Higgs Combination for Coupling Measurement and Beyond

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Higgs Combination in CEPC

- In pre-CDR, the Higgs coupling results were derived from the precision of individual Higgs channel
  - correlations between different signal modes were not taken into account
    - the ZH backgrounds in each channel should also enter the fit of the cross section and the constrain of the couplings
  - Systematics and their correlations are difficult to address

- A combination measurement with all Higgs channels would give a more precise coupling measurement
  - provides a uniformed statistical procedure and framework
  - can easily include necessary correlations and systematics
  - gives more potential for future interpretation of the results

Higgs Combination - Workspace

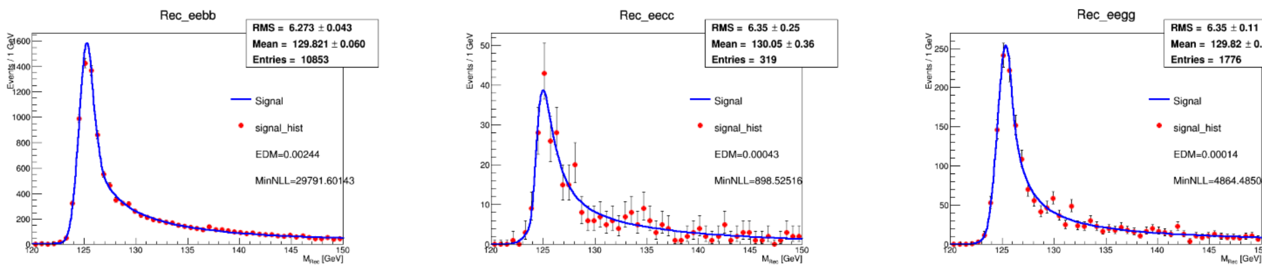
- Workspace – the container of the signal/background model, data, systematic parameters etc.. Its heart is the likelihood used in the final fit.

$$\mathcal{L}(N^{exp}, \varepsilon, \theta_\varepsilon, \theta_E, \theta_{lumi}) =$$
$$= P(N^{obs}|N^{exp}) \prod_c ((\sum_{i=sigal(i)} N^{exp}_{s,i} f_{s,i} + \sum_{j=bkg(j)} N^{exp}_{b,j} f_{b,j})/N^{exp}) \times G(\theta_\varepsilon)G(\theta_E)G(\theta_{lumi})$$

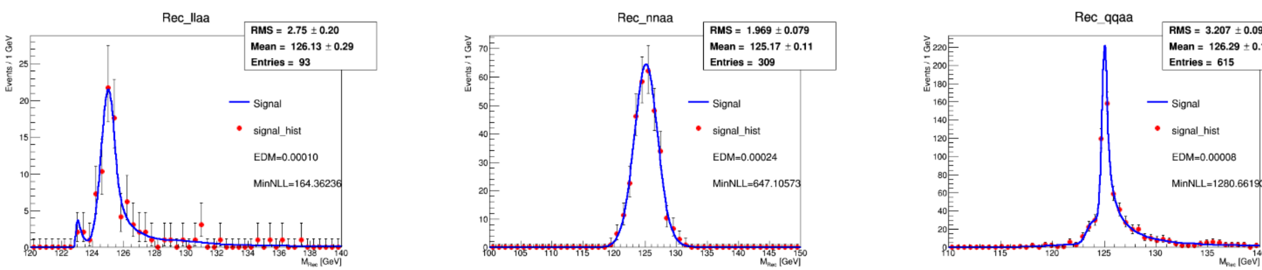
$N^{exp}_{s,i} f_{s,i} = \sigma \times Br(i) \times lumi(\theta_{lumi}) \times \varepsilon(\theta_\varepsilon \delta_\varepsilon) \times f_{s,i}(\theta_E)$, with signal process i.

