Higher twist effects in e^+e^- annihilation at high energies

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Content

Motivation

Process Involving Hadron States in Initial or Final State

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PDFs and FFs

- Transverse Momentum Dependence
- Spin Dependence

$\Leftarrow$

Experiments

- Azimuthal Asymmetries
- Spin Asymmetries

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$\Uparrow$

Collinear Expansion $\Rightarrow$ Higher Twist Terms

PDFs

R.K. Ellis, W. Furmanski and R. Petronzio

NPB 1982,1983

J.W. Qiu, G. Sterman NPB, 1991

TMD PDFs

Z.T. Liang and X.N. Wang

PRD, 2007

Collinear Expansion

$\swarrow$ $\downarrow$ $\searrow$

$e + p \rightarrow e + X$

$\Downarrow$

Gauge Invariant PDFs

Higher twist terms

Twist-4 in SIDIS

Y.K. Song,

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TMD PDFs

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TMD PDFs

Azimuthal Asymmetries

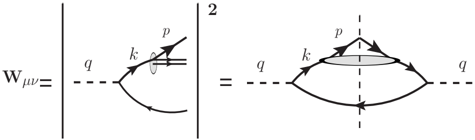
Fragmentation Functions (Leading Order Twist-3 Contributions)

$e^+e^-\rightarrow \mathrm{h}+X$	Tree Level	FFs	(PRD 89, (2014) 014024)
$e^+e^-\rightarrow \mathrm{h}+\bar{q}+X$	$\Rightarrow$	TMD FFs	(arXiv:1410.4314)

Collinear Expansion

$$d\sigma = \frac{g_Z^4}{32s} L_{\mu'\nu'}(l_1, l_2) D_F^{\mu'\mu}(q) D_F^{\nu'\nu*}(q) W_{\mu\nu}(q, p, S) \frac{d^3 p}{(2\pi)^2 2E_p}$$

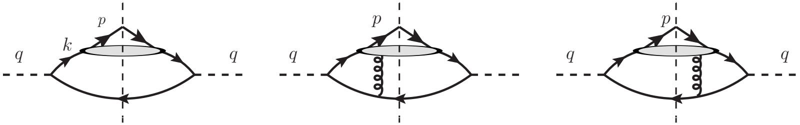
$$d\sigma = \frac{g_Z^4}{32s} L_{\mu'\nu'}(l_1,l_2) D_F^{\mu'\mu}(q) D_F^{\nu'\nu*}(q) W_{\mu\nu}(q,p,S) \frac{d^3p}{(2\pi)^2 2E_p}$$



$$W_{\mu\nu}^{(0)}(q,p,S) = \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\hat{H}_{\mu\nu}^{(0)}(k,q)\hat{\Pi}^{(0)}(k,p,S)\right]$$

$$\hat{H}_{\mu\nu}^{(0)}(k) = \Gamma_\mu^q (\not{q} - \not{k}) \Gamma_\nu^q (2\pi)\delta_+\left((q-k)^2\right)$$

$$\hat{\Pi}^{(0)}(k) = \frac{1}{2\pi} \sum_X \int d^4\xi\, e^{-ik\xi} \langle 0|\psi(0)|hX\rangle\langle hX|\bar{\psi}(\xi)|0\rangle$$



$$W_{\mu\nu} = W_{\mu\nu}^{(0)} + W_{\mu\nu}^{(1)} + W_{\mu\nu}^{(2)} + \cdots$$

Where,

$$W_{\mu\nu}^{(1,L)}(q,p,S) = \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\mathrm{Tr}[\hat{H}_{\mu\nu}^{(1,L)\rho}(k_1,k_2,q)\hat{\Pi}_{\rho}^{(1,L)}(k_1,k_2,p,S)]$$

$$\hat{H}_{\mu\nu}^{(1,L)\rho}(k_1,k_2) = \Gamma_{\mu}^{q}(\not{q}-\not{k}_1)\gamma^{\rho}\frac{\not{k}_2-\not{q}}{(k_2-q)^2-i\epsilon}\Gamma_{\nu}^{q}(2\pi)\delta_{+}\left((q-k_1)^2\right)$$

$$\hat{\Pi}_{\rho}^{(1,L)}(k_1,k_2) = \frac{1}{2\pi}\sum_X\int d^4\xi\, d^4\eta\, e^{-ik_1\xi}e^{-i(k_2-k_1)\eta}\langle 0|gA_{\rho}(\eta)\psi(0)|hX\rangle\langle hX|\bar{\psi}(\xi)|0\rangle$$

Collinear Expansion

Taylor Expansion

$$\hat{H}^{(0)}_{\mu\nu}(k) = \hat{H}^{(0)}_{\mu\nu}(z) + \frac{\partial \hat{H}^{(0)}_{\mu\nu}(k)}{\partial k_\rho}\bigg|_{k=p^+/z} \omega_\rho^{\ \rho'} k_{\rho'} + \cdots$$

$$= \hat{H}^{(0)}_{\mu\nu}(z) - (\hat{H}^{(1L)\rho}_{\mu\nu}(z,z) + \hat{H}^{(1R)\rho}_{\mu\nu}(z,z))\omega_\rho^{\ \rho'} k_{\rho'} + \cdots$$

$$\hat{H}^{(1L)\rho}_{\mu\nu}(k_1,k_2) = \hat{H}^{(1L)\rho}_{\mu\nu}(z_1,z_2) + \cdots$$

Gluon Fields

$$A_\rho = A^+ \bar{n}_\rho + \omega_\rho^{\ \rho'} A_{\rho'}$$

Ward Identities

$$p^+ \bar{n}_\rho \hat{H}^{(1L)\rho}_{\mu\nu}(z_1,z_2) = -\frac{z_1 z_2}{z_2 - z_1 - i\epsilon} \hat{H}^{(0)}_{\mu\nu}(z_1)$$

$$W^{(0)}_{\mu\nu} = H^{(0)}(k)\Phi_0 = H^{(0)}_{\mu\nu}(z) \times \phi_0 + H^{(1)}_{\mu\nu}(z) \times \phi_1 + \cdots$$

$$W^{(1)}_{\mu\nu} = H^{(1)}(k)\Phi_1 = H^{(0)}_{\mu\nu}(z) \times \phi_2 + H^{(1)}_{\mu\nu}(z) \times \phi_3 + \cdots$$

$$W_{\mu\nu} = W^{(0)}_{\mu\nu} + W^{(1)}_{\mu\nu} + W^{(2)}_{\mu\nu} + \cdots \qquad \Rightarrow \qquad W_{\mu\nu} = \tilde{W}^{(0)}_{\mu\nu} + \tilde{W}^{(1)}_{\mu\nu} + \tilde{W}^{(2)}_{\mu\nu} + \cdots$$

