High Precision Study of Higgs Physics

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理論物理

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Introduction

1. Thanks to the clean environment and the high luminosity in CEPC, many important properties of the Higgs boson can be measured to extremely high precisions.

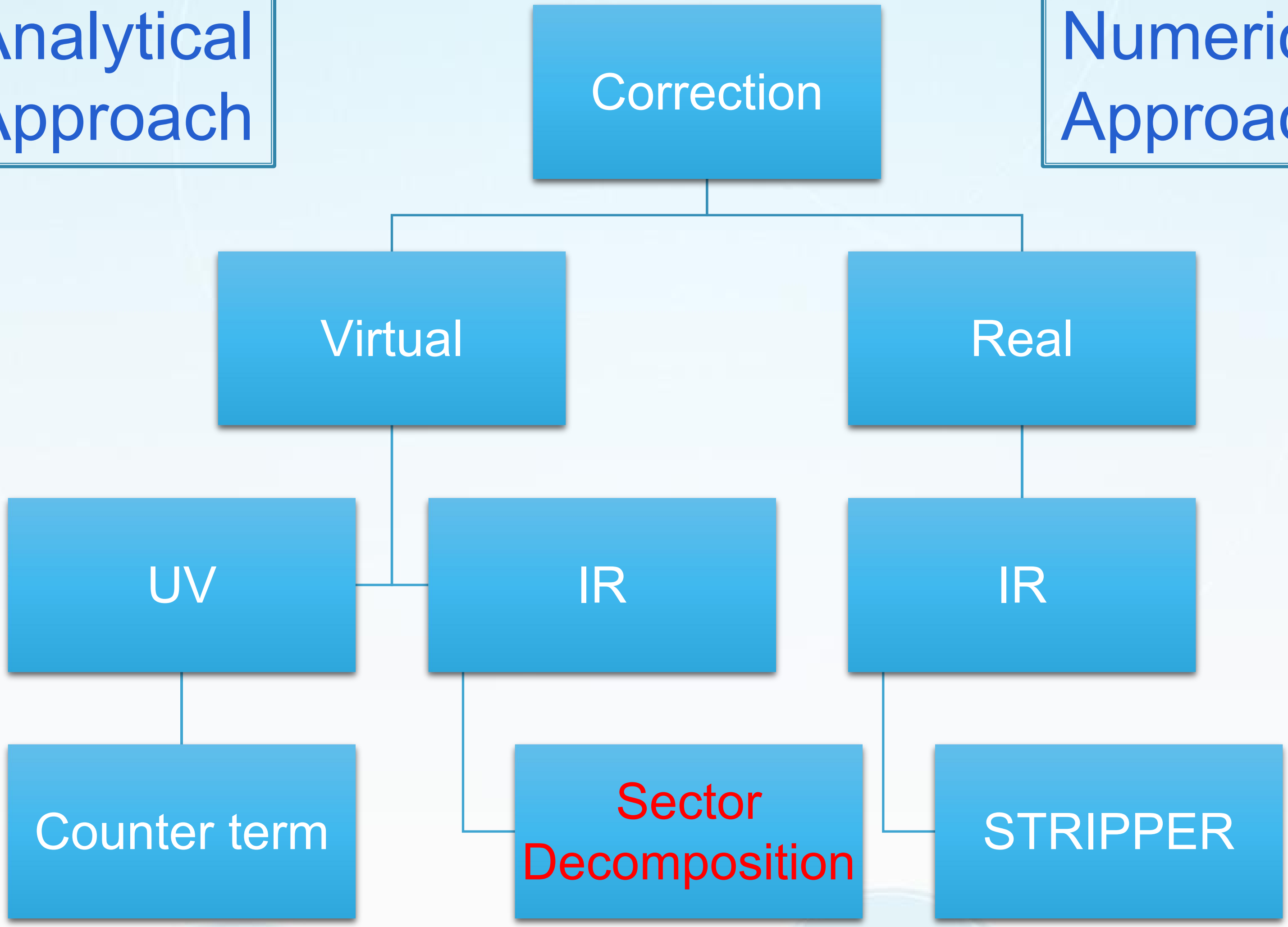
2. This will provide stringent tests of the Standard Model (SM) and has the potential to indirectly probe new physics beyond the SM which might exist at very high energy scales.

