High Precision Study of Higgs Physics

Gexing Li

Institute of High Energy Physics·CAS

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In collaboration with

Zhao Li, Yan Wang

Content

Introduction

1. Thanks to the clean environment and the high luminosity in CEPC, many important properties of the Higgs boson can be measured to extremely high precisions.

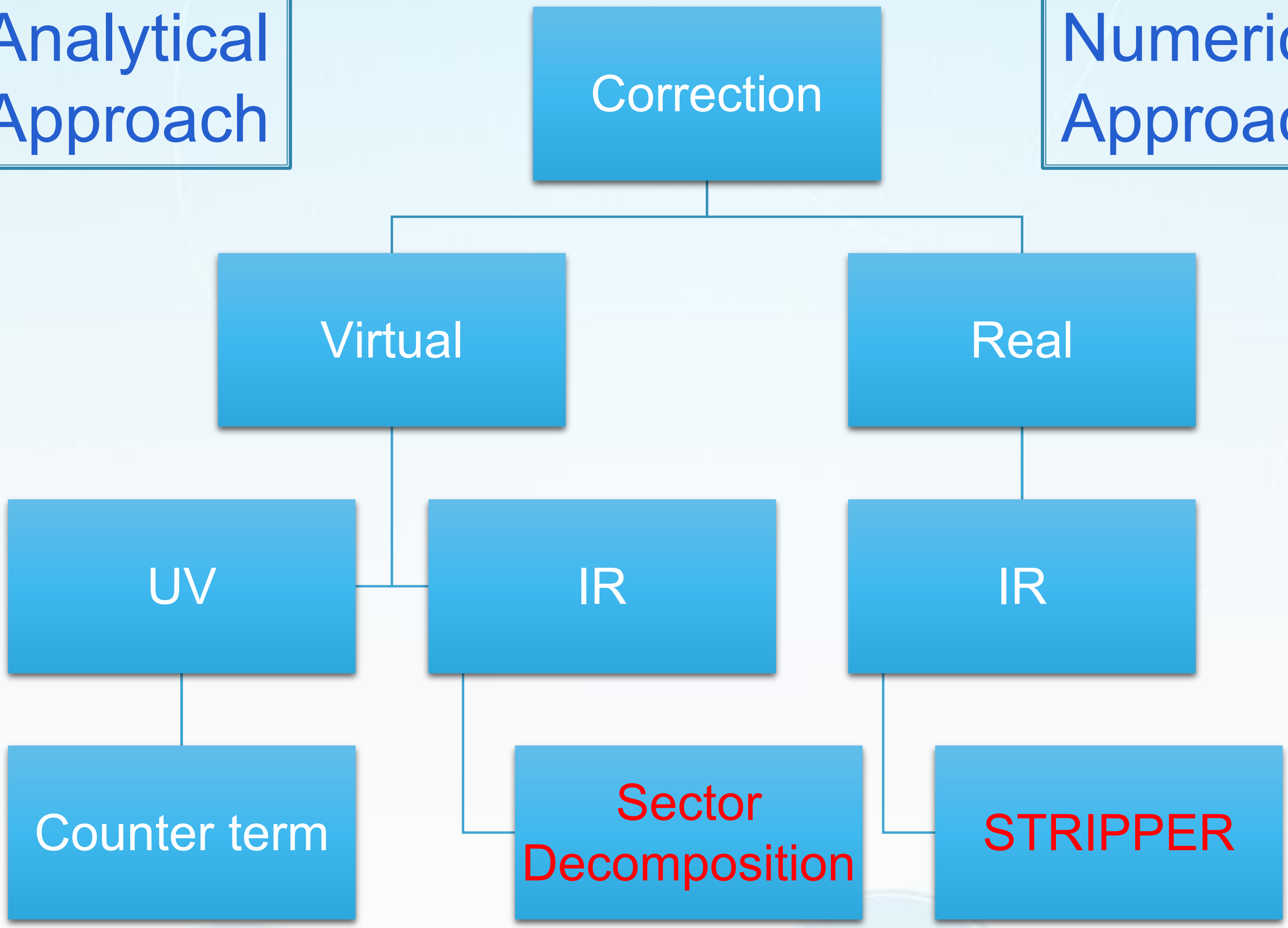
2. This will provide stringent tests of the Standard Model (SM) and has the potential to indirectly probe new physics beyond the SM which might exist at very high energy scales.