Inclusive Process

$$\tilde{W}^{(0)}_{\mu\nu} = \frac{1}{2}\mathrm{Tr}\left[\hat{\mathrm{h}}^{(0)}_{\mu\nu}\,\hat{\Xi}^{(0)}(z_B, p, S)\right] \qquad \Rightarrow \qquad \text{twist 2, twist 3, } \cdots$$

$$\tilde{W}^{(1L)}_{\mu\nu} = -\frac{1}{4p\cdot q}\mathrm{Tr}\left[\hat{\mathrm{h}}^{(1)\rho}_{\mu\nu}\,\omega_{\rho}^{\ \rho'}\hat{\Xi}^{(1)}_{\rho'}(z_B, p, S)\right] \qquad \Rightarrow \qquad \text{twist 3, } \cdots$$

Semi-inclusive Process

$$\tilde{W}^{(0,\mathrm{si})}_{\mu\nu} = \frac{1}{2}\mathrm{Tr}\left[\hat{\mathrm{h}}^{(0)}_{\mu\nu}\hat{\Xi}^{(0)}(z_B, k'_\perp, p, S)\right] \qquad \Rightarrow \qquad \text{twist 2, twist 3, } \cdots$$

$$\tilde{W}^{(1,L,\mathrm{si})}_{\mu\nu} = -\frac{1}{4p\cdot q}\mathrm{Tr}\left[\hat{\mathrm{h}}^{(1)\rho}_{\mu\nu}\,\omega_{\rho}^{\ \rho'}\hat{\Xi}^{(1,L)}_{\rho'}(z_B, k'_\perp, p, S)\right] \qquad \Rightarrow \qquad \text{twist 3, } \cdots$$

Collinear Expansion

Inclusive Process

$$\hat{\Xi}^{(0)}(z,p,S) = \sum_X \int \frac{p^+ d\xi^-}{2\pi} e^{-ik^+\xi^-} \langle 0|\mathcal{L}^\dagger(0^-,\infty)\psi(0)|\mathrm{hX}\rangle\langle \mathrm{hX}|\bar{\psi}(\xi^-)\mathcal{L}(\xi^-,\infty)|0\rangle$$

$$\hat{\Xi}^{(1)}_\rho(z,p,S) = \sum_X \int \frac{p^+ d\xi^-}{2\pi} e^{-ik^+\xi^-} \langle 0|\mathcal{L}^\dagger(0,\infty)[D_\rho(0)\psi(0)]|\mathrm{hX}\rangle\langle \mathrm{hX}|\bar{\psi}(\xi^-)\mathcal{L}(\xi^-,\infty)|0\rangle$$

Semi-inclusive Process

$$\hat{\Xi}^{(0)}(z,k_\perp,p,S) = \sum_X \int \frac{p^+ d\xi^- d^2\xi_\perp}{2\pi} e^{-ik^+\xi^- + ik_\perp\cdot\xi_\perp} \langle 0|\mathcal{L}^\dagger(0,\infty)\psi(0)|\mathrm{hX}\rangle\langle \mathrm{hX}|\bar{\psi}(\xi)\mathcal{L}(\xi,\infty)|0\rangle$$

$$\hat{\Xi}^{(1)}_\rho(z,k_\perp,p,S) = \sum_X \int \frac{p^+ d\xi^- d^2\xi_\perp}{2\pi} e^{-ik^+\xi^- + ik_\perp\cdot\xi_\perp} \langle 0|\mathcal{L}^\dagger(0,\infty)\hat{D}_\rho(0)\psi(0)|\mathrm{hX}\rangle\langle \mathrm{hX}|\bar{\psi}(\xi)\mathcal{L}(\xi,\infty)|0\rangle$$

$$W_0^{\mu\nu} \propto \mathrm{Tr}[\gamma^\mu \not{n} \gamma^\nu \hat{\Xi}^{(0)}]$$

Gamma Matrices Expansion

$$\hat{\Xi}^{(0)}(z,p,S,k_\perp) = \Xi_\alpha^{(0)}(z,p,S,k_\perp)\gamma^\alpha + \tilde{\Xi}_\alpha^{(0)}(z,p,S,k_\perp)\gamma_5\gamma^\alpha + ...$$
$$\hat{\Xi}_\rho^{(1)}(z,p,S,k_\perp) = \Xi_{\rho\alpha}^{(1)}(z,p,S,k_\perp)\gamma^\alpha + \tilde{\Xi}_{\rho\alpha}^{(1)}(z,p,S,k_\perp)\gamma_5\gamma^\alpha + ...$$

Parity Conservation

$$\Xi_\alpha^{(0)}(V,A) = \Xi^{(0)\alpha}(\tilde{V},-\tilde{A})$$
$$\tilde{\Xi}_\alpha^{(0)}(V,A) = -\tilde{\Xi}^{(0)\alpha}(\tilde{V},-\tilde{A})$$

$\mathbf{V}$: Vector

$\mathbf{A}$: Axis Vector

$\tilde{V}^\mu = V_\mu$

Spin Density Matrix (Particle Rest Frame)

Spin $\frac{1}{2}$ $\rho = \begin{pmatrix} \rho_{++} & \rho_{+-} \\ \rho_{-+} & \rho_{--} \end{pmatrix} = \frac{1}{2}(1 + S^i \sigma^i)$

Spin 1 $\rho = \begin{pmatrix} \rho_{++} & \rho_{+0} & \rho_{+-} \\ \rho_{0+} & \rho_{00} & \rho_{0-} \\ \rho_{-+} & \rho_{-0} & \rho_{--} \end{pmatrix} = \frac{1}{3} + \frac{1}{2} S^i \sigma^i + T^{ij} \Sigma^{ij}$

Relativistic Case $p^\rho = (p^+, M^2/2p^+, \vec{0}_\perp)$

$$S^\rho = \lambda_h \left(\frac{p^+}{M} \bar{n}^\rho - \frac{M}{2p^+} n^\rho \right) + S_\perp^\rho$$

$$T^{\mu\nu} = \frac{1}{2} \Big\{ \frac{4}{3} S_{LL} \Big[(\frac{p^+}{M})^2 \bar{n}^\mu \bar{n}^\nu + (\frac{M}{2p^+})^2 n^\mu n^\nu - \frac{1}{2}(\bar{n}^{\{\mu} n^{\nu\}} - g_\perp^{\mu\nu}) \Big]$$
$$+ [\frac{p^+}{M} \bar{n}^{\{\mu} - \frac{M}{2p^+} n^{\{\mu}] S_{LT}^{\nu\}} + S_{TT}^{\mu\nu} \Big\}$$

