# The Higgs Self-Coupling Measurement at $e^+e^-$ Colliders


Tim Barklow (SLAC)*


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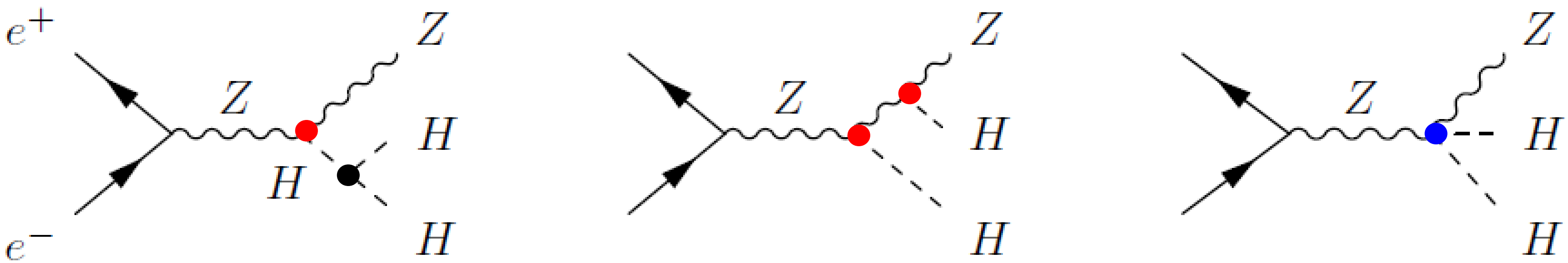
*collaborators: Keisuke Fujii (KEK), Sunghoon Jung (SLAC), Michael Peskin (SLAC), Junping Tian (U. Tokyo)

# Higgs Self Coupling Systematic Error Uncertainties for

## $g_{ZZH}$ $g_{ZZHH}$ (and other, BSM, couplings) in $\sigma(e^+e^- \to HHZ)$




We assume that $\sigma(e^+e^- \to HHZ)$ can be described by an effective field theory (EFT) containing a general $SU(2)\times U(1)$ gauge invariant Lagrangian with dimension-6 operators in addition to the SM.

Using the "Warsaw" basis, with the pure Higgs operators in the "SILH" basis, these are the 10 CP-conserving dim-6 operators relevant to this analysis:

$$\begin{aligned}
\Delta\mathcal{L} = & \frac{c_H}{2v^2}\partial^\mu(\Phi^\dagger\Phi)\partial_\mu(\Phi^\dagger\Phi) + \frac{c_T}{2v^2}(\Phi^\dagger \overleftrightarrow{D}^\mu \Phi)(\Phi^\dagger \overleftrightarrow{D}_\mu \Phi) - \frac{c_6\lambda}{v^2}(\Phi^\dagger\Phi)^3 \\
& + \frac{g^2 c_{WW}}{m_W^2}\Phi^\dagger\Phi W^a_{\mu\nu}W^{a\mu\nu} + \frac{4gg' c_{WB}}{m_W^2}\Phi^\dagger t^a \Phi W^a_{\mu\nu}B^{\mu\nu} \\
& + \frac{g'^2 c_{BB}}{m_W^2}\Phi^\dagger\Phi B_{\mu\nu}B^{\mu\nu} + \frac{g^3 c_{3W}}{m_W^2}\epsilon_{abc}W^a_{\mu\nu}W^{b\nu}{}_\rho W^{c\rho\mu} \\
& + i\frac{c_{HL}}{v^2}(\Phi^\dagger \overleftrightarrow{D}^\mu \Phi)(\overline{L}\gamma_\mu L) + +4i\frac{c'_{HL}}{v^2}(\Phi^\dagger t^a \overleftrightarrow{D}^\mu \Phi)(\overline{L}\gamma_\mu t^a L) \\
& + i\frac{c_{HE}}{v^2}(\Phi^\dagger \overleftrightarrow{D}^\mu \Phi)(\overline{e}\gamma_\mu e) \ .
\end{aligned}$$

## In addition there are 4 CP violating terms:

$$\Delta\mathcal{L}_{CP} = +\frac{g^2\tilde{c}_{WW}}{m_W^2}\Phi^\dagger\Phi W^a_{\mu\nu}\widetilde{W}^{a\mu\nu} + \frac{4gg'\tilde{c}_{WB}}{m_W^2}\Phi^\dagger t^a\Phi W^a_{\mu\nu}\widetilde{B}^{\mu\nu}$$
$$+\frac{g'^2\tilde{c}_{BB}}{m_W^2}\Phi^\dagger\Phi B_{\mu\nu}\widetilde{B}^{\mu\nu} + \frac{g^3\tilde{c}_{3W}}{m_W^2}\epsilon_{abc}W^a_{\mu\nu}W^{b\nu}{}_\rho\widetilde{W}^{c\rho\mu}$$

where the dual field strength tensors are defined by

$$\widetilde{B}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}B^{\rho\sigma}\ , \quad \widetilde{W}^k_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}W^{\rho\sigma k}$$

In summary there are 10 CP-conserving coefficients:

$c_{HL}$  $c'_{HL}$  $c_{HE}$  $c_T$  $c_{WB}$  $c_{3W}$

$c_H$  $c_{WW}$  $c_{BB}$

$c_6$

and 4 CP-violating coefficients:

$\tilde{c}_{WW}$  $\tilde{c}_{WB}$  $\tilde{c}_{BB}$  $\tilde{c}_{3W}$

After EWSB we have, $\Delta\mathcal{L} = \Delta\mathcal{L}_h + \Delta\mathcal{L}_{eehZ} + \Delta\mathcal{L}_{TGC}$ where

$$\Delta\mathcal{L}_h = -\eta_h \lambda_0 v_0 h^3 + \frac{\theta_h}{v_0} h\partial_\mu h \partial^\mu h + \eta_Z \frac{m_Z^2}{v_0} Z_\mu Z^\mu h + \frac{1}{2}\eta_{2Z}\frac{m_Z^2}{v_0^2} Z_\mu Z^\mu h^2$$
$$+\eta_W \frac{2m_W^2}{v_0} W_\mu^+ W^{-\mu} h + \eta_{2W}\frac{m_W^2}{v_0^2} W_\mu^+ W^{-\mu} h^2$$
$$+\frac{1}{2}\left(\zeta_Z \frac{h}{v_0} + \frac{1}{2}\zeta_{2Z}\frac{h^2}{v_0^2}\right)\hat{Z}_{\mu\nu}\hat{Z}^{\mu\nu} + \left(\zeta_W \frac{h}{v_0} + \frac{1}{2}\zeta_{2W}\frac{h^2}{v_0^2}\right)\hat{W}_{\mu\nu}^+ \hat{W}^{-\mu\nu}$$
$$+\frac{1}{2}\left(\zeta_A \frac{h}{v_0} + \frac{1}{2}\zeta_{2A}\frac{h^2}{v_0^2}\right)\hat{A}_{\mu\nu}\hat{A}^{\mu\nu} + \left(\zeta_{AZ} \frac{h}{v_0} + \zeta_{2AZ}\frac{h^2}{v_0^2}\right)\hat{A}_{\mu\nu}\hat{Z}^{\mu\nu} \ .$$

$$\Delta\mathcal{L}_{eehZ} = g_{LZh}(\overline{e}_L\gamma_\mu e_L)Z^\mu\left(\frac{h}{v_0} + \frac{1}{2}\frac{h^2}{v_0^2}\right) + g_{RZh}(\overline{e}_R\gamma_\mu e_R)Z^\mu\left(\frac{h}{v_0} + \frac{1}{2}\frac{h^2}{v_0^2}\right)$$

$$\Delta\mathcal{L}_{TGC} = ig_V\left\{g_{1V}V^\mu(\hat{W}_{\mu\nu}^- W^{+\nu} - \hat{W}_{\mu\nu}^+ W^{-\nu}) + \kappa_V W_\mu^+ W_\nu^- \hat{V}^{\mu\nu}\right.$$
$$\left.+\frac{\lambda_V}{m_W^2}\hat{W}_\mu^{-\rho}\hat{W}_{\rho\nu}^+\hat{V}^{\mu\nu}\right\}, \quad V = A, Z \qquad g_{1A} = 1 \qquad g_{1Z} = 1 + \Delta_g$$

$$\kappa_A = 1 + \Delta_\kappa \qquad \kappa_Z = 1 + \Delta_g - \frac{s_0^2}{c_0^2}\Delta_\kappa$$

$$\lambda_A = \Delta_\lambda \qquad \lambda_Z = \Delta_\lambda \ ,$$

In the SM at tree level $g_A = e \quad g_Z = gc_0 \quad g_{1V} = \kappa_V = \eta_H = \eta_Z = \eta_{2Z} = \eta_W = \eta_{2W} = 1$, and all others =0

And the CP violating piece $\Delta\mathcal{L}_{CP} = \Delta\mathcal{L}_{hCP} + \Delta\mathcal{L}_{3V_{CP}}$ where

$$\Delta\mathcal{L}_{hCP} = \frac{\tilde{\zeta}_Z}{v_0}\hat{Z}_{\mu\nu}\hat{\tilde{Z}}^{\mu\nu}h + \frac{1}{2}\frac{\tilde{\zeta}_{ZZ}}{v_0^2}\hat{Z}_{\mu\nu}\hat{\tilde{Z}}^{\mu\nu}h^2 + 2\frac{\tilde{\zeta}_W}{v_0}\hat{\tilde{W}}^{+}_{\mu\nu}\hat{W}^{-\mu\nu}h + \frac{\tilde{\zeta}_{WW}}{v_0^2}\hat{W}^{+}_{\mu\nu}\hat{\tilde{W}}^{-\mu\nu}h^2$$

$$\Delta\mathcal{L}_{3V_{CP}} = i\tilde{\kappa}_\gamma W^{\dagger}_{\mu}W_{\nu}\tilde{F}^{\mu\nu} + i\frac{\tilde{\lambda}_\gamma}{M_W^2}W^{\dagger}_{\lambda\mu}W^{\mu}_{\nu}\tilde{F}^{\nu\lambda} + i\tilde{\kappa}_z W^{\dagger}_{\mu}W_{\nu}\tilde{F}^{\mu\nu} + i\frac{\tilde{\lambda}_z}{M_W^2}W^{\dagger}_{\lambda\mu}W^{\mu}_{\nu}\tilde{F}^{\nu\lambda}$$

