Temperature dependent Energy Loss within HT formalism

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Outline

Introduction

Jet quenching observables in high-energy nuclear collisions has been proposed as a good probe of the dense medium produced in these collisions.

The medium induced photon and gluon radiation has been derived:

- up to twist-4 contribution in deep inelastic scattering (DIS) by HT formalism;
- through opacity expansion by GLV formalism [1];

Wang[2] introduced the detailed balance for the gluon radiation within opacity expansion formalism, where the medium temperature is included. [2] Here we will check the energy loss with detailed balance within twist-4 formalism.

The main motivation is to study the relationship between the jet transport coefficient $\hat{q}$ and the local temperature T.

[1]M. Gyulassy, P. Lévai, I. Vitev, Nucl. Phys. B **594** (2001) 371

[2]E. Wang, X.N. Wang, PRL **87**, 14 (2001)

about Higher Twist

We know that the mass term of the Lagrangian has dimension $[m\bar{\psi}\psi] = 4$, $[m] = 1$, the dimension of the ψ is $[\bar{\psi}] = [\psi] = \frac{3}{2}$, and the dimension of spin $[s_{\bar{\psi}}] = [s_\psi] = -\frac{1}{2}$.

Looking at the current tensor from $W_0^{A\mu\nu}$, with only $\bar{\psi}(y_0)$ and $\psi(0)$. The overall dimension is: $[\bar{\psi}] + [\psi] + [s_{\bar{\psi}}] + [s_\psi] = 2$. Which is called the twist-2 contribution (leading-twist).

Looking at the current tensor from $W_{(a)}^{A\mu\nu}$, with $\bar{\psi}(y_0)$, $\psi(0)$, $\not{A}^{a_1}$ and $\not{A}^{a_1'}$. We can treat $\not{A}$ as $F^{\mu\nu}$, such that $\not{A}\not{A}'$ has the dimension of $[F^{\mu\nu}F_{\mu\nu}] = 4$. With the bosonic spin $[s_{\not{A}}] = [s_{\not{A}'}] = -1$. Then the overall dimension is $2 + [\not{A}] + [\not{A}'] + [s_{\not{A}}] + [s_{\not{A}'}] = 4$. Which is called twist-4 contribution.

Photon Radiation

Consider the semi-inclusive process

$$e(L_1) + A(P_A) \to e(L_2) + q(l_q) + \gamma(l_\gamma) + X \qquad (1)$$

- where we've used the light-cone coordinate $p = [p^+, p^-, \mathbf{p}_\perp]$ with $p^\pm = (E \pm p_z)/\sqrt{2}$.
- The momentum for the virtual photon (γ^*) is $q = L_2 - L_1 = [-x_B p^+, q^-, \mathbf{0}_\perp]$, with $x_B = Q^2/(2p^+q^-)$.
- Each nucleon in nucleus A has momentum p such that $P_A = A[p^+, 0, \mathbf{0}]$.
- and the photon carries a fraction y of the quark's forward momentum $l_\gamma^- = yq^-$.

Photon Radiation cont.

The double differential cross section may be expressed as

$$E_{L_2} \frac{d\sigma}{d^3 L_2 d^3 l_q d^3 l_\gamma} = \frac{\alpha_e}{2\pi s} \frac{1}{Q^4} L_{\mu\nu} \frac{dW^{\mu\nu}}{d^3 l_q d^3 l_\gamma} \tag{2}$$

- the leptonic tensor is given by

$$L_{\mu\nu} = \frac{1}{2} Tr[\not{l}_1 \gamma_\mu \not{l}_2 \gamma_\nu] \tag{3}$$

- and the semi-inclusive hadronic tensor is define as

$$W^{\mu\nu} = \sum_X (2\pi)^4 \delta^4(q + P_A - P_X) \langle A|J^\mu(0)|X\rangle \langle X|J^\nu(0)|A\rangle \tag{4}$$

We will now only pay attention to the hadronic tensor.