Inclusive Process

$$\tilde{W}^{(0)}_{\mu\nu} = \frac{1}{2}\mathrm{Tr}\left[\hat{\mathrm{h}}^{(0)}_{\mu\nu}\gamma_\alpha\right]\Xi^{(0)\alpha}(z_B,p,S) + \frac{1}{2}\mathrm{Tr}\left[\hat{\mathrm{h}}^{(0)}_{\mu\nu}\gamma_5\gamma_\alpha\right]\tilde{\Xi}^{(0)\alpha}(z_B,p,S)$$

$$\tilde{W}^{(1L)}_{\mu\nu} = -\frac{1}{4p\cdot q}\mathrm{Tr}\left[\hat{\mathrm{h}}^{(1)\rho}_{\mu\nu}\gamma_\alpha\right]\omega_\rho^{\ \rho'}\Xi^{(1)\alpha}_{\rho'}(z_B,p,S) - \frac{1}{4p\cdot q}\mathrm{Tr}\left[\hat{\mathrm{h}}^{(1)\rho}_{\mu\nu}\gamma_5\gamma_\alpha\right]\omega_\rho^{\ \rho'}\tilde{\Xi}^{(1)}_{\rho'}(z_B,p,S)$$

$$\Xi^{(0)\alpha}(z,p,S) = \sum_X \int \frac{p^+ d\xi^-}{2\pi} e^{-ik^+\xi^-}\mathrm{Tr}[\gamma^\alpha \langle 0|\mathcal{L}^\dagger(0^-,\infty)\psi(0)|\mathrm{hX}\rangle\langle\mathrm{hX}|\bar{\psi}(\xi^-)\mathcal{L}(\xi^-,\infty)|0\rangle]$$

$$\Xi^{(1)\alpha}_\rho(z,p,S) = \sum_X \int \frac{p^+ d\xi^-}{2\pi} e^{-ik^+\xi^-}\mathrm{Tr}[\gamma^\alpha \langle 0|\mathcal{L}^\dagger(0,\infty)[D_\rho(0)\psi(0)]|\mathrm{hX}\rangle\langle\mathrm{hX}|\bar{\psi}(\xi^-)\mathcal{L}(\xi^-,\infty)|0\rangle]$$

$$\cdots$$

Lorentz Structures Up to Twist-3

Spin Independent	$z\Xi^{(0)\alpha} = p^\alpha D_1(z)$
Spin Vector	$z\Xi^{(0)\alpha} = M\epsilon_\perp^{\alpha\gamma} S_{\perp\gamma} D_T(z)$ $z\tilde{\Xi}^{(0)\alpha} = \lambda_\mathrm{h} p^\alpha \Delta D_{1L}(z) + MS_\perp^\alpha \Delta D_T(z)$ $z\Xi^{(1)\rho\alpha} = M\epsilon_\perp^{\rho\gamma} S_{\perp\gamma} p^\alpha \xi_{\perp S}^{(1)}(z)$ $z\tilde{\Xi}^{(1)\rho\alpha} = iMS_\perp^\rho p^\alpha \tilde{\xi}_{\perp S}^{(1)}(z)$
Spin Tensor	$z\Xi^{(0)\alpha} = S_{LL} p^\alpha D_{1LL}(z) + MS_{LT}^\alpha D_{LT}(z)$ $z\tilde{\Xi}^{(0)\alpha} = M\epsilon_\perp^{\alpha\gamma} S_{LT,\gamma} \Delta D_{LT}(z)$ $z\Xi^{(1)\rho\alpha} = MS_{LT}^\rho \xi_{LTS}^{(1)}(z) p^\alpha$ $z\tilde{\Xi}^{(1)\rho\alpha} = iM\epsilon_\perp^{\rho\gamma} S_{LT,\gamma} \tilde{\xi}_{LTS}^{(1)}(z) p^\alpha$

Leading twist fragmentation functions (possibility density)

$D_1(z)$ quark $\longrightarrow \sum_S$ hadron

$\Delta D_{1L}(z)$ longitudinally polarized quark $\longrightarrow$ longitudinally polarized hadron

$D_{1LL}(z)$ quark $\longrightarrow$ LL polarized hadron

Twist-3 fragmentation functions

Vector Polarized Hadron

$D_T(z)$, $\Delta D_T(z)$

Tensor Polarized Hadron

$D_{LT}(z)$, $\Delta D_{LT}(z)$

Gauge Invariant Hadronic Tensor

Spin-0

$$\tilde{W}_{\mu\nu} = -\frac{2}{z}\left(c_1^q d_{\mu\nu} + i c_3^q \epsilon_{\perp\mu\nu}\right) D_1(z)$$

Spin-$\frac{1}{2}$

$$\begin{aligned}\tilde{W}_{\mu\nu} = \frac{2}{z}\Big\{ & -\left(c_1^q d_{\mu\nu} + i c_3^q \epsilon_{\perp\mu\nu}\right) D_1(z) + \lambda_{\mathrm{h}} \left(c_3^q d_{\mu\nu} + i c_1^q \epsilon_{\perp\mu\nu}\right) \Delta D_{1L}(z) \\ & + \frac{M}{p\cdot q}\left[c_1^q (q-2p/z)_{\{\mu}\epsilon_{\perp\nu\}\gamma} S_\perp^\gamma + i c_3^q (q-2p/z)_{[\mu} S_{\perp\nu]}\right] D_T(z) \\ & - \frac{M}{p\cdot q}\left[c_3^q (q-2p/z)_{\{\mu} S_{\perp\nu\}} - i c_1^q (q-2p/z)_{[\mu}\epsilon_{\perp\nu]\gamma} S_\perp^\gamma\right] \Delta D_T(z)\Big\}\end{aligned}$$

Spin-1

$$\begin{aligned}\tilde{W}_{\mu\nu} = \frac{2}{z}\Big\{ & -\left(c_1^q d_{\mu\nu} + i c_3^q \epsilon_{\perp\mu\nu}\right) [D_1 + S_{LL} D_{1LL}] + \lambda_{\mathrm{h}} \left(c_3^q d_{\mu\nu} + i c_1^q \epsilon_{\perp\mu\nu}\right) \Delta D_{1L} \\ & + \frac{M}{p\cdot q}\left[c_1^q (q-2p/z)_{\{\mu}\epsilon_{\perp\nu\}\gamma} S_\perp^\gamma + i c_3^q (q-2p/z)_{[\mu} S_{\perp\nu]}\right] D_T(z) \\ & + \frac{M}{p\cdot q}\left[c_1^q (q-2p/z)_{\{\mu} S_{LT,\nu\}} - i c_3^q (q-2p/z)_{[\mu}\epsilon_{\perp\nu]\gamma} S_{LT}^\gamma\right] D_{LT}(z) \\ & - \frac{M}{p\cdot q}\left[c_3^q (q-2p/z)_{\{\mu} S_{\perp\nu\}} - i c_1^q (q-2p/z)_{[\mu}\epsilon_{\perp\nu]\gamma} S_\perp^\gamma\right] \Delta D_T(z) \\ & - \frac{M}{p\cdot q}\left[c_3^q (q-2p/z)_{\{\mu}\epsilon_{\perp\nu\}\gamma} S_{LT}^\gamma + i c_1^q (q-2p/z)_{[\mu} S_{LT,\nu]}\right] \Delta D_{LT}(z)\Big\}\end{aligned}$$