- $N^{exp}_{s,i}$: expected number of different signal composition I
- $f_{s,i}$: signal PDF
- $G(\theta_\varepsilon)G(\theta_E)G(\theta_{lumi})$: constrains on systematic uncertainties like efficiency/acceptance, energy scale/resolution, luminosity.
- $\sigma \times Br(i)$: one of the parameter of interest

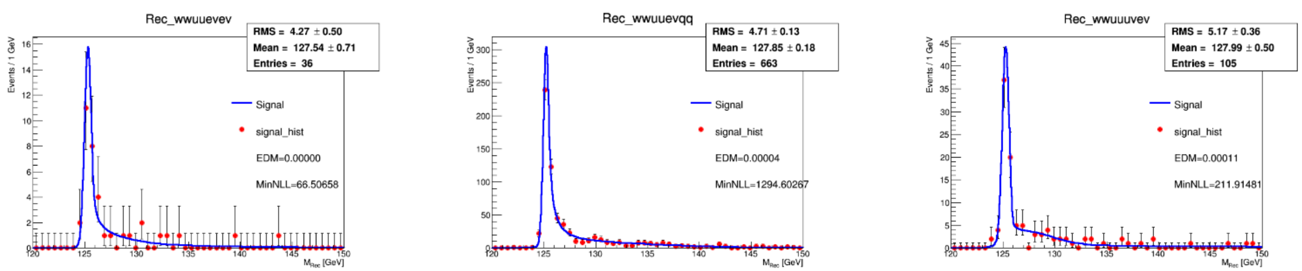
Signal shapes examples



ZH, H->jj



ZH, H->aa



ZH, H->WW

Higgs Combination – Systematics and Correlations

- Introduce nuisance parameters with response function and constrains terms in likelihood
  - need detailed tables about all the systematics effects on acceptances, efficiencies and shapes of signals and backgrounds, channel by channel. E.g. object energy scale, energy resolution, identification efficiency uncertainty, uncertainties due to limited statistics of Monte Carlo samples, uncertainties on background estimation etc..
  - PDFS with response function like $(1+\theta_{lumi}\delta_{lumi})$.

$$N_{s,i}^{exp} f_{s,i} = \sigma \times Br(i) \times (lumi \times (1+\theta_{lumi}\delta_{lumi})) \times (\varepsilon \times (1+\theta_{\varepsilon}\delta_{\varepsilon})) \times f_{s,i}(\theta_E)$$

$$N_{b,j}^{exp} f_{b,j} = \sigma_{b,j}^{SM}(\theta_{xs\_b}) \times Br_{b,j}^{SM}(\theta_{Br\_b}) \times lumi(\theta_{lumi}) \times \varepsilon(\theta_{\varepsilon}) \times f_{s,j}(\theta_E)$$

  - constrains

$$G(\theta_{\varepsilon})G(\theta_E)G(\theta_{lumi})G(\theta_{xs\_b})G(\theta_{Br\_b})$$

- Correlated systematics share the same name in the fit

Example codes


```
// ADD luminosity uncertainty 0.1%
Float_t uncer_lumi = 0.001;
char sigma_lumi[9];  sprintf(sigma_lumi, "%4.9f", uncer_lumi);
wspace->factory("sum::uncertainty_lumi(1, prod::uncer_lumi(nuis_lumi[ 0 , -5 , 5 ],"+(TString)sigma_lumi+"))");
wspace->factory("RooGaussian::constraint_lumi(nuis_lumi,global_nuis_lumi[0,-5,5],1)");

nuispara->add(*wspace->var("nuis_lumi"));
globobs->add(*wspace->var("global_nuis_lumi"));
uncertainty->add(*wspace->function("uncertainty_lumi"));
constraints->add(*wspace->pdf("constraint_lumi"));


RooProdPdf constraint("constraint","constraint",*constraints);
wspace->import(constraint);

RooProduct prod_uncertainty_common("prod_uncertainty_common","prod_uncertainty_common",*uncertainty);
wspace->import(prod_uncertainty_common);
```