leading-twist

The hadronic tensor for photon radiation at the zero-th order (without scattering with the medium from the quark jet) can be written as:

$$
\begin{aligned}
W_0^{A\mu\nu} &= \sum_q Q_q^4 e^2 \int \frac{d^4 l_\gamma}{(2\pi)^4}(2\pi)\delta(l_\gamma^2) \int \frac{d^4 l_q}{(2\pi)^4}(2\pi)\delta(l_q^2) \\
&\times \int d^4 y_0 e^{iq\cdot y_0} \int d^4 z \int d^4 z' \int \frac{d^4 q_1}{(2\pi)^4} \int \frac{d^4 q_1'}{(2\pi)^4} \\
&\times e^{-iq_1\cdot(y_0-z)} e^{-iq_1'\cdot(z'-y_0')} e^{-il_q\cdot(z-z')} e^{-il_\gamma\cdot(z-z')} \\
&\times \langle A|\bar{\psi}(y_0)\gamma^\mu \frac{\not{q}_1}{q_1^2 - i\epsilon}\gamma^\alpha \not{l}_q \gamma^\beta \frac{\not{q}_1'}{q_1'^2 - i\epsilon}\gamma^\nu \psi(0)|A\rangle \\
&\times G_{\alpha\beta}(l_\gamma) \qquad (5)
\end{aligned}
$$

leading-twist cont.

In the high energy and collinear limit, one can use the following approximation

$$\langle A|\bar{\psi}(y_0)\hat{O}\psi(0)|A\rangle \approx AC_p^A\langle p|\bar{\psi}(y_0)\frac{\gamma^+}{2}\psi(0)|p\rangle Tr[\frac{\gamma^-}{2}\hat{O}] \qquad (6)$$

where C_p^A represents the probability of finding a nucleon state with momentum p in a nucleus with A nucleons (nucleon distribution function). Also, we have:

$$f_q(x) = \int \frac{dy_0^-}{2\pi} e^{-ix(p^+\cdot y_0^-)}\langle p|\bar{\psi}(y_0^-)\frac{\gamma^+}{2}\psi(0)|p\rangle \qquad (7)$$

where $f_q(x)$ represents the parton distribution function of a quark with flavour q and momentum fraction x. Then the hadronic tensor:

$$\begin{aligned} W_0^{A\mu\nu} \quad = \quad & AC_p^A\sum_q Q_q^4\frac{\alpha_e}{2\pi}\int dy\int\frac{d^2l_{\gamma\perp}}{\pi l_{\gamma\perp}^2}\frac{(2\pi)f_q(x_B+x_L)}{8p^+(q^-)^2x_L} \\ & \times Tr[\frac{\gamma^-}{2}\gamma^\mu \not{q}_1\gamma^\alpha \not{l}_q\gamma^\beta \not{q}_1'\gamma^\nu]G_{\alpha\beta}(l_\gamma) \qquad (8) \end{aligned}$$

leading-twist cont.

The trace combined with $G_{\alpha\beta}$ can be carried out by using the commutation relations of γ matrices. Then the differential hadronic tensor is now

$$\frac{dW_0^{A\mu\nu}}{d^2 l_{\gamma\perp} dy} = AC_p^A \sum_q Q_q^4 \frac{\alpha_e}{2\pi} \frac{P(y)}{\pi l_{\gamma\perp}^2} (-g_\perp^{\mu\nu})(2\pi) f_q(x_B + x_L) \tag{9}$$

- where $P(y) = [1 + (1-y)^2]/y$ is the quark-to-photon splitting function.
- $g_\perp^{\mu\nu} = g^{\mu\nu} - g^{\mu-}g^{\nu+} - g^{\mu+}g^{\nu-}$ is the projection tensor.
- $x_L = l_{\gamma\perp}^2/[2p^+q^-y(1-y)]$ is the momentum fraction that is related to the formation time $\tau_{form} = [x_L p^+]^{-1}$.