3. The theoretical prediction has to be known with a similar or even higher precision than the experimental.

Introduction

Analytical Approach

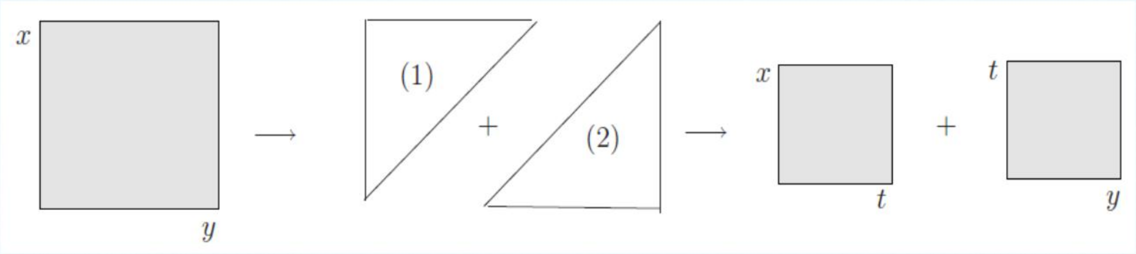
Numerical Approach



Sector Decomposition

Gudrun Heinrich (arXiv:0803.4177[hep-ph])

$$I = \int_0^1 dx \int_0^1 dy\, x^{-1-a\varepsilon} y^{-b\varepsilon} (x + (1-x)y)^{-1}$$



$$I = \int_0^1 dx \int_0^1 dy\, x^{-1-a\varepsilon} y^{-b\varepsilon} (x + (1-x)y)^{-1} [\theta(x-y) + \theta(y-x)]$$

We substitute y = x t in sector (1) and x = y t in sector (2) to remap the integration range to the unit square and obtain

$$I = \int_0^1 dx\, x^{-1-(a+b)\varepsilon} \int_0^1 dt\, t^{-b\varepsilon} (1 + (1-x)t)^{-1}$$

$$+ \int_0^1 dy\, y^{-1-(a+b)\varepsilon} \int_0^1 dt\, t^{-1-a\varepsilon} (1 + (1-y)t)^{-1}$$

Feynman parameter integrals

A general Feynman graph in D dimensions at L loops with N propagators and R loop momenta in the numerator

$$G^{\mu_1\ldots\mu_R}_{l_1\ldots l_R} = \int \prod_{l=1}^{L} \mathrm{d}^D\kappa_l \, \frac{k_{l_1}^{\mu_1}\ldots k_{l_R}^{\mu_R}}{\prod_{j=1}^{N} P_j^{\nu_j}(\{k\},\{p\},m_j^2)}$$

Feynman parameterization

$$G^{\mu_1\ldots\mu_R}_{l_1\ldots l_R} = \frac{\Gamma(N_\nu)}{\prod_{j=1}^{N}\Gamma(\nu_j)} \int_0^\infty \prod_{j=1}^{N} \mathrm{d}x_j \; x_j^{\nu_j-1}\,\delta\big(1-\sum_{i=1}^{N} x_i\big) \int \mathrm{d}^D\kappa_1\ldots\mathrm{d}^D\kappa_L$$
$$k_{l_1}^{\mu_1}\ldots k_{l_R}^{\mu_R}\left[\sum_{i,j=1}^{L} k_i^{\mathrm{T}}\,M_{ij}\,k_j - 2\sum_{j=1}^{L} k_j^{\mathrm{T}}\cdot Q_j + J + i\,\delta\right]^{-N\nu},$$

Momentum integration

Primary sectors

As the basic algorithm is the same for tensor integration, we will set R=0,

$$G = (-1)^{N_\nu} \frac{\Gamma(N_\nu - LD/2)}{\prod_{j=1}^{N} \Gamma(\nu_j)} \int_0^\infty \prod_{j=1}^{N} dx_j \, x_j^{\nu_j - 1} \, \delta(1 - \sum_{l=1}^{N} x_l) \frac{\mathcal{U}^{N_\nu - (L+1)D/2}}{\mathcal{F}^{N_\nu - LD/2}} .$$

Decompose the integration range into N sectors

$$\int_0^\infty d^N x = \sum_{l=1}^{N} \int_0^\infty d^N x \prod_{\substack{j=1 \\ j \neq l}}^{N} \theta(x_l \geq x_j)$$

Substitute

$$x_j = \begin{cases} x_l t_j & \text{for} \quad j < l \\ x_l & \text{for} \quad j = l \\ x_l t_{j-1} & \text{for} \quad j > l \end{cases}$$

$$G_l = \int_0^1 \prod_{j=1}^{N-1} \mathrm{d}t_j \, t_j^{\nu_j - 1} \frac{\mathcal{U}_l^{N_\nu - (L+1)D/2}(\vec{t})}{\mathcal{F}_l^{N_\nu - LD/2}(\vec{t})} , \quad l = 1, \ldots, N .$$

