Vorticity in Heavy-Ion Collisions

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Non-relativistic case

The vorticity (pseudo) vector is defined by,

$$\boldsymbol{\omega}_1(\boldsymbol{x}, t) = \boldsymbol{\nabla} \times \boldsymbol{v}$$

The Euler equation for ideal fluid,

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v} \cdot \boldsymbol{\nabla}) = -\frac{1}{\rho}\mathrm{grad}p$$

It comes from the equation of motion $\rho \frac{d\boldsymbol{v}}{dt} = -\mathrm{grad}p$,

- The change during dt in the velocity at a point fixed in space,

  $\frac{\partial \boldsymbol{v}}{\partial t} dt$

- The difference between the velocities (at the same instant) at two points $d\boldsymbol{r}$ apart,

$$dx\frac{\partial \boldsymbol{v}}{\partial x} + dy\frac{\partial \boldsymbol{v}}{\partial y} + dz\frac{\partial \boldsymbol{v}}{\partial z} = (d\boldsymbol{r} \cdot \boldsymbol{\nabla})\,\boldsymbol{v}$$

Thus $d\boldsymbol{v} = \frac{d\boldsymbol{v}}{\partial t}dt + (d\boldsymbol{r} \cdot \boldsymbol{\nabla})\,\boldsymbol{v}$ dividing both sides by dt.

(1) Circulation conservation

$$\frac{d}{dt}\oint \boldsymbol{v}\cdot d\boldsymbol{x} = 0$$

(2) Helicity (pseudoscalar)field

$$h_f(\boldsymbol{x}, t) = \boldsymbol{v}\cdot\boldsymbol{\omega}_1 \rightarrow \text{helicity density}$$

$$\mathcal{H}_f = \int d^3\boldsymbol{x}\, h_f \rightarrow \text{total helicity} \ \rightarrow \text{conserved}$$

$$\frac{\partial\boldsymbol{\omega}_1}{\partial t} = \boldsymbol{\nabla}\times(\boldsymbol{v}\times\boldsymbol{\omega}_1)$$

Relativistic case

A natural extension of the definition to relativistic fluid is,

$$\omega^{\mu} = \epsilon^{\mu\nu\rho\sigma} u_{\nu} \partial_{\rho} u_{\sigma} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} u_{\nu} \omega_{\rho\sigma}$$

$$\omega_{\mu\nu} = \partial_{\mu} u_{\nu} - \partial_{\nu} u_{\mu} \text{ (kinematic vorticity tensor)}$$

It is worth writing down the components of ω_2^{μ}

- Spatial components

$$\boldsymbol{\omega}_2 = \gamma^2 \boldsymbol{\omega}_1 + \gamma^2 \mathbf{v} \times \partial_t \mathbf{v}$$

- time component

$$\omega_2^{\mu} = \gamma^2 \mathbf{v} \cdot \boldsymbol{\omega}_1 = \mathbf{v} \cdot \boldsymbol{\omega}_2$$

Thus, in the non-relativistic limit, $\omega_2^0 \to (h_f, \boldsymbol{\omega}_1)$

In order to maintain the circulation conservation and helicity conservation,another definition of vorticity tensor has been introduced as,

$$\Omega_{\mu\nu} = \partial_\mu(Tu_\nu) - \partial_\nu(Tu_\mu)$$

The relativistic Euler equation for ideal fluid,

$$(\varepsilon + P)\frac{d}{d\tau}u^\mu = \nabla^\mu P$$

can be deduced to

$$\frac{d}{d\tau}(Tu^\mu) = \partial^\mu T$$

Thus, the relativistic circulation conservation,

$$\frac{d}{d\tau}\oint Tu_\mu dx^\mu = \oint \partial_\mu T dx\mu = 0$$

The vorticity(pseudo) vector corresponding to $\Omega_{\mu\nu}$

$$\omega^{\mu} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma} T u_{\nu} \Omega_{\rho\sigma} = T^2 \omega_2^{\mu}$$

Therefore, the integral of Ω^0 over space is conserved and we can identify Ω^0 as the conserved helicity density
If the fluid carries a conserved charge,one can define the vorticity tensor as

$$\tilde{\Omega}_{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\partial_{\rho}(\beta u_{\sigma})$$
$$\omega^{\mu} = \tilde{\Omega}^{\mu\nu} u_{\nu}$$

Setup of the numerical simulations

Definition of the velocity field

Velocity:

- Use program of Hydro to generate a velocity field
- Use a non-boost-invariant solution of relativistic hydrodynamics in 1+3 dimensions

$$u_t = -\frac{1}{t^2 - x_\perp^2}\left(\frac{t^2\left(t^2 - x_\perp^2 + L^2 + z^2\right)}{\sqrt{\left(t^2 - x_\perp^2 - L^2 - z^2\right)^2 + 4L^2\left(t^2 - x_\perp^2\right)}} - \alpha x_\perp^2\right)$$

$$\vec{u}_\perp = \frac{t\vec{x}_\perp}{t^2 - x_\perp^2}\left(\frac{t^2 - x_\perp^2 + L^2 + z^2}{\sqrt{\left(t^2 - x_\perp^2 - L^2 - z^2\right)^2 + 4L^2\left(t^2 - x_\perp^2\right)}} - \alpha\right)$$

$$u_z = \frac{2zt}{\sqrt{\left(t^2 - x_\perp^2 - L^2 - z^2\right)^2 + 4L^2\left(t^2 - x_\perp^2\right)}}$$

Velocity Field

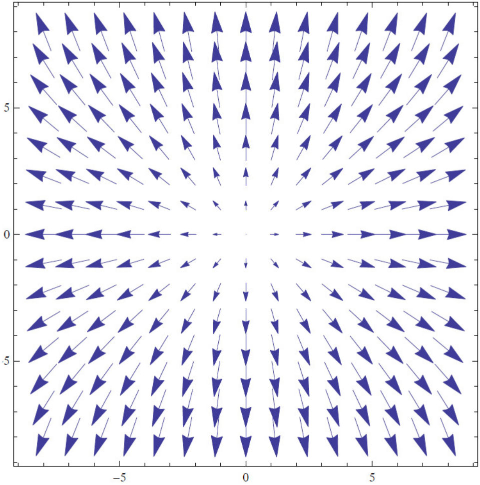

Figure: (v_z, v_x)in xoz plane

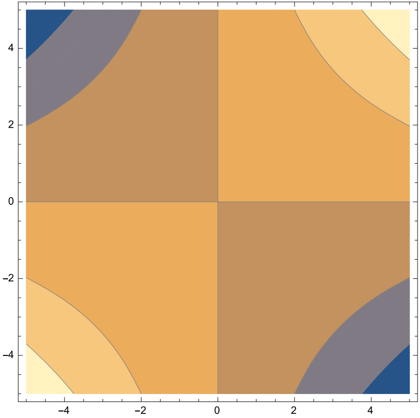

Figure: $\omega_y$in xoz plane

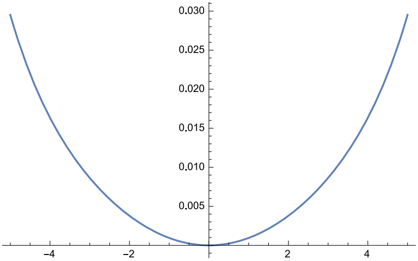

Figure: $\omega_y$when x=z


THANK YOU