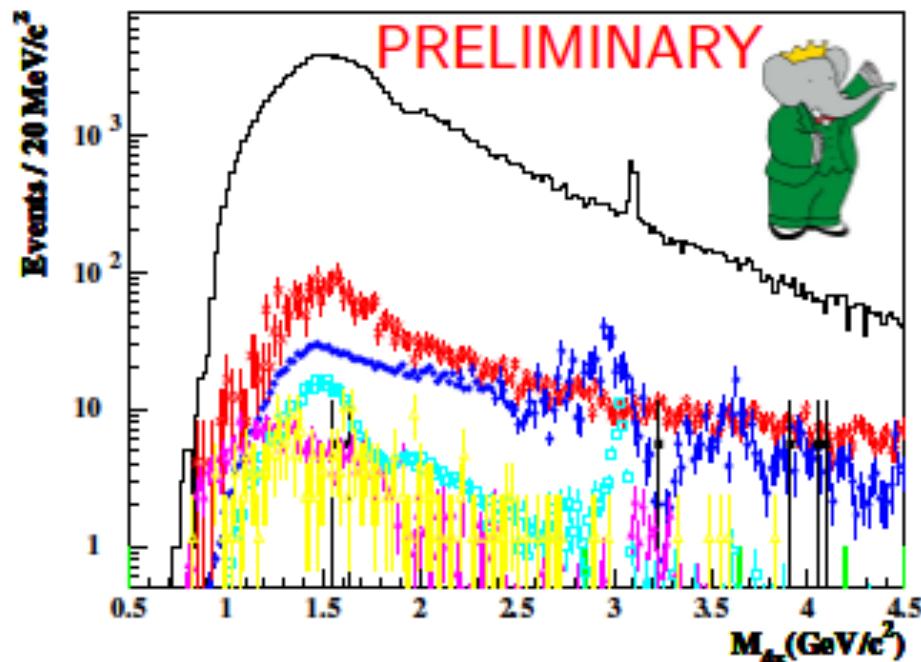
## Main Selection Requirements

- exactly 2 charged tracks
- $\geq 5$  photons
- $E_\gamma^{\text{lab}} > 0.05 \text{ GeV}$
- $|M_{\pi^0}^{\text{reco}} - M_{\pi^0}^{\text{PDG}}| < 0.03 \text{ GeV}$
- $E_{\gamma \text{ISR}} > 3 \text{ GeV}$
- 6C kinematic fit:  $\chi^2_{2\pi 2\pi^0\gamma} < 30$
- reject other hypotheses
- Muon and Kaon PID



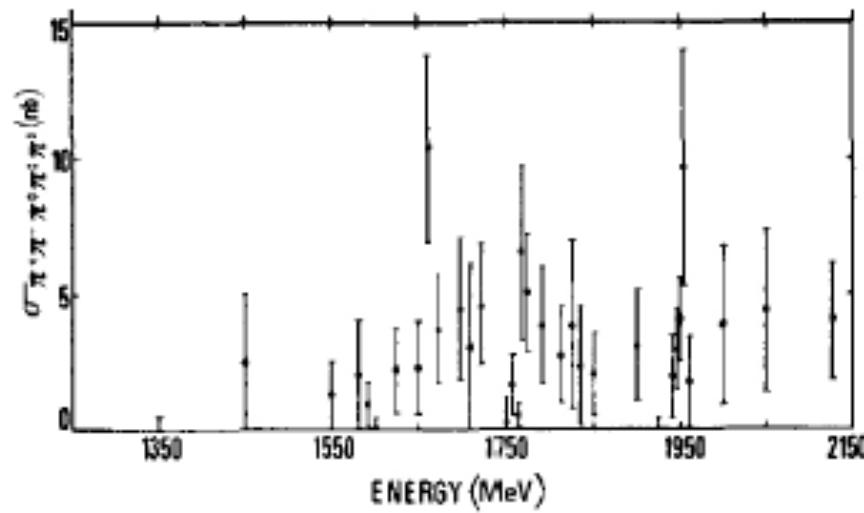
$$e^+ e^- \rightarrow \pi^+ \pi^- \pi^0 \pi^0$$