In the SM at tree level all $\tilde{\zeta}_x$ , $\tilde{\zeta}_{xx}$ , $\tilde{\kappa}_x$ , $\tilde{\lambda}_x$ =0

## The couplings $\theta_h$, $\eta_x$ $\zeta_x$ $g_{xZH}$, TGC's & EWPO's take the following form in our EFT:

$$\eta_h = (1 - c'_{HL} - \frac{1}{2}c_H + c_6)$$
$$\theta_h = c_H$$
$$\eta_Z = (1 - c_T - \frac{1}{2}c_H - c'_{HL})$$
$$\eta_{2Z} = (1 - 5c_T - c_H - 2c'_{HL})$$
$$\eta_W = (1 - \frac{1}{2}c_H - c'_{HL})$$
$$\eta_{2W} = (1 - c_H - c'_{HL}) \ .$$
$$\zeta_W = \zeta_{2W} = 8(c_{WW})$$
$$\zeta_Z = \zeta_{2Z} = 8(c_0^2 c_{WW} + 2s_0^2 c_{WB} + \frac{s_0^4}{c_0^2}c_{BB})$$
$$\zeta_{AZ} = \zeta_{2AZ} = 8(s_0 c_0 c_{WW} - s_0 c_0(1 - \frac{s_0^2}{c_0^2})c_{WB} - \frac{s_0^3}{c_0}c_{BB})$$
$$\zeta_A = \zeta_{2A} = 8s_0^2(c_{WW} - 2c_{WB} + c_{BB}) \ .$$

$c_6$ is uniquely accessible through the Higgs self coupling measurement. The other 9 EFT parameters appear in several places.

Note the relationship between the SM *hzz* & *hhzz* couplings $\eta_Z$ & $\eta_{2Z}$

Precise measurement of $\Gamma(H \to \gamma\gamma)$ from LHC+ILC will be used to constrain $c_{WW} + c_{BB}$

$$g_{LZh} = -\frac{e_0}{c_0 s_0}(c_{HL} + c'_{HL})$$
$$g_{RZh} = -\frac{e_0}{c_0 s_0}(c_{HE})$$

$$\Delta_g = \frac{1}{c_0^2 - s_0^2}(\frac{1}{2}c_T - c'_{HL} - 8\frac{s_0^2}{c_0^2}c_{WB})$$
$$\Delta_\kappa = +8c_{WB}$$
$$\Delta_\lambda = -6\frac{e_0^2}{s_0^2}c_{3W}$$

The TGC's depend on $c_T$ , $c'_{HL}$ , $c_{WB}$ through the Goldstone bosons that are eaten by the *Z* and *W* fields

$$m_W^2/m_Z^2 = c_0^2 + \frac{c_0^2}{c_0^2 - s_0^2}(c_0^2 c_T - 2s_0^2(c'_{HL} + 8c_{WB})$$
$$s_*^2 = s_0^2 + \frac{s_0^2}{c_0^2 - s_0^2}(c'_{HL} + 8c_{WB} - c_0^2 c_T) - \frac{1}{2}c_{HE} - s_0^2(c_{HL} - c_{HE})$$
$$g_L - g_R = \frac{1}{2}\frac{e_0}{s_0 c_0}(1 + c_{HL} - c_{HE} + \frac{1}{2}c_T)$$

EWPO's also depend on many of the Higgs-related operator coefficients, again through EWSB.

Basic set of observables: $\alpha \equiv \alpha_*(M_Z)$ , $G_F$ , $m_Z$ , $m_h$

$$e_0^2 = 4\pi\alpha \ , \qquad v_0^2 = 1/\sqrt{2}G_F \ , \qquad \lambda_0 = m_h^2/2v_0^2 \ ,$$

$$4s_0^2c_0^2 = \frac{4\pi\alpha}{\sqrt{2}G_F m_Z^2} \qquad\qquad g_0^2 = e_0^2/s_0^2,\ g_0'^2 = e_0^2/c_0^2$$

The coefficients $c_{HL}$ $c'_{HL}$ $c_{HE}$ $c_T$ $c_{WB}$ $c_{3W}$

are determined by the 3 EWPO's as follows:

$$m_W^2/m_Z^2 = c_0^2 + \frac{c_0^2}{c_0^2 - s_0^2}(c_0^2 c_T - 2s_0^2(c'_{HL} + 8c_{WB})$$

$$s_*^2 = s_0^2 + \frac{s_0^2}{c_0^2 - s_0^2}(c'_{HL} + 8c_{WB} - c_0^2 c_T) - \frac{1}{2}c_{HE} - s_0^2(c_{HL} - c_{HE})$$

$$g_L - g_R = \frac{1}{2}\frac{e_0}{s_0 c_0}(1 + c_{HL} - c_{HE} + \frac{1}{2}c_T)$$

and the 3 TGC's

$$\Delta_g = \frac{1}{c_0^2 - s_0^2}(\frac{1}{2}c_T - c'_{HL} - 8\frac{s_0^2}{c_0^2}c_{WB})$$

$$\Delta_\kappa = +8c_{WB}$$

$$\Delta_\lambda = -6\frac{e_0^2}{s_0^2}c_{3W}$$

At ILC with the full H-20 scenario the error on the TGC's are

$$\Delta(\Delta_\kappa) = 2 \times 10^{-4}$$

$$\Delta(\Delta_g) = 8 \times 10^{-4}$$

Through EWPOs and ILC measurements of TGC's the number of independent EFT parameters has been reduced from 10 to just 4: $c_H$ $c_{WW}$ $c_{BB}$ $c_6$

