Scattering studies with the DATURA beam telescope

Probing Highland & applications

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The DATURA beam telescope

- Used extensively in sensor R&D
- Located at DESY TB hall 21
- Six Mimosa26 sensors
- NI-based DAQ system
- EUDET Trigger Logic Unit
  - Input: 4 scintillators
  - Output: Trigger to DAQ systems
- Connect multiple DUTs or additional reference sensors
- Available: x-y-phi stage for Device Under Test (DUT)

Goal:
→ Measure electron tracks passing DUTs
→ Perform track fits for multitude of studies
Mimosa 26 pixel sensors

- AMS 350 nm CMOS
- Dimensions:
  - 10 mm x 20 mm / 50 um
  - 18.4 um x 18.4 um
  - 1152 x 576 pixels
- HR epitaxial layer of 20 um thickness
- Binary read-out (no charge information)
- Theoretical binary resolution: 5.3 um
- Measured intrinsic resolution: 3.24 um * (mean CS = 3.28 $\mu\text{m}$)
- Protected by 25 um Kapton on each side
- Material budget of sensor plus Kapton: $\varepsilon_{M26} = x / X_0 = 7.5e^{-4}$
Measurement geometry

- Plane spacing $dz = 20\ \text{mm}$, $dz_{\text{SUT}} = 15\ \text{mm}$
- Total material budget telescope: $\varepsilon(M26 + \text{air}) = 4.8 \times 10^{-3}$
Data analysis flow

Analysis done with EUTelescope *

• Conversion of Mimosa26 raw data to LCIO format
• Hot pixel search
• Cluster formation, remove clusters with hot pixels
• Construct triplets for up- and downstream plane
• Isolation cut on triplets
• Match up- and downstream triplets in the centre → six-tuple from physical track
• Feed six-tuple to Millepede for alignment

* http://eutelescope.web.cern.ch/
General Broken Lines

- GBL track model allows for kinks at scatterers
- Calculating corrections to an initial simple seed track
- Perform $\chi^2$ minimisation to find track parameters
- Simple track model: no bremsstrahlung, no non-Gaussian tails, no non-linear effects
- Inputs: Measurement + error, geometry, scattering estimate
- Outputs: residuals, residual width, kinks, track resolution


Multiple scattering

- Variance predicted by Highland at a single scatterer:
  \[ \Theta^2_0 = \left( \frac{13.6 \text{ MeV}}{\beta cp} \cdot z \right)^2 \cdot \varepsilon \cdot (1 + 0.038 \cdot \ln(\varepsilon))^2 \]

- For a composition of scatterers
  \[ \varepsilon = \sum \varepsilon_i \]
  Highland predicts variance after \textit{last} scatterer

- For individual scatterer within composition:
  \[ \Theta^2_{0,i} \equiv \frac{\varepsilon_i}{\varepsilon} \cdot \Theta^2_0 = \left( \frac{13.6 \text{ MeV}}{\beta cp} \cdot z \right)^2 \cdot \varepsilon_i \cdot (1 + 0.038 \cdot \ln(\varepsilon))^2 \]
Unbiased kinks

- Last slides: scatterers of *known* material budget → constrained kink angle in $\chi^2$ (biased)

- Goal: kink for *unknown* scatterer (unbiased) → introduce free local parameters in track model → dedicated track model for unbiased kinks
Targets and measurements

- Homogeneous targets
  - aluminium sheets of thicknesses:
    - empty, 25, 50, 100, 200, 1000 um
  - energies: 1 – 5 GeV

- Inhomogeneous target
  - coaxial connector

- Excellent angular resolution
  → measure kink angle precisely
  → calculate material budget

- Excellent position resolution
  → measure impact position on sample
  → position-resolved material budget
Kink angles

- Kink angle distribution for various energies

→ Measurable difference for 100 um aluminium

→ Clear energy-dependence

→ Large statistics → populated tails
Kink angles II

- Measurement of aluminium includes “empty measurement” → apply correction

- Results within ~10% of Highland prediction for 1 – 3 GeV

- Energy-dependence to be understood (work in progress)

- Method yields reasonable kink estimates
2D analysis – events

- Map of homogeneous sample 1 GeV, 1 mm alu

→ Beam spot at centre of sample
2D analysis – mean

- 2D map of homogeneous sample 1 GeV, 1 mm alu
- Mean values of bins mostly ~zero
2D analysis – width

- 2D map of homogeneous sample 1 GeV, 1 mm alu
- Mean values of bins mostly ~zero
- Widths of bins show slight trend from left → right
2D analysis – width II