response function + Gaussian constrain

```
wspace->factory("prod::nsig_bb(n_bb, prod_uncertainty_common, mu, mu_bb, mu_JJ)");
wspace->factory("prod::nsig_cc(n_cc, prod_uncertainty_common, mu, mu_cc, mu_JJ)");
wspace->factory("prod::nsig_gg(n_gg, prod_uncertainty_common, mu, mu_gg, mu_JJ)");
wspace->factory("prod::nsig_zz(n_zz, prod_uncertainty_common, mu, mu_zz)")
wspace->factory("prod::nsig_ww(n_ww, prod_uncertainty_common, mu, mu_ww)")

wspace->factory("SUM::modelSB(nsig_bb*signalPdf_bb + nsig_cc*signalPdf_cc + nsig_gg*signalPdf_gg + nsig_zz*signalPdf_zz + nsig_ww*signalPdf_ww, nbkg*bkgPdf");

wspace->factory("PROD::model(modelSB,constraint)");
```

different signal composition + final likelihood

Higgs Combination - Coupling

- Re-parameterize parameters of interests to coupling modifiers in workspace

The deviations from the SM are implemented as scale factors (κ's) of Higgs couplings relative to their SM values:

$$g_{Hff} = \kappa_f \cdot g_{Hff}^{\rm SM} = \kappa_f \cdot \frac{m_f}{v} \quad \text{and} \quad g_{HVV} = \kappa_V \cdot g_{HVV}^{\rm SM} = \kappa_V \cdot \frac{2m_V^2}{v}$$

such that $\kappa_f = 1$ and $\kappa_V = 1$ in SM. For example, at the LHC the $gg \to H \to \gamma\gamma$ rate can be written as

$$\sigma \times {\rm BR}(gg \to H \to \gamma\gamma) = \sigma_{\rm SM}(gg \to H) \cdot {\rm BR}_{\rm SM}(H \to \gamma\gamma) \cdot \frac{\kappa_g^2 \cdot \kappa_\gamma^2}{\kappa_H^2},$$

where κ_g and κ_γ are effective scale factors for the Hgg and $H\gamma\gamma$ couplings through loops. Additionally, κ_H^2 is the scale factor for the Higgs width:

$$\kappa_H^2 = \sum_X \kappa_X^2 {\rm BR}_{\rm SM}(H \to X)\,,$$

where κ_X is the scale factor for the HXX coupling and ${\rm BR}_{\rm SM}(H \to X)$ is the SM value of the $H \to X$ decay branching ratio.

Higgs Combination – κ parameterization

The kappa framework – the dictionary

Factors depend on
- *Assumed value m_H,*
- *Calculations of σ,Γ*
- *Kinematic selections*

Production	Loops	Interference	Multiplicative factor
$\sigma(ggF)$	✓	$b-t$	$\kappa_g^2 \sim 1.06\cdot\kappa_t^2 + 0.01\cdot\kappa_b^2 - 0.07\cdot\kappa_t\kappa_b$
$\sigma(VBF)$	–	–	$\sim 0.74\cdot\kappa_W^2 + 0.26\cdot\kappa_Z^2$
$\sigma(WH)$	–	–	$\sim \kappa_W^2$
$\sigma(qq/qg \to ZH)$	–	–	$\sim \kappa_Z^2$
$\sigma(gg \to ZH)$	✓	$Z-t$	$\sim 2.27\cdot\kappa_Z^2 + 0.37\cdot\kappa_t^2 - 1.64\cdot\kappa_Z\kappa_t$
$\sigma(ttH)$	–	–	$\sim \kappa_t^2$
$\sigma(gb \to WtH)$	–	$W-t$	$\sim 1.84\cdot\kappa_t^2 + 1.57\cdot\kappa_W^2 - 2.41\cdot\kappa_t\kappa_W$
$\sigma(qb \to tHq)$	–	$W-t$	$\sim 3.4\cdot\kappa_t^2 + 3.56\cdot\kappa_W^2 - 5.96\cdot\kappa_t\kappa_W$
$\sigma(bbH)$	–	–	$\sim \kappa_b^2$
Partial decay width			
Γ^{ZZ}	–	–	$\sim \kappa_Z^2$
Γ^{WW}	–	–	$\sim \kappa_W^2$
$\Gamma^{\gamma\gamma}$	✓	$W-t$	$\kappa_\gamma^2 \sim 1.59\cdot\kappa_W^2 + 0.07\cdot\kappa_t^2 - 0.66\cdot\kappa_W\kappa_t$
$\Gamma^{\tau\tau}$	–	–	$\sim \kappa_\tau^2$
Γ^{bb}	–	–	$\sim \kappa_b^2$
$\Gamma^{\mu\mu}$	–	–	$\sim \kappa_\mu^2$
Total width for $BR_{BSM} = 0$			
Γ_H	✓	–	$\kappa_H^2 \sim 0.57\cdot\kappa_b^2 + 0.22\cdot\kappa_W^2 + 0.09\cdot\kappa_g^2 + 0.06\cdot\kappa_\tau^2 + 0.03\cdot\kappa_Z^2 + 0.03\cdot\kappa_c^2 + 0.0023\cdot\kappa_\gamma^2 + 0.0016\cdot\kappa_{Z\gamma}^2 + 0.0001\cdot\kappa_s^2 + 0.00022\cdot\kappa_\mu^2$

