GLV - formalism

Chen Lin

Institute of Particle Physics
CCNU, Wuhan

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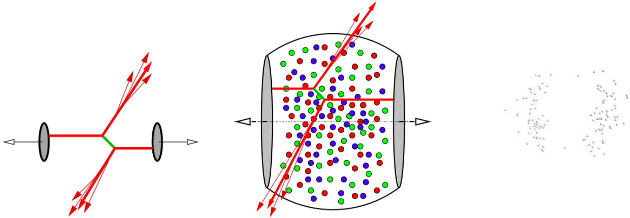
Outline

Introduction

In an ultra relativistic heavy ion collision, partons produced from hard collision processes travel through a dense matter previously predicted as Quark-Gluon Plasma (QGP) and losses energy in the surrounding medium. Although collisional energy loss were predicted to be moderate ($dE_{coll}/dx \ll 1$ GeV/fm) [1] , radiative energy loss were expected to be significantly large ($dE_{rad}/dx \gg$ few GeV/fm) [2] This radiative energy loss phenomena is called "Jet Quenching", and it is one of the signatures for QGP production in RHI-Collisions observed in RHIC(BNL) and LHC(CERN).



[1] M.H.Thoma and M. Gyulassy, Nucl. Phys. B **351** (1991) 491

[2] J.F. Gunion and G. Bertsch, Phys. Rev. D **25** (1982) 746

Introduction

Jet energy loss schemes available in the market [3]:

- Modeling of medium
  - Static scattering centers (BDMPS, Zakharov, GLV, ASW)
  - Thermally equilibrated, perturbative medium (AMY)
  - Nuclear medium with short correlation length (Higher Twist)
- Resummation schemes
  - Sum over all possible soft interactions (BDMPS, AMY)
  - Path integral of hard parton propagation (Zakharov, ASW)
  - Opacity expansion (GLV)
- Evolution scheme (multiple emissions)
  - Poisson ansatz(BDMPS, GLV, ASW)
  - Rate equations (AMY)
  - Modified DGLAP equations (Higher Twist)

[3] G.Y. Qin, pres. *Jet Quenching in Nuclear Collisions* (2010)

Basic idea (radiative energy loss)

Radiative energy loss is given as sum over all radiated gluon energy [4]

$$\Delta E_{rad} = \int d\omega \frac{dN}{d\omega}\omega \; \theta(E-\omega) \tag{1}$$

The gluon number distribution is proportional to the phase space integral on the radiation amplitude squared

$$\frac{dN}{d^2\vec{k}_\perp dy d\omega} = \frac{1}{d_R}\sum_{color} |\mathcal{R}_{rad}|^2 \tag{2}$$

Where it can be extracted from the scattering amplitude of every diagram

$$\mathcal{M}_{rad} = \mathcal{M}_{ela} \; i \; \mathcal{R}_{rad} \tag{3}$$

[4] photon radiation is ignored since QED coupling is much less giving rise to negligible cross-section.

Gyulassy, Lévai, Vitev model

One begins with the Static Color Screening Yukawa Potential from the GW [5] [6] model:

$$V_n = 2\pi\delta(q_n^0)V(\vec{q_n})e^{-i\vec{q_n}\cdot\vec{x_n}}T_{an}(R)\otimes T_{an}(n), \quad V(\vec{q_n}) = \frac{4\pi\alpha_s}{\vec{q_n}^{\,2}+\mu^2} \tag{4}$$

For small transverse momentum transfer, the elastic cross-section between the jet and the target parton is:

$$\frac{d\sigma_{el}}{d^2\vec{q}_\perp} = \frac{C_R C_2(T)}{d_A}\frac{|v(\vec{q}_\perp)|^2}{(2\pi)^2} \tag{5}$$

Where the color bookkeeping techniques are described by:

$$\begin{aligned} Tr(T_a(R)T_b(R)) &= \delta_{ab}C_R D_R/D_A & (6)\\ D_A &= N_c^2-1 & (7)\\ Tr(T_a(i)T_b(j)) &= \delta_{ab}\delta_{ij}C_2(i)D_i/D_A & (8)\\ Tr(T_a(R)) &= 0 & (9) \end{aligned}$$

[5] M. Gyulassy, X. N. Wang, Nucl. Phys. B **420** 583 (1994)

[6] X. N. Wang, M. Gyulassy, M. Plümer, Phys. Rev. D **51** 3436 (1995)

Feynman rules (example)

From the given potential above, one has the Hamiltonian:

$$
\begin{aligned}
\mathcal{H}_I(t) &= \int dt\ \mathcal{L} = \int dt(I - V) \\
&\int d^3\vec{x} \sum_{i=1}^{N} V(\vec{x} - \vec{x}_i) T_a(i) \phi^\dagger(\vec{x}, t) T_a(R) \hat{D}(t) \phi(\vec{x}, t) \quad (10)
\end{aligned}
$$

where $\hat{D}(t) = i\overleftrightarrow{\partial_t}$, $A\overleftrightarrow{\partial_t}B = A(\partial_t B) - (\partial_t A)B$. Consider a simple scattering diagram:

$$
\begin{aligned}
i\mathcal{M} &= \left\langle p' \left| (-i) T \exp\left[\int_{-T}^{T} dt \mathcal{H}_I(t)\right] \right| p \right\rangle \\
&= (-i)(E_p + E_{p'}) \times \bar{u}(p') u(p) \\
&\quad \times \int d^4x \sum_i V(\vec{x} - \vec{x}_i) \cdot e^{i(p'-p)\cdot\vec{x}_i} T_a(i) \otimes T_a(R) \quad (11)
\end{aligned}
$$

Feynman rules (cont.)

Then one has:

$$
\begin{aligned}
i\mathcal{M} &= (-i)(E_p + E_{p'}) \times \bar{u}(p')u(p) \\
&\quad \times (2\pi)\delta(E_{p'} - E_p) \sum_i \tilde{V}(\vec{q}) \cdot e^{-i\vec{q}\cdot\vec{x}_i} T_a(i) \otimes T_a(R) \qquad (12)
\end{aligned}
$$

Separating the potential and the Dirac spinors, one can see that the Feynman rule for scattering vertex is given as $(-i)(E_p + E_{p'})$.
One can derive the following Feynman rules from the given potential accordingly:

$$
\begin{aligned}
\text{Quark scattering vertex} &= -i(2p^0 - q^0) \qquad (13) \\
\text{Quark propagator} &= i/(p^2 + i\varepsilon) \qquad (14) \\
\text{Gluon propagator} &= -ig^{\mu\nu}/(k^2 + i\varepsilon) \qquad (15) \\
\text{Emission vertex} &= ig_s(2p + k)^\mu \cdot \epsilon_\mu T_c \qquad (16)
\end{aligned}
$$

Assumptions and Approximations

- Targets are distributed with density:

$$\bar{\rho}(z_1, \cdots, z_n) = \prod_{j=1}^{N} \frac{\theta(\Delta z_j)}{L_e(N)} e^{-\frac{\Delta z_j}{L_e(N)}} \tag{17}$$