Single scattering(γ-rad)

We now write out the hadronic tensor for one of (10-2=8) Feynman diagrams as an example (0110).

$$\begin{aligned}
W^{A\mu\nu}_{(a)} &= \sum_q Q_q^4 e^2 g^2 \frac{1}{N_c} Tr[T^{a_1} T^{a_1'}] \int \frac{d^4 l_\gamma}{(2\pi)^4} 2\pi\delta(l_\gamma^2) \int \frac{d^4 l_q}{(2\pi)^4} 2\pi\delta(l_q^2) \\
&\times \int d^4 y_0 e^{iq\cdot y_0} \int d^4 y_1 \int d^4 y_1' \int d^4 z \int d^4 z' \\
&\times \int \frac{d^4 q_1}{(2\pi)^4} e^{-iq_1\cdot(y_0-z)} \int \frac{d^4 \bar{q}_1}{(2\pi)^4} e^{-i\bar{q}_1\cdot(z-y_1)} e^{-il_\gamma\cdot(z-z')} \\
&\times e^{-il_q\cdot(y_1-y_1')} \int \frac{d^4 \bar{q}_1'}{(2\pi)^4} e^{-i\bar{q}_1'\cdot(y_1'-z')} \int \frac{d^4 q_1'}{(2\pi)^4} e^{-iq_1'\cdot(z'-y_0')} \\
&\times \langle A|\bar{\psi}(y_0)\gamma^\mu \frac{\not{q}_1}{q_1^2 - i\epsilon}\gamma^\alpha \left(\frac{\not{\bar{q}}_1}{\bar{q}_1^2 - i\epsilon}\not{A}^{a_1}(y_1)\right) \not{l}_q \left(\not{A}^{a_1'}(y_1')\frac{\not{\bar{q}}_1'}{\bar{q}_1'^2 + i\epsilon}\right) \\
&\times \gamma^\beta \frac{\not{q}_1'}{q_1'^2 + i\epsilon}\gamma^\nu \psi(0)|A\rangle G_{\alpha\beta}(l_\gamma) \qquad (10)
\end{aligned}$$

Single scattering(γ-rad)

Including the rest of the diagrams

- central cutting: (1001), (0101), (1010)
- non-central cutting: (0200), (0020), (2000), (0002)
  Note that (1100) and (0011) results in vanishing phase factor.

The double differential hadronic tensor for single scattering is now:

$$W_{11}^{A\mu\nu} = (-g_{\perp}^{\mu\nu}) A C_p^A \sum_q Q_q^4 \frac{\alpha_e}{2\pi} \frac{P(y)}{l_{\gamma\perp}^2} f_q(x_B + x_L)\{\cdots\} \qquad (11)$$

Gluon radiation

Consider the semi-inclusive process [3] [4]

$$e(L_1) + A(P_A) \to e(L_2) + q(l_q) + g(l_g) + X \tag{12}$$

Similar to the photon radiation. We will only pay attention to the hadronic tensor.
We now write out the hadronic tensor for one of (21-4=17) diagrams.

- $3 \times 3 = 9$ for central cutting
- $3 \times 2 = 6$ for non-central double scattering
- $3 \times 2 = 6$ for non-central mixed scattering

Note that 4 of the mixed scattering diagram vanishes due to their vanishing phase factor.

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[3] B.W. Zhang, X.N. Wang, NPA **720** (2003) 429-451
[4] E. Wang, X.N. Wang, PRL **89**(2002) 162301

single scattering(g-rad)

$$
\begin{aligned}
W^{A\mu\nu}_{(a)} = & \frac{g^4}{N_c}\int \frac{d^4 l_g}{(2\pi)^4} 2\pi\delta(l_g^2) \int \frac{d^4 l_q}{(2\pi)^4} 2\pi\delta(l_q^2) \\
& \times \int d^4 y_0 e^{iq\cdot y_0} \int d^4 z \int d^4 z' \int d^4 z_1 \int d^4 z_1' \\
& \times \int \frac{d^4 q_1}{(2\pi)^4} e^{-iq_1\cdot(y_0-z)} \int \frac{d^4 l_1}{(2\pi)^4} e^{-il_1\cdot(z-z_1)} e^{-il_g\cdot(z_1-z_1')} \\
& \times e^{-il_q\cdot(z-z')} \int \frac{d^4 l_1'}{(2\pi)^4} e^{-il_1'\cdot(z_1'-z')} \int \frac{d^4 q_1'}{(2\pi)^4} e^{-iq_1'\cdot(z'-y_0')} \\
& \times \langle A|\bar{\psi}(y_0)\gamma^\mu \frac{\not q_1}{q_1^2 - i\epsilon}\gamma_{\alpha_0} T^{d_0} \not l_q G^{\alpha_0\beta_1}_{d_0 b_1}(l_1) \\
& \times \Gamma^{b_1 c_1 d_1}_{\beta_1\gamma_1\alpha_1}(-l_1, -k_1, l_g) A^{\gamma_1}_{c_1}(z_1) \tilde{G}^{\alpha_1\alpha_1'}_{d_1 d_1'}(l_g) \Gamma^{d_1' c_1' b_1'}_{\alpha_1'\gamma_1'\beta_1'}(-l_g', k_1', l_1') A^{\gamma_1'}_{c_1'}(z_1') \\
& \times G^{\beta_1'\alpha_0'}_{b_1' d_0'}(l_1')\gamma_{\alpha_0'} T^{d_0'} \frac{\not q_1'}{q_1'^2 + i\epsilon}\gamma^\nu \psi(0)|A\rangle \qquad (13)
\end{aligned}
$$

