A few certainties and many uncertainties about multiquarks

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A. Valcarce, J.-M. Richard
Experiment: The first charmed “exotic” states (1974)

**BNL**
Experimental Observation of a Heavy Particle \( J^+ \)

- \( M = 3.1 \text{ GeV} \)
- \( \Gamma < 1.3 \text{ MeV} \)

**SLAC**
Discovery of a Second Narrow Resonance in \( e^+ e^- \) Annihilation

- \( M = 3.105 \text{ GeV} \)
- \( \Gamma = 2.7 \text{ MeV} \)

**SLAC**
Discovery of a Narrow Resonance in \( e^+ e^- \) Annihilation

- \( M = 3.695 \text{ GeV} \)
- \( \Gamma = 0 \text{ MeV} \)
The interpretation of the narrow boson resonances at 3.1 and 3.7 GeV as charmed quark-antiquark bound states implies the existence of other states. Some of these should be copiously produced in the radiative decays of the 3.7-GeV resonance. We estimate the masses and decay rates of these states and emphasize the importance of γ-ray spectroscopy.

tum numbers and estimate masses and decay widths of these states. Their existence should be revealed by γ-ray transitions among them.

Observation of a Narrow Meson State Decaying to $D_\pi \pi^0$ at a Mass of 2.32 GeV/c²

We have observed a narrow state near 2.32 GeV/c² in the inclusive $D_\pi \pi^0$ invariant mass distribution from $e^+e^-$ annihilation data at energies near 10.6 GeV. The observed width is consistent with the experimental resolution. The small intrinsic width and the quantum numbers of the final state indicate that the decay violates isospin conservation. The state has natural spin-parity and the low mass suggests a $D_s^*$ assignment. The data sample corresponds to an integrated luminosity of 91 fb⁻¹ recorded by the BABAR detector at the SLAC PEP-II asymmetric-energy $e^+e^-$ storage ring.

$D_{s0}^*(2317)$, $J^P=0^+$, $\Gamma<3.8$ MeV

$D_{s1}(2460)$, $J^P=1^+$, $\Gamma<3.5$ MeV

This mass value and the absence of a strong signal in the $\gamma X$ decay channel are in some disagreement with potential model expectations for the $^3D_2$ charmonium state. The mass is within errors at the $D^0 D_{s0}^*$ mass threshold (3871.1 ± 1.0 MeV [9]), which is suggestive of a loosely bound $D D^*$ multiquark “molecular state,” as

$X(3872)$, $\Gamma<2.3$ MeV
Are all these resonances (if they really exist!) multiquark states and/or hadron-hadron molecules?

Back to 1974: A challenge for theory!!

Predictions: An experimental challenge!!
Many speculations about the stability of \((Q_1Q_2Q_3Q_4)\): \((cccc)\), \((bbcc)\), \((bccc)\), …

- Extrapolation of quarkonium dynamics to higher configurations
- New color substructures: 3- or 4-body forces / Role of antisymmetry

### Chromoelectric (CE) limit
(Two-body forces and color as a global operator)

Limit of very heavy constituents: Neglect chromomagnetic terms \(\propto (m_i m_j)^{-1}\)

\[
H = \sum_i \frac{\vec{p}_i^2}{2 m_i} - \frac{16}{3} \sum_{i<j} \vec{x}_i \cdot \vec{x}_j V(r_{ij})
\]

\(\Leftrightarrow\)

\[
H = \sum_i \frac{\vec{p}_i^2}{2 m_i} - \sum_{i<j} \frac{e_i e_j}{r_{ij}}
\]

### QED

- \((e^+ e^+ e^- e^-) \equiv Ps_2\) positronium molecule, stable although with tiny binding
- \((p \ p e^- e^-) \equiv H_2\) hydrogen molecule, stable with a comfortable binding
- \((M^+ M^+ m^- m^-)\) more stable than \((m^+ m^+ m^- m^-)\). Stability depends critically on the masses involved
- But \((M^+ m^+ M^- m^-)\) unstable if \(M/m \geq 2.2\)

### QCD

- \((QQqq)\) stable for large \(M/m\) ratio
- \((QQQQ)\) unstable in naive CE limit
- Delicate four-body problem
- Approximations, like diquarks or restricting the Hilbert space, **artificially** favor binding
Improved chromoelectric model: Many-body confining forces

\[ V_s = \min(V_f, V_b). \]