Simulated background channels:



$q\bar{q}$ ,  $3\pi$ ,  $4\pi 2\pi^0$ ,  $K_s K\pi$ ,  $K^+ K^- 2\pi^0$ ,  $\tau\tau$ ,  
 $\pi^+ \pi^- 3\pi^0$

Main issue: background from  
 $e^+ e^- \rightarrow \pi^+ \pi^- 3\pi^0$



Only little data [3] and no full simulation available

$$e^+ e^- \rightarrow \pi^+ \pi^- \pi^0 \pi^0$$

$M(\pi^+ \pi^- 2\pi^0)$ (GeV/c <sup>2</sup> )	< 1.2	1.2 – 2.7	2.7 – 3.2	> 3.2
Tracking eff.	0.8%	0.8%	0.8%	0.8%
$\gamma$ eff.	0.4%	0.4%	0.4%	0.4%
$2\pi^0$ eff.	2.0%	2.0%	2.0%	2.0%
$\chi^2_{4\pi\gamma}$ eff.	0.4%	0.4%	0.4%	0.4%
Resonances in AfkQED	0.4%	0.4%	0.4%	0.4%
Mass res.	0.3%	0.3%	0.3%	0.3%
FSR	1.0%	1.0%	1.0%	1.0%
NLO ISR	0.5%	0.5%	0.5%	0.5%
ISR luminosity	1.0%	1.0%	1.0%	1.0%
continuum Bkg	1.0%	1.0%	1.0%	2.0%
ISR Background	1 – 100%	1.0%	6.0%	6.0%
Kaon PID	0.5%	0.5%	0.5%	0.5%
Muon PID	0%	0%	0%	2.0%
total	3 – 100%	3.1%	6.7%	7.2%

# $e^+ e^- \rightarrow \pi^+ \pi^- \eta$

Source	Correction, %	Systematic uncertainty, %
Selection criteria		2.5
Background subtraction		
$m_{\pi^+\pi^-\eta} < 1.35$		9
$1.35 < m_{\pi^+\pi^-\eta} < 1.80$		2
$1.80 < m_{\pi^+\pi^-\eta} < 2.50$		5
$2.50 < m_{\pi^+\pi^-\eta} < 3.10$		10.5
$3.10 < m_{\pi^+\pi^-\eta} < 3.50$		11
Trigger and filters	-1.5	1.6
$\eta$ reconstruction	-2	1.0
ISR photon efficiency	-1.1	1.0
Track reconstruction	-1.1	1.0
Radiative correction		1.0
Luminosity		1.0
Total		
$m_{\pi^+\pi^-\eta} < 1.35$	-5.7	10
$1.35 < m_{\pi^+\pi^-\eta} < 1.80$	-5.7	4.5
$1.80 < m_{\pi^+\pi^-\eta} < 2.50$	-5.7	6.5
$2.50 < m_{\pi^+\pi^-\eta} < 3.10$	-5.7	11
$3.10 < m_{\pi^+\pi^-\eta} < 3.50$	-5.7	12



Source	Correction (%)	Systematic uncertainty (%)
$\pi^0$ reconstruction	+2.0	1.0
$K^\pm, \pi^\pm$ reconstruction	+1.6	2.0
$K_S^0$ reconstruction	+1.1	1.0
PID efficiency	0.0	2.0
$\chi^2$ selection	+3.7	4.6
Background subtraction	—	2.5, < 2.0 GeV 4.2, 2.0-3.0 GeV 10.0, > 3.0 GeV
Model acceptance	—	0.5
Luminosity and Rad.Corr.	—	1.4
Total	+8.6	6.3, < 2.0 GeV 7.1, 2.0-3.0 GeV 11.5, > 3.0 GeV