Notice the triple gluon vertex Γ factor in the current tensor.

single scattering(g-rad)

The double differential hadronic tensor from all 17 cut diagrams contributions is given as:

$$\frac{dW^{A\mu\nu}}{dy dl_{g\perp}^2} = (-g_{\perp}^{\mu\nu}) A C_p^A C_A \frac{\alpha_s}{2\pi} \frac{P(y)}{l_{g\perp}^2} f_q(x_B + x_L) \times \int_0^{L^-} dZ^- \{\cdots\} \tag{14}$$

Then, the gluon radiation spectrum is:

$$\frac{dN_g}{dy dl_{g\perp}^2} = C_A \frac{\alpha_s}{2\pi} \frac{P(y)}{l_{g\perp}^2} \left[1 + \int_0^{L^-} dZ^- \{\cdots\} \right] \tag{15}$$

where the kernel is same for zero-th order.

Transport coefficient

We now look at the complete expression of the gluon radiation spectrum.

$$\begin{aligned}
\frac{dN_g}{dy dl_\perp^2} = \; & C_A \frac{\alpha_s}{2\pi} \frac{P(y)}{l_{g\perp}^2} \left[1 + \int_0^{L^-} dZ^- \left\{ \frac{1}{[1+(1-y)^2]} \right. \right. \\
& \times \left[\left(-\frac{4-3y^2+y^3}{2} f(x_L p^+ Z^-) - \frac{C_F}{C_A} y^2 (1-y)[2 - f(x_L p^+ Z^-)] \right) \frac{D_{L1}}{y q^-} \right. \\
& \left. + \left(\frac{8-y^2+y^3-2y^4}{4} f(x_L p^+ Z^-) + \frac{C_F}{C_A} y^3 \{ y - (1-y)[2 - f(x_L p^+ Z^-)] \} \right) \frac{D_{L2}}{(y q^-)^2} \right] \\
& \left. \left. + 2 \left(f(x_L p^+ Z^-) - \frac{1}{2} y f(x_L p^+ Z^-) + \frac{C_F}{C_A} y^2 \right) \frac{D_{T2}}{l_{g\perp}^2} \right\} \right]
\end{aligned} \tag{16}$$

where we have:

- $C_A P(y) = C_A P(y)_{q \to q\gamma} = \frac{C_A}{C_F} P(y)_{q \to qg}$
- D_{L1} and D_{L2} are the longitudinal drag $\hat{e}$ and longitudinal momentum diffusion $\hat{e}_2$.
- $D_{T2} = g^2 \frac{C_F}{N_c^2 - 1} \int dz^- \frac{\rho}{2p^+} \langle p | \partial_\perp A^+(z^-) \partial_\perp A^+(0) | p \rangle = \frac{1}{2} \hat{q}_q$ is the transverse momentum diffusion (transport coefficient), $\frac{C_A}{C_F} \hat{q}_q = \hat{q}_g$
- $f(x) = 2 - 2\cos(x) = 4 \sin^2 \left(\frac{Z^-}{2\tau_f} \right)$ is related to the formation time.