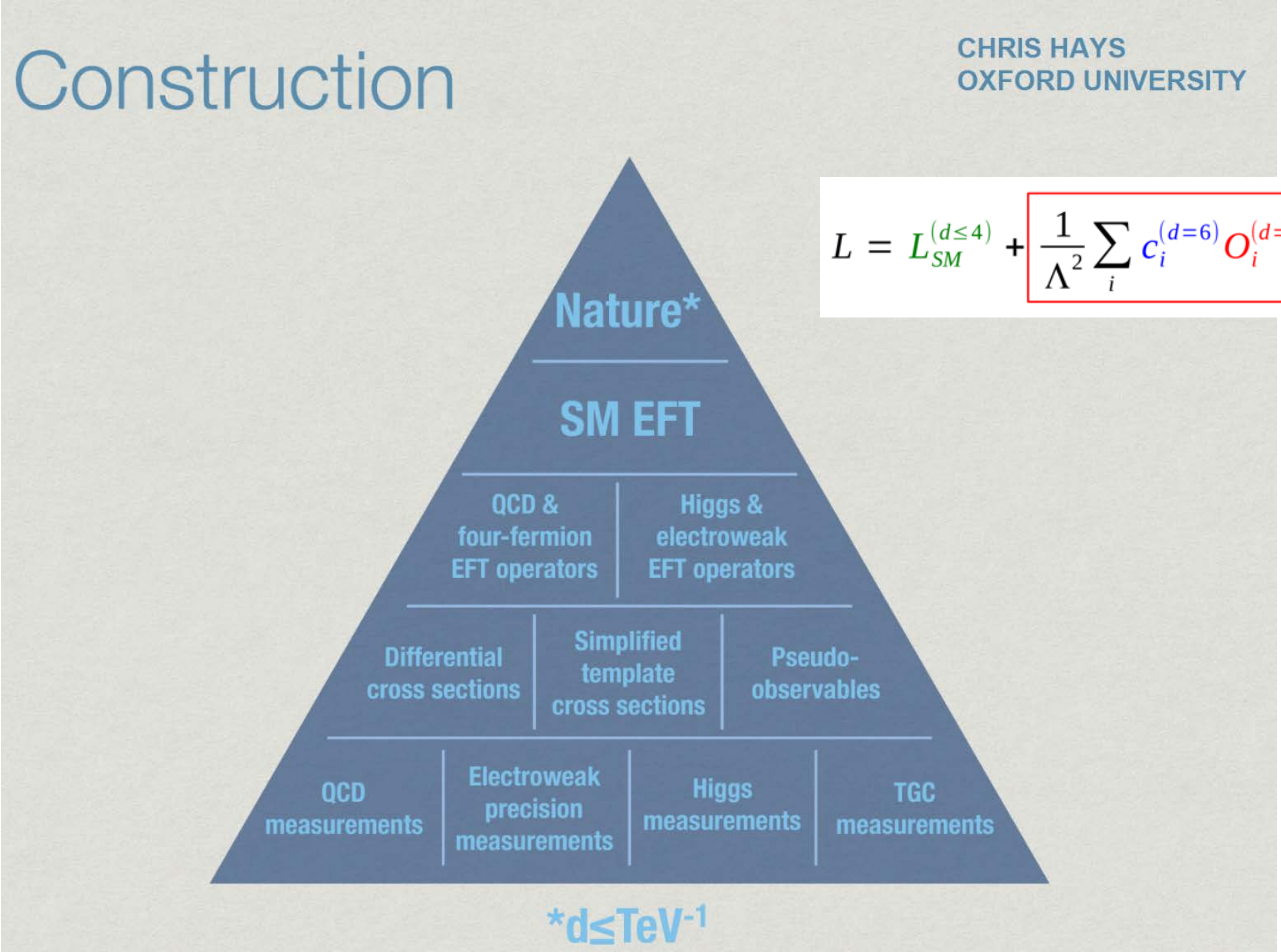
example in codes

```
wchannel->factory("sum::kappa_Signal_ZH_square(kappa_Signal_z_square)");

wchannel->factory("sum::kappa_Signal_gamma_square(1.59*kappa_Signal_w_square, 0.07*kappa_Signal_t_square, -0.66*kappa_Signal_w_times_kappa_Signal_t)");

wchannel->factory("sum::kappa_Signal_H_square(0.57*kappa_Signal_t_square, 0.22*kappa_Signal_w_square, 0.09*kappa_Signal_g_square, 0.06*kappa_Signal_tau_square, 0.03*kappa_Signal_z_square, 0.03*kappa_Signal_c_square, 0.0023*kappa_Signal_gamma_square, 0.0016*kappa_Signal_Zgamma_square, 0.0001*kappa_Signal_s_square, 0.00022*kappa_Signal_mu_square)");
```