3. The theoretical prediction has to be known with a similar or even higher precision than the experimental.

Introduction

Analytical Approach

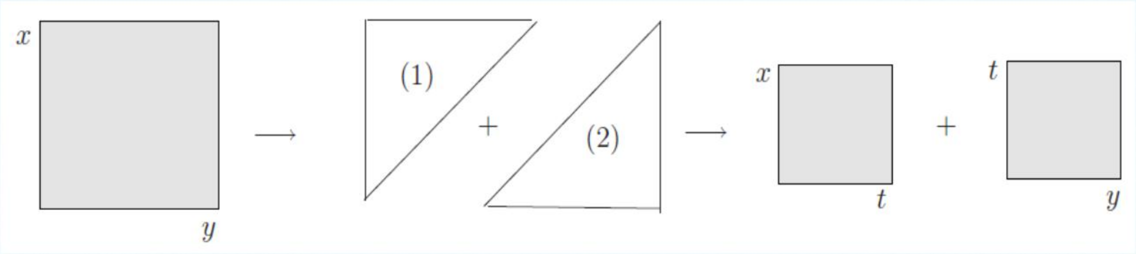
Numerical Approach



Sector Decomposition

Gudrun Heinrich (arXiv:0803.4177[hep-ph])

$$I = \int_0^1 dx \int_0^1 dy\, x^{-1-a\varepsilon} y^{-b\varepsilon} (x + (1-x)y)^{-1}$$



$$I = \int_0^1 dx \int_0^1 dy\, x^{-1-a\varepsilon} y^{-b\varepsilon} (x + (1-x)y)^{-1} [\theta(x-y) + \theta(y-x)]$$

We substitute y = x t in sector (1) and x = y t in sector (2) to remap the integration range to the unit square and obtain

$$I = \int_0^1 dx\, x^{-1-(a+b)\varepsilon} \int_0^1 dt\, t^{-b\varepsilon} (1 + (1-x)t)^{-1}$$

$$+ \int_0^1 dy\, y^{-1-(a+b)\varepsilon} \int_0^1 dt\, t^{-1-a\varepsilon} (1 + (1-y)t)^{-1}$$

Feynman parameter integrals

A general Feynman graph in D dimensions at L loops with N propagators and R loop momenta in the numerator

$$G^{\mu_1\ldots\mu_R}_{l_1\ldots l_R} = \int \prod_{l=1}^{L} \mathrm{d}^D\kappa_l \, \frac{k_{l_1}^{\mu_1}\ldots k_{l_R}^{\mu_R}}{\prod_{j=1}^{N} P_j^{\nu_j}(\{k\},\{p\},m_j^2)}$$

where, $\mathrm{d}^D\kappa_l = \frac{\mu^{4-D}}{i\pi^{\frac{D}{2}}}\mathrm{d}^D k_l \, , \; P_j(\{k\},\{p\},m_j^2) = (q_j^2 - m_j^2 + i\delta)$

Feynman parameterization

$$\frac{1}{\prod_{j=1}^{N} P_j^{\nu_j}} = \frac{\Gamma(N_\nu)}{\prod_{j=1}^{N}\Gamma(\nu_j)} \int_0^\infty \prod_{j=1}^{N} \mathrm{d}x_j \; x_j^{\nu_j-1} \, \delta(1-\sum_{i=1}^{N} x_i) \frac{1}{\left[\sum_{j=1}^{N} x_j P_j\right]^{N_\nu}}$$

where, $N_\nu = \sum_{j=1}^{N} \nu_j$

$$G^{\mu_1\ldots\mu_R}_{l_1\ldots l_R} = \frac{\Gamma(N_\nu)}{\prod_{j=1}^{N}\Gamma(\nu_j)} \int_0^\infty \prod_{j=1}^{N} \mathrm{d}x_j \; x_j^{\nu_j-1} \, \delta(1-\sum_{i=1}^{N} x_i) \int \mathrm{d}^D\kappa_1 \ldots \mathrm{d}^D\kappa_L$$

$$k_{l_1}^{\mu_1}\ldots k_{l_R}^{\mu_R} \left[\sum_{i,j=1}^{L} k_i^{\mathrm{T}} M_{ij} \, k_j - 2\sum_{j=1}^{L} k_j^{\mathrm{T}} \cdot Q_j + J + i\,\delta\right]^{-N\nu},$$