Subsectors

$$G_l = \int_0^1 \prod_{j=1}^{N-1} \mathrm{d}t_j \, t_j^{\nu_j - 1} \frac{\mathcal{U}_l^{N_\nu - (L+1)D/2}(\vec{t})}{\mathcal{F}_l^{N_\nu - LD/2}(\vec{t})} \quad , \quad l = 1, \ldots, N \, .$$

Decompose the corresponding r-cube into r subsectors

$$\prod_{j=1}^{r} \theta(1 \geq t_{\alpha_j} \geq 0) = \sum_{k=1}^{r} \prod_{\substack{j=1 \\ j \neq k}}^{r} \theta(t_{\alpha_k} \geq t_{\alpha_j} \geq 0)$$

$$t_{\alpha_j} \rightarrow \begin{cases} t_{\alpha_k} t_{\alpha_j} & \text{for} \quad j \neq k \\ t_{\alpha_k} & \text{for} \quad j = k \end{cases}$$

The resulting subsector integrals have the general form

$$G_{lk} = \int_0^1 \left(\prod_{j=1}^{N-1} \mathrm{d}t_j \, t_j^{a_j - b_j \epsilon} \right) \frac{\mathcal{U}_{lk}^{N_\nu - (L+1)D/2}}{\mathcal{F}_{lk}^{N_\nu - LD/2}} \, , \quad k = 1, \ldots, r \, .$$

The iteration stops if the functions contain a constant term

$$\begin{aligned} \mathcal{U}_{lk_1k_2\ldots} &= 1 + u(\vec{t}) \\ \mathcal{F}_{lk_1k_2\ldots} &= -s_0 + \sum_\beta (-s_\beta) f_\beta(\vec{t}) \end{aligned}$$

Extraction of the poles

For a particular t_j

$$\boxed{I_j = \int_0^1 dt_j \, t_j^{(a_j - b_j \epsilon)} \mathcal{I}(t_j, \{t_{i\neq j}\}, \epsilon)}$$

where, $\mathcal{I} = \mathcal{U}_{lk}^{N_\nu - (L+1)D/2} / \mathcal{F}_{lk}^{N_\nu - LD/2}$

Expand into a Taylor series around $t_j = 0$:

$$\mathcal{I}(t_j, \{t_{i\neq j}\}, \epsilon) = \sum_{p=0}^{|a_j|-1} \mathcal{I}_j^{(p)}(0, \{t_{i\neq j}\}, \epsilon) \frac{t_j^p}{p!} + R(\vec{t}, \epsilon)$$

$$\boxed{I_j = \sum_{p=0}^{|a_j|-1} \frac{1}{a_j + p + 1 - b_j \epsilon} \frac{\mathcal{I}_j^{(p)}(0, \{t_{i\neq j}\}, \epsilon)}{p!} + \int_0^1 dt_j \, t_j^{a_j - b_j \epsilon} R(\vec{t}, \epsilon)}$$

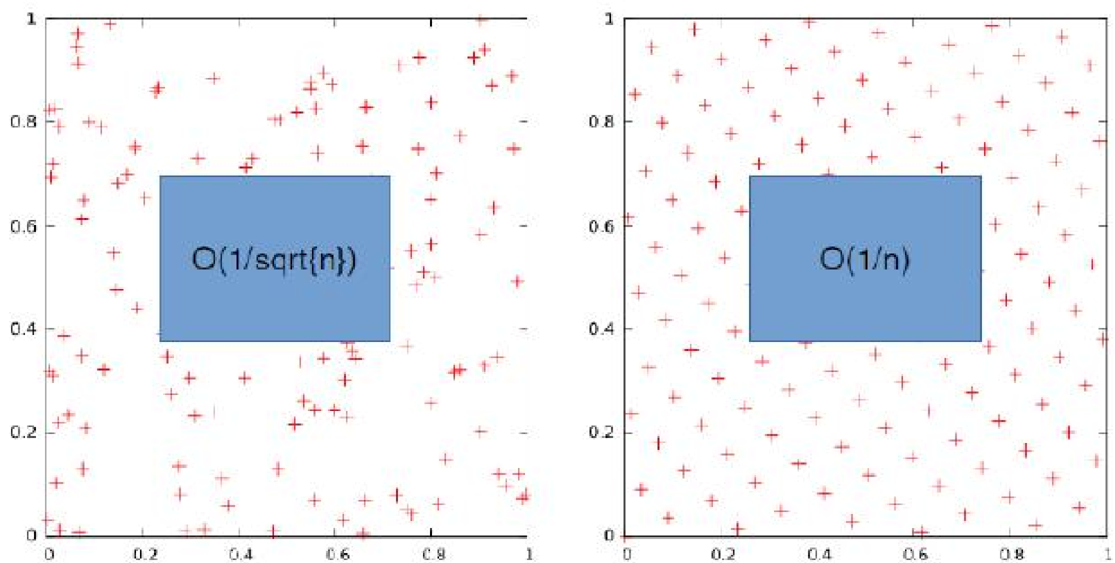
Therefore, the integrands can be expanded by small ϵ, and the coefficients of the Laurent series in ϵ can be evaluated numerically order by order.

