Spectroscopy of the Fully-Heavy Tetraquarks

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will appear soon on arXiv

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Outline

- Motivation & History
- An Effective Field Theory (Nonrelativistic)
- Relativized Diquark Model
- Result Comparison
- Mass Inequality Relations
- Summary

Motivation & History

- Does Nature allows the formation of fully-heavy tetraquarks?

  An open question…!
- Stability is in ICU currently, discussed by Richard et. al. PRD 95, 054019 (2017)
- $cc\bar{q}\bar{q}$ & $bb\bar{q}\bar{q}$ are argued as stable against strong decays Lipkin PLB 172 (1986)
- $bb\bar{b}\bar{b}$ or $cc\bar{c}\bar{c}$ are quite similar to Polyelectrons, $(e^+e^+e^-e^-)$
- In 1945, Wheeler speculated the existence of Positronium molecule, P_{S2}, with very tiny binding Wheeler, ANYA Sci. 48, 219 (1946)
- Soon later, Ore put a question mark on the stability of P_{S2} Ore, PR 70, 90 (1946)
- Ore & Hylleraas came up with an elegant prescription of solving equal-mass four-body problem and proved the stability of P_{S2}

  Ore & Hylleraas, PR 70, 90 (1946)
- Finally, debate ended on experimental observation

  Cassidy & Mills, Nature 449, 195 (2007)

EFT Approach

- Lowest-lying heavy $Q\bar{Q}$ treated as weekly-coupled system

- When $mv_Q \gg \Lambda_{\rm QCD}$, dynamics is dominant by short-range interactions
  Brambilla & Vairo, PRD 62, 094019 (2000)

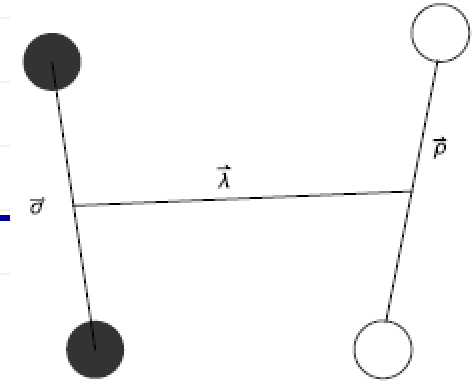
- First few Υ and B_c considered as weekly-coupled systems
  Titard & Yndurain PRD 49, 6007 (1994)
  Brambilla et. al. PLB 513, 381 (2001)

- Doubly- and triply-heavy baryons as weekly-coupled bound states
  Brambilla et. al. PRD 72, 034021 (2005) & Yu Jia, JHEP 10, 073 (2006)

- Lowest $bb\bar{b}\bar{b}$ is also expected to be a weekly-coupled bound, dominant interaction is short-range

- mass scale $\sim m_b$ chromomegnectic interactions are tiny

Nonrelativistic EFT



- Weinstein and Isgur four equal-mass quark Hamiltonian

$$\mathcal{H}^{\rm NR} = \sum_{i=1}^{4}\left[m_i + \frac{\mathbf{p}_i^2}{2m_i}\right] + \sum_{i<j}\left[V_{\rm SI}(\mathbf{r}_{ij}) + V_{\rm hyp}(\mathbf{r}_{ij})\right]$$

- At the leading order, short-distance interaction is taken OGE/color Coulomb

$$V_{\rm SI}(\mathbf{r}_{ij}) = \sum_{i<j}\frac{\lambda_i}{2}\cdot\frac{\lambda_j}{2}\frac{\alpha_s}{|\mathbf{r}_i - \mathbf{r}_j|}$$

NLO

$$V_{\rm hyp}(\mathbf{r}_{ij}) = \sum_{i<j} -\frac{8\pi\alpha_s}{3m_i m_j}\tilde{\delta}(\mathbf{r}_{ij})\frac{\lambda_i}{2}\cdot\frac{\lambda_j}{2}\mathbf{S}_i\cdot\mathbf{S}_j$$

- K.E matrix elements

$$T = \frac{\mathbf{p}_\sigma^2}{2m_1} + \frac{\mathbf{p}_\rho^2}{2m_2} + \frac{\mathbf{p}_\lambda^2}{2m_3} = \frac{1}{m_b}\left(\mathbf{p}_\sigma^2 + \mathbf{p}_\rho^2 + 2\mathbf{p}_\lambda^2\right)$$

Red. masses

- Variational method → trail wavefunction (spatial) $\psi(\sigma,\rho,\lambda)_{\rm spatial} = \mathcal{N}^{-1}\prod_{i=1}^{3}\exp\left[-\frac{1}{2}\beta^2\xi_i^2\right]$