With $c_T$ and $c'_{HL}$ tightly constrained by EWPO's &TGC's , $c_H$ is obtained through a measurement of $\eta_Z$ using a combination of the $\sigma_{ZH}$ measurement and an angular analysis of $e^+e^- \rightarrow ZH$ , Z $\rightarrow e^+e^-, \mu^+\mu^-, q\bar{q}$ (more on this later).

$$\eta_Z = (1 - c_T - \frac{1}{2}c_H - c'_{HL})$$

With $c_{WB}$ tightly constrained by EWPO's &TGC's. $c_{WW}$ & $c_{BB}$ are obtained through measurements of $\Gamma(\mathrm{H} \rightarrow \gamma\gamma)$ (from LHC+ILC) and the HZZ Lorentz structure parameter $\zeta_Z$ measured at the ILC with the angular analysis of $e^+e^- \rightarrow ZH$ . The relationship between these two measurements and the coefficients $c_{WW}$ & $c_{BB}$ is given by

$$\zeta_A = \zeta_{2A} = 8s_0^2(c_{WW} - 2c_{WB} + c_{BB})$$

Combining LHC and ILC gives $\Delta g_{H\gamma\gamma} = 0.01$

$$\zeta_Z = \zeta_{2Z} = 8(c_0^2 c_{WW} + 2s_0^2 c_{WB} + \frac{s_0^4}{c_0^2} c_{BB})$$

At LCWS16 T. Ogawa obtained $\Delta\zeta_Z = 0.001$

Let's now rewrite the Lagrangian using our measured variables

$\eta_Z$ $\zeta_Z$ and the one remaining unconstrained EFT parameter $c_6$

$$\mathcal{L} = -\lambda_0 v_0 (\eta_Z - c_6) h^3 + \frac{2}{v_0}(1-\eta_Z) h\partial_\mu h \partial^\mu h + \eta_Z \frac{M_Z^2}{v_0} Z_\mu Z^\mu h + (2\eta_Z - 1)\frac{M_Z^2}{2v_0^2} Z_\mu Z^\mu h^2$$

$$+ \frac{\zeta_Z}{2v_0} Z_{\mu\nu} Z^{\mu\nu} h + \frac{\zeta_Z}{4v_0^2} Z_{\mu\nu} Z^{\mu\nu} h^2$$

In this EFT approach all of the couplings in the calculation of $\sigma(e^+e^- \to HHZ)$ are tightly constrained by the other Higgs coupling measurements, TGC measurements, and EWPT's. The only unconstrained parameter is $c_6$. If the best match to the measured $\sigma(e^+e^- \to HHZ)$ is $c_6 = 0$ within sys+stat errors then we have observed SM Higgs self coupling.

ILC 250+350+500 GeV with 2000+200+4000 $\text{fb}^{-1}$ (H-20 scenario full run $\Rightarrow$ 20.2 yrs)

$$\frac{\Delta\sigma(ZHH)}{\sigma(ZHH)} = 16.8\% \qquad \frac{\Delta\eta_Z}{\eta_Z} = 0.005 \quad \Delta\zeta_Z = 0.001$$

$\sigma(ZHH)$ can be measured with a precision of 16.8% at the ILC

## Prospects for the Full ILC Running Scenario

$\sqrt{s} = 500$ GeV, $\mathcal{L} = 4$ ab$^{-1}$, $P(e^+e^-) = (\pm 0.3, \mp 0.8)$

**Measurement prospects for $\lambda_{SM}$**

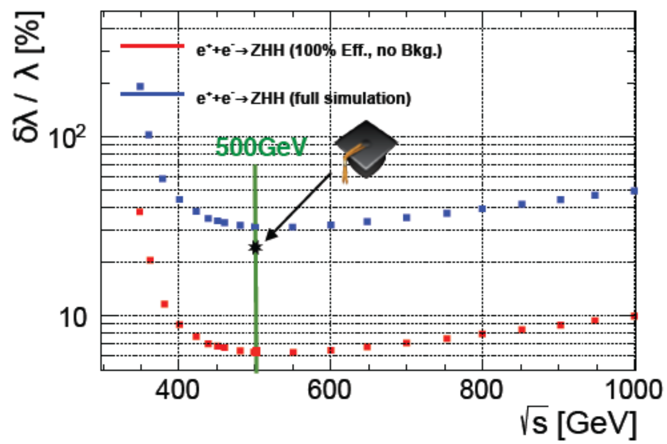
- for HH → bbbb

$$\frac{\Delta\sigma(ZHH)}{\sigma(ZHH)} = 21.1\% \rightarrow 5.9\sigma \text{ discovery}$$

- combined with HH → bbWW*

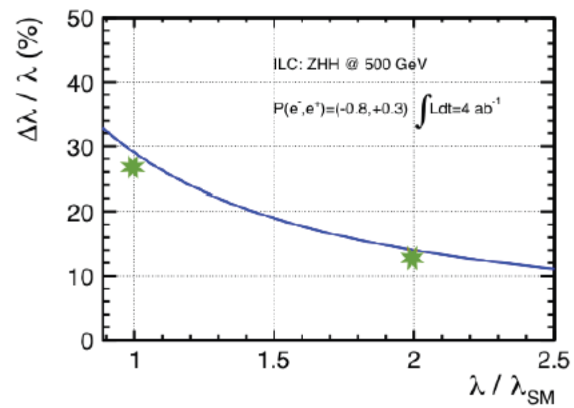
$$\frac{\Delta\sigma(ZHH)}{\sigma(ZHH)} = 16.8\% \rightarrow 8.0\sigma \text{ discovery}$$