\( V_f \) stands for the so-called “flip-flop” model

\[ V_f = \lambda \min(r_{13} + r_{24}, r_{23} + r_{14}). \]

\( V_b \) is the butterfly-like configuration,

\[ V_b = \lambda \min_{k, \ell}(r_{1k} + r_{2k} + r_{k\ell} + r_{\ell3} + r_{\ell4}). \]

*Flip-flop*

*Butterfly*

\[ (M^+ M^+ m^- m^-) \equiv QQqq \]

\[ (M^+ m^+ M^- m^-) \equiv QQqq \]

\[ M/m \]

\[ u \]

---


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Improved chromoelectric model: Many-body confining forces

\[ V = -\frac{3}{16} \sum_{i<j} \tilde{\lambda}_i \cdot \tilde{\lambda}_j v(r_{ij}). \]

\[ \Psi = \psi_T |T\rangle + \psi_M |M\rangle, \]

A ⇒ String model with $|T\rangle$
B ⇒ String model with $|T\rangle$ and $|M\rangle$
C ⇒ Pairwise with $|T\rangle$ and $|M\rangle$
D ⇒ Adiabatic limit of C

A,D (adiabatic) bound !! BUT B,C (color+antisymmetry) unbound
(bbbb) and (cccc) are **unstable** in serious 4-body estimates in naive **CE models**. They follow the trends of (++---) in QED, but less favorable due to the non-Abelian algebra of charges.

(bcbc) might have some opportunities as compared to (bbbb) and (cccc)

\[(bc)(cb) \equiv MM\]

**BUT** (bcbc) there are two different thresholds

\[(bb)(cc) \equiv \Upsilon J/\psi\]

**THUS** it may present **metastability** below the MM threshold

(bbcc) although more delicate:

- Benefits from symmetry breaking
- There is a single threshold
- For non-identical quarks and antiquarks, string potentials offer good opportunities
- \((QQqq)\) favored in the CE limit due to the striking \(M/m\) dependence
Similar findings: Baryonia ($Q^3q^3$) and dibaryons ($Q^3q^3$)

Baryonia

Dibaryons

Unbound

Bound

Dibaryons ($Q^3q^3$)

Baryonia ($Q^3q^3$)

Chromomagnetic term: Dibaryons (qqqq’QQ’)

\[ V = -\frac{3}{16} \sum_{i<j} \tilde{\lambda}_i \cdot \tilde{\lambda}_j \left( -\frac{a}{r_{ij}} + br_{ij} \right) + \frac{c}{m_i m_j} \left( \frac{\mu}{\pi} \right)^{3/2} \exp(-\mu r_{ij}^2) \sigma_i \cdot \sigma_j \]

\[ S_1 = (000), \quad S_2 = (011), \quad S_3 = (101), \quad S_4 = (110), \quad S_5 = (111). \]

\[ C_1 = (666), \quad C_2 = (633), \quad C_3 = (363), \quad C_4 = (336), \quad C_5 = (333). \]

Conflict between CE and CM

It goes against binding

Color + Antisymmetry

\[ J = 0^+ \]

<table>
<thead>
<tr>
<th>Color-spin vector ( C_i S_j )</th>
<th>( J^P = 0^+ )</th>
<th>( E ) (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_1 S_1 )</td>
<td></td>
<td>3.079</td>
</tr>
<tr>
<td>( C_2 S_1 )</td>
<td></td>
<td>2.829</td>
</tr>
<tr>
<td>( C_3 S_4 )</td>
<td></td>
<td>2.831</td>
</tr>
<tr>
<td>( C_2 S_2 )</td>
<td></td>
<td>3.030</td>
</tr>
<tr>
<td>( C_3 S_3 )</td>
<td></td>
<td>3.030</td>
</tr>
<tr>
<td>( C_3 S_5 )</td>
<td></td>
<td>2.908</td>
</tr>
<tr>
<td>( C_1 S_2 )</td>
<td></td>
<td>2.995</td>
</tr>
<tr>
<td>( C_4 S_3 )</td>
<td></td>
<td>2.835</td>
</tr>
<tr>
<td>( C_4 S_4 )</td>
<td></td>
<td>3.080</td>
</tr>
<tr>
<td>( C_4 S_5 )</td>
<td></td>
<td>3.016</td>
</tr>
<tr>
<td>( C_5 S_3 )</td>
<td></td>
<td>2.891</td>
</tr>
<tr>
<td>( C_5 S_4 )</td>
<td></td>
<td>2.997</td>
</tr>
<tr>
<td>( C_5 S_5 )</td>
<td></td>
<td>3.034</td>
</tr>
<tr>
<td>Coupled</td>
<td></td>
<td>2.767</td>
</tr>
</tbody>
</table>

Thresholds

\[ 2.570 \quad 2.630 \]

TABLE III: Probabilities of the different six-body channels contributing to the \( J^P = 0^+ \) six-quark state.