Spin-0 Hadrons - $e^+e^- \to Z_0 \to \mathrm{hX}$

$$\frac{d^2\sigma}{dzdy} = \sum_q \frac{2\pi\alpha^2}{Q^2}\chi T_0^q(y) D_1^{q\to\mathrm{h}}(z).$$

$$D_1^{q\to\mathrm{h}}(z) = \frac{z}{4}\sum_X \int \frac{d\xi^-}{2\pi} e^{-ip^+\xi^-/z}\mathrm{Tr}\left[\gamma^+\langle 0|\mathcal{L}^\dagger(0,\infty)\psi(0)|hX\rangle\langle hX|\bar{\psi}(\xi^-)\mathcal{L}(\xi^-,\infty)|0\rangle\right]$$

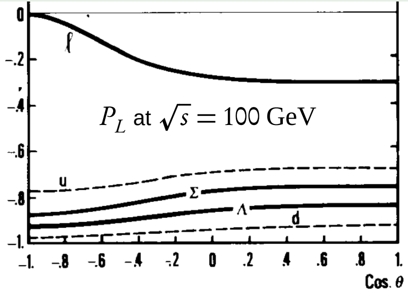
For $e^+e^- \to \gamma^* \to q\bar{q} \to \mathrm{hX}$,

$$\frac{d\sigma^{\mathrm{em}}}{dz} = \sum_q \frac{4\pi\alpha^2}{3Q^2} e_q^2 D_1^{q\to\mathrm{h}}(z)$$

Spin-1/2 Hadrons - $e^+e^- \to Z_0 \to \mathrm{hX}$

$$\frac{d^2\sigma}{dz dy} = \chi \frac{2\pi\alpha^2}{Q^2}\Big\{\big[T_0(y)D_1(z) + \lambda_{\mathrm{h}} T_1(y)\Delta D_{1L}(z)\big] + \frac{4M}{zQ^2}\big[T_2(y)D_T(z)\epsilon_\perp^{l_\perp S_\perp} + T_3(y)\Delta D_T(z) l_\perp \cdot S_\perp\big]\Big\}$$

$$P_{L\mathrm{h}}(z,y) = \frac{\sum_q P_q(y) T_0^q(y) \Delta D_{1L}^{q\to \mathrm{h}}(z)}{\sum_q T_0^q(y) D_1^{q\to \mathrm{h}}(z)}$$



NPB, 1980, J.E. Augustin, F.M. Renard

P_q: Longitudinal polarization of quark

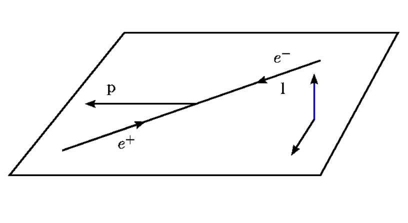
$\Delta D_{1L}^{q\to \mathrm{h}}(z)$: Longitudinally polarized quark $\longrightarrow$ Longitudinally polarized hadron

Spin-1/2 Hadrons - $e^+e^- \to Z_0 \to \mathrm{hX}$

$$\frac{d^2\sigma}{dzdy} = \chi \frac{2\pi\alpha^2}{Q^2}\Big\{\big[T_0(y)D_1(z) + \lambda_\mathrm{h} T_1(y)\Delta D_{1L}(z)\big] + \frac{4M}{zQ^2}\big[T_2(y)D_T(z)\epsilon_\perp^{l_\perp S_\perp} + T_3(y)\Delta D_T(z) l_\perp \cdot S_\perp\big]\Big\}$$

$$P_{\mathrm{hx}}(z,y) = -\frac{4M}{zQ}\frac{\sum_q \tilde{T}_3^q(y)\Delta D_T^{q\to\mathrm{h}}(z)}{\sum_q T_0^q(y)D_1^{q\to\mathrm{h}}(z)}$$

$$P_{\mathrm{hy}}(z,y) = \frac{4M}{zQ}\frac{\sum_q \tilde{T}_2^q(y) D_T^{q\to\mathrm{h}}(z)}{\sum_q T_0^q(y)D_1^{q\to\mathrm{h}}(z)}$$



P_q: Longitudinal polarization of quark

$\Delta D_{1L}^{q\to\mathrm{h}}(z)$: Longitudinally polarized quark $\longrightarrow$ Longitudinally polarized hadron

Spin-1/2 Hadrons - $e^+e^- \to \gamma^* \to \mathrm{hX}$

$$\frac{d\sigma^{\mathrm{em}}}{dzdy} = \frac{2\pi\alpha^2 e_q^2}{Q^2}\left\{A(y)D_1(z) + \frac{4M}{zQ^2}B(y)D_T(z)\epsilon_\perp^{l_\perp S_\perp}\right\}.$$

$$P_{L\mathrm{h}}(z,y) = 0 \qquad P_{\mathrm{hx}}^{\mathrm{em}}(z,y) = 0$$

$$P_{\mathrm{hy}}^{\mathrm{em}}(z,y) = \frac{4M}{zQ}\frac{\sqrt{y(1-y)}(1-2y)}{(1-y)^2+y^2}\frac{\sum_q e_q^2 D_T^{q\to\mathrm{h}}(z)}{\sum_q e_q^2 D_1^{q\to\mathrm{h}}(z)}$$

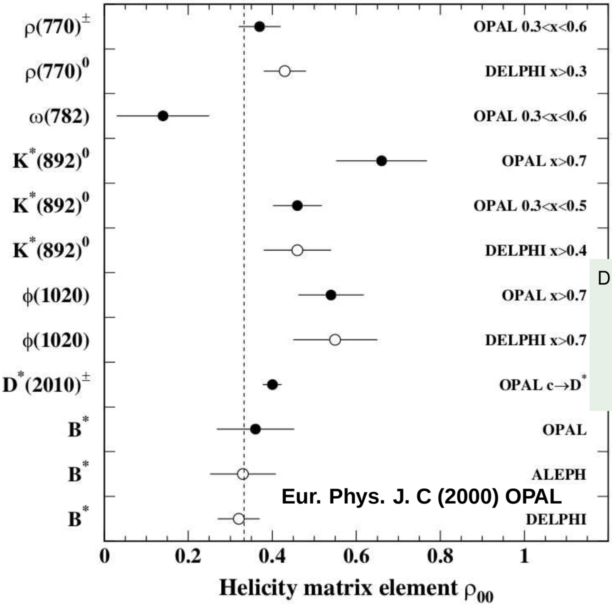
$$MS_\perp^2 D_T(z) = \frac{z}{4}\sum_X \int \frac{p^+ d\xi^-}{2\pi} e^{-ip^+\xi^-/z}\epsilon_\perp^{\alpha\gamma} S_{\perp\gamma}\mathrm{Tr}[\gamma_\alpha \langle 0|\mathcal{L}^\dagger(0,\infty)\psi(0)|\mathrm{hX}\rangle\langle \mathrm{hX}|\bar{\psi}(\xi^-)\mathcal{L}(\xi^-,\infty)|0\rangle]$$