- results in 26.6% precision on $\lambda_{SM}$
- advanced reconstruction gives 10% improvement
- combined with WW fusion @ 1TeV → 10% precision on $\lambda_{SM}$




**Measurement prospects for $\lambda \neq \lambda_{SM}$**

- $\sigma_{ZHH}$ enhanced compared to SM
- less affected by additional diagrams
- e. g. $\lambda = 2\lambda_{SM}$ results in 13% precision on $\lambda$



# Full Simulation Study of Anomalous VVH Couplings at ILC

2016/12/08
LCWS2016 @ Morioka

Tomohisa Ogawa (SOKENDAI)
Keisuke Fujii (KEK)
Junping Tian (U-Tokyo)

international linear collider

International Workshop on Future Linear Colliders
LCWS2016
5-9 DECEMBER, 2016
Alina Center & MALIOS,
MORIOKA CITY, IWATE, JAPAN

# Introduction

- Our convention for anomalous coupling analysis
- Relevant terms of the general Lagrangian which describes the couplings between the Higgs and W, Z bosons are parametrized using dimension-5 operators.

$$\mathcal{L}_{VVH} = 2M_V^2 \frac{1}{\Lambda}\left(\frac{\Lambda}{v} + a\right) H V_\mu^+ V^{-\mu} + C_V \frac{b}{\Lambda} H V_{\mu\nu}^+ V^{-\mu\nu} + C_V \frac{\tilde{b}}{\Lambda} H \epsilon^{\mu\nu\rho\sigma} V_{\mu\nu}^+ \tilde{V}_{\rho\sigma}^- \quad \text{(arXiv:1011.5805)}$$

$$(\Lambda = 1\ \text{TeV},\ \ \mathrm{C_V} = \frac{1}{2}, 1\ \ \text{for V} = \mathrm{Z, W})$$

  - "a" is a simple normalization parameter which affects the overall cross section of processes. (just rescales the SM-coupling)
  - "b" has a different spin structure which affects momentum spectra and changes the ratio of couplings to transverse or longitudinal components.
  - "bt" is a CP-violating parameter which affects angular/spin correlations.
- The strategy to estimate sensitivity to the anomalous parameters is to use kinematical information of final state particles such as momentum spectra and spin correlations.
  - Precise reconstruction is needed to catch slight deviations from predictions of the SM.

# Strategy of Shape Analysis

- **The definition of $\chi 2$ function to estimate sensitivity. ( 2-dimensional distribution )**

$$\chi^2 = \sum_{i=1}^{n}\sum_{j=1}^{n}\left[\frac{N^{SM}(x_{ij})\cdot f_{ij} - N^{BSM}(x_{ij};a,b,\tilde{b})\cdot f_{ij}}{\delta N^{SM}(x_{ij})}\right]^2 + \left[\frac{N^{SM}\cdot\epsilon - N^{BSM}\cdot\epsilon}{\delta\sigma\cdot N^{SM}\cdot\epsilon}\right]^2$$

- ***$f_{ij}$* is overall acceptance which includes the detector response function.**
- **$\delta$N is an error of remaining signals for each bin ( kinematical shapes are binned. ) which are estimated by full simulation.**

  — Background suppression —

  - Any angular variables are not applied for background suppression.
  - Momentum variables are also restricted.
  - Every cut values are optimized to get the best significance point.

$$Signif = \frac{N_{sig}}{\sqrt{N_{sig}+N_{bkg}}}$$

- N is the expected/calculated number of events.
- $\varepsilon$ is selection efficiency of a signal process.
- $\delta\sigma$ is an error of a cross section of a signal process, which is estimated by full simulation.
  - $\delta\sigma$ is set to 2.0% for a 250 GeV process and 3.0% for 500 GeV process.

- Three parameters are completely free
- Scale to H20 senario
- Considering processes are only ZZH

| $\sqrt{s}$ | (-,+) $[\mathrm{fb}^{-1}]$ | (+,-) $[\mathrm{fb}^{-1}]$ |
|---|---|---|
| 250 GeV | 1350 | 450 |
| ~~350 GeV~~ | ~~135~~ | ~~45~~ |
| 500 GeV | 1600 | 1600 |

$$\mathcal{L}_{VVH} = 2M_V^2 \frac{1}{\Lambda}\left(\frac{\Lambda}{v} + a\right) H V_\mu^+ V^{-\mu} + C_V \frac{b}{\Lambda} H V_{\mu\nu}^+ V^{-\mu\nu} + C_V \frac{\tilde{b}}{\Lambda} H \epsilon^{\mu\nu\rho\sigma} V_{\mu\nu}^+ \tilde{V}_{\rho\sigma}^-$$

$$(\Lambda = 1\ \mathrm{TeV},\ \ \mathrm{C_V} = \frac{1}{2}, 1\ \ \mathrm{for\ V = Z, W})$$