- 2D map of homogeneous sample 1 GeV, 1 mm alu
- Mean values of bins mostly ~zero
- Widths of bins show slight trend from left → right

Projection on X

3 % over 18 mm
2D analysis – width II

- 2D map of homogeneous sample 1 GeV, 1 mm alu
- Mean values of bins mostly ~zero
- Widths of bins show slight trend from left → right
Inhomogeneous sample

- Can we resolve structured samples?
  → electron-illuminated a coax connector

![Coax Connector Diagram]

- Chrome plated brass
- Nickel plated brass
- Teflon
Inhomogeneous sample

- Can we resolve structured samples?
  → electron-illuminated a coax connector
Inhomogeneous sample

- Can we resolve structured samples? → electron-illuminated a coax connector

Reconstruct tomographic image
Inhomogeneous sample

• Can we resolve structured samples?
  → electron-illumined a coax connector

Reconstruct tomographic image

Paul Schuetze, Tuesday 9:36, R3 Medical Imaging, security and other applications
Potential

- For known thickness, homogeneous sample
  - measure kink width
  → for known E → calculate $X_0$ from Highland
  → for known $X_0$ → measure beam E to %-level
  - measure position-resolved kink width
  → probe homogeneity of measurement for corrections

- For inhomogeneous sample
  - measure position-resolved kink width
  → material budget map

- For sample with `internal structure'
  - measure position-resolved kink width
  - repeat for different sample angles
    → Tomography

\[ \Theta_0^2 = \left( \frac{13.6 \text{ MeV}}{\beta c p} \cdot z \right)^2 \cdot c \cdot (1 + 0.038 \cdot \ln (c))^2 \]
Conclusion

- Performed scattering study with DATURA beam telescope
- Precise tool:
  - few um track resolution
  - few tens urad angular resolution of kinks
- Implemented GBL tracking with dedicated track model for unbiased kink angle
- Measure position-resolved material budget
- Large range of applications

**Dedicated BTTB workshop for test beam users**

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6th Beam Telescopes and Test Beams Workshop

Zurich, Switzerland

January 16th – 19th, 2018

cern egroup:
BeamTelescopesAndTestBeams-Announcements@cern.ch

bttb-ws@desy.de
Back-up
A word on thick scatterers

- Assume a non-homogeneous scatterer along z
- Describe with three parameters:
  length $s$, mean $\bar{s}$, variance $\Delta s^2$

$$\theta^2 = \sum_i \theta_i^2, \quad \bar{s} = \frac{1}{\theta^2} \sum_i s_i \theta_i^2, \quad \Delta s^2 = \frac{1}{\theta^2} \sum_i (s_i - \bar{s})^2 \theta_i^2$$

- Find a toy scatterer composed of two thin scatterers resembling the thick scatterer; $s_1$, $s_2$, $\Theta_1$, $\Theta_2$.
  - e.g. for homogeneous scatterer
    - $s_1 = \bar{s} - d/\sqrt{12}$
    - $s_2 = \bar{s} + d/\sqrt{12}$
    - $\Theta_1 = \Theta_2 = \Theta/2$
  - e.g. for inhomogeneous scatterer

$$s_1 = s_0, \quad s_2 = \bar{s} + \frac{\Delta s^2}{\bar{s} - s_1}, \quad \theta_1^2 = \theta^2 \frac{\Delta s^2}{\Delta s^2 + (\bar{s} - s_1)^2}, \quad \theta_2^2 = \theta^2 \frac{(\bar{s} - s_1)^2}{\Delta s^2 + (\bar{s} - s_1)^2}$$
Offline analysis and reconstruction

- EUTelescope is based on the ILCSoft framework:
  - generic data model (LCIO)
  - geometry description (GEAR)
  - central event processor (Marlin)
- Marlin allows for modular composition of analysis chain
- Build-in job submission framework
- Steering of analysis via XML files loaded at runtime
- EUTelescope provides processors for full track reco including:
  - Alignment with Millepede-II
  - General Broken Lines track fitter
  - many more
Biased residuals III

→ Average intrinsic resolution: \( \sigma_{M26} = (3.24 \pm 0.09) \mu m \)
Multiple scattering

- Average deflection predicted by Highland

\[ \Theta_0 = \frac{13.6 \text{ MeV}}{\beta_c p} \cdot z\sqrt{\varepsilon} \cdot (1 + 0.038 \ln(\varepsilon)) \]