- The opacity defined by:

$$\bar{n} = \frac{L}{\lambda} = \frac{N\sigma_{el}}{A_\perp} \tag{18}$$

- Energy of jet is high compare to potential screening scale:

$$E^+ \approx 2E \gg \mu \tag{19}$$

- distance between source and scattering center are larger than interaction range:

$$z_i - z_0 \gg \frac{1}{\mu} \tag{20}$$

- One defines the jet with momentum p:

$$M_0 = ie^{ip\cdot x_0} J(p) \times \mathbb{1} \tag{21}$$

Light-cone kinematics

One can define the following light-cone coordinates:

$$
\begin{aligned}
k &= \left[2\omega, \frac{\vec{k}_\perp^2}{2\omega}, \vec{k}_\perp\right] &(22)\\
\epsilon(k) &= \left[0, \frac{\vec{k}_\perp \cdot \vec{\epsilon}_\perp}{\omega}, \vec{\epsilon}_\perp\right] &(23)\\
p &= \left[2(E-\omega), \frac{(\vec{Q}_\perp - \vec{k}_\perp)^2}{2(E-\omega)}, (\vec{Q}_\perp - \vec{k}_\perp)^2\right] &(24)\\
Q &= \left[0, \frac{\vec{k}_\perp^2}{2\omega}(\frac{\omega}{E-\omega}+1), \vec{Q}_\perp\right] &(25)
\end{aligned}
$$

and their corresponding dot products. We can use the assumption: $(E \gg \omega \gg Q)$ to simplify our calculation.

Gluon tree matrices

It is best to work out the matrices below to simplify the calculations follow:

$$
\begin{aligned}
\Gamma^{\alpha}(k;q_1) &= \Gamma^{\alpha 0\gamma}(k;q_1)\cdot\epsilon_{\gamma}(k)\\
\Gamma_1 &= (2p+k-q_1)_{\alpha}\Gamma^{\alpha}(k;q_1)\\
\Lambda_1 &= \Gamma_1(ig_s t_a)T_b(1)\\
&= -2g_s[2E\vec{\epsilon}_{\perp}\cdot(\vec{k}_{\perp}-\vec{q}_{1\perp})+\omega(\vec{\epsilon}_{\perp}\cdot\vec{q}_{1\perp})][c,b]T_b(1)
\end{aligned}
$$

$$
\begin{aligned}
\Gamma^{\alpha}(k;q_1;q_2) &= \Gamma^{\alpha 0\mu}(k-q_2;q_1)g_{\mu\nu}\Gamma^{\nu 0\gamma}(k;q_2)\cdot\epsilon_{\gamma}(k)\\
\Gamma_{12} &= (2p+k-q_1-q_2)_{\alpha}\Gamma^{\alpha}(k;q_1;q_2)\\
\Lambda_{12} &= \Gamma_{12}(ig_s t_a)T_{a_1}(1)T_{a_2}(2)\\
&= -ig_s 4\omega[2E\vec{\epsilon}_{\perp}\cdot(\vec{k}_{\perp}-\vec{q}_{1\perp}-\vec{q}_{2\perp})+\omega\vec{\epsilon}_{\perp}\cdot(\vec{q}_{1\perp}+2\vec{q}_{2\perp})]\\
&\quad\times[[c,a_2],a_1]T_{a_1}(1)T_{a_2}(2)
\end{aligned}
$$

Diagrammatic approach

$$dN \propto \int Tr|t^0 \cdot \mathcal{R}^{(0)} + t^1 \cdot \mathcal{R}^{(1)} + t^2 \cdot \mathcal{R}^{(2)} + \cdots|^2 \qquad (26)$$

where $t^n \propto T_a^n$, with $Tr(T_a^{odd}) = 0$, and $Tr(T_a^{even}) = \left(\frac{C_2(T)d_T}{d_A}\right)^{n/2}$. For opacity order $= n/2$, one has the following opacity expansions:

$$dN^{(0)} \propto \int Tr|\mathcal{R}^{(0)}|^2 \qquad (27)$$

$$\begin{aligned} dN^{(1)} &\propto \int Tr|\mathcal{R}^{(0)} + t^1 \cdot \mathcal{R}^{(1)} + t^2 \cdot \mathcal{R}^{(2)}|^2 \\ &= dN^{(0)} + \left(\frac{C_2(T)d_T}{d_A}\right) \times \int Tr[\mathcal{R}^{(1)2} + 2Re(\mathcal{R}^{(0)\dagger}\mathcal{R}^{(2)})] \qquad (28) \end{aligned}$$

$$\begin{aligned} dN^{(2)} &\propto \int Tr|\mathcal{R}^{(0)} + t^1 \cdot \mathcal{R}^{(1)} + t^2 \cdot \mathcal{R}^{(2)} + t^3 \cdot \mathcal{R}^{(3)} + t^4 \cdot \mathcal{R}^{(4)}|^2 \\ &= dN^{(1)} + \left(\frac{C_2(T)d_T}{d_A}\right)^2 \times \int Tr[\mathcal{R}^{(2)2} + 2Re(\mathcal{R}^{(1)\dagger}\mathcal{R}^{(3)} + \mathcal{R}^{(0)\dagger}\mathcal{R}^{(4)})] \qquad (29) \end{aligned}$$

Self-Quenching

The scattering matrix:

$$\begin{aligned}\mathcal{M}^{(0)}_{rad} &= iJ(p+k)e^{i(p+k)\cdot x_0}(ig_s)(2p+k)_\mu \epsilon^\mu i\Delta(p+k)c \\ &= iJ(p+k)e^{i(p+k)\cdot x_0}(-2g_s)\frac{E-\omega}{E}\frac{\vec{\epsilon}_\perp \cdot \vec{k}_\perp}{\vec{k}^2_\perp}c \\ &= \mathcal{M}^{(0)}_{el} i\mathcal{R}^{(0)}_{rad} \qquad (30)\end{aligned}$$

The radiation amplitude squared:

$$\begin{aligned}\frac{1}{d_R}\sum_i\sum_j |\mathcal{R}^{(0)}_{rad}|^2 &= \frac{1}{d_R} Tr|\mathcal{R}^{(0)}_{rad}|^2 \\ &= 16\pi\alpha_s C_R\left(\frac{E-\omega}{E}\right)^2\frac{1}{\vec{k}^2_\perp} \qquad (31)\end{aligned}$$

Absorption (optional)

In the QGP heat bath, the jet parton can either emit or absorb a gluon, one will take into account the Bose enhancement and absorption factor $N(|\vec{k}|) = (e^{|\vec{k}|/T} - 1)^{-1}$ in the phase space integration. [7]

$$d\Phi = \frac{d^3|\vec{k}|}{(2\pi)^3}\frac{1}{2|\vec{k}|}\begin{cases} 1 + N(|\vec{k}|) & : \text{if } k^0 = |\vec{k}| \text{ for emission} \\ N(|\vec{k}|) & : \text{if } k^0 = |\vec{k}| \text{ for absorption} \end{cases} \tag{32}$$

$$dN = \frac{1}{d_R} Tr|\mathcal{R}^{(0)}|^2 d\Phi \tag{33}$$

Then:

$$\frac{dN_1^{(0)}}{dyd\omega} = \frac{2C_R\alpha_s}{\pi}\int\frac{d|\vec{k}_\perp|}{|\vec{k}_\perp|}\left(\frac{E-\omega}{E}\right)^2 \times\left[(1+N(|\vec{k}|))\delta(\omega-|\vec{k}|) + N(|\vec{k}|)\delta(\omega+|\vec{k}|)\right] \tag{34}$$