Higgs Combination - EFT



$$L = L_{SM}^{(d\leq 4)} + \frac{1}{\Lambda^2}\sum_i c_i^{(d=6)} O_i^{(d=6)} + \frac{1}{\Lambda^4}\sum_i c_i^{(d=8)} O_i^{(d=8)} + \ldots$$

Higgs Combination - EFT

$$O_{\delta\lambda_3} = -\frac{1}{v^2}(H^\dagger H)^3, \qquad [O_{\delta y_f}]_{ij} = -\frac{\sqrt{2m_{f_i}m_{f_j}}}{v^3} H^\dagger H \bar{f}_{L,i} H f_{R,j} + \text{h.c.},$$

$$O_{c_{gg}} = \frac{g_s^2}{4v^2} H^\dagger H\, G^a_{\mu\nu} G^a_{\mu\nu}$$

$$O_{\delta c_z} = -\frac{1}{v^2}\left[\partial_\mu (H^\dagger H)\right]^2 + \frac{3\lambda}{v^2}(H^\dagger H)^3 + \left(\sum_f \frac{\sqrt{2}m_{f_i}}{v^3} H^\dagger H \bar{f}_{L,i} H f_{R,i} + \text{h.c.}\right),$$

$$O_{c_{z\Box}} = \frac{ig^3}{v^2(g^2-g'^2)}\left(H^\dagger \sigma^i \overleftrightarrow{D}_\mu H\right) D_\nu W^i_{\mu\nu} - \frac{ig^2 g'}{v^2(g^2-g'^2)}\left(H^\dagger \overleftrightarrow{D}_\mu H\right)\partial_\nu B_{\mu\nu},$$