**Preliminary**

$$a = 0 \pm 0.021$$
$$b = 0 \pm 0.0050$$
$$\tilde{b} = 0 \pm 0.0057$$

$$\begin{pmatrix} 1 & -0.8 & 0.01 \\ -0.8 & 1 & 0.02 \\ 0.01 & 0.02 & 1 \end{pmatrix}$$

correlation matrix

$$\eta_Z = 2\left(1 + \frac{v_0}{\Lambda} a\right) \qquad \frac{\Delta\eta_Z}{\eta_Z} = \frac{v_0}{\Lambda}\Delta a = \frac{1}{4}\Delta a = 0.005$$

$$\zeta_Z = \frac{v_0}{\Lambda} b \qquad \Delta\zeta_Z = \frac{v_0}{\Lambda}\Delta b = \frac{1}{4}\Delta b = 0.001$$

Let's include operator coefficients constrained by TGC's & EWPO's.

$$\mathcal{L} = -\lambda_0 v_0 \left(\eta_Z + c_T - c_6\right) h^3 + \frac{2}{v_0}\left(1 - \eta_Z - c_T - c'_{HL}\right) h\partial_\mu h \partial^\mu h + \eta_Z \frac{M_Z^2}{v_0} Z_\mu Z^\mu h$$

$$+ (2\eta_Z - 1 - 3c_T)\frac{M_Z^2}{2v_0^2} Z_\mu Z^\mu h^2 + \frac{\zeta_Z}{2v_0} Z_{\mu\nu} Z^{\mu\nu} h + \frac{\zeta_Z}{4v_0^2} Z_{\mu\nu} Z^{\mu\nu} h^2$$

$$- \frac{e_0}{c_0 s_0}\left[(c_{HL} + c'_{HL})(\overline{e}_L \gamma_\mu e_L) + c_{HE}(\overline{e}_R \gamma_\mu e_R)\right] Z^\mu \left(\frac{h}{v_0} + \frac{1}{2}\frac{h^2}{v_0^2}\right)$$

$$m_W^2/m_Z^2 = c_0^2 + \frac{c_0^2}{c_0^2 - s_0^2}(c_0^2 c_T - 2s_0^2(c'_{HL} + 8c_{WB})$$

$$s_*^2 = s_0^2 + \frac{s_0^2}{c_0^2 - s_0^2}(c'_{HL} + 8c_{WB} - c_0^2 c_T) - \frac{1}{2}c_{HE} - s_0^2(c_{HL} - c_{HE})$$

$$g_L - g_R = \frac{1}{2}\frac{e_0}{s_0 c_0}(1 + c_{HL} - c_{HE} + \frac{1}{2}c_T)$$

$$\Delta_g = \frac{1}{c_0^2 - s_0^2}(\frac{1}{2}c_T - c'_{HL} - 8\frac{s_0^2}{c_0^2}c_{WB})$$

$$\Delta_\kappa = +8c_{WB}$$

$c_T$ , $c'_{HL}$ , $c_{HL}$ , $c_{HE}$ are linear combinations of $M_W^2 / M_Z^2$ , $s_*^2$ , $(g_L - g_R)$ , $\Delta_g$ , $\Delta_\kappa$

$$\frac{\Delta(M_W^2)}{M_Z^2} = 0.00029 \qquad \Delta s_*^2 = 0.00035$$

$$\frac{\Delta(g_L - g_R)}{g_L - g_R} = \frac{1}{2}\left[\left(\frac{\Delta\Gamma(Z \to e^+e^-)}{\Gamma(Z \to e^+e^-)}\right)^2 + \left(\frac{\Delta s_*^2}{s_*^2}\right)^2\right] = 0.00015$$