Evolution equation

The medium-modified DGLAP fragmentation function is then:

$$\frac{\partial \tilde{D}(z, q^-, Q^2)}{\partial \ln Q^2} = \frac{\alpha_s}{2\pi} \int \frac{dy}{y} P(y) \Big[\tilde{D}(z/y, q^-, Q^2) + \int d\zeta K(\zeta, y, q^-, Q^2) \tilde{D}(z/y, q^- y, Q^2) \Big] \quad (17)$$

where we have used:

- the kernel $K = \left(\frac{4\hat{q}}{Q^2}\right) \sin^2\left(\frac{\zeta}{2\tau_f}\right)$; $\zeta = Z^-$;
- transverse momentum $l_{g\perp}^2 = Q^2$;
- formation time $\tau_{form} = \frac{2q^- y(1-y)}{l_{g\perp}^2}$

Detailed Balance

We now try to include the detailed balance equation by introducing the Bose enhancement and absorption factor into the gluon radiation spectrum. where

$$\left.\begin{array}{ll} 1+N(\omega) & : \text{ for emission} \\ N(\omega) & : \text{ for absorption} \end{array}\right\} \omega = l_g^0 \tag{18}$$

with $N(\omega) = [e^{\omega/T} - 1]^{-1}$, T is local temperature. Knowing that the gluon radiation spectrum is an odd function:

$$\frac{dN_g^{abs}(\omega)}{d\omega dl_{g\perp}^2} = -\frac{dN_g^{rad}(-\omega)}{d\omega dl_{g\perp}^2} \tag{19}$$

We can work out the total gluon spectrum.

Detailed Balance cont.

Assume that ω represents the absolute energy of the radiated gluon, taking and values of $(0, +\infty)$,
and the momentum fraction z takes the value of $(-\infty, +\infty)$, where $z < 0$ represents absorption and $z > 0$ represents radiation. Then the total gluon spectrum is:

$$\begin{aligned} \frac{dN_g^{tot}}{d\omega} &= \int dz \int dl_\perp^2 \int d\tau \frac{dN_g^{rad}}{dzdl_\perp^2\, d\tau}(z, l_\perp^2, \tau) \\ &\times\{[1 + f_g(\omega)]\delta(\omega - zE)\theta(1 - z) + f_g(\omega)\delta(\omega + zE)\} \end{aligned} \tag{20}$$

Then the energy loss:

$$\begin{aligned} \Delta E &= \int_0^\infty d\omega \frac{dN_g^{tot}}{d\omega}\omega \\ &= \int dz \int dl_\perp^2 \int d\tau \int_0^\infty d\omega \frac{dN_g^{rad}}{dzdl_\perp^2\, d\tau}(z, l_\perp^2, \tau)\omega[1 + f_g(\omega)]\delta(\omega - zE)\theta(1 - z) \\ &+ \int dz \int dl_\perp^2 \int d\tau \int_0^\infty d\omega \frac{dN_g^{rad}}{dzdl_\perp^2\, d\tau}(z, l_\perp^2, \tau)\omega\ f_g(\omega)\delta(\omega + zE) \end{aligned} \tag{21}$$

Detailed Balance cont.

where the first term is the radiation effect and the second term is the absorption effect.
Since $\omega = (0, +\infty)$, integrating $\int_0^\infty d\omega\delta(\omega + zE)$ force z to take only negative values, i.e. $\theta(-z)$.

$$\begin{aligned}
\Delta E &= \int dz \int dl_\perp^2 \int d\tau \frac{dN_g^{rad}}{dzdl_\perp^2 d\tau}(z, l_\perp^2, \tau)(zE)[1 + f_g(zE)]\theta(1 - z) \\
&\quad + \int dz \int dl_\perp^2 \int d\tau \frac{dN_g^{rad}}{dzdl_\perp^2 d\tau}(z, l_\perp^2, \tau)(-zE) f_g(-zE)\theta(-z) \\
&= \int_0^1 dz \int dl_\perp^2 \int d\tau \frac{dN_g^{rad}}{dzdl_\perp^2 d\tau}(z, l_\perp^2, \tau)(zE)[1 + f_g(zE)] \\
&\quad + \int_{-\infty}^0 dz \int dl_\perp^2 \int d\tau \frac{dN_g^{rad}}{dzdl_\perp^2 d\tau}(z, l_\perp^2, \tau)(-zE) f_g(-zE) \qquad (22)
\end{aligned}$$

We can change the integration variable for the absorption term from dz to dz', i.e. $z' = -z$ by the following substitution:

$$\int_{-\infty}^0 dz \int dz' \delta(z + z') \Rightarrow -\int_\infty^0 dz' = \int_0^\infty dz' \qquad (23)$$

Detailed Balance cont.