[7] E. Wang, X.N. Wang, Phys. Rev. Lett. **87**, 142301 (2001)

Divergence and virtual correction

Virtual processes:

$$\frac{dN_2^{(0)}}{dyd\omega} = -\frac{2\alpha_s C_R}{\pi}\int \frac{d|\vec{k}_\perp|}{|\vec{k}_\perp|}\left(\frac{E^2-|\vec{k}|^2}{E^2}\right)[1+2N]\delta(\omega) \tag{35}$$

Then, the gluon spectrum is:

$$\begin{aligned}\frac{dN^{(0)}}{dyd\omega} &= \frac{dN_1^{(0)}+dN_2^{(0)}}{dyd\omega} \\ &= \frac{2\alpha_s C_R}{\pi}\int \frac{d|\vec{k}|}{|\vec{k}|}\left[\left(\frac{E-\omega}{E}\right)^2(1+N)\delta(\omega-|\vec{k}|)\right. \\ &\quad + \left(\frac{E-\omega}{E}\right)^2 N\delta(\omega+|\vec{k}|) \\ &\quad \left. - \left(\frac{E^2-|\vec{k}|^2}{E^2}\right)^2(1+2N)\delta(\omega)\right]\end{aligned} \tag{36}$$

Using the gluon number spectrum, one can then calculate the energy loss:

$$\Delta E_{rad}^{(0)} = \int dy d\omega \frac{dN^{(0)}}{dy d\omega} \omega \theta(E-\omega) \tag{37}$$

Note that $\int d\omega \delta(\omega)\omega = 0$, which means that the virtual gluon does not contribute to the total energy loss. Then:

$$\begin{aligned} \Delta E_{rad}^{(0)} &= \frac{2\alpha_s C_R}{\pi} E \int \frac{d|\vec{k}_\perp|}{|\vec{k}_\perp|} \int dx [(1-x)^2 \theta(1-x) \\ &\quad -4xN\theta(1-x) - (1+x)^2 N\theta(x-1)] \end{aligned} \tag{38}$$

Energy loss (Analysis)

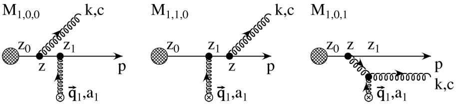
We can write the energy loss in three terms:

$$\Delta E_{rad}^{(0)} = \Delta E_{rad}^{(0)a} - \Delta E_{rad}^{(0)b} - \Delta E_{rad}^{(0)c} \tag{39}$$

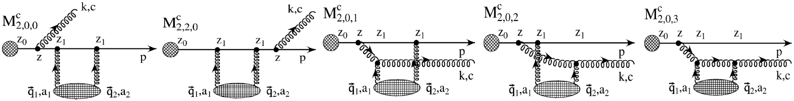
Note that $\Delta E_{rad}^{(0)a}$ is the energy loss from emission at $T = 0$. $\Delta E_{rad}^{(0)b}$ and $\Delta E_{rad}^{(0)c}$ are the energy absorption at finite temperature. looking at the ratio of $\Delta E_{rad}^{(0)b}/\Delta E_{rad}^{(0)a} = 12T/E$, which means that if $E < 12T$, anti-self-quenching happens. E.g. at SPS($T = 150MeV$), jet with $E < 1.8GeV$ will absorb energy instead of quenching. $\Delta E_{rad}^{(0)c}$ is negligible for $E \gg T$ and becomes significant when $E \ll T$. However, the zeroth order self-quenching calculations over-estimates the energy loss at high-energy collisions and thus will not be use in the future, but demonstrates how to systematically calculate jet-quenching.

First order Feynman diagrams

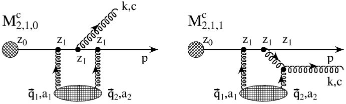
Single direct scattering:



Double Born scattering (contact limit):



No contribution:



Summation at finite temperature

When the jet parton rescatter off the target parton with static potential, the q_0 integration should be replaced by the summation at finite temperature field theory. From quantum field theory to the finite temperature, the replacement rule is the following [8]:

$$q_0 \rightarrow iv_n = i2\pi nT, \quad n = 0, \pm 1, \pm 2 \cdots \tag{40}$$

$$\int \frac{dq_0}{(2\pi)} \frac{d^3\vec{q}}{(2\pi)^3} \rightarrow iT \sum_{n=-\infty}^{\infty} \int \frac{d^3\vec{q}}{(2\pi)^3} \tag{41}$$

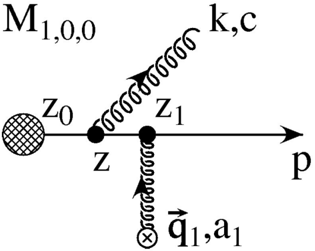
$$2\pi\delta(q_1^0 + q_2^0) \rightarrow \frac{1}{iT}\delta_{v_{n_1}+v_{n_2},0} \tag{42}$$

$$\Delta(q) = \frac{1}{q^2 - m^2} \rightarrow \frac{1}{(iv_n)^2 - \vec{q}^2 - m^2} \tag{43}$$

However, one can show that at static potential case the above replacement is not needed at finite temperature field theory for the calculation of the rescattering amplitude.

[8] C.W. Bernard PRD **9** (1974) 3312

Single Direct Rescattering - $\mathcal{M}_{100}$



The scattering amplitude:

$$
\begin{aligned}
\mathcal{M}_{rad}^{(1)a} \quad = \quad & \int \frac{d^4 q_1}{(2\pi)^4} iJ(p+k-q_1) e^{i(p+k-q_1)\cdot x_0} (-i) D(2p-q_1) \\
& \times V(q_1) e^{iq_1 \cdot x_1} (ig_s c)(2p+k-2q_1)^{\mu} \epsilon_{\mu} \\
& \times i\Delta(p-q_1) i\Delta(p+k-q_1) \qquad (44)
\end{aligned}
$$

Diagram numbering

Quantum cascading Feynman diagrams can be very complex and requires a systematic way of numbering.

$$M_{n,m,l}$$

- n - number of scattering centres(potentials).
- m - gluon radiation position(after the m^{th} scattering center).
- l - gluon, potential interaction structure.

where

$$l = \frac{1}{2^m}(\sum_{j=1}^{n} \sigma_j 2^{j-1})$$

Numerator Algebra

For calculations under the color-screening potential of the $SU(N)$ group, one requires necessary techniques for color factor manipulation as given below:

$$
\begin{aligned}
[a, b] &= if^{abc} c && (45) \\
Tr(ab) &= C(r)\delta^{ab} = C_R d_R && (46) \\
aa &= C_R \cdot \mathbb{1} && (47) \\
Tr(a) &= 0 && (48)
\end{aligned}
$$

With these identities, one can derive the following:

$$
\begin{aligned}
a_1 c c a_1 &= C_R^2 \cdot \mathbb{1} && (49) \\
a_1 c\, [a_1, c] &= -\tfrac{1}{2} C_R C_A \cdot \mathbb{1} && (50) \\
[c, a_1]\, c a_1 &= -\tfrac{1}{2} C_R C_A \cdot \mathbb{1} && (51) \\
[c, a_1]\, [a_1, c] &= C_R C_A \cdot \mathbb{1} && (52)
\end{aligned}
$$

Residue Theorem

Whenever we perform integration on a propagator, we need to find singularities on the denominator and use residue integration to get rid of the poles.