$$O_{c_{zz}} = \frac{ig(g^2+g'^2)}{2v^2(g^2-g'^2)}\left(H^\dagger \sigma^i \overleftrightarrow{D}_\mu H\right) D_\nu W^i_{\mu\nu} - \frac{ig'(g^2+g'^2)}{2v^2(g^2-g'^2)}\left(H^\dagger \overleftrightarrow{D}_\mu H\right)\partial_\nu B_{\mu\nu}$$
$$- \frac{ig}{v^2}\left(D_\mu H^\dagger \sigma^i D_\nu H\right) W^i_{\mu\nu} - \frac{ig'}{v^2}\left(D_\mu H^\dagger D_\nu H\right) B_{\mu\nu},$$

$$O_{c_{z\gamma}} = -\frac{2igg'^2}{v^2(g^2+g'^2)}\left(D_\mu H^\dagger \sigma^i D_\nu H\right) W^i_{\mu\nu} + \frac{2ig'g^2}{v^2(g^2+g'^2)}\left(D_\mu H^\dagger D_\nu H\right) B_{\mu\nu},$$

$$O_{c_{\gamma\gamma}} = -\frac{igg'^4}{2v^2(g^4-g'^4)}\left(H^\dagger \sigma^i \overleftrightarrow{D}_\mu H\right) D_\nu W^i_{\mu\nu} + \frac{ig'^5}{2v^2(g^4-g'^4)}\left(H^\dagger \overleftrightarrow{D}_\mu H\right)\partial_\nu B_{\mu\nu}$$
$$- \frac{igg'^4}{v^2(g^2+g'^2)^2}\left(D_\mu H^\dagger \sigma^i D_\nu H\right) W^i_{\mu\nu} + \frac{ig'^3(2g^2+g'^2)}{(g^2+g'^2)^2 v^2}\left(D_\mu H^\dagger D_\nu H\right) B_{\mu\nu}$$
$$+ \frac{g'^2}{4v^2} H^\dagger H\, B_{\mu\nu} B_{\mu\nu},$$

Higgs Basis

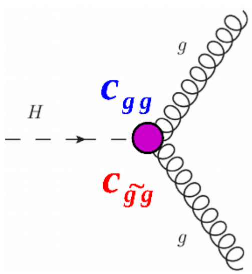
from CERN Yellow Report 4
https://arxiv.org/abs/1610.07922

Higgs Combination - EFT

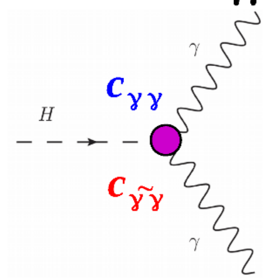
⇒ κ framework with effective (κ_g,κ_γ) + CP-odd couplings + modified HZZ structure

→ 17 operators (10 CP-even + 7 CP-odd) involving the Higgs boson

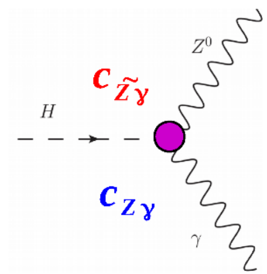
Tree-level ggH (κ_g)



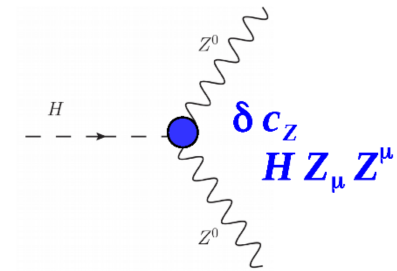
Tree-level Hγγ (κ_γ)



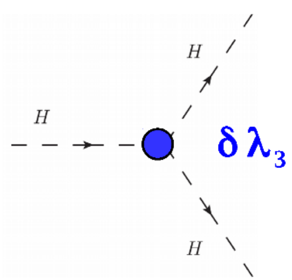
Tree-level HZγ



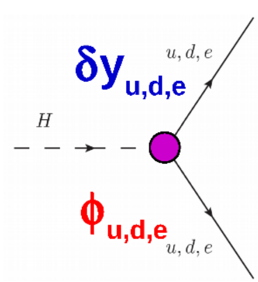
HZZ coupling modifier (κ_Z)