<table>
<thead>
<tr>
<th>Channel</th>
<th>( C_1 S_2 )</th>
<th>( C_2 S_1 )</th>
<th>( C_3 S_4 )</th>
<th>( C_4 S_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.004</td>
<td>0.539</td>
<td>0.456</td>
<td>0.001</td>
</tr>
</tbody>
</table>

TABLE I: Energy (in GeV) of the baryons involved in the thresholds within the model (1). \( \Sigma \) stands for a baryon where the first two quarks are in a spin 1 state, and \( \Lambda \) in a spin 0 state.

<table>
<thead>
<tr>
<th>( q\bar{q}Q(\Sigma) )</th>
<th>( q\bar{q}Q'(\Lambda) )</th>
<th>( qqqq(\Sigma) )</th>
<th>( QQ'q(\Sigma) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.372</td>
<td>1.258</td>
<td>1.461</td>
<td>1.109</td>
</tr>
</tbody>
</table>

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What about pentaquarks?: \((QQqqq)\)

\[ V(r) = -\frac{3}{16} \frac{\lambda_i \cdot \lambda_j}{r} \left[ \lambda r - \frac{\kappa}{r} - \Lambda + \frac{V_{SS}(r)}{m_i m_j} \vec{\sigma}_i \cdot \vec{\sigma}_j \right], \]

\[ V_{SS} = \frac{2\pi \kappa'}{3} \frac{1}{\pi^{3/2} r_0^3} \exp\left(-r^2 / r_0^2\right), \quad r_0(m_i, m_j) = A \left(\frac{2m_i m_j}{m_i + m_j}\right)^{-B} \]

<table>
<thead>
<tr>
<th>(I = 1/2)</th>
<th>(J)</th>
<th>(1/2)</th>
<th>(3/2)</th>
<th>(5/2)</th>
<th>Mass</th>
<th>(I = 3/2)</th>
<th>(J)</th>
<th>(1/2)</th>
<th>(3/2)</th>
<th>(5/2)</th>
<th>Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>(N\eta_c)</td>
<td>(S)</td>
<td>(D)</td>
<td>(D)</td>
<td>(4.001)</td>
<td>(\Delta\eta_c)</td>
<td>(D)</td>
<td>(S)</td>
<td>(D)</td>
<td>(4.312)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(N\Lambda/\psi)</td>
<td>(S)</td>
<td>(S)</td>
<td>(D)</td>
<td>(4.097)</td>
<td>(D\Sigma_c)</td>
<td>(S)</td>
<td>(D)</td>
<td>(D)</td>
<td>(4.329)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(D\Lambda_c)</td>
<td>(S)</td>
<td>(D)</td>
<td>(D)</td>
<td>(4.154)</td>
<td>(D\Sigma^*_c)</td>
<td>(S)</td>
<td>(D)</td>
<td>(D)</td>
<td>(4.408)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(D^*\Lambda_c)</td>
<td>(S)</td>
<td>(D)</td>
<td>(D)</td>
<td>(4.308)</td>
<td>(\Delta J/\psi)</td>
<td>(S)</td>
<td>(S)</td>
<td>(S)</td>
<td>(4.408)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(D\Sigma_c)</td>
<td>(S)</td>
<td>(D)</td>
<td>(D)</td>
<td>(4.329)</td>
<td>(D^*\Sigma_c)</td>
<td>(S)</td>
<td>(S)</td>
<td>(S)</td>
<td>(4.483)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(D\Sigma^*_c)</td>
<td>(D)</td>
<td>(S)</td>
<td>(D)</td>
<td>(4.408)</td>
<td>(D^<em>\Sigma^</em>_c)</td>
<td>(S)</td>
<td>(S)</td>
<td>(S)</td>
<td>(4.562)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(D^*\Sigma_c)</td>
<td>(S)</td>
<td>(S)</td>
<td>(D)</td>
<td>(4.483)</td>
<td>(D^<em>\Sigma^</em>_c)</td>
<td>(S)</td>
<td>(S)</td>
<td>(S)</td>
<td>(4.562)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Not so Stable! Above D-wave threshold and below S-wave one.