$$\oint_c f(z)dz = 2\pi i \sum_{j=1}^{k} Res_{z=z_j}\ f(z)$$

We have the following:

$$\begin{aligned}
\Delta(p-q_1) &= [(q_{1z}-\bar{q}_1)(q_{1z}-\bar{q}_2)]^{-1} \\
\Delta(p+k-q_1) &= [(q_{1z}-\bar{q}_3)(q_{1z}-\bar{q}_4)]^{-1} \\
\Delta(k-q_1) &= [(q_{1z}-\bar{q}_5)(q_{1z}-\bar{q}_6)]^{-1}
\end{aligned}$$

Where:

$$\bar{q}_1 = 2(E-\omega)+i\varepsilon \quad , \quad \bar{q}_2 = -i\varepsilon \tag{53}$$

$$\bar{q}_3 = 2E+i\varepsilon \quad , \quad \bar{q}_4 = -\omega_0 - i\varepsilon \tag{54}$$

$$\bar{q}_5 = 2\omega-\omega_1'+i\varepsilon \quad , \quad \bar{q}_6 = -\omega_0'+\omega_1'+i\varepsilon \tag{55}$$

Single Direct Rescattering(cont.) - $\mathcal{M}_{100}$

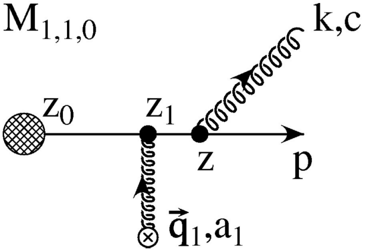
Substituting it back to the scattering amplitude, one has:

$$\begin{aligned} \mathcal{M}_{rad}^{(1)a} &= iJ(p+k)e^{i(p+k)\cdot x_0}(-i)\int \frac{d^2\vec{q}_{1\perp}}{(2\pi)^2}e^{-1\vec{q}_{1\perp}\cdot\vec{b}_1} \\ &\times V(o,\vec{q}_{1\perp})(2g_s)\frac{E-\omega}{E}\frac{\vec{\epsilon}_\perp\cdot\vec{k}_\perp}{\vec{k}_\perp^2} \\ &\times[e^{i\omega_0(z_1-z_0)}-1]T_{a_1}a_1c \end{aligned} \tag{56}$$

Thus, the radiation amplitude for $\mathcal{M}_{100}$:

$$\mathcal{R}_{rad}^{(1)a} = (-2ig_s)\frac{E-\omega}{E}\frac{\vec{\epsilon}_\perp\cdot\vec{k}_\perp}{\vec{k}_\perp^2}[e^{i\omega_0(z_1-z_0)}-1]a_1c \tag{57}$$

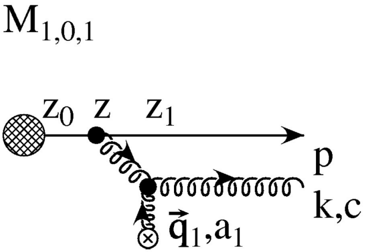
Single Direct Rescattering - $\mathcal{M}_{110}$



Similar to the the above calculation, the radiation amplitude for $\mathcal{M}_{110}$ is given as:

$$\mathcal{R}_{rad}^{(1)b} = (2ig_s)\frac{E-\omega}{E}\frac{\vec{\epsilon}_\perp \cdot \vec{k}_\perp}{\vec{k}_\perp^2} e^{i\omega_0(z_1-z_0)} c a_1 \tag{58}$$

Single Direct Rescattering - $\mathcal{M}_{101}$



And $\mathcal{M}_{101}$ is given as:

$$\mathcal{R}_{rad}^{(1)c} = (-2ig_s)\frac{E-\omega}{E}\frac{[2E\vec{\epsilon}_\perp\cdot(\vec{k}_\perp-\vec{q}_{1\perp})-\omega\vec{\epsilon}_\perp\cdot\vec{q}_{1\perp}]}{2E(\vec{k}_\perp-\vec{q}_{1\perp})^2} \times e^{i\omega_0(z_1-z_0)}(1-e^{-i\omega_1(z_1-z_0)})[c,a_1] \quad (59)$$

Single Direct Rescattering

We now add the total radiation amplitude for single direct rescattering:

$$\begin{aligned}
\mathcal{R}_{rad}^{(1)S} &= \mathcal{R}_{rad}^{(1)a} + \mathcal{R}_{rad}^{(1)b} + \mathcal{R}_{rad}^{(1)c} \\
&= (2ig_s)\frac{E-\omega}{E}[\vec{\epsilon}_\perp \cdot \vec{H}\; a_1 c + \vec{\epsilon}_\perp \cdot \vec{B}_1 e^{i\omega_0(z_1-z_0)}[c, a_1] \\
&\quad + \vec{\epsilon}_\perp \cdot \vec{C}_1 e^{i(\omega_0-\omega_1)(z_1-z_0)}[c, a_1]] \qquad (60)
\end{aligned}$$

Where we have:

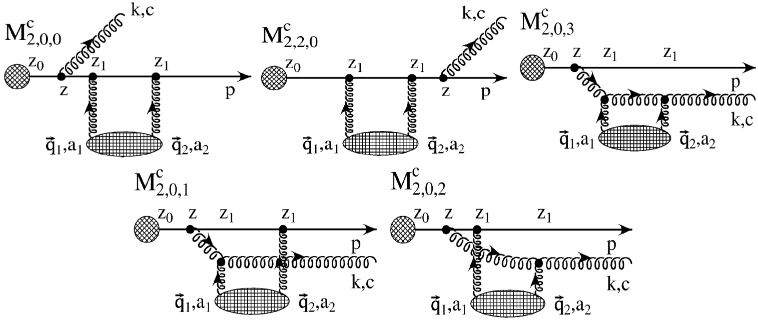
$$\vec{H} = \frac{\vec{k}_\perp}{\vec{k}_\perp^2} \qquad (61)$$

$$\vec{C}_1 = \frac{\vec{k}_\perp - \vec{q}_{1\perp}}{(\vec{k}_\perp - \vec{q}_{1\perp})^2} \qquad (62)$$

$$\vec{B}_1 = \vec{H} - \vec{C}_1 \qquad (63)$$

Double Born "virtual" interaction

The double Born "virtual" interaction corresponds to the contact-limit of double direct rescattering. Double Born scattering (contact limit):

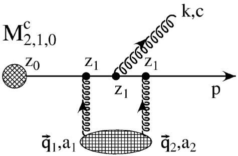


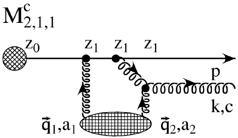
Double Born "virtual" interaction(cont.)