- Literature offers other models, too, HL most popular
- Distribution assumed to be Gaussian centrally
- Non-Gaussian tails
- MS and intrinsic resolution defines *track resolution*, i.e. uncertainty in space of a track along the track
Biased residuals and pulls

- Biased residual = (measurement – fit) including all 6 planes
- Normalise residual by expected residual width

\[ \text{pull}_b \equiv p_b = \frac{r_b}{\sqrt{\sigma^2_{\text{int}} - \sigma^2_{t,b}}} \]

- Pull is N(0,1) if
  - estimate for intrinsic resolution matches true value
  - material budget and scattering is accurately described

→ **Iterate** track fit with updated \( \sigma_{\text{int}} \) and \( \sigma_{t,b} \) using the pull width
→ \( \text{pull}_b \rightarrow N(0,1) \) and \( \sigma_{\text{int}} \) converges against true value
→ Use results from narrow and wide set-up for cross validation
Quoted is a Gaussian width (95%), but actually RMS is within 1% of this value
Kink angles II

- Measurement of aluminium includes “empty measurement” → apply correction
Track cleaning

- Cut on tracks: prob < 0.01 (0.1) for 20 mm (150 mm) - model less valid for larger amount of material budget
- Use robust statistics (down-weighting of out-layers) only if you don't have enough data (and if you know what you are doing)
- If track collection is not cleaned, “bad” tracks affect the measured intr. reso.
Prob biased vs unbiased

GBL fit probability

<table>
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<th>gblprb</th>
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<tbody>
<tr>
<td>Entries</td>
</tr>
<tr>
<td>Mean</td>
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<tr>
<td>RMS</td>
</tr>
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</table>
Residuals

- Residual = Measurement - Fit
- Biased (use all measurements) and unbiased (leave one out) tracks are different!

- Use track fits for residual and pull distribution

\[
\begin{align*}
    r_{b}^{2}(z) &= \sigma_{\text{int}}^{2}(z) - \sigma_{t,b}^{2}(z) \\
    r_{u}^{2}(z) &= \sigma_{\text{int}}^{2}(z) + \sigma_{t,u}^{2}(z)
\end{align*}
\]
Pulls

- Normalise residual by expected residual width

\[
pull_b \equiv p_b = \frac{r_b}{\sqrt{\sigma_{int}^2 - \sigma_{t,b}^2}}
\]

- Pull is \(N(0,1)\) if
  - estimate for intrinsic resolution matches true value
  - material budget and scattering is accurately described

→ **Iterate** track fit with updated \(\sigma_{int}\) using the pull width
→ \(\pull_b \rightarrow N(0,1)\) and \(\sigma_{int}\) converges against true value
→ Use results from narrow and wide set-up for cross validation
Pulls and track resolution

- Normalise residual by expected residual width

\[
pull_u \equiv \rho_u = \frac{r_u}{\sqrt{\sigma^2_{\text{int}} + \sigma^2_{t,u}}}
\]

Pull is $N(0,1)$ if
- estimate for intrinsic resolution matches true value
- material budget is accurate
- deflection due to multiple Coulomb scattering is accurately described

→ repeat track fit varying $\sigma_{\text{int}}$ by pull width
→ pull $\rightarrow N(0,1)$ and $\sigma_{\text{int}}$ converges
Pulls and track resolution II

- One example of an iteration step:

\[ \begin{array}{c|c}
\text{BIASED} & \text{UNBIASED} \\
\end{array} \]

→ Increase \( \sigma_{\text{int}} \) by 6%, re-fit the tracks
Pulls and track resolution III

- Residual estimate as function of intr. resolution:

- Systematics affect unbiased track reso. relatively equal

- But $\sigma_{t,b} < \sigma_{t,u}$

  $\rightarrow$ absolute error smaller

  $\rightarrow$ what about the residual?
Intrinsic resolution

- The iterative method converges i.e. estimator for $\sigma_{\text{int}}$ converges against the true value
- We find energy independent value of
  \[ \sigma_{\text{int}} = 3.24 \pm 0.5\% \text{ (stat.)} \pm 3\% \text{ (syst.)} \] (cf. last slide)

- Control sys. uncert. further by comparing set-ups
- Increases for lower thresholds (more noise hits)
- Increases for higher thresholds (smaller clusters)
- Optimum is 5 – 6, probably a tune of 5.5
Systematics

- Estimate systematic uncertainties of intrinsic resolution based on the input uncertainties