Improvement on Numerical approach

Z.Li, J.Wang, Q.Yan and X.Zhao
Chinese Physics C, Vol. 40, No. 3 (2016) 033103

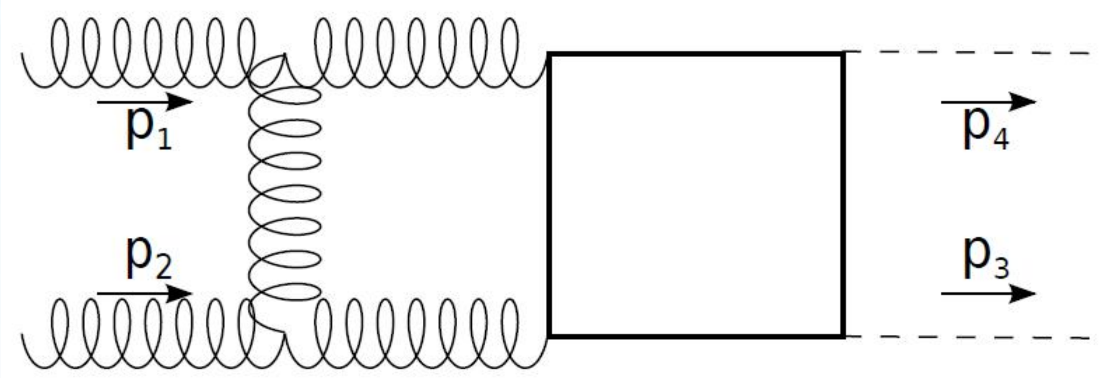
Improving the efficiency of numerical integration in sector decomposition by implementing a quasi-Monte Carlo method associated with the CUDA/GPU technique.

$$I(f) = \int_0^1 d^s x f(\vec{x}) \qquad I_{estimate}(f) = \sum_{i=0}^{n-1} f(\vec{x}_i)$$



Improvement on Numerical approach

Planar two-loop double box Feynman diagram for Higgs pair production via gluon fusion:

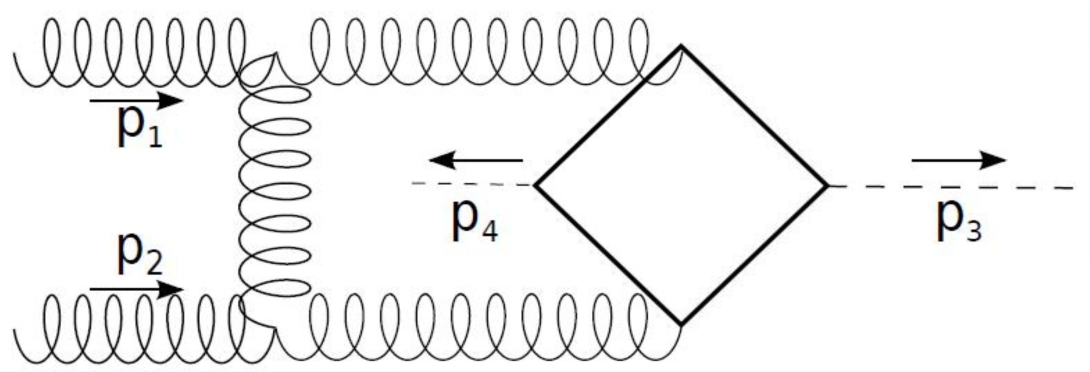


$$I_C = e^{-2\epsilon\gamma_E} s^{-3-2\epsilon} \sum_{i=0}^{i=2} \frac{P_i}{\epsilon^i}$$

	Vegas/CPU	QMC/GPU
P_2	$-7.959 \pm 0.009 - 10.586i \pm 0.009i$	$-7.949 \pm 0.003 - 10.585i \pm 0.005i$
P_1	$3.9 \pm 0.1 - 28.1i \pm 0.1i$	$3.831 \pm 0.005 - 28.022i \pm 0.005i$
P_0	$-3.9 \pm 0.8 + 92.3i \pm 0.8i$	$-4.63 \pm 0.07 + 92.13i \pm 0.07i$
Integration Time	45540s	19s