The radiation amplitude for double Born interactions is given as:

$$\mathcal{R}_{rad}^{(1)D} = (2ig_s)c\left(\frac{E-\omega}{E}\right)e^{i\omega_0(z_1-z_0)}\vec{\epsilon}_\perp \cdot \left[-\frac{C_R+C_A}{2}\vec{H}e^{-i\omega_0(z_1-z_0)} + \frac{C_A}{2}\vec{B}_1 + \frac{C_A}{2}\vec{C}_1 e^{-i\omega_1(z_1-z_0)}\right] \quad (64)$$

Note that the following diagrams gives no contribution under the contact-limit:





Energy loss - First order

We again perform phase space integration on the radiation amplitude to get the gluon number distribution for self-quenching, single direct rescattering and double Born interaction. One will see that the gluon spectrum is infrared divergent, we can introduce the virtual gluon exchange processes to cancel the divergences, but for the calculation of energy loss, the contribution for virtual gluons are not included. One will then arrive at the following energy loss expression:

$$\begin{aligned}\Delta E^{(1)}_{rad} &= \frac{2\alpha_s}{\pi}\frac{C_R C_A C_2(T)}{d_A}\frac{N}{A_\perp}\int dz_1\rho(z_1)\int\frac{d^2\vec{q}_{1\perp}}{(2\pi)^2}V^2(0,\vec{q}_{1\perp})\\ &\times\int d|\vec{k}_\perp||\vec{k}_\perp|(-2\vec{B}_1\cdot\vec{C}_1)\int\frac{dx}{x}(xE)\\ &\times\Big\{(1-x)^2 Re(1-e^{i\omega_{11}z_{10}})[1+N]\theta(1-x)\\ &\quad -(1+x)^2 Re(1-e^{i\omega_{12}z_{10}})N\theta(1+x)\Big\} \qquad (65)\end{aligned}$$

Energy loss - First order(cont.)

We can separate it in three terms as before:

$$\Delta E^{(1)} = \Delta E^{(1)a} - \Delta E^{(1)b} - \Delta E^{(1)c} \tag{66}$$

where

$$\begin{aligned}\Delta E^{(1)a} &= \frac{4\alpha_s C_R}{\pi^2}\mu_{eff}^2 \frac{L}{\lambda_g} E \\ &\times \int_0^1 dx(1-x)^2 \int_{|\vec{k}_\perp|_{min}}^{|\vec{k}_\perp|_{max}} d|\vec{k}_\perp| \int_0^{|\vec{q}_\perp|_{max}} d|\vec{q}_\perp| \frac{|\vec{q}_\perp|^2}{(\vec{q}_\perp + \mu^2)^2} \\ &\times \int_0^{2\pi} d\psi \frac{\cos\psi(|\vec{k}_\perp| - |\vec{q}_\perp|)^2 L^2}{16E^2x^2(1-x)^2 + (|\vec{k}_\perp| - |\vec{q}_\perp|)^4 L^2}\end{aligned} \tag{67}$$

with

$$(|\vec{k}_\perp| - |\vec{q}_\perp|)^2 = |\vec{k}_\perp|^2 - 2|\vec{k}_\perp||\vec{q}_\perp|\cos\psi + |\vec{q}_\perp|^2$$

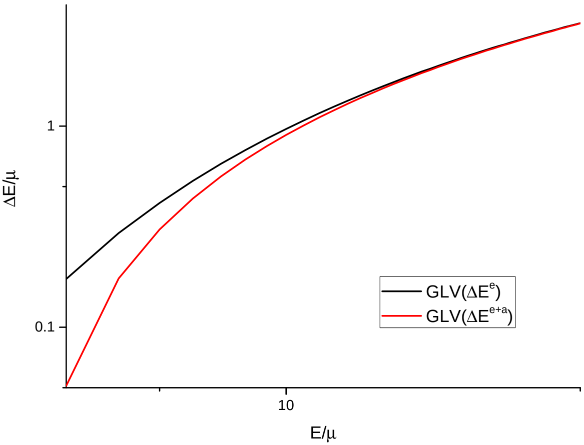
Energy loss - First order(cont.)

$$\Delta E^{(1)b} = \frac{4\alpha_s C_R}{\pi^2}\mu_{eff}^2 \frac{L}{\lambda_g} E \int_0^1 dx \int_{|\vec{k}_\perp|_{min}}^{|\vec{k}_\perp|_{max}} d|\vec{k}_\perp| \int_0^{|\vec{q}_\perp|_{max}} d|\vec{q}_\perp| \frac{|\vec{q}_\perp|^2}{(\vec{q}_\perp + \mu^2)^2}$$
$$\times \int_0^{2\pi} d\psi \frac{\cos\psi(|\vec{k}_\perp| - |\vec{q}_\perp|)^2 L^2}{e^{x\beta E} - 1}$$
$$\times \left[\frac{(1+x)^2}{16E^2x^2(1+x)^2 + (|\vec{k}_\perp| - |\vec{q}_\perp|)^4 L^2} - \frac{(1-x)^2}{16E^2x^2(1-x)^2 + (|\vec{k}_\perp| - |\vec{q}_\perp|)^4 L^2}\right] \quad (68)$$

$$\Delta E^{(1)c} = \frac{4\alpha_s C_R}{\pi^2}\mu_{eff}^2 \frac{L}{\lambda_g} E \int_1^\infty dx \int_{|\vec{k}_\perp|_{min}}^{|\vec{k}_\perp|_{max}} d|\vec{k}_\perp| \int_0^{|\vec{q}_\perp|_{max}} d|\vec{q}_\perp| \frac{|\vec{q}_\perp|^2}{(\vec{q}_\perp + \mu^2)^2}$$
$$\times \int_0^{2\pi} d\psi \frac{(1+x)^2}{e^{x\beta E} - 1} \left[\frac{\cos\psi(|\vec{k}_\perp| - |\vec{q}_\perp|)^2 L^2}{16E^2x^2(1+x)^2 + (|\vec{k}_\perp| - |\vec{q}_\perp|)^4 L^2}\right] \quad (69)$$

Note that we began with $E \gg \omega$ assumption to take absorption into account. For pure emission, as in the GLV(2001) paper, $\Delta E^{(1)a}$ is suffice.

Numerical Results



at fixed temperature $T = 500 MeV$ for opacity $L/\lambda_g = 3$, $\alpha_s = 0.3$, $\mu = 0.5 GeV$, $\lambda_g = 5 GeV^{-1}$.

Recursive approach (Reaction operator)

- What happens when we have n scattering centres?
- How do we calculate upto n^{th} order opacity expansion?

We begin by defining the following:

$$Q_i = \sum_{k=i}^{n} q_k = (q_i + q_{i+1} + \cdots + q_{n-1} + q_n) \tag{70}$$

For simplification, absorption is not considered in this method. i.e. $E \gg \omega$.

Sequential Multiple Scattering

Consider a quark jet undergoing n scatterings, the amplitude:

$$\mathcal{M}_n^0 = ie^{i(p-Q_1)\cdot x_0} J(p-Q_1) \prod_i \int \frac{d^4 q_i}{(2\pi)^4} (-i)(2p^0 - Q_i^0) \times \frac{1}{(q-Q_i)^2 + i\varepsilon} 2\pi\delta(q_i^0) V(\vec{q}_i) e^{iq_i\cdot x_i} \times Col(0) \tag{71}$$

where the color factor $Col(0)$ is:

$$Col(0) = a_n a_{n-1} \cdots a_1 T(a_n) T(a_{n-1}) \cdots T(a_1) \tag{72}$$

We now proceed to work on the propagator.