<table>
<thead>
<tr>
<th></th>
<th>$E$ ± 5%</th>
<th>$\Theta_0$ ± 3%</th>
<th>$\sigma_{\sigma_{\text{int}}}$ in %</th>
<th>$\sqrt{\sum(x_i)^2}$</th>
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<tr>
<td>6 GeV</td>
<td></td>
<td></td>
<td>per plane fit range all planes</td>
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<td>20 mm</td>
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<td>+1.76</td>
<td>1.57</td>
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<td>+0.21</td>
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<td>-1.27</td>
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<td>150 mm</td>
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<td>+0.65</td>
<td>+0.23</td>
<td>3.1</td>
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<td>3.1</td>
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<td>+10.2</td>
<td>+8.0</td>
<td>0.82</td>
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<td>+15.7</td>
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<td>-23.9</td>
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<td>2 GeV</td>
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</table>
Threshold dependency

Towards higher threshold:
- → cut signal
- → smaller clustersize
- → worse resolution

Towards lower threshold:
- → more noise hits
- → worse resolution

→ Optimum at threshold 5 to 6
Track resolution predictions

- Using 6 planes, assuming DUT in the centre
Track resolution predictions

- Using 6 planes, assuming DUT in the centre

\[ \text{→ } dz_{\text{DUT}} \text{ as small as possible} \]

\[ \text{→ Thick DUT: use wide set-up} \]

\[ \text{Thin DUT: use narrow set-up} \]
Track resolution predictions

- Using 6 planes, assuming DUT in the centre
- Wide set-up offers superior track resolution with thicker DUTs and vice versa.
- Intersection is function of material budget
  → Optimise resolution prior to your test beam
Looking even closer ...

Fold occurrence into one pixel for intra-pixel studies

→ Density of recon. track position is non-uniform, it depends on cluster size
→ Populated areas differ in size
→ Resolution is CS dependent
   → Calculate differential intrinsic resolution
Looking even closer …

Fold occurrence into one pixel for intra-pixel studies

→ Density of recon. track position is non-uniform, it depends on cluster size
→ Populated areas differ in size
→ Resolution is CS dependent
→ Calculate differential intrinsic resolution
Looking even closer ...

CS 1  GBL in-pixel occurrence of CS1

CS 2  GBL in-pixel occurrence of CS2

CS 3  GBL in-pixel occurrence of CS3

GBL intra-pixel occurrence of CS1-4
CS-dependent quantities

- Repeat iterative pull method for each cluster size → differential intrinsic resolution

<table>
<thead>
<tr>
<th>Cluster Size</th>
<th>Resolution (μm)</th>
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<tbody>
<tr>
<td>CS1</td>
<td>3.60</td>
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<tr>
<td>CS2</td>
<td>3.16</td>
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<tr>
<td>CS3</td>
<td>2.86</td>
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<tr>
<td>CS4</td>
<td>3.40</td>
</tr>
<tr>
<td>CS5</td>
<td>2.53</td>
</tr>
<tr>
<td>CS6</td>
<td>2.70</td>
</tr>
<tr>
<td>CS&gt;6</td>
<td>4.17</td>
</tr>
</tbody>
</table>

- Resulting $\sigma_x$ vs $x$ within a pixel per cluster size:

\[
\langle \sigma_x \rangle [\mu m]
\]

\[
\begin{align*}
x & [\mu m] \\
0 & 2 \\
4 & 6 \\
8 & 10 \\
10 & 12 \\
12 & 14 \\
14 & 16 \\
16 & 18 \\
\end{align*}
\]

CS = 1
CS-dependent quantities

- Repeat iterative pull method differentially for each clustersize → differential intrinsic resolution
- Resulting $\sigma_x$ vs $x$ within a pixel per clustersize:

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</table>
CS-dependent quantities

- Repeat iterative pull method differentially for each clustersize → differential intrinsic resolution

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<tr>
<td>CS&gt;6</td>
<td>4.17</td>
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</tbody>
</table>

- Resulting $\sigma_x$ vs $x$ within a pixel per clustersize:

![](image.png)

CS = 3
CS-dependent quantities

- Repeat iterative pull method for each cluster size → differential intrinsic resolution

CS1: 3.60 μm  
CS2: 3.16 μm  
CS3: 2.86 μm  
CS4: 3.40 μm  
CS5: 2.53 μm  
CS6: 2.70 μm  
CS>6: 4.17 μm

- Resulting $\sigma_x$ vs x within a pixel per cluster size:
Horizontal beam spread

- After spectral magnet

The deflection angle $\theta$ for particles with an energy between 2.95 and 3.05 GeV

<table>
<thead>
<tr>
<th>Theta_singleE</th>
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<tbody>
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<td>Entries</td>
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<td>Sigma</td>
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