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Study of Deuteron System in the Hypercentral Approach

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1. Abstract

In this work, we study the six-quark deuteron system in the non-relativistic limit. We solve the equation of the system using the hypercentral approach and obtain the ground and excited states wave functions as well as the corresponding energy eigenvalues of the deuteron system. The considered potentials are a combination of the confinement, one gluon exchange and Goldstone boson exchange interactions.

2. The Model

We consider the deuteron as a bound state of three diquarks. Therefore the difficult six-body problem is simplified to three-body problem. At first step, we calculate the diquark masses and then considering the diquarks as point particles, we study the deuteron system. The Schrodinger equation of two-body diquark is as

$$\left[\frac{d^2}{dx^2}+\frac{2}{x}\frac{d}{dx}-\frac{\gamma(\gamma+1)}{x^2}\right]\psi_\gamma(x)=-2m\left[E_\gamma-V(x)\right]\psi_\gamma(x)$$

Where $\psi_\gamma(x)$, E_γ, x and γ are the wave function, energy eigenvalues, distance between two particles, and angular quantum number respectively. m is the reduced mass and potential $V(x)$ is definded as follows

$$V(x)=V_{Conf}+V_{OGE}+V_{OBE}$$

Where the confining, One-gluon exchange and One-boson exchange interactions are respectively defined as

$$V_{Conf}(x)=ax^2$$

$$V_{qq}^{OGE}(x)=\frac{\alpha_s}{4}\lambda_i^c.\lambda_j^c\left[\frac{1}{x}-\frac{\pi}{m_q^2}\left(1+\frac{2}{3}(\sigma_i\cdot\sigma_j)\right)\times\delta(x)\right]$$

$$V_{qq}^{OBE}(x)=\frac{g^2}{4\pi}\frac{1}{12m_q^2}\vec{\lambda}_i^F.\vec{\lambda}_j^F\vec{\sigma}_i.\vec{\sigma}_j\left\{\mu^2\frac{e^{-\mu x}}{x}-4\pi\delta(x)\right\}$$

One-boson exchange interaction is considered as perturbation. The second term of V_{qq}^{OGE} is known as color-coulombic potential and considered as perturbation. Hence, the form of $V(x)$ in Schrodinger equation is as

$$V(x)=ax^2-c/x$$

Where a and c are constant. We assume the wave function to be as

$$\psi_\gamma(x)=x^{-1}\varphi_\gamma(x)=x^{-1}\exp\left[-\frac{1}{2}\alpha x^2+\delta\ln x\right]$$

Using the introduced wave function, Schrodinger equation can be solved and the eigenfunctions and eigenvalues of the diquark systems are obtained as

$$\psi_\gamma=N_\gamma x^{-\frac{1}{2}}\varphi_\gamma=N_\gamma x^\gamma\exp\left(-\frac{m\omega}{2}x^2-\frac{2mc}{(2\gamma+3)}x\right),\qquad E_\gamma=(2\gamma+3)\frac{\omega}{2}-\frac{2mc^2}{(2\gamma+3)^2}.$$

Where $\omega=\sqrt{2a/m}$ and N_γ and c are the normalization constant.

By using the obtained results and perturbative energy, the mass of diquark is obtained as

$$M=m_{q1}+m_{q2}+E_{0\gamma}+\langle H_{CM}\rangle+\langle H_{OBE}\rangle$$

Now considering the three diquarks as point particles, we can study the deuteron system. The three-body Schrodinger equation in the hypercentral approach is obtained as

$$\left[\frac{d^2}{dx^2}+\frac{5}{x}\frac{d}{dx}-\frac{\gamma(\gamma+1)}{x^2}\right]\psi_\gamma(x)=-2m\left[E_\gamma-V(x)\right]\psi_\gamma(x)$$

In which m is the reduced mass and x is the hyerradious and definded in terms of the Jacobi coordinates:

$$m=\frac{2m_\rho m_\lambda}{m_\rho+m_\lambda},\qquad x=\sqrt{\vec{\rho}^2+\vec{\lambda}^2},$$

We assume the wave function to be in the form of

$$\psi_\gamma(x)=x^{\frac{-5}{2}}\exp\left[-\frac{1}{2}\alpha x^2+\delta\ln x\right]$$

By substituting the new function in Schrodinger equation we can get the wave function and energy of the deuteron:

$$\psi_\gamma=N_\gamma x^{-\frac{5}{2}}\varphi_\gamma=N_\gamma x^\gamma\exp\left(-\frac{m\omega}{2}x^2-\frac{2mc}{(2\gamma+5)}x\right),\qquad E_\gamma=(2\gamma+6)\frac{\omega}{2}-\frac{2mc^2}{(2\gamma+5)^2}.$$

For the hyperfine interaction, we consider the new form of exchange interactions which are defined so that the perturbed energy can be computed in the hypercentral model:

$$H_{CM}=V(x)\vec{\sigma}_i.\vec{\sigma}_j,\qquad H_{OBE}\approx-\sum_{i<j}V(x)(\vec{\tau}_i.\vec{\tau}_j)(\vec{\sigma}_i.\vec{\sigma}_j)$$

Where τ_i is the isospin of ith diquark and $V(x)$ is defined as follows:

$$V(x)=A_\chi\left(\frac{1}{\sqrt{\pi}\sigma_\chi}\right)^3\exp(-x^2/\sigma_\chi^2),$$

A_χ and σ_χ are constant and have different values for OGE and OBE interactions. By using the obtained results and perturbative spin- and isospin-dependent energy, the mass of the deuteron can be obtained as

$$M_D=m_{diq1}+m_{diq2}+m_{diq3}+E_{0\gamma}+\langle H_{CM}\rangle+\langle H_{OBE}\rangle$$

3. References

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