Feynman parameter integrals

Momentum integration

$$G^{\mu_1 \ldots \mu_R}_{l_1 \ldots l_R} = (-1)^{N_\nu} \frac{1}{\prod_{j=1}^{N} \Gamma(\nu_j)} \int_0^\infty \prod_{j=1}^{N} dx_j \; x_j^{\nu_j - 1} \, \delta(1 - \sum_{l=1}^{N} x_l)$$

$$\sum_{m=0}^{[R/2]} (-\frac{1}{2})^m \Gamma(N_\nu - m - LD/2) \left[(\tilde{M}^{-1} \otimes g)^{(m)} \, \tilde{l}^{(R-2m)} \right]^{\Gamma_1, \ldots, \Gamma_R}$$

$$\times \frac{\mathcal{U}^{N_\nu - (L+1)D/2 - R}}{\mathcal{F}^{N_\nu - LD/2 - m}}$$

where,

$$\mathcal{F}(\vec{x}) = \det(M) \left[\sum_{j,l=1}^{L} Q_j \, M^{-1}_{jl} \, Q_l - J - i\,\delta \right]$$

$$\mathcal{U}(\vec{x}) = \det(M)$$

$$\tilde{M}^{-1} = \mathcal{U} M^{-1} , \quad \tilde{l} = \mathcal{U}\, v$$

Primary sectors

As the basic algorithm is the same for tensor integration, we will set R=0,

$$G = (-1)^{N_\nu} \frac{\Gamma(N_\nu - LD/2)}{\prod_{j=1}^{N} \Gamma(\nu_j)} \int_0^\infty \prod_{j=1}^{N} dx_j \; x_j^{\nu_j - 1} \, \delta(1 - \sum_{l=1}^{N} x_l) \, \frac{\mathcal{U}^{N_\nu - (L+1)D/2}}{\mathcal{F}^{N_\nu - LD/2}} \, .$$

Decompose the integration range into N sectors

$$\int_0^\infty d^N x = \sum_{l=1}^{N} \int_0^\infty d^N x \prod_{\substack{j=1 \\ j \neq l}}^{N} \theta(x_l \geq x_j)$$

The θ -function is defined as

$$\theta(x \geq y) = \begin{cases} 1 & \text{if } x \geq y \text{ is true} \\ 0 & \text{otherwise.} \end{cases}$$

Primary sectors

Substitute

$$x_j = \begin{cases} x_l t_j & \text{for} \quad j < l \\ x_l & \text{for} \quad j = l \\ x_l t_{j-1} & \text{for} \quad j > l \end{cases}$$

$$G_l = \int_0^1 \prod_{j=1}^{N-1} \mathrm{d}t_j \, t_j^{\nu_j - 1} \, \frac{\mathcal{U}_l^{N_\nu - (L+1)D/2}(\vec{t})}{\mathcal{F}_l^{N_\nu - LD/2}(\vec{t})} \quad , \quad l = 1, \ldots, N \, .$$

Eliminate the δ distribution in such a way that the remaining integrations are from 0 to 1. It preserves the feature that singularities only occur at special points at the boundary of parameter space.

Subsectors

$$G_l = \int_0^1 \prod_{j=1}^{N-1} \mathrm{d}t_j \, t_j^{\nu_j - 1} \, \frac{\mathcal{U}_l^{N_\nu - (L+1)D/2}(\vec{t})}{\mathcal{F}_l^{N_\nu - LD/2}(\vec{t})} \quad , \quad l = 1, \ldots, N \, .$$

Determine a set of parameters

$$\mathcal{S} = \{t_{\alpha_1}, \ldots, t_{\alpha_r}\}$$

Decompose the corresponding r-cube into r subsectors

$$\prod_{j=1}^{r} \theta(1 \geq t_{\alpha_j} \geq 0) = \sum_{k=1}^{r} \prod_{\substack{j=1 \\ j \neq k}}^{r} \theta(t_{\alpha_k} \geq t_{\alpha_j} \geq 0)$$

$$t_{\alpha_j} \to \begin{cases} t_{\alpha_k} t_{\alpha_j} & \text{for} \quad j \neq k \\ t_{\alpha_k} & \text{for} \quad j = k \end{cases}$$

Subsectors

The resulting subsector integrals have the general form

$$G_{lk} = \int_0^1 \left(\prod_{j=1}^{N-1} \mathrm{d}t_j \; t_j^{a_j - b_j \epsilon} \right) \frac{\mathcal{U}_{lk}^{N_\nu - (L+1)D/2}}{\mathcal{F}_{lk}^{N_\nu - LD/2}} , \quad k = 1, \ldots, r \; .$$

The iteration stops if the functions contain a constant term

$$\begin{aligned} \mathcal{U}_{lk_1k_2\ldots} &= 1 + u(\vec{t}) \\ \mathcal{F}_{lk_1k_2\ldots} &= -s_0 + \sum_\beta (-s_\beta) f_\beta(\vec{t}) \end{aligned}$$

Extraction of the poles

$$G_{lk} = \int_0^1 \left(\prod_{j=1}^{N-1} \mathrm{d}t_j \; t_j^{a_j - b_j \epsilon} \right) \frac{\mathcal{U}_{lk}^{N_\nu - (L+1)D/2}}{\mathcal{F}_{lk}^{N_\nu - LD/2}} , \quad k = 1, \ldots, r \, .$$