Spin-1 Hadrons - $e^+e^- \to Z_0 \to \mathrm{hX}$

$$\frac{d\sigma}{dzdy} = \chi \frac{2\pi\alpha^2}{Q^2}\Big\{\big[T_0(y)D_1(z) + T_0(y)S_{LL}D_{1LL}(z) + \lambda_{\mathrm{h}}T_1(y)\Delta D_{1L}(z)\big]$$
$$+ \frac{4M}{zQ^2}\big[T_2(y)\epsilon_\perp^{l_\perp S_\perp}D_T(z) + T_3(y)l_\perp \cdot S_\perp \Delta D_T(z)\big]$$
$$+ \frac{4M}{zQ^2}\big[T_2(y)l_\perp \cdot S_{LT}D_{LT}(z) + T_3(y)\epsilon_\perp^{l_\perp S_{LT}}\Delta D_{LT}(z)\big]\Big\}$$

Spin alignment

$$\rho_{00} = \frac{1}{3} - \frac{1}{3}\frac{\sum_q t_0^q D_{1LL}^{q\to \mathrm{h}}(z)}{\sum_q t_0^q D_1^{q\to \mathrm{h}}(z)}$$



Eur. Phys. J. C (2000) OPAL

$)\Delta D_{1L}(z)]$

DELPHI Data for k^{*0}

0.1 $\sim$ 0.3	0.33 $\pm$ 0.05
z $\geq$ 0.3	0.41 $\pm$ 0.07
z $\geq$ 0.4	0.46 $\pm$ 0.08
z $\geq$ 0.5	0.47 $\pm$ 0.10

PLB (1997) DELPHI

Spin-1 Hadrons - $e^+e^- \to \gamma^* \to \mathrm{hX}$

$$\frac{d\sigma^{\mathrm{em}}}{dzdy} = \frac{2\pi\alpha^2 e_q^2}{Q^2}\Big\{A(y)\big[D_1(z) + S_{LL}D_{1LL}(z)\big] + \frac{M}{zQ}\sqrt{y(1-y)}B(y)\big[|\vec{S}_\perp|\sin\phi_s D_T(z) - |\vec{S}_{LT}|\cos\phi_{LT}D_{LT}(z)\big]\Big\}.$$

Spin alignment

$$\rho_{00}^{\mathrm{em}} = \frac{1}{3} - \frac{1}{3}\frac{\sum_q e_q^2 D_{1LL}^{q\to\mathrm{h}}(z)}{\sum_q e_q^2 D_1^{q\to\mathrm{h}}(z)}.$$

Semi-inclusive Process

$$\tilde{W}^{(0)}_{\mu\nu} = \frac{1}{2}\mathrm{Tr}\left[\hat{\mathrm{h}}^{(0)}_{\mu\nu}\gamma_\alpha\right]\Xi^{(0)\alpha}(z_B, p, S, k'_\perp) + \frac{1}{2}\mathrm{Tr}\left[\hat{\mathrm{h}}^{(0)}_{\mu\nu}\gamma_5\gamma_\alpha\right]\tilde{\Xi}^{(0)\alpha}(z_B, p, S, k'_\perp)$$

$$\tilde{W}^{(1L)}_{\mu\nu} = -\frac{1}{4p\cdot q}\mathrm{Tr}\left[\hat{\mathrm{h}}^{(1)\rho}_{\mu\nu}\gamma_\alpha\right]\omega_\rho^{\ \rho'}\Xi^{(1)\alpha}_{\rho'}(z_B, p, S, k'_\perp) - \frac{1}{4p\cdot q}\mathrm{Tr}\left[\hat{\mathrm{h}}^{(1)\rho}_{\mu\nu}\gamma_5\gamma_\alpha\right]\omega_\rho^{\ \rho'}\tilde{\Xi}^{(1)}_{\rho'}(z_B, p, S, k'_\perp)$$

$$\Xi^{(0)\alpha}(z, k_\perp, p, S) = \sum_X \int \frac{p^+ d\xi^- d^2\xi_\perp}{2\pi} e^{-ik^+\xi^- + ik_\perp\cdot\xi_\perp}$$
$$\mathrm{Tr}[\gamma^\alpha \langle 0|\mathcal{L}^\dagger(0,\infty)\psi(0)|\mathrm{hX}\rangle\langle \mathrm{hX}|\bar{\psi}(\xi)\mathcal{L}(\xi,\infty)|0\rangle]$$

$$\Xi^{(1)\alpha}_\rho(z, k_\perp, p, S) = \sum_X \int \frac{p^+ d\xi^- d^2\xi_\perp}{2\pi} e^{-ik^+\xi^- + ik_\perp\cdot\xi_\perp}$$
$$\mathrm{Tr}[\gamma^\alpha \langle 0|\mathcal{L}^\dagger(0,\infty)\hat{D}_\rho(0)\psi(0)|\mathrm{hX}\rangle\langle \mathrm{hX}|\bar{\psi}(\xi)\mathcal{L}(\xi,\infty)|0\rangle]$$

$$\cdots$$

New Lorentz Structures up to twist-3

Spin Independent

$$z\Xi^{(0)}_{\alpha}(z,k_\perp,p) = k_{\perp\alpha}\hat{D}^\perp(z,k_\perp)$$
$$z\tilde{\Xi}^{(0)}_{\alpha}(z,k_\perp,p) = \epsilon_{\perp\alpha k_\perp}\Delta\hat{D}^\perp(z,k_\perp)$$
$$z\Xi^{(1)}_{\rho\alpha}(z,k_\perp,p) = p_\alpha k_{\perp\rho}\,\xi^{(1)}_\perp(z,k_\perp)$$
$$z\tilde{\Xi}^{(1)}_{\rho\alpha}(z,k_\perp,p) = i\,p_\alpha\epsilon_{\perp\rho k_\perp}\,\tilde{\xi}^{(1)}_\perp(z,k_\perp)$$