- Thanks to Ore & Hylleraas prescription of $(e^+e^+e^-e^-)$

$$\psi_{\rm spatial}(bb) = \psi_{\rm spatial}(\bar{b}\bar{b})$$

reduces integration variables

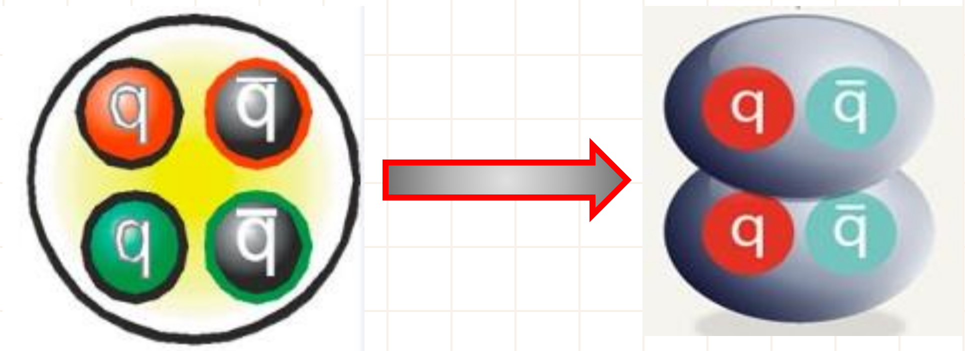
Color Representations

- Physical color wavefunction of $bb\bar{b}\bar{b}$ system

$$|\psi_c\rangle = \alpha\,|\bar{3}_{12}3_{34}\rangle + \beta\,|6_{12}\bar{6}_{34}\rangle$$

- $|1_{12}1_{34}>$ or $|8_{12}8_{34}>$ form an independent basis $|\bar{3}_{12}3_{34}>$ and $|6_{12}\bar{6}_{34}>$
- We work in $|\bar{3}_{12}3_{34}>$ basis

- A transition



Flip-flop transition, Okiharu et. al. PRD 72, 014505 (2005)

- Might possible for $Q\bar{Q}q\bar{q}$, but not likely to possible in the case of lowest fully-heavy $QQ\bar{Q}\bar{Q}$, mass < threshold

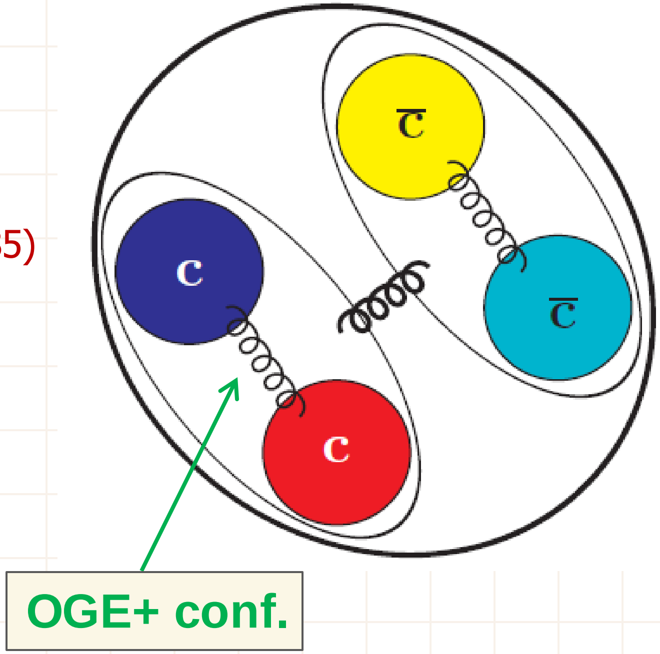
Relativized Diquark Model

- Effective d-o-f is diquark, D_{bb} or $D_{\bar{b}\bar{b}}$
- Interaction b/w bb $(\bar{b}\bar{b})$, OGE + confined pot.
  of GI model, Godfrey & Isgur, PRD 32, 189-231 (1985)
- No spatial excitations b/w bb or $\bar{b}\bar{b}$
- Due to same flavor, lowest axial-vector D_{bb}

$$M_{bb} = 9845 \text{ MeV}$$

- 0^{++} $1S$ fully-bottom tetraquark mass

$$M_{bb\bar{b}\bar{b}} = 18.748 \text{ GeV}$$



- All model parameters are fixed from earlier studies, GI
- For $cc\bar{q}\bar{q}$ & $c\bar{c}q\bar{q}$ study is in progress for the spectrum, will available soon
- For the decays into open-bottom/hidden-bottom meson(s) or singly b-baryons, the diquark configurations is not favorable

$\Longrightarrow$ Favorable for doubly b-baryonic decays

Result Comparison

Reference	Mass [GeV]	Binding [GeV]
This Work (NR)	18.641	-0.559
This Work (REL)	18.748	-0.942
Karliner *et. al.* [37]	18.862 ± 0.025	-0.856
Bai *et. al.* [36]	18.690 ± 0.03	-0.330
Berezhnoy *et. al.* [42]	18.754	–
Chen *et. al.* [40]	18.462 ± 0.15	–
Wu *et. al.* [38]	$18.462 \sim 18.568$	–
Wang [41]	18.84 ± 0.09	–

Why so large B. E ?

TABLE I: Different theoretical predictions of masses and binding energies for 0^{++} $X_{bb\bar{b}\bar{b}}$. The symbol – indicates that the prediction is not available. All physical quantities are in unit of GeV. The acronyms NR and REL refer to nonrelativistic and relativized results, respectively.