For a particular t_j

$$I_j = \int_0^1 dt_j \, t_j^{(a_j - b_j \epsilon)} \, \mathcal{I}(t_j, \{t_{i \neq j}\}, \epsilon)$$

where, $\mathcal{I} = \mathcal{U}_{lk}^{N_\nu - (L+1)D/2} / \mathcal{F}_{lk}^{N_\nu - LD/2}$

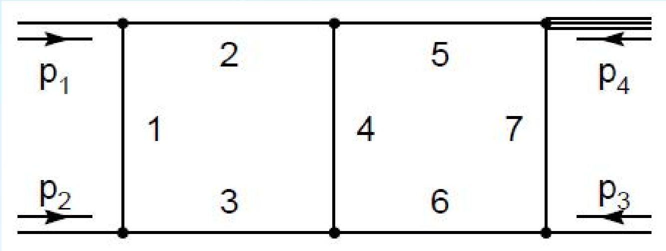
Expand into a Taylor series around $t_j = 0$:

$$\mathcal{I}(t_j, \{t_{i \neq j}\}, \epsilon) = \sum_{p=0}^{|a_j|-1} \mathcal{I}_j^{(p)}(0, \{t_{i \neq j}\}, \epsilon) \frac{t_j^p}{p!} + R(\vec{t}, \epsilon)$$

where, $\mathcal{I}_j^{(p)}(0, \{t_{i \neq j}\}, \epsilon) = \partial^p \mathcal{I}(t_j, \{t_{i \neq j}\}, \epsilon) / \partial t_j^p \Big|_{t_j = 0}$

$$I_j = \sum_{p=0}^{|a_j|-1} \frac{1}{a_j + p + 1 - b_j \epsilon} \frac{\mathcal{I}_j^{(p)}(0, \{t_{i \neq j}\}, \epsilon)}{p!} + \int_0^1 dt_j \, t_j^{a_j - b_j \epsilon} R(\vec{t}, \epsilon)$$

Example



$$p_1^2 = p_2^2 = p_3^2 = 0, p_4^2 \neq 0$$

$$\begin{aligned}
\mathcal{U} &= x_{123}x_{567} + x_4 x_{123567} \\
\mathcal{F} &= (-s_{12})(x_2x_3x_{4567} + x_5x_6x_{1234} + x_2x_4x_6 + x_3x_4x_5) \\
&\quad +(-s_{23})x_1x_4x_7 + (-p_4^2)x_7(x_2x_4 + x_5x_{1234}) \ ,
\end{aligned}$$

$$\begin{aligned}
DB_{m4} &= \int \frac{d^Dk_1}{i\pi^{\frac{D}{2}}} \int \frac{d^Dk_2}{i\pi^{\frac{D}{2}}} \times \\
&\quad \frac{1}{k_1^2\, k_2^2\, (k_1+p_1)^2\, (k_1+p_1+p_2)^2\, (k_2+p_1+p_2)^2\, (k_2-p_4)^2\, (k_1-k_2)^2} \\
&= \Gamma(1+\epsilon)^2 \left(\frac{P_4}{\epsilon^4} + \frac{P_3}{\epsilon^3} + \frac{P_2}{\epsilon^2} + \frac{P_1}{\epsilon} + P_0 \right) .
\end{aligned}$$

STRIPPER

M.Czakon and D.Heymes (arXiv:1408.2500[hep-ph])

Phase space decomposition

For the single-unresolved process, the selector functions are

$$\mathcal{S}_{i,k} = \frac{1}{D_1\, d_{i,k}}$$

where, $D_1 = \sum_{ik} \frac{1}{d_{i,k}}$ $d_{i,k} = \left(\frac{E_i}{\sqrt{\hat{s}}}\right)^{\alpha} (1 - \cos\theta_{ik})^{\beta}$

For the double-unresolved process, there are two types of selector functions:

triple-collinear $\mathcal{S}_{ij,k} = \frac{1}{D_2\, d_{ij,k}}$

double-collinear $\mathcal{S}_{i,k;j,l} = \frac{1}{D_2\, d_{i,k} d_{j,l}}$

where, $D_2 = \sum_{ij} \left[\sum_{k} \frac{1}{d_{ij,k}} + \sum_{kl} \frac{1}{d_{i,k} d_{j,l}} \right]$

$$d_{ij,k} = \left(\frac{E_i}{\sqrt{\hat{s}}}\right)^{\alpha_i} \left(\frac{E_j}{\sqrt{\hat{s}}}\right)^{\alpha_j} \left[(1 - \cos\theta_{ij})(1 - \cos\theta_{ik})(1 - \cos\theta_{jk})\right]^{\beta}$$