Spin Vector

$$z\Xi^{(0)}_{\alpha} = p_\alpha\frac{\epsilon_\perp^{k_\perp S_\perp}}{M}\hat{D}^\perp_{1T} + k_{\perp\alpha}\frac{\epsilon_\perp^{k_\perp S_\perp}}{M}\hat{D}^\perp_T + \lambda_h\epsilon_{\perp\alpha k_\perp}\hat{D}^\perp_L$$
$$z\tilde{\Xi}^{(0)}_{\alpha} = p_\alpha\frac{k_\perp\cdot S_\perp}{M}\Delta\hat{D}^\perp_{1T} + \frac{\epsilon_\perp^{k_\perp S_\perp}}{M}\epsilon_{\perp\alpha k_\perp}\Delta\hat{D}^\perp_T + \lambda_h k_{\perp\alpha}\Delta\hat{D}^\perp_L$$
$$z\Xi^{(1)}_{\rho\alpha} = p_\alpha\left[k_{\perp\rho}\frac{\epsilon_\perp^{k_\perp S_\perp}}{M}\xi^{(1)\perp}_T + \lambda_h\epsilon_{\perp\rho k_\perp}\xi^{(1)\perp}_L\right]$$
$$z\tilde{\Xi}^{(1)}_{\rho\alpha} = i\,p_\alpha\left[\frac{\epsilon_\perp^{k_\perp S_\perp}}{M}\epsilon_{\perp\rho k_\perp}\tilde{\xi}^{(1)\perp}_T + \lambda_h k_{\perp\rho}\tilde{\xi}^{(1)\perp}_L\right]$$

New Lorentz Structures (Spin Tensor)

$$z\Xi_\alpha^{(0)} = p_\alpha \frac{S_{LT}\cdot k_\perp}{M}\hat{D}_{1LT}^\perp + p_\alpha \frac{k_{\perp\gamma}k_{\perp\delta}S_{TT}^{\gamma\delta}}{M^2}\hat{D}_{1TT}^\perp$$

$$+ k_{\perp\alpha}S_{LL}\hat{D}_{LL}^\perp + k_{\perp\alpha}\frac{k_\perp\cdot S_{LT}}{M}\hat{D}_{LT}^\perp + S_{TT\alpha\beta}k_\perp^\beta\hat{D}_{TT}^{\perp A} + k_{\perp\alpha}\frac{k_{\perp\gamma}k_{\perp\delta}S_{TT}^{\gamma\delta}}{M^2}\hat{D}_{TT}^{\perp C}$$

$$z\tilde{\Xi}_\alpha^{(0)} = p_\alpha \frac{\epsilon_\perp^{k_\perp S_{LT}}}{M}\Delta\hat{D}_{1LT}^\perp + p_\alpha \frac{\epsilon_{\perp k_\perp \gamma}k_{\perp\delta}S_{TT}^{\gamma\delta}}{M^2}\Delta\hat{D}_{1TT}^\perp$$

$$+\epsilon_{\perp\alpha k_\perp}\left[S_{LL}\Delta\hat{D}_{LL}^\perp + \frac{k_\perp\cdot S_{LT}}{M}\Delta\hat{D}_{LT}^\perp + \frac{k_{\perp\gamma}k_{\perp\delta}S_{TT}^{\gamma\delta}}{M^2}\Delta\hat{D}_{TT}^{\perp C}\right] + \epsilon_{\perp\alpha\beta}S_{TT}^{\beta\gamma}k_{\perp\gamma}\Delta\hat{D}_{TT}^{\perp A}$$

$$z\Xi_{\rho\alpha}^{(1)} = p_\alpha\left[k_{\perp\rho}S_{LL}\xi_{LL}^\perp + k_{\perp\rho}\frac{k_\perp\cdot S_{LT}}{M}\xi_{LT}^\perp + S_{TT\rho\beta}k_\perp^\beta\xi_{TT}^{\perp A} + k_{\perp\rho}\frac{k_{\perp\gamma}k_{\perp\delta}S_{TT}^{\gamma\delta}}{M^2}\xi_{TT}^{\perp C}\right]$$

$$z\tilde{\Xi}_{\rho\alpha}^{(1)} = i\,p^\alpha\left[\epsilon_{\perp\rho k_\perp}S_{LL}\tilde{\xi}_{LL}^\perp + \epsilon_{\perp\rho k_\perp}k_\perp\cdot S_{LT}\,\tilde{\xi}_{LT}^\perp + \epsilon_{\perp\rho\beta}S_{TT}^{\beta\gamma}k_{\perp\gamma}\tilde{\xi}_{TT}^{\perp A} + \epsilon_{\perp\rho k_\perp}\frac{k_{\perp\gamma}k_{\perp\delta}S_{TT}^{\gamma\delta}}{M^2}\tilde{\xi}_{TT}^{\perp C}\right]$$

New leading twist fragmentation functions (possibility density)

$\hat{D}_{1T}^{\perp}(z, k_\perp)$ quark $\longrightarrow$ transversely polarized hadron

$\Delta\hat{D}_{1T}^{\perp}(z, k_\perp)$ longitudinally polarized quark $\longrightarrow$ transversely polarized hadron

$\hat{D}_{1LT}^{\perp}(z, k_\perp)$ quark $\longrightarrow$ LT polarized hadron

$\Delta\hat{D}_{1LT}^{\perp}(z, k_\perp)$ longitudinally polarized quark $\longrightarrow$ LT polarized hadron

$\hat{D}_{1TT}^{\perp}(z, k_\perp)$ quark $\longrightarrow$ TT polarized hadron

$\Delta\hat{D}_{1TT}^{\perp}(z, k_\perp)$ longitudinally polarized quark $\longrightarrow$ TT polarized hadron

New Twist-3 fragmentation functions

unpolarized hadron: 2; polarization vector: 4; polarization tensor: 8;

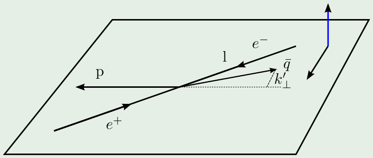
Spin-0 Hadrons - $e^+e^- \to Z_0 \to \mathrm{h} + \bar{\mathrm{q}} + \mathrm{X}$

$$\frac{d\sigma^{(\mathrm{si,unp})}}{dy\,dz\,d^2k'_\perp} = \frac{\alpha^2\chi}{2\pi Q^2}\Big\{T_0^q(y)\hat{D}_1(z,k'_\perp) + \frac{4}{zQ^2}\Big[T_2^q(y)l_\perp\cdot k'_\perp\hat{D}^\perp(z,k'_\perp) + T_3^q(y)\epsilon_\perp^{l_\perp k'_\perp}\Delta\hat{D}^\perp(z,k'_\perp)\Big]\Big\}$$

Azimuthal Asymmetries,

$$A_{\mathrm{unp}}^{\cos\varphi} = -\frac{2|\vec{k}'_\perp|}{zQ}\frac{\sum_q \tilde{T}_2^q(y)\hat{D}^{\perp q\to h}}{\sum_q T_0^q(y)\hat{D}_1^{q\to h}}$$

$$A_{\mathrm{unp}}^{\sin\varphi} = \frac{2|\vec{k}'_\perp|}{zQ}\frac{\sum_q \tilde{T}_3^q(y)\Delta\hat{D}^{\perp q\to h}}{\sum_q T_0^q(y)\hat{D}_1^{q\to h}}$$