Large Binding??? Diquark Model

- Four-body problem is approximated by two-body system
- Six interactions → one effective interaction
- All interactions contributing –ive to B.E in $|\bar{3}_{12}3_{34}>$ color configuration, so in diquark model B.E is large
- B.E of the order of $\approx mv_Q$ is not irrational
- If we turn on all the interactions among constituents, it is a direct consequence of decreasing B.E
- Over estimation of B.E may be a drawback of diquark model

Mass Inequality (MI) Relations

- Using Nussinov's prescription of baryon-meson mass inequality

$$M_{\mathrm{Baryon}} \geq \frac{3}{2} M_{\mathrm{Meson}}$$

Nussinov, PRL 51, 2081 (1983)

$cc\bar{q}\bar{q}$ and $bb\bar{q}\bar{q}$ Lipkin PLB 172, 241 (1986)

- Extension to $bb\bar{b}\bar{b}$ system

$$\begin{aligned} H_4(q_1 q_2 \bar{q}_3 \bar{q}_4) &= T_1(q_1) + T_2(q_2) + T_3(\bar{q}_3) + T_4(\bar{q}_4) + V_{q_1 q_2} + V_{\bar{q}_3 \bar{q}_4} \\ &+ V_{q_1 \bar{q}_3} + V_{q_1 \bar{q}_4} + V_{q_2 \bar{q}_3} + V_{q_2 \bar{q}_4} . \end{aligned}$$

- Using $V_{qq} = \frac{1}{2} V_{q\bar{q}}$

$$\begin{aligned} 3H_4(q_1 q_2 \bar{q}_3 \bar{q}_4) &= \left(T_1 + T_2 + \tfrac{3}{2} V^{q\bar{q}}_{q_1 q_2}\right) + \left(T_3 + T_4 + \tfrac{3}{2} V^{q\bar{q}}_{\bar{q}_3 \bar{q}_4}\right) + \left(T_1 + T_3 + 3V_{q_1 \bar{q}_3}\right) \\ &+ \left(T_1 + T_4 + 3V_{q_1 \bar{q}_4}\right) + \left(T_2 + T_3 + 3V_{q_2 \bar{q}_3}\right) + \left(T_2 + T_4 + 3V_{q_2 \bar{q}_4}\right) \\ &= H_{12} + H_{34} + H_{13} + H_{14} + H_{23} + H_{24} . \end{aligned}$$

- Projecting it to $bb\bar{b}\bar{b}$ bound system

$$3B_{bb\bar{b}\bar{b}} \geq 2\left(\frac{3}{2} B_{b\bar{b}}\right) + 4(3B_{b\bar{b}}) \quad \boxed{\Rightarrow B_{bb\bar{b}\bar{b}} \geq 5B_{b\bar{b}}}$$

MI Relations

$m_b = 4.8$ GeV	$m_c = 1.585$ GeV	$\alpha_s = 0.29$
$B_{b\bar{b}} = 0.152$ GeV	$B_{c\bar{c}} = 0.102$ GeV	$\alpha_s' = 0.412$

- Relations are somewhat model independent,

  However, the numerical values are model dependent

State	Expression for Mass	Lowest Mass (GeV)
$X_{bb\bar{b}\bar{b}}$	$\approx 4m_b + 5B_{b\bar{b}}$	≥ 18.435
$X_{bb\bar{b}\bar{c}}$	$\approx 3m_b + m_c + \frac{5}{2}(B_{b\bar{b}} + B_{b\bar{c}})$	≥ 15.578
$X_{bb\bar{c}\bar{c}}/X_{cc\bar{b}\bar{b}}$	$\approx 2m_b + 2m_c + \frac{1}{2}(B_{b\bar{b}} + B_{c\bar{c}}) + 4B_{b\bar{c}}$	≥ 12.399
$X_{b\bar{b}c\bar{c}}$	$\approx 2m_b + 2m_c + B_{b\bar{b}} + B_{c\bar{c}} + 3B_{b\bar{c}}$	≥ 12.333
$X_{cc\bar{c}\bar{b}}$	$\approx 3m_c + m_b + \frac{5}{2}(B_{c\bar{c}} + B_{c\bar{b}})$	≥ 9.147
$X_{cc\bar{c}\bar{c}}$	$\approx 4m_c + 5B_{c\bar{c}}$	≥ 5.83

- For $X_{bb\bar{b}\bar{b}}$ all the theoretical predictions are above this minima

Summary

- We argue that there is not any Natural restriction on the existence of $bb\bar{b}\bar{b}$ bound systems
- NREFT with OGE potential → results are more reliable than the diquark models
- Decays of $bb\bar{b}\bar{b}$ are an interesting and itself a vast topic, study is ongoing, will be available soon on arXiv
- A single experimental observation provides a good test to presented MI relations
- It is a potential need to scan the energy regime 18.5 ~ 19 GeV, a fully-heavy system will definitely deepen our understanding of QCD
- Looking forward to have more surprises from LHC and long-waiting Belle II

Thanks for Your Attention