STRIPPER

Phase space parameterization

The phase space integrals

$$\int \mathrm{d}\mathbf{\Phi}_{n+n_u} = \int \mathrm{d}\mathbf{\Phi}_{\substack{\text{reference}\\\text{unresolved}}} \int \mathrm{d}\mathbf{\Phi}_{n-n_{fr}}(Q)$$

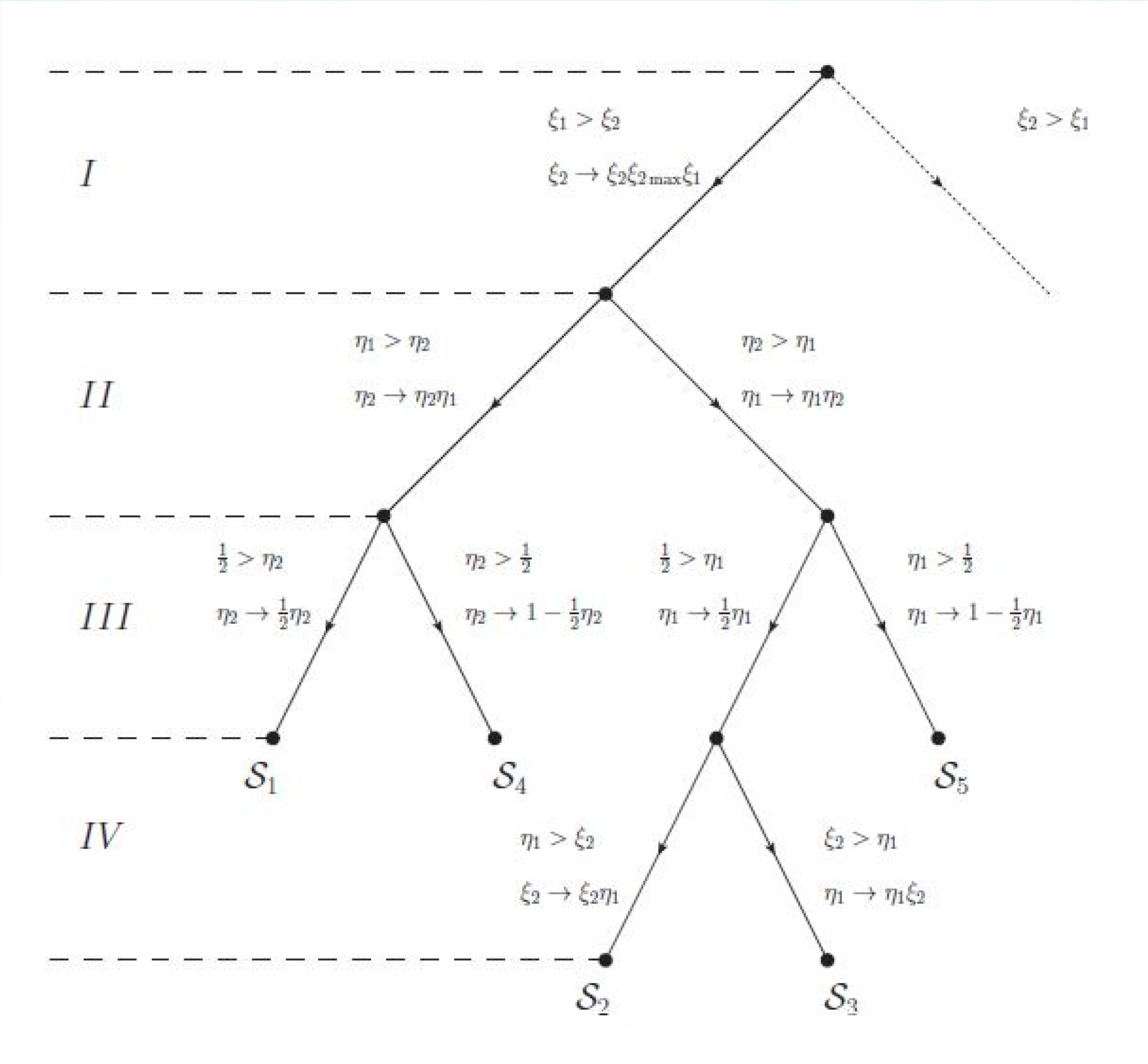
For the single-unresolved process, the unresolved phase space integrals contain

$$\iint_0^1 \mathrm{d}\eta\,\mathrm{d}\xi\,\eta^{a_1-b_1\epsilon}\,\xi^{a_2-b_2\epsilon}$$