Stable! Below S- and D-wave thresholds.
\[ V(r) = -\frac{3}{16} \tilde{\lambda}_i \cdot \tilde{\lambda}_j \left[ \lambda r - \frac{\kappa}{r} - \Lambda + \frac{V_{SS}(r)}{m_i m_j} \tilde{\sigma}_i \cdot \tilde{\sigma}_j \right], \]

\[ V_{SS} = \frac{2 \gamma' \kappa'}{3} \frac{1}{r_0^{3/2} r_0^3} \exp(-r^2/r_0^2), \quad r_0(m_i, m_j) = A \left( \frac{2m_i m_j}{m_i + m_j} \right)^{-B}. \]
Summary of four-quark states: BCN and CQC

<table>
<thead>
<tr>
<th></th>
<th>( \text{Q} Q_n ) (Non exotic)</th>
<th>( \text{Q} Q_{nn} ) (Exotic)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Compact</strong></td>
<td>No states</td>
<td>( J^P = 1^+ )</td>
</tr>
<tr>
<td><strong>Molecular</strong></td>
<td>( (I)J^{PC} = (0)1^{++} \Rightarrow X(3872) )</td>
<td>( (I)J^{PC} = (1)2^{++} ) (Exotic)</td>
</tr>
</tbody>
</table>

**How the molecular \( Q_n Q_n \) states are formed in a quark model framework?**
Coupled channel effect ↔ Hidden color vectors

FIG. 1: Experimental masses of the different two meson systems made of a heavy and a light quark and their corresponding antiquarks, $QnQ\bar{n}$ with $Q = s, c,$ or $b$, for several sets of quantum numbers, $J^{PC}$. We have set as our origin of energies the $KK$, $DD$ and $BB$ masses for the hidden strange, charm and bottom sectors, respectively.
4. Unravelling the pattern of the XYZ mesons

There should not be a partner of the X(3872) in the bottom sector.

There should be a $J^P = 1^+$ bound state in the exotic bottom sector.

\( Q \equiv c \)  
\( Q \equiv b \)
A quark-model mechanism for the XYZ mesons

\[ (QnQn) \]

\[ (QQ)(nn) \]

\[ (Qn)(nQ) \]

Central

\[ M_{QQ} + M_{nn} \leq M_{Qn} + M_{nQ} \]

\[ \uparrow \alpha_{Qn} \]

\[ (Qn)(nQ) \]

\[ (QQ)(nn) \]

\[ (bnbn) \]

\[ L=0, S=1, C=+1, P=+1, I=0 \]

\[ (bnbn) \]

\[ L=0, S=1, C=+1, P=+1, I=1 \]
In multiquark studies, both CE and CM effects have to be included, color may generate conflicts between the preferred configurations.

All-heavy tetraquarks are unstable in naive CE models.

\((QqQq)\) might have some opportunities of metastability below the MM threshold, which is extremely important for the existence of the \(X(3872)\) in the charm sector.

\((QQqq)\) are definitively the best candidates for stable multiquark states due to the striking \(M/m\) dependence in the CE limit. Besides, for non-identical quarks and antiquarks, string potentials offer good opportunities.

Hidden heavy flavor pentaquarks are predicted in the chromomagnetic limit due to hidden-color components dynamics.

Hidden flavor components (unquenching the quark model) offer a possible explanation of new experimental data and old problems in the meson and baryon spectra. There is not a proliferation of multiquarks, they are very rare.

We have presented a plausible mechanism explaining the origin of the XYZ mesons: based on coupled-channel effects.