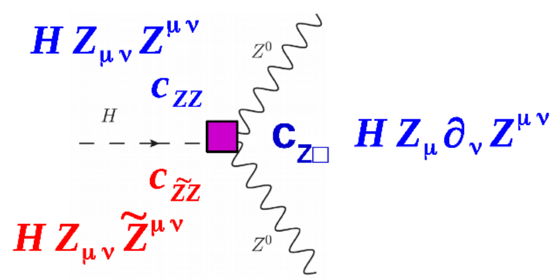
Modified H^3 Coupling



Modified Yukawa coupling magnitudes (κ_f) and phases



Modified HZZ interaction with derivative couplings



Higgs Combination – CP Measurement

- CP property of Higgs could be probed from combination of different Higgs production and Higgs decay in CPEC
  - a generic optimal observable method (adapted from **Markus Schumacher**)

- The matrix element with additional contribution to the SM coupling from CP odd interactions:

$$|\mathcal{M}|^2 = |\mathcal{M}_{SM}|^2 + \tilde{d} \cdot 2\Re(\mathcal{M}^*_{SM}\mathcal{M}_{CP\text{-}odd}) + \tilde{d}^2 \cdot |\mathcal{M}_{CP\text{-}odd}|^2$$

- A non-zero interference term indicates the CP violation

$$\tilde{d} \cdot 2\,\Re(\mathcal{M}^*_{SM}\,\mathcal{M}_{CPodd})$$

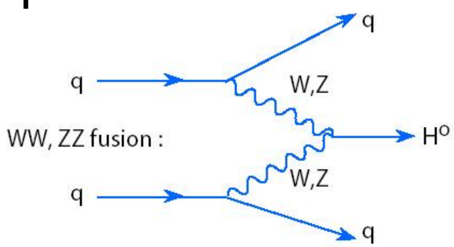
- Optimal observable is given by:

$$O := \frac{d\sigma_{nonSM}}{d\sigma_{SM}} \simeq \frac{2\,\Re(\mathcal{M}^*_{SM}\,\mathcal{M}_{CPodd})}{|\mathcal{M}_{SM}|^2}$$

- It combines the information of the seven-dimensional phase space into one single observable [D. Atwood, A. Soni; Phys. Rev. D45 (1992)] , [M. Davier et al.; Phys. Lett. B306 (1993)] , [M. Diehl, O. Nachtmann; Z. Phys. C62 (1994)] would give the highest statistical sensitivity)

Calculation of Matrix Elements

- The matrix elements depend on the 4-momentum of the initial and final state particles:



$$\mathcal{M} = \mathcal{M}(p_{p1}^{\mu}, p_{p2}^{\mu}, p_{k1}^{\mu}, p_{k2}^{\mu}, p_{H}^{\mu})$$

- The initial parton 4 momentums can be calculated from the final state:

$$p_{p1}^{\mu} = x_1 \frac{\sqrt{s}}{2}(1,0,0,1) \quad \text{with} \quad x_1 = \frac{M_{final}}{\sqrt{s}} e^{+y_{final}}$$

$$p_{p2}^{\mu} = x_2 \frac{\sqrt{s}}{2}(1,0,0,-1) \quad \text{with} \quad x_2 = \frac{M_{final}}{\sqrt{s}} e^{-y_{finale}}$$

- The fortran routines for the matrix elements are extracted from the public HAWK2.0 code.
  - calculate the Bjorken x1 and x2 and get 4 momentums of incoming partons
  - also pass final states jet1, jet2, and higgs 4 vectors to fortran subroutine

Weighted Sum over flavours

- As we do not know flavours of incoming and outgoing partons
  - calculate PDF weighted sum over flavour configuration separately for matrix elements in numerator and denominator of the optimal observable