At ILC with the full H-20 scenario

$\Delta(\Delta_g) = 0.0008$ $\quad$ $\Delta\eta_Z = 0.005$

$\Delta(\Delta_\kappa) = 0.0002$ $\quad$ $\Delta\zeta_Z = 0.001$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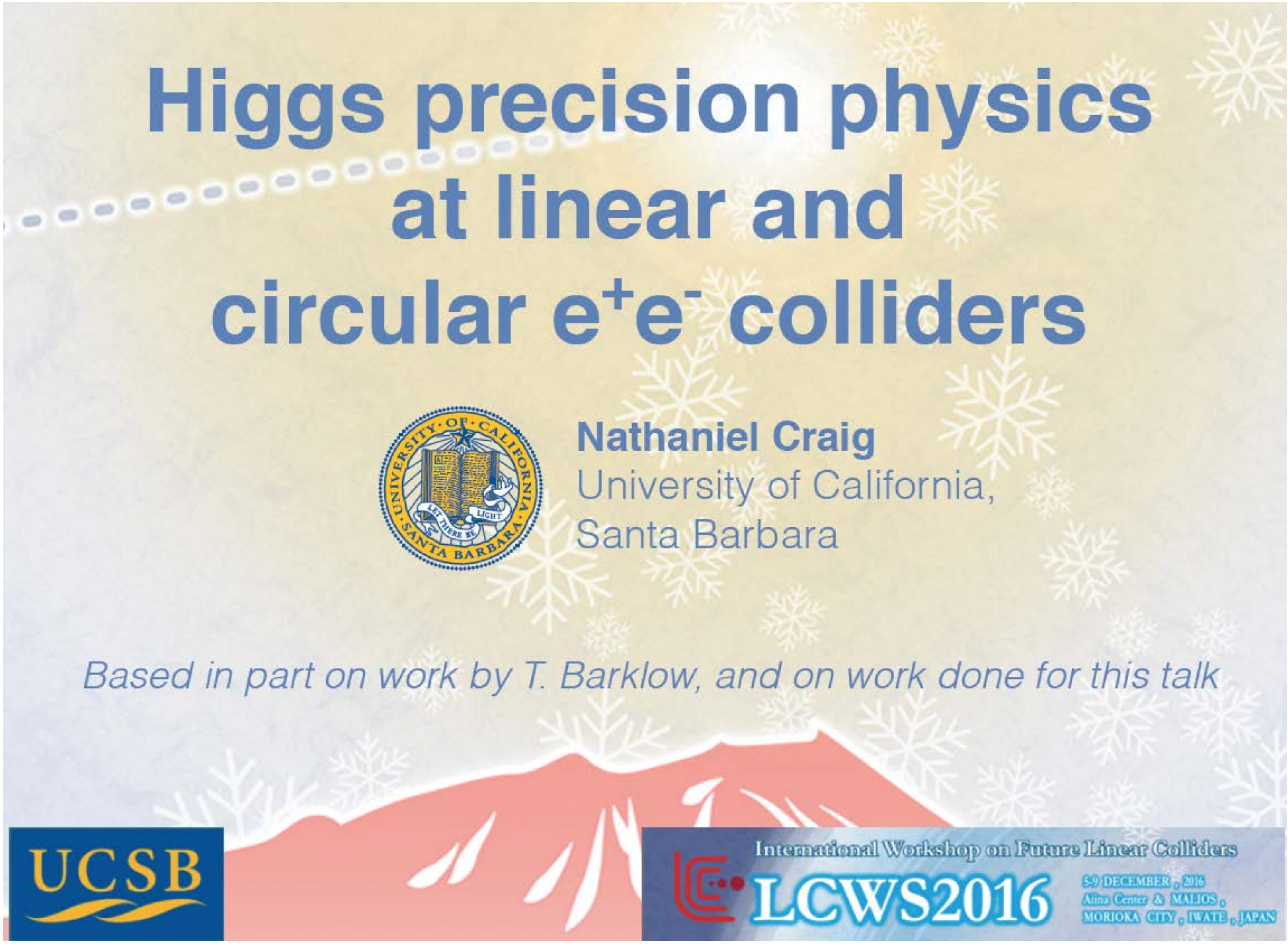
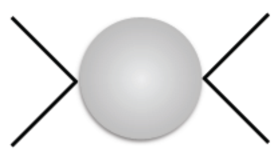

Higgs precision physics
at linear and
circular e+e- colliders
Nathaniel Craig
University of California,
Santa Barbara
Based in part on work by T. Barklow, and on work done for this talk
UCSB
International Workshop on Future Linear Colliders
LCWS2016
5-9 DECEMBER, 2016
Aiina Center & MALIOS,
MORIOKA CITY, IWATE, JAPAN

# Higgs Force Observables

If new particles are light, look for them directly via on-shell or off-shell Higgs.

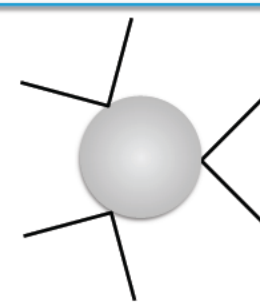
If new particles are heavy, integrate out and go to dim-6 EFT

*Integrating out singlet physics only generates $O_H$, $O_6$ at dim-6*



only f(∂|H|²)

$$\mathcal{O}_H = \frac{1}{2}\left(\partial_\mu |H|^2\right)^2$$



only f(|H|²)

$$\mathcal{O}_6 = |H|^6$$

So $O_H$, $O_6$ completely characterize Higgs coupling deviations from Standard Model singlets at dim-6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# Bounding the Higgs Force

*Measure Zh, ZZh, keep track of $c_6$, $c_H$ everywhere*

Already understand effects in Zh

Need to track of $c_6$, $c_H$ effects at ILC in ZZh

**Measurement prospects for $\lambda_{SM}$**

- for HH → bbbb

$$\frac{\Delta\sigma(\text{ZHH})}{\sigma(\text{ZHH})} = 21.1\% \rightarrow 5.9\sigma \text{ discovery}$$

- combined with HH → bbWW*

$$\frac{\Delta\sigma(\text{ZHH})}{\sigma(\text{ZHH})} = 16.8\% \rightarrow 8.0\sigma \text{ discovery}$$

From C. Dürig's talk

*$c_H$ shows up in $h^3$ and $h\partial h\partial h$*

$$\frac{\sigma(e^+e^- \to Zhh)}{\sigma_{SM}} = 1 - 3.6\,c_H + 7.4\,(16 c_{WW}) + 0.56\,c_6$$