We do not find evidence for charged and bottom partners of the \(X(3872)\). To answer this question is a keypoint to advance in the study of hadron spectroscopy.
\[ \Delta = E[(Qn)(nQ)] - E[(QQ)(nn)] \]

**X axis**

- \( \Delta = 0 \) \( \Rightarrow \) \( E[(Qn)(nQ)] = E[(QQ)(nn)] \) : Degeneracy
- \( \Delta > 0 \) \( \Rightarrow \) \( E[(Qn)(nQ)] > E[(QQ)(nn)] \) : Normal ordering
- \( \Delta < 0 \) \( \Rightarrow \) \( E[(Qn)(nQ)] < E[(QQ)(nn)] \) : Reversed levels

**Y axis**

\[ E_{K=22}[(QnQn)]/[E(M_1)+E(M_2)] \]

- \( \Delta = 0 \) \( \Rightarrow \) \( E[(QnQn)] = [E(M_1)+E(M_2)] \) : Threshold
- \( \Delta > 0 \) \( \Rightarrow \) \( E[(QnQn)] > [E(M_1)+E(M_2)] \) : Continuum state
- \( \Delta < 0 \) \( \Rightarrow \) \( E[(QnQn)] < [E(M_1)+E(M_2)] \) : Bound state

\[ L=0, S=1, C=+1, P=+1, I=0 \]
Chromoelectric central potential:

\[ V(r) = -\frac{4}{3} \frac{\alpha_s(r)}{r} + br \]

Chromomagnetic spin-spin correction:

\[ \frac{4\alpha_s(r)}{3m_i m_j} \left\{ \frac{8\pi}{3} \left( \vec{S}_i \cdot \vec{S}_j \delta^3(\vec{r}_{ij}) \right) + \frac{1}{r_{ij}^3} \left[ \frac{3\vec{S}_i \cdot \vec{r}_{ij} \vec{S}_j \cdot \vec{r}_{ij} - \vec{S}_i \cdot \vec{S}_j}{r_{ij}^2} \right] \right\} \]

\[ 1S \quad 1^3S_1 J/\psi \]

\[ 1^1S_0 \eta_c \]

Chromomagnetic spin-orbit term:

\[ H_{ij}^{s.o. (cm)} = \frac{4\alpha_s(r)}{3r_{ij}^3} \left( \frac{1}{m_i} + \frac{1}{m_j} \right) \left( \frac{\vec{S}_i}{m_i} + \frac{\vec{S}_j}{m_j} \right) \cdot \vec{L} \]

\[ H_{ij}^{s.o. (tp)} = -\frac{1}{2r_{ij}} \frac{\partial V(r)}{\partial r_{ij}} \left( \frac{\vec{S}_i}{m_i^2} + \frac{\vec{S}_j}{m_j^2} \right) \cdot \vec{L} \]

\[ 1P \quad \chi_2(1^3P_2) \]

\[ \chi_1(1^3P_1) \]

\[ \chi_0(1^3P_0) \]

---

**Table: Predictions**

<table>
<thead>
<tr>
<th>Multiplet</th>
<th>State</th>
<th>Expt.</th>
<th>Input (NR)</th>
<th>Theor.</th>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>NR</td>
</tr>
<tr>
<td>1S</td>
<td>( J/\psi(1^3S_1) )</td>
<td>3096.87 ± 0.04</td>
<td>3097</td>
<td>3090</td>
</tr>
<tr>
<td></td>
<td>( \eta_c(1^1S_0) )</td>
<td>2979.2 ± 1.3</td>
<td>2979</td>
<td>2982</td>
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<tr>
<td>2S</td>
<td>( \psi(2^3S_1) )</td>
<td>3685.96 ± 0.09</td>
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<td>3672</td>
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<td>( \eta_c(2^1S_0) )</td>
<td>3637.7 ± 4.4</td>
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<tr>
<td>3S</td>
<td>( \psi(3^3S_1) )</td>
<td>4040 ± 10</td>
<td>4040</td>
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<tr>
<td>4S</td>
<td>( \psi(4^3S_1) )</td>
<td>4415 ± 6</td>
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<td>( \eta_c(4^1S_0) )</td>
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<td>1P</td>
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<td>( \chi_1(1^3P_1) )</td>
<td>3510.51 ± 0.12</td>
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<tr>
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<td>( \chi_0(1^3P_0) )</td>
<td>3415.3 ± 0.4</td>
<td>3415</td>
<td>3424</td>
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<td></td>
<td>( h_c(1^1P_1) )</td>
<td>see text</td>
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<td>3972</td>
<td>3979</td>
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<td>( \chi_1(2^3P_1) )</td>
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<td>( h_c(2^1P_1) )</td>
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<td>1D</td>
<td>( \psi_2(1^3D_3) )</td>
<td>3806</td>
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<td>3806</td>
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<tr>
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<td>3800</td>
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<td>( \psi_1(1^3D_1) )</td>
<td>3785</td>
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<tr>
<td></td>
<td>( \eta_{c2}(1^1D_2) )</td>
<td>3799</td>
<td>3837</td>
<td>3799</td>
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<td>2D</td>
<td>( \psi_2(2^3D_3) )</td>
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<td>4217</td>
<td>4167</td>
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<td>( \psi_2(2^3D_2) )</td>
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<td>( \psi_2(2^3D_1) )</td>
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<td>( \eta_{c2}(2^1D_2) )</td>
<td>4158</td>
<td>4208</td>
<td>4158</td>
</tr>
</tbody>
</table>
Symmetry breaking