Spin-0 Hadrons - $e^+e^- \to \gamma^* \to \mathrm{h} + \bar{\mathrm{q}} + \mathrm{X}$

$$\frac{d\sigma^{(\mathrm{si,unp,em})}}{dy\,dz\,d^2k'_\perp} = \frac{\alpha^2 e_q^2}{2\pi Q^2}\left\{A(y)\hat{D}_1(z,k'_\perp) + \frac{4l_\perp \cdot k'_\perp}{zQ^2}B(y)\hat{D}^\perp(z,k'_\perp)\right\}$$

Azimuthal Asymmetries,

$$A^{\cos\varphi}_{\mathrm{unp,em}}(z,y,k'_\perp) = -\frac{2|\vec{k}'_\perp|}{zQ}\frac{\tilde{B}(y)\sum_q e_q^2 D^{\perp q\to h}(z,k'_\perp)}{A(y)\sum_q e_q^2 D_1^{q\to h}(z,k'_\perp)}$$

$$A^{\sin\varphi}_{\mathrm{unp,em}}(z,y,k'_\perp) = 0$$

Spin-1/2 Hadrons - $e^+e^- \to Z_0 \to \mathrm{h} + \bar{\mathrm{q}} + \mathrm{X}$

$$\frac{d\sigma^{(\mathrm{si},1/2)}}{dydzd^2k'_\perp} = \frac{d\sigma^{(\mathrm{si,unp})}}{dydzd^2k'_\perp} + \frac{d\sigma^{(\mathrm{si,Vpol})}}{dydzd^2k'_\perp}$$

$$\begin{aligned}\frac{d\sigma^{(\mathrm{si,Vpol})}}{dydzd^2k'_\perp} = \frac{\alpha^2\chi}{2\pi Q^2}\Big\{ & T_0^q(y)\frac{\epsilon_\perp^{k'_\perp S_\perp}}{M}\hat{D}_{1T}^\perp(z,k'_\perp) + T_1^q(y)\big[\lambda_h \Delta\hat{D}_{1L}(z,k'_\perp) + \frac{k'_\perp\cdot S_\perp}{M}\Delta\hat{D}_{1T}^\perp(z,k'_\perp)\big] \\ & + \frac{4\lambda_h}{zQ^2}\big[T_2^q(y)\epsilon_\perp^{l_\perp k'_\perp}\hat{D}_L^\perp(z,k'_\perp) + T_3^q(y) l_\perp\cdot k'_\perp \Delta\hat{D}_L^\perp(z,k'_\perp)\big] \\ & + \frac{4\epsilon_\perp^{k'_\perp S_\perp}}{zMQ^2}\big[T_2^q(y) l_\perp\cdot k'_\perp \hat{D}_T^\perp(z,k'_\perp) + T_3^q(y)\epsilon_\perp^{l_\perp k'_\perp}\Delta\hat{D}_T^\perp(z,k'_\perp)\big] \\ & + \frac{4M}{zQ^2}\big[T_2^q(y)\epsilon_\perp^{l_\perp S_\perp}\hat{D}_T(z,k'_\perp) + T_3^q(y) l_\perp\cdot S_\perp \Delta\hat{D}_T(z,k'_\perp)\big]\Big\}.\end{aligned}$$

Polarization

$$P_{\mathrm{Lh}}(y,z,k'_\perp) = \frac{\sum_q T_0^q(y) P_q(y) \Delta\hat{D}_{1L}(z,k'_\perp)}{\sum_q T_0^q(y)\hat{D}_1(z,k'_\perp)}$$

$$P_{\mathrm{hn}}(y,z,k'_\perp) = -\frac{|\vec{k}'_\perp|}{M} \frac{\sum_q T_0^q(y)\hat{D}_{1T}^\perp(z,k'_\perp)}{\sum_q T_0^q(y)\hat{D}_1(z,k'_\perp)}$$

$$P_{\mathrm{ht}}(y,z,k'_\perp) = -\frac{|\vec{k}'_\perp|}{M} \frac{\sum_q P_q(y) T_0^q(y)\Delta\hat{D}_{1T}^\perp(z,k'_\perp)}{\sum_q T_0^q(y)\hat{D}_1(z,k'_\perp)}$$

Polarization

$$P_{\mathrm{Lh}}(y,z,k'_\perp) = \frac{\sum_q T_0^q(y) P_q(y) \Delta\hat{D}_{1L}(z,k'_\perp)}{\sum_q T_0^q(y)\hat{D}_1(z,k'_\perp)} \left[1 + \frac{M}{Q}\hat{\Delta}(y,z,k'_\perp)\right] + \frac{4}{zQ}\frac{\sum_q \left[\tilde{T}_2^q(y) k'_y \hat{D}_L^\perp(z,k'_\perp) - \tilde{T}_3^q(y) k'_x \Delta\hat{D}_L^\perp(z,k'_\perp)\right]}{\sum_q T_0^q(y)\hat{D}_1(z,k'_\perp)}$$

$$P_{\mathrm{hn}}(y,z,k'_\perp) = -\frac{|\vec{k}'_\perp|}{M}\frac{\sum_q T_0^q(y)\hat{D}_{1T}^\perp(z,k'_\perp)}{\sum_q T_0^q(y)\hat{D}_1(z,k'_\perp)} + \cdots$$

$$P_{\mathrm{ht}}(y,z,k'_\perp) = -\frac{|\vec{k}'_\perp|}{M}\frac{\sum_q P_q(y) T_0^q(y)\Delta\hat{D}_{1T}^\perp(z,k'_\perp)}{\sum_q T_0^q(y)\hat{D}_1(z,k'_\perp)} + \cdots$$

Spin-1/2 Hadrons - $e^+e^- \to \gamma^* \to \mathrm{h} + \bar{\mathrm{q}} + \mathrm{X}$

$$\frac{d\sigma^{(\mathrm{si,Vpol,em})}}{dy\,dz\,d^2k'_\perp} = \frac{\alpha^2 e_q^2}{2\pi Q^2}\Big\{A(y)\frac{\epsilon_\perp^{k'_\perp S_\perp}}{M}\hat{D}_{1T}^\perp(z,k'_\perp)$$
$$+\frac{4B(y)}{zMQ^2}\left[\lambda_h M\epsilon_\perp^{l_\perp k'_\perp}\hat{D}_L^\perp(z,k'_\perp)+\epsilon_\perp^{k'_\perp S_\perp} l_\perp\cdot k'_\perp \hat{D}_T^\perp(z,k'_\perp)+M^2\epsilon_\perp^{l_\perp S_\perp}\hat{D}_T(z,k'_\perp)\right]\Big\}$$