For the double-unresolved process, the unresolved phase space integrals contain

$$\iiiint_0^1 \mathrm{d}\eta_1\mathrm{d}\eta_2\mathrm{d}\xi_1\mathrm{d}\xi_2\,\eta_1^{a_1-b_1\epsilon}\,\eta_2^{a_2-b_2\epsilon}\,\xi_1^{a_3-b_3\epsilon}\,\xi_2^{a_4-b_4\epsilon}$$

STRIPPER



I
II
III
IV
$\xi_1 > \xi_2$
$\xi_2 \to \xi_2 \xi_{2\,\max} \xi_1$
$\xi_2 > \xi_1$
$\eta_1 > \eta_2$
$\eta_2 \to \eta_2 \eta_1$
$\eta_2 > \eta_1$
$\eta_1 \to \eta_1 \eta_2$
$\frac{1}{2} > \eta_2$
$\eta_2 \to \frac{1}{2}\eta_2$
$\eta_2 > \frac{1}{2}$
$\eta_2 \to 1 - \frac{1}{2}\eta_2$
$\frac{1}{2} > \eta_1$
$\eta_1 \to \frac{1}{2}\eta_1$
$\eta_1 > \frac{1}{2}$
$\eta_1 \to 1 - \frac{1}{2}\eta_1$
$\mathcal{S}_1$
$\mathcal{S}_4$
$\mathcal{S}_5$
$\eta_1 > \xi_2$
$\xi_2 \to \xi_2 \eta_1$
$\xi_2 > \eta_1$
$\eta_1 \to \eta_1 \xi_2$
$\mathcal{S}_2$
$\mathcal{S}_3$

STRIPPER

Generation of subtraction and integrated subtraction terms

For the single unresolved process:

$$\hat{\sigma}^{\mathrm{R}} = \sum_{ik} \iint_0^1 \frac{\mathrm{d}\eta}{\eta^{1+\epsilon}} \frac{\mathrm{d}\xi}{\xi^{1+2\epsilon}} f_{i,k}(\eta,\xi)$$

$$f_{i,k}(\eta,\xi) = \frac{E_{\max}^2}{16\pi^3 \hat{s} N_{ab}} \left(\frac{\pi \mu_R^2 e^{\gamma_E}}{4E_{\max}^2 (1-\eta)} \right)^{\epsilon} \int_{S_1^{1-2\epsilon}} \mathrm{d}\Omega(\phi, \rho_1, \ldots) \int \mathrm{d}\Phi_n(p_1 + p_2 - u)\, \mathcal{S}_{i,k} \left[\eta\, \xi^2 \langle \mathcal{M}_{n+1}^{(0)} | \mathcal{M}_{n+1}^{(0)} \rangle \right] \mathrm{F}_{n+1}$$

The Laurent expansion can be derived using the master formula:

$$\frac{1}{x^{1+a\epsilon}} = -\frac{1}{a\epsilon}\delta(x) + \left[\frac{1}{x^{1+a\epsilon}} \right]_+ \qquad \int_0^1 \mathrm{d}x \left[\frac{1}{x^{1+a\epsilon}} \right]_+ f(x) = \int_0^1 \mathrm{d}x\, \frac{f(x) - f(0)}{x^{1+a\epsilon}}$$

where x is either η or ξ .

These are needed to evaluate is:

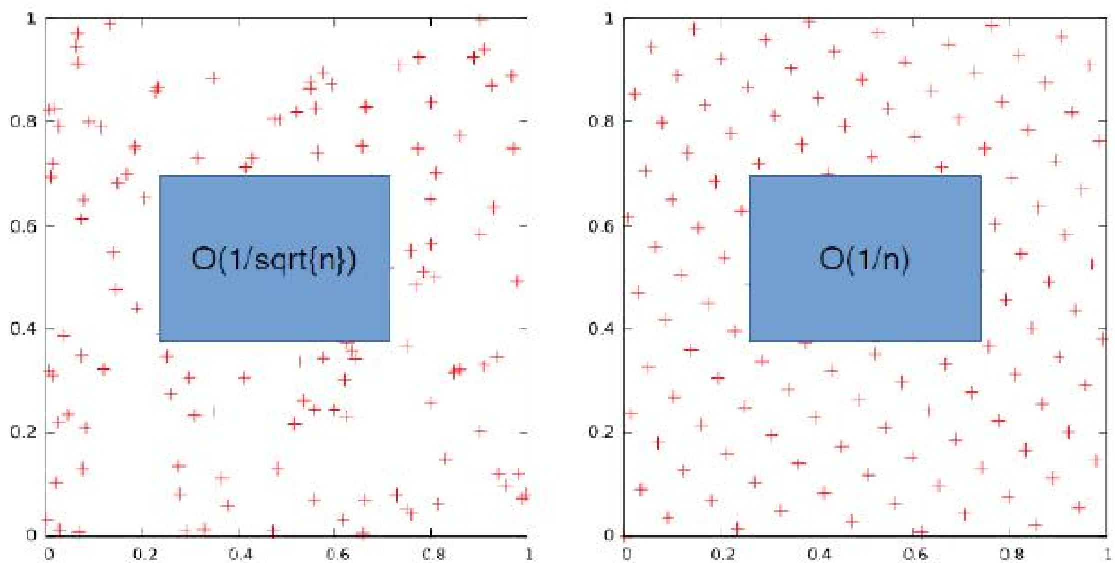
$$\lim_{\eta \to 0} \left[\eta\, \xi^2 \langle \mathcal{M}_{n+1}^{(0)} | \mathcal{M}_{n+1}^{(0)} \rangle \right], \quad \lim_{\xi \to 0} \left[\eta\, \xi^2 \langle \mathcal{M}_{n+1}^{(0)} | \mathcal{M}_{n+1}^{(0)} \rangle \right], \quad \lim_{\eta \to 0} \lim_{\xi \to 0} \left[\eta\, \xi^2 \langle \mathcal{M}_{n+1}^{(0)} | \mathcal{M}_{n+1}^{(0)} \rangle \right]$$

Improvement on Numerical approach

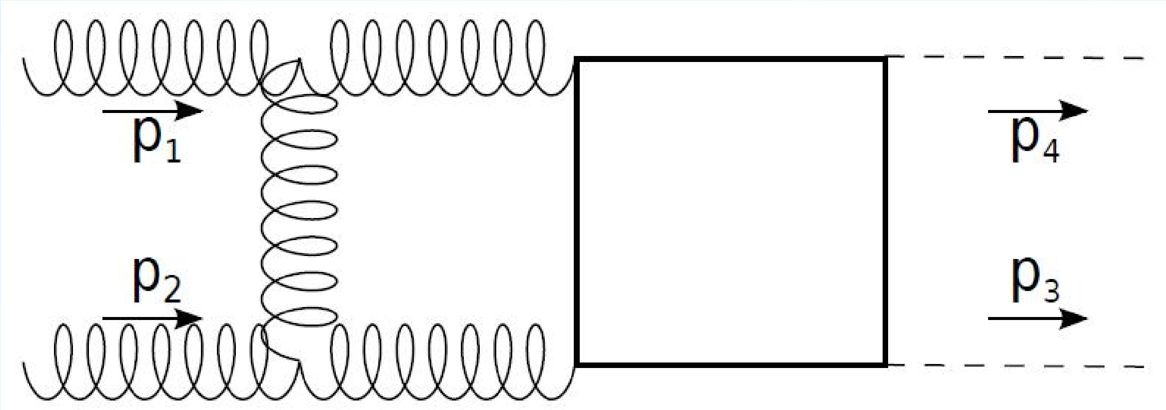
Z.Li, J.Wang, Q.Yan and X.Zhao
Chinese Physics C, Vol. 40, No. 3 (2016) 033103

Improving the efficiency of numerical integration in sector decomposition by implementing a quasi-Monte Carlo method associated with the CUDA/GPU technique.

$$I(f) = \int_0^1 d^s x f(\vec{x}) \qquad I_{estimate}(f) = \sum_{i=0}^{n-1} f(\vec{x}_i)$$