QM \Rightarrow \text{Min} \ (p^2 + x^2 + \lambda x) < \text{Min} \ (p^2 + x^2) \Rightarrow \text{Min} \ (H_{\text{even}} + H_{\text{odd}}) < \text{Min} \ (H_{\text{even}})

Min \ (M^+ M^+ m^- m^-) < Min \ (\mu^+ \mu^+ \mu^- \mu^-), \ 2\mu^{-1} = M^{-1} + m^{-1}

- They have the same threshold

\[ \frac{\bar{p}_1^2}{2M} + \frac{\bar{p}_2^2}{2M} + \frac{\bar{p}_3^2}{2m} + \frac{\bar{p}_4^2}{2m} + V = \left[ \sum_i \frac{\bar{p}_i^2}{2\mu} + V \right] + \left( \frac{1}{4M} - \frac{1}{4m} \right) [\bar{p}_1^2 + \bar{p}_2^2 - \bar{p}_3^2 - \bar{p}_4^2] \]

\Rightarrow \text{Min} \ (H_{\text{C-even}} + H_{\text{C-odd}}) < \text{Min} \ (H_{\text{C-even}})

\Rightarrow \ H_2 \text{ is more stable than } Ps_2

Breaking particle symmetry

- Thus \ (M^+ m^+ M^- m^-) more stable than \ (\mu^+ \mu^+ \mu^- \mu^-) ????

\[ \frac{\bar{p}_1^2}{2M} + \frac{\bar{p}_2^2}{2m} + \frac{\bar{p}_3^2}{2M} + \frac{\bar{p}_4^2}{2m} + V = \left[ \sum_i \frac{\bar{p}_i^2}{2\mu} + V \right] + \left( \frac{1}{4M} - \frac{1}{4m} \right) [\bar{p}_1^2 + \bar{p}_3^2 - \bar{p}_2^2 - \bar{p}_4^2] \]

\Rightarrow \ \text{Symmetry breaking benefits more to } (M^+ M^-) + (m^+ m^-)

\Rightarrow \ \text{One may expect some kind of metastability below } (M^+ m^-) + (m^+ M^-)

In short: \ (Un)favorable symmetry breaking can (spoil)generate stability
Symmetry breaking

✓ Equal-mass case: Asymmetry in the potential energy

\[
H = \sum_i \frac{p_i^2}{2m} + \sum_{i<j} g_{ij} V(r_{ij}) , \quad \sum_{i<j} g_{ij} = 2
\]

⇒ Equal g_{ij} gives the highest energy
⇒ The broader the distribution of g_{ij} gives the lower energy

<table>
<thead>
<tr>
<th>(abcd)</th>
<th>V(r_{ij})</th>
<th>g_{ij}</th>
<th>q</th>
<th>Δg</th>
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<tbody>
<tr>
<td>Threshold (1,3)+(2,4)</td>
<td>-1/r_{ij}, r_{ij}</td>
<td>{0,0,1,0,1,0}</td>
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<td>0.52</td>
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<tr>
<td>Ps_2</td>
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<td>1.03</td>
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<td>{1/2,1/2,1/4,1/41/4,1/4}</td>
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<tr>
<td></td>
<td>M⟩ ≡ [(qq)_6(qq)_6]</td>
<td>-1/r_{ij}, r_{ij}</td>
<td>{-1/4,-1/4,5/8,5/8,5/8,5/8}</td>
<td>1/3</td>
</tr>
</tbody>
</table>

⇒ Ps_2 favored compared to quark models
⇒ Mixing effects do not help much