Improvement on Numerical approach

Non-planar two-loop double box Feynman diagram for the Higgs pair production via gluon fusion:



$$I_D = e^{-2\epsilon\gamma_E} s^{-3-2\epsilon} \sum_{i=0}^{i=2} \frac{P_i}{\epsilon^i}$$

	Vegas/CPU	QMC/GPU
P_2	$-3.848 \pm 0.004 + 0.0005i \pm 0.003i$	$-3.8482 \pm 0.0007 + 0.0004i \pm 0.0003i$
P_1	$3.81 \pm 0.03 - 6.41i \pm 0.03i$	$3.83 \pm 0.02 - 6.40i \pm 0.02i$
P_0	$77.2 \pm 0.2 + 20.1i \pm 0.2i$	$77.2 \pm 0.1 + 19.9i \pm 0.1i$
Integration Time	54290s	20s

Mixed QCD-EW corrections for Higgs boson production at e+e- colliders

Y.Gong, Z.Li, X.Xu, L.Yang and X.Zhao
arXiv:1609.03955[hep-ph]

Investigate the production of the Higgs boson at such an e+e-collider and calculate for the first time the mixed QCD-electroweak corrections to the total cross sections.

We obtain an approximate analytic formula for the cross section by sering expansion in m_t^{-1}. The approximate analytic formula evaluates much faster than the sector decomposition method. And it reproduces the exact numeric results rather well for collider energies up to 350 GeV.

Find that the corrections amount to a 1.3% increase of the cross section for a center-of-mass energy around 250 GeV. This is significantly larger than the expected experimental accuracy.

Mixed QCD-EW corrections for Higgs boson production at e+e- colliders

We choose the input parameters as:

m_t = 173.3 GeV, m_H = 125.1 GeV, m_Z = 91.1876 GeV, m_W = 80.385 GeV,

$\alpha(m_Z)$ = 1/127.94 , $\alpha_s(m_Z)$ = 0.118 .

Series expansion in m_t^{-1} and perform the expansion up to order m_t^{-4} .

$\sqrt{s}$ (GeV)	$\sigma_{\rm LO}$ (fb)	$\sigma_{\rm NLO}$ (fb)	$\sigma_{\rm NNLO}$ (fb)	$\sigma_{\rm NNLO}^{\rm exp.}$ (fb)
240	256.3(9)	228.0(1)	230.9(4)	230.9(4)
250	256.3(9)	227.3(1)	230.2(4)	230.2(4)
300	193.4(7)	170.2(1)	172.4(3)	172.4(3)
350	138.2(5)	122.1(1)	123.9(2)	123.6(2)
500	61.38(22)	53.86(2)	54.24(7)	54.64(10)

Mixed QCD-EW corrections for Higgs boson production at e+e- colliders

The mixed QCD-electroweak corrections increase the NLO cross section by about 1.3% for all 3 collider energies below the $\bar{t}t$ threshold of about 346 GeV. This is significantly larger than the expected experimental accuracy and has to be included for extracting the properties of the Higgs boson from the measurements of the cross sections in the future.

The variations of the NLO cross sections are too small to cover the higher order corrections, the NNLO cross sections exhibit larger scale variations than the NLO. We use this to estimate that the size of even higher order corrections amounts to about 0.2%.

Below the threshold: The analytical results approximates the numerical results remarkably well.

Above the threshold: The approximate formula is still valid with good precisions.

Mixed QCD-EW corrections for Higgs boson production at e+e- colliders

The analytical results have good convergence of the m_t^{-1} expansion as long as the energies are below or even slightly above the $\bar{t}\,t$ threshold. For center-of-mass energy $\sqrt{s}$ = 500 GeV which is far beyond the threshold, the power series tends to diverge as expected.

$\sqrt{s}$ (GeV)	$\mathcal{O}(m_t^2)$	$\mathcal{O}(m_t^0)$	$\mathcal{O}(m_t^{-2})$	$\mathcal{O}(m_t^{-4})$
240	81.8%	16.2%	1.4%	0.4%
250	81.7%	16.1%	1.5%	0.5%
300	80.0%	15.2%	2.1%	1.1%
350	69.7%	12.6%	2.7%	2.1%
500	137%	18.6%	17.3%	31.1%

Conclusion

- Make a brief review of sector decomposition.

- Compare the MC and QMC, QMC has a faster speed and higher accuracy, which makes the direct numerical approach viable for precise investigation of higher order effects in multi-loop processes.

- Mixed QCD-EW corrections for Higgs boson production at e+e- colliders amount to a 1.3% increase of the cross section for a center-of-mass energy around 250 GeV. And obtain an approximate analytical formula for the cross section, Thanks!