Propagator Integration (Residue Theorem)

$$I_{iz}^0 = \int \frac{dq_{iz}}{2\pi} \frac{1}{(p - Q_i)^2 + i\varepsilon} V(\vec{q}_i) e^{-iq_{iz}(z_i - z_0)} \quad (73)$$

Using the residue theorem and assume small momentum transfer, we have:

$$q_{iz} = 2E + i\varepsilon' \quad , \quad I_{iz}^0 = \frac{-i}{2E} V(0, \vec{q}_{i\perp}) \quad (74)$$

We then have:

$$\mathcal{M}_n^0 = iJ(p)e^{ip\cdot x_0} \prod_i^n (-i) \int \frac{d^2\vec{q}_{i\perp}}{(2\pi)^2} V(0, \vec{q}_{i\perp}) e^{-i\vec{q}_{i\perp}(\vec{x}_{i\perp} - \vec{x}_{0\perp})} Col(0) \quad (75)$$

Induced Gluon Emission

Consider a gluon has been radiated between the q_j and q_{j+1} potential. The amplitude:

$$\begin{aligned}
\mathcal{M}_n^j &= iJ(p-Q_1+k)e^{i(p-Q_1+k)\cdot x_0} \\
&\times \left[\prod_{i=1}^{j}\int \frac{d^4q_i}{(2\pi)^4}(-i)2p^0\frac{i}{(p-Q_i+k)^2+i\varepsilon}2\pi\delta(\vec{q}_i)V(\vec{q}_i)e^{iq_i\cdot x_i}\right] \\
&\times \left[\int \frac{d^4q_{j+1}}{(2\pi)^4}(-i)2p^0\frac{i}{(p-Q_{j+1})^2+i\varepsilon}(2ig_s\epsilon\cdot p)\right. \\
&\quad \left.\times \frac{i}{(p-Q_{j+1}+k)^2+i\varepsilon}2\pi\delta(q_{j+1}^0)V(\vec{q}_{j+1})e^{iq_{j+1}\cdot x_{j+1}}\right] \\
&\times \left[\prod_{i=j+2}^{n}\int \frac{d^4q_i}{(2\pi)^4}(-i)2p^0\frac{i}{(p-Q_i)^2+i\varepsilon}2\pi\delta(q_i^0)V(\vec{q}_i)e^{iq_i\cdot x_i}\right] \qquad (76)
\end{aligned}$$

Propagator Integration

Use residue theorem to evaluate the denominators:

$$q_{iz} = -\omega_0 - Q_{i+1,z} - i\varepsilon \tag{77}$$

We can use partial fraction to separate the denominators:

$$2\frac{i}{(p-Q_{j+1})^2 + i\varepsilon}\frac{i}{(p-Q_{j+1}+k)^2 + i\varepsilon} \approx \frac{1}{k\cdot p}\left[\frac{i}{(p-Q_{j+1})^2 + i\varepsilon} - \frac{i}{(p-Q_{j+1}+k)^2 + i\varepsilon}\right] \tag{78}$$

Which gives:

$$\Delta(p-Q_i+k)_{1\to j}\Delta(p-Q_i)_{j+1\to n} - \Delta(p-Q_i+k)_{1\to j+1}\Delta(p-Q_i)_{j+2\to n} \tag{79}$$

Then, the integrals:

$$I_{iz}^{10} = \frac{-i}{2E} \quad , \quad I_{iz}^{11} = \frac{-i}{2E-\omega_0} \approx \frac{-i}{2E} \tag{80}$$

Phase factor analysis

From experience:

$$\prod_{i=1}^{n} e^{-iq_{iz}(z_i - z_0)} = e^{-iq_{1z}(z_1 - z_0)} e^{-iq_{2z}(z_2 - z_0)} \cdots e^{-iq_{nz}(z_n - z_0)} \tag{81}$$

Then, we will use the following lemma [9] to rewrite the phase factor:

$$\sum_{i=1}^{n} q_i(z_i - z_0) = \sum_{j=1}^{n} Q_j(z_j - z_{j-1}) \tag{82}$$

Substitute in the residues, and quite a lot of cancellation, the phase factor for the two propagators are given as:

$$e^{i\omega_0(z_{j+1} - z_0)} - e^{i\omega_0(z_j - z_0)} \tag{83}$$

[9] We can proof this by mathematical induction

Induced Gluon Emission(cont.)

Taking the initial phase into account, we have the overall phase factor:

$$e^{i\omega_0 z_{j+1}} - e^{i\omega_0 z_j} \tag{84}$$

The color matrix is still a simple expression:

$$Col(1) = a_n \cdots a_{j+1}\ c\ a_j \cdots a_1\, T_{a_n} \cdots T_{a_1} \tag{85}$$

Substituting everything in, we have:

$$\begin{aligned} \mathcal{M}_n^{1j} &= iJ(p)e^{ip\cdot x_0} \prod_{i=1}^{n}(-i) \int \frac{d^2\vec{q}_{i\perp}}{(2\pi)^2} V(\vec{q}_i) e^{-i\vec{q}_{i\perp}\cdot\vec{b}_i} \\ &\quad \times \frac{2g_s \vec{\epsilon}_\perp \cdot \vec{k}_\perp}{\vec{k}_\perp^2} \left[e^{i\omega_0 z_{j+1}} - e^{i\omega_0 z_j}\right] Col(1) \end{aligned} \tag{86}$$

Gluon radiation with Quantum Cascading

We now consider the full diagram where the jet and the radiated gluon both undergoes multiple rescattering. We need an effective parameter to correctly describe the scattering centres. We define the following:

$$\vec{\sigma} = (\sigma_1 = 0, \cdots, \sigma_m = 0, \sigma_{m+1}, \cdots, \sigma_n) \tag{87}$$

Where σ_i takes the value of 0 if the potential is interacting on the jet, and the value 1 if the potential is interacting on the radiated gluon[10]. and Assume the gluon is radiated after the m^{th} and before the $(m+1)^{th}$ potential. Thus, the value of σ_i from $i = 1$ to $i = m$ is 0.

[10] a binary representation

Propagator Integration

We now separate the diagram into two parts, jet line and the gluon line. Since we did the quark propagator analysis before, we will focus here the gluon propagator of the j^{th} potential:

$$-i\Delta(k - q_j - \sum_{i=j+1}^{n} \sigma^i q_i) \tag{88}$$

Rewriting the denominator and find the pole:

$$q_{jz} = -k^- - \sigma^i q_{iz} + \frac{(\vec{k}_\perp - \sigma^i \vec{q}_{i\perp} - \vec{q}_{j\perp})^2}{2\omega} - i\varepsilon \tag{89}$$

and the residue:

$$Res = \frac{1}{k^+ + k^- + \frac{(\vec{k}_\perp - \sigma^i \vec{q}_{i\perp} - \vec{q}_{j\perp})^2}{\omega}} \approx \frac{1}{k^+} \tag{90}$$

Gluon momentum analysis

We begin by writing out the momentum part (without the color factor), i.e. Γ, and notice the pattern when considering multiple potentials.

$$(2p + k - q_m)_\alpha \Gamma^\alpha_m = 4E\vec{\epsilon}_\perp \cdot [\vec{k}_\perp - \vec{q}_{m\perp}] + \mathcal{O}(\vec{k}^2_\perp) \quad (91)$$

$$(2p + k - q_m - q_n)_\alpha \Gamma^\alpha_{nm} = 8E\omega\vec{\epsilon}_\perp \cdot [\vec{k}_\perp - \vec{q}_{m\perp} - \vec{q}_{n\perp}] + \mathcal{O}(\vec{k}^2_\perp) \quad (92)$$

$$(2p + k - q_l - q_m - q_n)_\alpha \Gamma^\alpha_{nml} = 16E\omega^2\vec{\epsilon}_\perp \cdot [\vec{k}_\perp - \vec{q}_{l\perp} - \vec{q}_{m\perp} - \vec{q}_{n\perp}] + \mathcal{O}(\vec{k}^2_\perp) \quad (93)$$

We can generalize this expression to n_g potentials:

$$(2p + k - \sum_{i=1}^{n_g} q_i)_\alpha \Gamma^\alpha_{1,\cdots,n_g} = 2E^+(k^+)^{n_g-1}\vec{\epsilon}_\perp \cdot (\vec{k}_\perp - \sum^{n_g} \vec{q}_{i\perp}) + \mathcal{O}(\vec{k}^2_\perp) \quad (94)$$

Note that this is simplified under the small momentum transfer approximation.