Then the energy loss:

$$\Delta E = \int_0^1 dz \int dl_\perp^2 \int d\tau \frac{dN_g^{rad}}{dz dl_\perp^2 d\tau}(z, l_\perp^2, \tau)(zE)[1 + f_g(zE)] + \int_0^\infty dz' \int dl_\perp^2 \int d\tau \frac{dN_g^{rad}}{dz' dl_\perp^2 d\tau}(-z', l_\perp^2, \tau)(z'E) f_g(z'E) \tag{24}$$

Where we have:

$$\begin{aligned} \frac{dN_g^{rad}}{dz dl_\perp^2 d\tau}(z, l_\perp^2, \tau) &= C_A \frac{2\alpha_s}{\pi} \left[\frac{1 + (1-z)^2}{z}\right] \frac{\hat{q}}{l_\perp^2 (l_\perp^2 + \mu^2)} \sin^2\left[\frac{l_\perp^2 \tau}{4Ez(1-z)}\right] \\ \frac{dN_g^{rad}}{dz' dl_\perp^2 d\tau}(-z', l_\perp^2, \tau) &= -C_A \frac{2\alpha_s}{\pi} \left[\frac{1 + (1+z')^2}{z'}\right] \frac{\hat{q}}{l_\perp^2 (l_\perp^2 + \mu^2)} \sin^2\left[\frac{-l_\perp^2 \tau}{4Ez'(1+z')}\right] \end{aligned} \tag{25}$$

Detailed Balance cont.

Finally, we can rewrite this as:

$$\Delta E = \Delta E_a + \Delta E_b + \Delta E_c \tag{26}$$

Where

$$\begin{aligned}
\Delta E_a &= \int_0^1 dz \int dl_\perp^2 \int d\tau \frac{dN_g^{rad}}{dz dl_\perp^2 d\tau}(z, l_\perp^2, \tau)(zE) \\
\Delta E_a &= \int_0^1 dz \int dl_\perp^2 \int d\tau \frac{dN_g^{rad}}{dz dl_\perp^2 d\tau}(z, l_\perp^2, \tau)(zE) f_g(zE) \\
\Delta E_c &= \int_0^\infty dz \int dl_\perp^2 \int d\tau \frac{dN_g^{rad}}{dz dl_\perp^2 d\tau}(-z, l_\perp^2, \tau)(zE) f_g(zE)
\end{aligned} \tag{27}$$

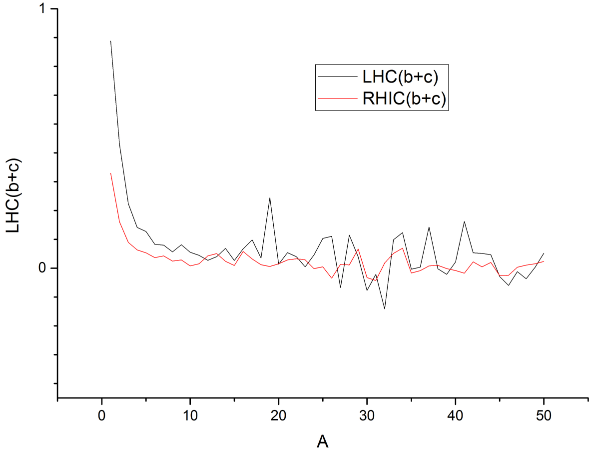
Work in progress

- currently working on implementing the detailed balanced gluon spectrum in the software
- testing whether numerical results will fit experimental data (R_{AA})
- using RHIC and LHC kinematics at various centrality.
- radiative and collisional energy loss included.
- using parton evolution at both vaccum and radiation medium.

Thank you.

Effects of detailed balance

We wanted to see how the detailed balance relation (temperature dependent) can affect the overall energy loss mechanism.



Effects of detailed balance cont.