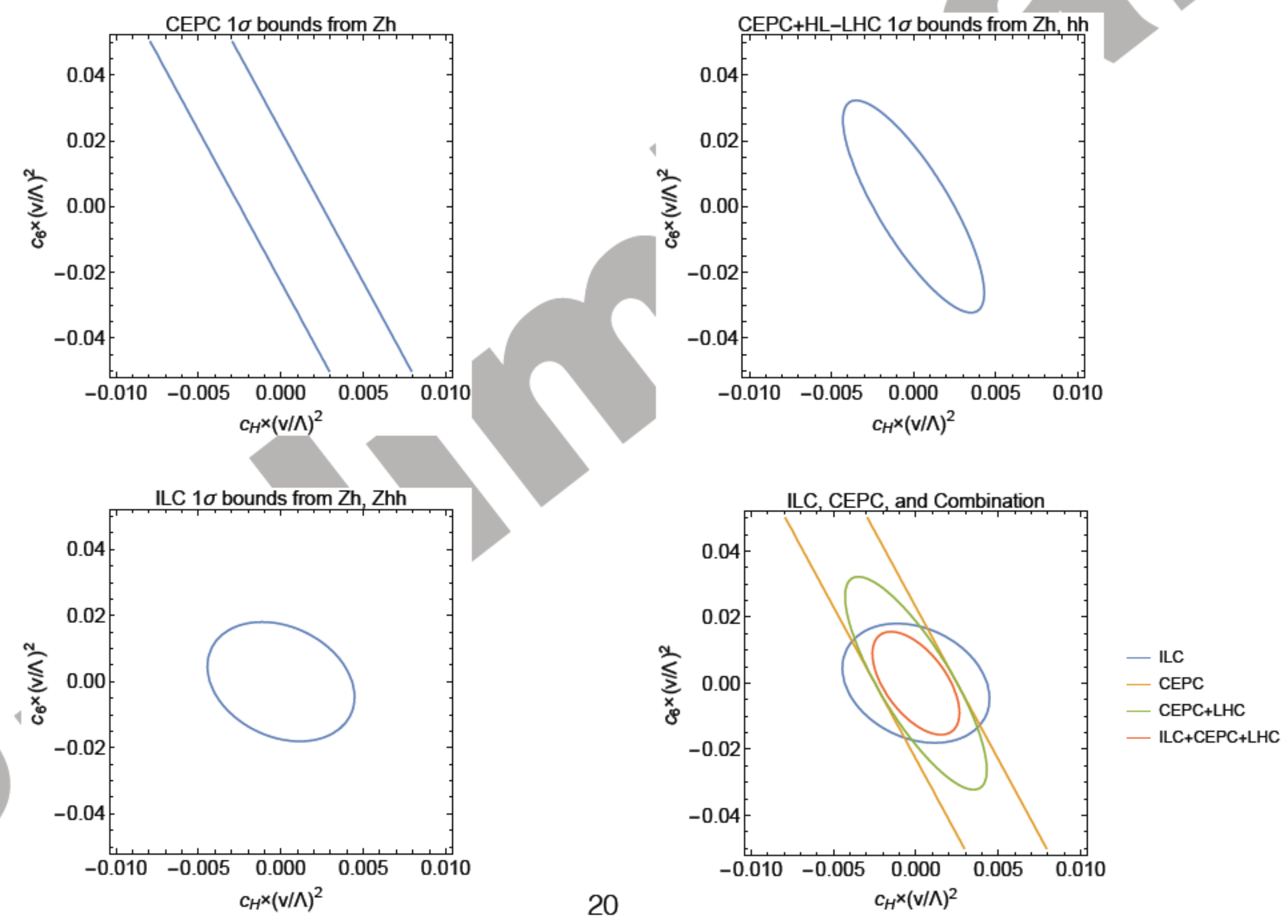
Higgs wavefunction renormalization & new vertex $\Delta\mathcal{L} = \frac{c_H}{v_0} h\partial_\mu h\partial^\mu h$

dim-6 vertices enhanced by $(s/m_Z^2)$

Barklow et al., from T. Tanabe's talk @ HC2016

*Note: their conventions for $c_H$, $c_6$.*
*Convert to mine, can check that I reproduce quoted trilinear bound.*

# Bounding the Higgs Force

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# Summary

- In an EFT approach all but one of the couplings in the calculation of $\sigma(e^+e^- \to HHZ)$ are tightly constrained by the other Higgs coupling measurements, TGC measurements, and EWPT's.

- The unmeasured ZZHH quartic coupling is related to the HZZ couplings, which are measured to 0.1% – 0.5% at the ILC in the H-20 scenario. The systematic error due to the unmeasured quartic coupling is therefore very small. A full error analysis is underway to propagate the errors due to all EWPO, TGC and Higgs measurements.

- A simplified similar analysis using $e^+e^- \to ZH$ only at CEPC indicates, on the one hand, that a measurement of double Higgs production from someplace else is required to close the limit contours in $(c_6, c_H)$ space. But it also demostrates that the CEPC improves the $(c_6, c_H)$ limits compared to HL-LHC or ILC alone.

- It is not sufficient, in the context of EFT's, to measure $\sigma$(ZH) in order to obtain the hZZ coupling. The Higgsstrahlung cross section $\sigma(\text{ZH}) = f(\eta_Z, \zeta_Z)$ and so one must also perform the angular analysis of $e^+e^- \to ZH$ , $\text{Z} \to e^+e^-, \mu^+\mu^-, q\bar{q}$ in order to distinguish $\eta_Z$ from $\zeta_Z$.
  This angular analysis is therefore not some backwater BSM search, but rather a Higgs measurement of fundamental importance at $\sqrt{s} = 250$ GeV.