Phase factor Analysis

The initial charge phase factor is given as:

$$e^{i\omega_0 z_0} = e^{i\omega_0' z_0} \cdot e^{i\sum_{i=1}^{n}\sigma^i q_i^- \cdot z_0} \tag{95}$$

where $\omega_0' = \frac{(\vec{k}_\perp - \sum_{i=1}^{n}\sigma^i \vec{q}_{i\perp})^2}{2\omega}$. Phase factor for quark propagator:

$$\begin{aligned} & e^{i\omega_0' z_0}[e^{i\omega_0'(z_{j+1}-z_0)} - e^{i\omega_0'(z_j - z_0)}] \\ = \quad & e^{iz_{j+1}(\vec{k}_\perp - \sum_{i=1}^{n}\sigma^i \vec{q}_{i\perp})^2/2\omega} - e^{iz_j(\vec{k}_\perp - \sum_{i=1}^{n}\sigma^i \vec{q}_{i\perp})^2/2\omega} \end{aligned} \tag{96}$$

where the gluon is radiated after the j^{th} and before the $(j+1)^{th}$ potential.

Phase factor Analysis(cont.)

The gluon cascade phase factor is given as:

$$\prod_{j=1}^{n_g} e^{-iq_{jz}(z_j-z_0)} = e^{\sum_{j=1}^{n} i\sigma^j[(\vec{k}_\perp-\sigma^i\vec{q}_{i\perp})^2-(\vec{k}_\perp-\sigma^i\vec{q}_{i\perp}-\vec{q}_{j\perp})^2]\frac{(z_j-z_0)}{2\omega}} \tag{97}$$

Notice that if we isolate the z_0 part out of this exponential, it cancels with the second term from the initial charge phase factor. Thus, we only have the z_j part left. We now have the total phase factor:

$$\left[e^{iz_{m+1}(\vec{k}_\perp-\sum_{i=1}^{n}\sigma^i\vec{q}_{i\perp})^2/2\omega} - e^{iz_m(\vec{k}_\perp-\sum_{i=1}^{n}\sigma^i\vec{q}_{i\perp})^2/2\omega}\right]$$
$$\times \quad \prod_{i=1}^{n} e^{i\sigma^i z_i[(\vec{k}_\perp-\sum_{l=i+1}^{n}\sigma^l\vec{q}_{l\perp})^2-(\vec{k}_\perp-\sum_{l=i+1}^{n}\sigma^l\vec{q}_{l\perp}-\vec{q}_{i\perp})^2]/2\omega} \tag{98}$$

Colour Factor Analysis

We label the color factor as follows:

- a_1 to a_m for the potentials before the radiated gluon.
- c_1 to c_{n_g} for the potentials that interacts with the gluon.
- b_1 to b_{n-m-n_g} for the potentials that did not interact with the gluon after a_m.

colour factor for gluon cascade:

$$f^{d_{n_g} c_{n_g} c} f^{d_{n_g-1} c_{n_g-1} d_{n_g}} \dots f^{d_2 c_2 d_3} f^{d_1 c_1 d_2} t_a = (-i)^{n_g} [\cdots [c, c_{n_g}], c_{n_g-1}], \cdots, c_1] \tag{99}$$

Therefore, the total color factor:

$$Col(2) = (i)^{n_g} b_{n-m-n_g} \cdots b_1 [\cdots [c, c_{n_g}], \cdots, c_1] a_m \cdots a_1 T_{a_n} \cdots T_{a_1} \tag{100}$$

Gluon radiation with Quantum Cascading

We now include the rest of the factors:

- gluon propagator $(1/k^+)^{n_g}$
- cascade vertices $2E^+(k^+)^{n_g-1}\vec{\epsilon}_\perp \cdot (\vec{k}_\perp - \sum_{i=1}^{n_g} \vec{q}_{i\perp})$
- gluon radiation coupling $ig_s E(k^+)^{-1}\vec{\epsilon}_\perp \cdot (\vec{k}_\perp - \sum_{i=1}^{n_g} \vec{q}_{i\perp})$
- partial fraction factor $\frac{1}{k\cdot p} = \frac{k^+ E^{-1}}{(\vec{k}_\perp - \sum_{i=1}^{n_g} \vec{q}_{i\perp})^2}$

Counting all the i and include everything, we have:

$$
\begin{aligned}
\mathcal{M}_{nl}^m &= iJ(p)e^{ip\cdot x_0}\prod_{i=1}^{n}(-i)\int \frac{d^2\vec{q}_{i\perp}}{(2\pi)^2} e^{-i\vec{q}_{i\perp}\cdot \vec{b}_i} V(0,\vec{q}_{i\perp}) \\
&\times 2g_s \frac{\vec{\epsilon}_\perp \sqrt{2\omega\omega_0'}}{\omega_0'}\left[e^{iz_{m+1}\omega_0'} - e^{iz_m\omega_0'}\right] \\
&\times \prod_{i=1}^{n} e^{i\sigma^i z_i[\omega_{0,i+1}' - \omega_{0,i}']} \times Col(2) \qquad (101)
\end{aligned}
$$

Recursive Analysis

We then analyse what happens when we add potential under different situations. Some of the parameters that needs to be define are:

- we define the following quantities to simplify the expression:

$$H = \frac{\vec{k}}{\vec{k}^2} \ , \ C_{(i_1,\cdots,i_m)} = \frac{(\vec{k}_\perp - \sum_{j=1}^{m} \vec{q}_{ij\perp})}{(\vec{k}_\perp - \sum_{j=1}^{m} \vec{q}_{ij\perp})^2}$$

$$B_i = H - C_i \ , \ B_{(i_1,\cdots,i_m)(j_1,\cdots,j_m)} = C_{(i_1,\cdots,i_m)} - C_{(j_1,\cdots,j_m)}$$

- define $\hat{D}_n$ as the operator that adds a direct interaction on diagram with $n-1$ potentials.
- define $\hat{V}_n$ as the operator that adds a virtual interaction on diagram with $n-1$ potentials.
- the position of the last potential that interacts with the quark line before the n^{th} potential is denoted as z_f.
- define the amplitude of the diagram with $n-1$ potentials as $\mathcal{A}_{1\cdots i_{n-1}}(x, k, c)$.