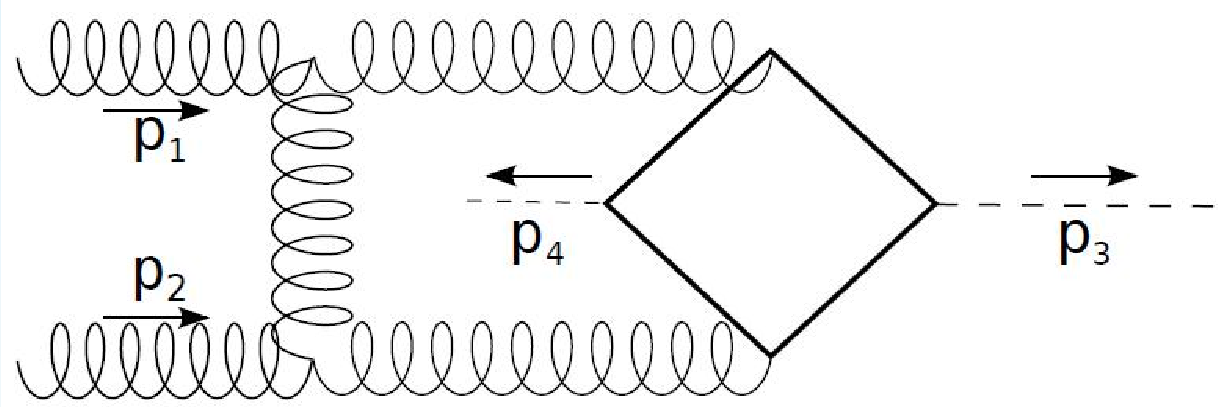


Improvement on Numerical approach



	Vegas/CPU	QMC/GPU
P_2	$-7.959 \pm 0.009 - 10.586i \pm 0.009i$	$-7.949 \pm 0.003 - 10.585i \pm 0.005i$
P_1	$3.9 \pm 0.1 - 28.1i \pm 0.1i$	$3.831 \pm 0.005 - 28.022i \pm 0.005i$
P_0	$-3.9 \pm 0.8 + 92.3i \pm 0.8i$	$-4.63 \pm 0.07 + 92.13i \pm 0.07i$
Integration Time	45540s	19s

Improvement on Numerical approach



	Vegas/CPU	QMC/GPU
P_2	$-3.848 \pm 0.004 + 0.0005i \pm 0.003i$	$-3.8482 \pm 0.0007 + 0.0004i \pm 0.0003i$
P_1	$3.81 \pm 0.03 - 6.41i \pm 0.03i$	$3.83 \pm 0.02 - 6.40i \pm 0.02i$
P_0	$77.2 \pm 0.2 + 20.1i \pm 0.2i$	$77.2 \pm 0.1 + 19.9i \pm 0.1i$
Integration Time	54290s	20s

Mixed QCD-EW corrections for Higgs boson production at e+e- colliders

Y.Gong, Z.Li, X.Xu, L.Yang and X.Zhao
arXiv:1609.03955[hep-ph]

Investigate the production of the Higgs boson at such an e+e-collider and calculate for the first time the mixed QCD-electroweak corrections to the total cross sections.

Provide an approximate analytic formula for the cross section and show that it reproduces the exact numeric results rather well for collider energies up to 350 GeV.

Find that the corrections amount to a 1.3% increase of the cross section for a center-of-mass energy around 250 GeV. This is significantly larger than the expected experimental accuracy and has to be included for extracting the properties of the Higgs boson from the measurements of the cross sections in the future.

Mixed QCD-EW corrections for Higgs boson production at e+e- colliders

Numerical Approach

We choose the input parameters as:

m_t = 173.3 GeV, m_H = 125.1 GeV, m_Z = 91.1876 GeV, m_W = 80.385 GeV,
$\alpha(m_Z)$ = 1/127.94 , $\alpha_s(m_Z)$ = 0.118 .

$\sqrt{s}$ (GeV)	σ_{LO} (fb)	σ_{NLO} (fb)	σ_{NNLO} (fb)
240	256.3(9)	228.0(1)	230.9(4)
250	256.3(9)	227.3(1)	230.2(4)
300	193.4(7)	170.2(1)	172.4(3)
350	138.2(5)	122.1(1)	123.9(2)
500	61.38(22)	53.86(2)	54.24(7)

Mixed QCD-EW corrections for Higgs boson production at e+e- colliders

Analytical Approach

The method involves an series expansion in m_t^{-1} , which is expected to converge for $\sqrt{s} < 2m_t$, perform the expansion up to order m_t^{-4} . In this way we obtain an approximate analytic formula for the cross section.

The analytical results approximates the numerical results remarkably well for the 3 energies.

$\sqrt{s}$ (GeV)	σ_{LO} (fb)	σ_{NLO} (fb)	σ_{NNLO} (fb)	$\sigma_{\mathrm{NNLO}}^{\mathrm{exp.}}$ (fb)
240	256.3(9)	228.0(1)	230.9(4)	230.9(4)
250	256.3(9)	227.3(1)	230.2(4)	230.2(4)
300	193.4(7)	170.2(1)	172.4(3)	172.4(3)

Conclusion

- Introduce a popular method -- sector decomposition -- to extract the poles from the virtual correction.
- Apply the STRIPPER to deal with the infrared singular of real correction.
- Compare the MC and QMC, QMC has a faster speed and higher accuracy, which makes the direct numerical approach viable for precise investigation of higher order effects in multi-loop processes.
- Mixed QCD-EW corrections for Higgs boson production at e+e- colliders amount to a 1.3% increase of the cross section for a center-of-mass energy around 250 GeV.

Thanks!