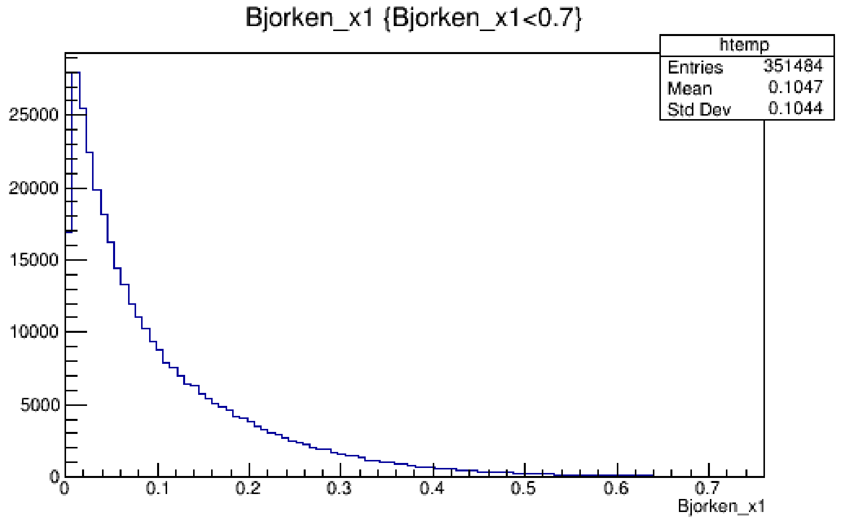
$$|\mathcal{M}_{SM}|^2 = \sum_{i,j,k,l} f_i(x_1) f_j(x_2) |\mathcal{M}(ij \to klH)_{SM}|^2$$

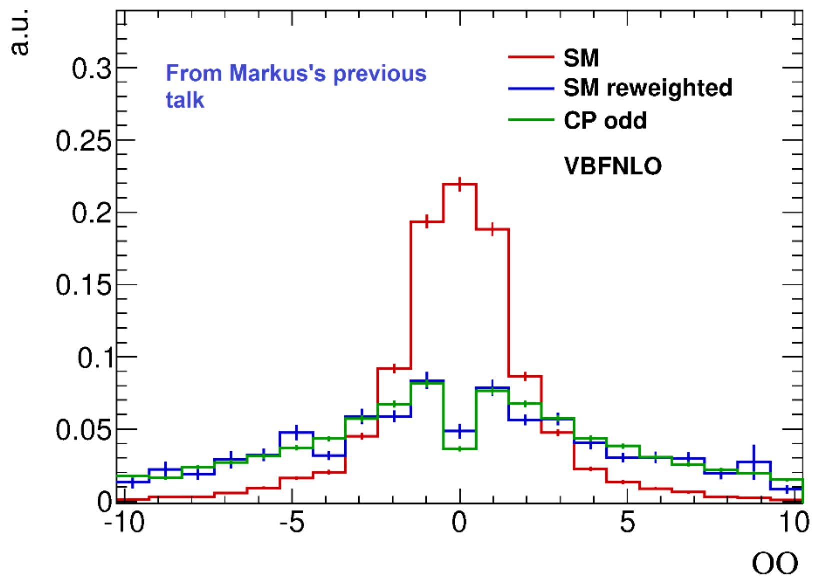
  - where the f_i is the pdf of parton i calculated using the LHAPDF package with the CT10 PDF set
    - install and setup LHAPDF
    - get double PDF value from CT10, depending on the momentum fraction and flavor
    - pass PDF array from c++ to fortran, loop and sum over flavors
    - compile c++ codes and fortran with correct libraries

Bjorken x and Optimal Observable

- Reco level Bjorken x distribution could be compare with truth
  - gives the incoming particle momentum

- OO distribution that distinguishes different Higgs CP hypothesis





Conclusion

- Developing the Higgs combination for a better interpretation of the measurements
  - take into account the correlations between different signal modes
  - systematics and their correlations could be easily addressed
  - push a more uniformed statistical procedure and framework for different Higgs analyses

- Higgs combination now and future
  - coupling measurement
  - probe the CP property of the Higgs
  - constrain EFT operators
    - give direct hints on the new physics
    - guidance for future CEPC development