Interaction added on quark line

- gluon radiate before z_f:
  - phase: no change.
  - color: $Col(A^0) \to a_n Col(A^0)$
- gluon radiate between z_f and z_n:
  - phase: $-e^{i\omega_0 z_f} \to e^{i\omega_0 z_n} - e^{i\omega_0 z_f}$
  - color: $c(a_f \cdots a_1) \to a_n c(a_f \cdots a_1)$
- gluon radiate after z_n:
  - phase: $-e^{i\omega_0 z_n}$
  - color: $ca_n(a_f \cdots a_1)$

Then, the amplitude when adding a direct interaction on quark line becomes:

$$\mathcal{A}^{(q)} = a_n \mathcal{A} + e^{i\omega_0 z_0}[c, a_n](a_f \cdots a_1) \tag{102}$$

Interaction added on gluon line

- gluon radiate before z_f:
  - phase: phase$(k_\perp - q_{n\perp}, \sigma_i) \times e^{i(\omega_0-\omega_m)z_n}(e^{i\omega_m z_{m+1}} - e^{i\omega_m z_m})$
  - color: $\mathcal{A}^0(c) \to \mathcal{A}^0 c[c, a_n]$
- gluon radiate between z_f and z_n that interacts with q_n:
  - phase: $-e^{i\omega_0 z_f} \to e^{i\omega_0 z_n} - e^{i(\omega_0-\omega_n)z_n} e^{i\omega_n z_f}$
  - color: $c(a_f \cdots a_1) \to [c, a_n] a_f \cdots a_1$

We then have the amplitude when adding a direct interaction on gluon line, with some simplification with the previous result:

$$\mathcal{A}^{(q)} = a_n \mathcal{A} + e^{i\omega_0 z_0}[c, a_n]\mathcal{A}' \tag{103}$$

We then have ($\frac{1}{2}$ comes from unitarity):

$$\begin{aligned}\hat{D}_n \mathcal{A}(\vec{k}_\perp, c) \quad = \quad & a_n \mathcal{A}(\vec{k}_\perp, c) + e^{1(\omega_0-\omega_n)z_n}\mathcal{A}(\vec{k}_\perp - \vec{q}_{n\perp}, [c, a_n]) \\ & -(1/2)^{N_v)} B_n e^{i\omega_0 z_n}[c, a_n] T_{el}(\mathcal{A})\end{aligned} \tag{104}$$

Contact Interaction

With the same analysis as above, we have the $\hat{V}_n$ operator:

$$\begin{aligned}\hat{V}_n\mathcal{A}(\vec{k}_\perp, c) &= -\frac{1}{2}(C_R + C_A)\mathcal{A}(\vec{k}_\perp, c) \\ &\quad - e^{i(\omega_0-\omega_n)z_n} a_n \mathcal{A}(\vec{k}_\perp - \vec{q}_{n\perp}, [c, a_n]) \\ &\quad - (-\frac{1}{2})^{N_v} \frac{C_A}{2} B_n e^{i\omega_0 z_n} c T_{el}(\mathcal{A}) \end{aligned} \tag{105}$$

We can simplify these two equations by defining $\hat{S}$ and $\hat{B}$. Then:

$$\hat{D}_n\mathcal{A}_{i_1,\cdots,i_{n-1}}(\vec{k}, c) \equiv (a_n + \hat{S}_n + \hat{B}_n) \tag{106}$$

$$\begin{aligned}\hat{V}_n\mathcal{A}_{i_1,\cdots,i_{n-1}}(\vec{k}, c) &\equiv -\frac{1}{2}(C_A + C_R) - a_n\hat{S}_n - a_n\hat{B}_n \\ &= -a_n\hat{D}_n - \frac{1}{2}(C_A - C_R)\end{aligned} \tag{107}$$

Reaction Operator

After all the derivations, we now arrive at the radiation probability:

$$P_n = \bar{\mathcal{A}}^{i_1,\cdots,i_{n-1}} \hat{R}_n \mathcal{A}_{i_1,\cdots,i_{n-1}} \tag{108}$$

where the reaction operator is given as:

$$\begin{aligned} \hat{R}_n &= \hat{D}_n^\dagger \hat{D}_n + \hat{V}_n + \hat{V}_n^\dagger \\ &= (\hat{S}_n + \hat{B}_n)^\dagger (\hat{S}_n + \hat{B}_n) - C_A \end{aligned} \tag{109}$$

Recursive approach (Gluon spectrum)

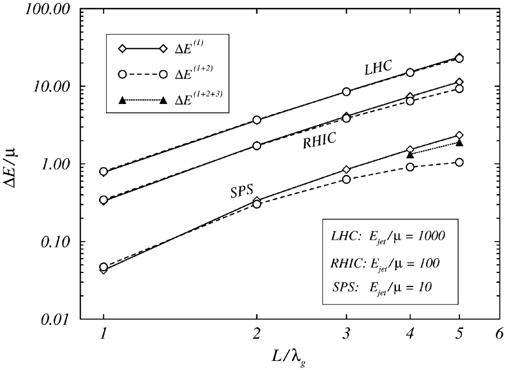
The gluon radiation probability:

$$P_n = -2C_R C_A^n \; Re \sum_{i=1}^{n} \left[\prod_{j=i+1}^{n} (e^{iq_{j\perp}\cdot\vec{b}} - 1) \right] \hat{B}_i \cdot e^{iq_{i\perp}\cdot\vec{b}} e^{-i\omega_0 z_i} \times \left[\prod_{m=1}^{i-1} (e^{i(\omega_0-\omega_m)z_m} e^{iq_{m\perp}\cdot\vec{b}} - 1) \right] \hat{H}(e^{i\omega_0 z_1} - e^{i\omega_0 z_0}) \tag{110}$$

Which gives the gluon number distribution as:

$$x\frac{dN^{(n)}}{dx d^2 k_\perp} = \frac{C_R \alpha_s}{\pi^2} \frac{1}{n!} \left(\frac{L}{\lambda_g(1)} \right)^n \int \prod_{i=1}^{n} \left(d^2 q_{i\perp} \left(\frac{\lambda_g(1)}{\lambda_g(i)} \right) [\bar{v}_i^2(q_{i\perp}) - \delta^2(q_{i\perp})] \right) \times \left(-2\hat{C}_{(1,\cdots,n)} \cdot \sum_{m=1}^{n} \hat{B}_{(m+1,\cdots,n)(m,\cdots,n)} \times \left[\cos\left(\sum_{k=2}^{m} \omega_{(k,\cdots,n)} \Delta z_k \right) - \cos\left(\sum_{k=1}^{m} \omega_{(k,\cdots,n)} \Delta z_k \right) \right] \right) \tag{111}$$

Energy loss as a function of opacity



A generalized induced energy loss equation: [11]

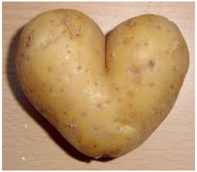
$$\Delta E^{(ind)} = \frac{C_R \alpha_s}{N(E)} \frac{L^2 \mu^2}{\lambda_g} \log \frac{E}{\mu} \tag{112}$$

[11] M. Gyulassy, P. Lévai, I. Vitev, Nucl. Phys. B **594** (2001) 371

Summary

- The GLV model was explained and a set of assumptions and approximations were given, with a set of Feynman rules and light-cone kinematics.
- The graphical approach was described.
- The reaction operator approach was described.
- during the derivation, we take note that absorption can be taken into account by including finite temperature field theory parameters, while introducing the temperature dependence to the equation.
- necessary high energy limit was made during the derivation to simplify the expression.

Thank you!



Here's a potato.