Polarization (Leading Twist)

$$P_{\mathrm{Lh}}^{(0)(\mathrm{em})}(y,z,k'_\perp)=P_{\mathrm{ht}}^{(0)(\mathrm{em})}(y,z,k'_\perp)=0,\quad P_{\mathrm{hn}}^{(0)(\mathrm{em})}(y,z,k'_\perp)=-\frac{|\vec{k}'_\perp|}{M}\frac{\sum_q e_q^2\hat{D}_{1T}^\perp(z,k'_\perp)}{\sum_q e_q^2\hat{D}_1(z,k'_\perp)}$$

Twist-3

$$P_{\mathrm{Lh}}^{(\mathrm{em})}(y,z,k'_\perp)=\frac{4k'_y}{zQ}\frac{\tilde{B}(y)}{A(y)}\frac{\sum_q e_q^2\hat{D}_L^\perp(z,k'_\perp)}{\sum_q e_q^2\hat{D}_1(z,k'_\perp)}$$

Spin-1 Hadrons - $e^+e^- \to Z_0 \to \mathrm{h} + \bar{\mathrm{q}} + \mathrm{X}$

$$\frac{d\sigma^{(\mathrm{si},1)}}{dydzd^2k'_\perp} = \frac{d\sigma^{(\mathrm{si,unp})}}{dydzd^2k'_\perp} + \frac{d\sigma^{(\mathrm{si,Vpol})}}{dydzd^2k'_\perp} + \frac{d\sigma^{(\mathrm{si,Tpol})}}{dydzd^2k'_\perp}$$

$$\frac{d\sigma^{(\mathrm{si,Tpol})}}{dydzd^2k'_\perp} = \frac{d\sigma^{(\mathrm{si,LL})}}{dydzd^2k'_\perp} + \frac{d\sigma^{(\mathrm{si,LT})}}{dydzd^2k'_\perp} + \frac{d\sigma^{(\mathrm{si,TT})}}{dydzd^2k'_\perp}$$

$$[\mathbf{LL}] = \frac{\alpha^2\chi}{2\pi Q^2} S_{LL}\Big\{T_0^q(y)\hat{D}_{1LL}(z,k'_\perp) + \frac{4}{zQ^2}\big[T_2^q(y)(l_\perp\cdot k'_\perp)\hat{D}_{LL}^\perp(z,k'_\perp) + T_3^q(y)\epsilon_\perp^{l_\perp k'_\perp}\Delta\hat{D}_{LL}^\perp(z,k'_\perp)\big]\Big\}$$

Spin Alignment

$$\rho_{00} = \frac{1}{3} - \frac{1}{3}\frac{\sum_q T_0^q(y)\hat{D}_{1LL}(z,k'_\perp)}{\sum_q T_0^q(y)\hat{D}_1(z,k'_\perp)}\Big[1 + \frac{M}{Q}\hat{\Delta}(y,z,k'_\perp)\Big]$$
$$- \frac{4}{3}\frac{\sum_q\Big[\tilde{T}_2^q(y)k'_x\hat{D}_{LL}^\perp(z,k'_\perp) - \tilde{T}_3^q(y)k'_y\Delta\hat{D}_{LL}^\perp(z,k'_\perp)\Big]}{zQ\sum_q T_0^q(y)\hat{D}_1(z,k'_\perp)}.$$

Semi-inclusive Process $e^+ + e^- \to \mathrm{h} + \bar{\mathrm{q}} + \mathrm{X}$

$$[\text{LT Terms}] = \frac{\alpha^2\chi}{2\pi Q^2} S^{\alpha}_{LT}\Big\{T_0^q(y)\frac{k'_{\perp\alpha}}{M}\hat{D}^{\perp}_{1LT}(z,k'_\perp) + T_1^q(y)\frac{\epsilon_{\perp k'\alpha}}{M}\Delta\hat{D}^{\perp}_{1LT}(z,k'_\perp)$$

$$+\frac{4}{zQ^2}T_2^q(y)\big[(l_\perp\cdot k'_\perp)\frac{k'_{\perp\alpha}}{M}\hat{D}^{\perp}_{LT}(z,k'_\perp) + M l_{\perp\alpha}\hat{D}_{LT}(z,k'_\perp)\big]$$

$$+\frac{4}{zQ^2}T_3^q(y)\big[\epsilon_\perp^{l_\perp k'_\perp}\frac{k'_{\perp\alpha}}{M}\Delta\hat{D}^{\perp}_{LT}(z,k'_\perp) + M\epsilon_{\perp l\alpha}\Delta\hat{D}_{LT}(z,k'_\perp)\big]\Big\}$$

$$[\text{TT Terms}] = \frac{\alpha^2\chi}{2\pi Q^2} S^{\alpha\beta}_{TT}\Big\{T_0^q(y)\frac{k'_{\perp\alpha}k'_{\perp\beta}}{M^2}\hat{D}^{\perp}_{1TT}(z,k'_\perp) + T_1^q(y)\frac{\epsilon_{\perp k'\alpha}k'_{\perp\beta}}{M^2}\Delta\hat{D}^{\perp}_{1TT}(z,k'_\perp)$$

$$+\frac{4}{zQ^2}T_2^q(y)\big[(l_\perp\cdot k'_\perp)\frac{k'_{\perp\alpha}k'_{\perp\beta}}{M^2}\hat{D}^{\perp C}_{TT}(z,k'_\perp) + l_{\perp\alpha}k'_{\perp\beta}\hat{D}^{\perp A}_{TT}(z,k'_\perp)\big]$$

$$+\frac{4}{zQ^2}T_3^q(y)\big[\epsilon_\perp^{l_\perp k'_\perp}\frac{k'_{\perp\alpha}k'_{\perp\beta}}{M^2}\Delta\hat{D}^{\perp C}_{TT}(z,k'_\perp) + \epsilon_{\perp l\alpha}k'_{\perp\beta}\Delta\hat{D}^{\perp A}_{TT}(z,k'_\perp)\big]\Big\}$$

Summary

Inclusive $e^+e^- \to Z_0 \to \mathrm{h} + X$

- There is a leading twist longitudinal polarization for spin-1/2 hadrons and also spin alignment ($\rho_{00} \neq 1/3$) for vector mesons.
- On the twist-3 level, there are transverse polarizations for spin-1/2 hadrons that in and perpendicular to the leptonic plane.

Semi-inclusive $e^+e^- \to Z_0 \to \mathrm{h} + \bar{q}(jet) + X$

- For spin-0 hadrons, there are two azimuthal asymmetries on twist-3 level.
- For spin-1/2 hadrons, there is a longitudinal polarization and also transverse polarizations that in and transverse to the production plane on the leading twist.
- For vector mesons, all five tensor polarizations have leading twist contributions. S_{LL}, S_{LT}^n, S_{LT}^t, S_{TT}^{nn} and S_{TT}^{nt}

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Thanks for your attention!