# Study Long-distance Processes Using LQCD: from Flavor Physics to Nuclear Physics

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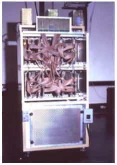
# Introduction to lattice QCD

- Invented by Kenneth G. Wilson in 1973
- 1st numerical implementation by M. Creutz in 1979
- QCD computers 1983 – 2011

Matrix Multiplier

1Mflops 1983

16-Node



256 Mflops 1985

64-Node


1.0 Gflops 1987

256-Node

16 Gflops 1989

QCDSP

600 Gflops 1998

QCDOC


20 Tflops 2005

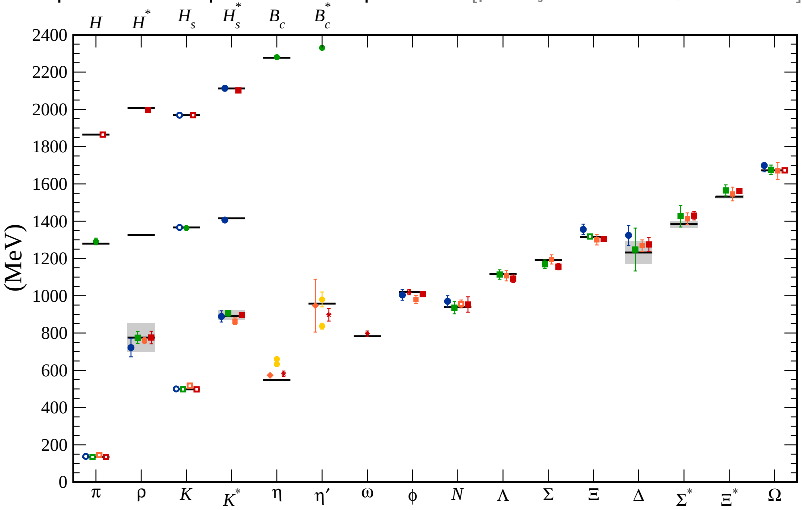
LLNL Sequoia, IBM

20 Pflops 2011

QCD computers start to enter in the Eflops generation ($10^{18}$ operations/second)

## Hadron spectrum from lattice QCD

- Input: $\alpha_s$, quark masses; set by $\pi$, $K$, $\cdots$ (empty symbols in the plot)
- Output: hadron spectrum vs experiment [plot by A. Kronfeld, 1209.3468]



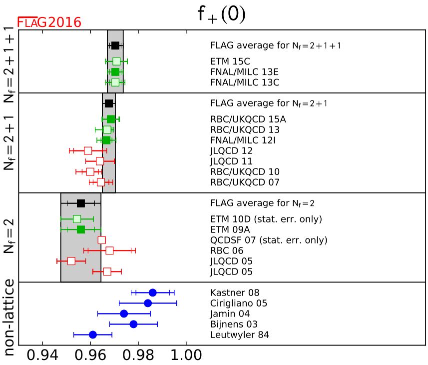

**Progress on exotic hadrons** [Y. Chen's plenary talk]

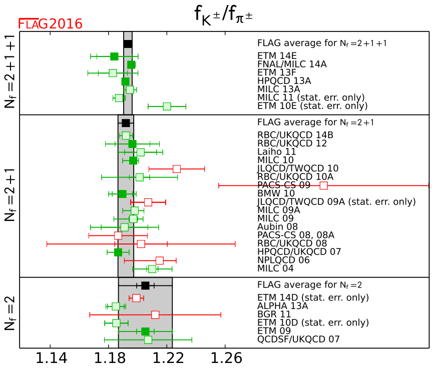
# Milestone: $f_+^{K\pi}(0)$ and $f_{K^\pm}/f_{\pi^\pm}$

## Flavor Lattice Averaging Group (FLAG) average, updated in Nov. 2016

$$f_+^{K\pi}(0) = 0.9706(27) \quad \Rightarrow \quad 0.28\% \text{ error}$$

$$f_{K^\pm}/f_{\pi^\pm} = 1.1933(29) \quad \Rightarrow \quad 0.25\% \text{ error}$$



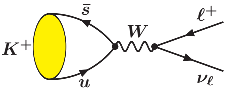

## Experimental information [arXiv:1411.5252, 1509.02220]

$$K_{\ell 3} \quad \Rightarrow \quad |V_{us}|f_+(0) = 0.2165(4) \quad \Rightarrow \quad |V_{us}| = 0.2231(7)$$

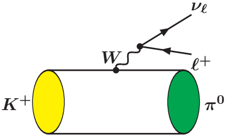
$$K_{\mu 2}/\pi_{\mu 2} \quad \Rightarrow \quad \left|\frac{V_{us}}{V_{ud}}\right| \frac{f_{K^\pm}}{f_{\pi^\pm}} = 0.2760(4) \quad \Rightarrow \quad \left|\frac{V_{us}}{V_{ud}}\right| = 0.2313(7)$$

# "Standard" and "non-standard" observables

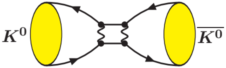
**Lattice QCD is powerful for "standard" observables $\langle f|O|i\rangle$** with




$K_{\ell 2} \to f_K$




$K_{\ell 3} \to f_+(0)$

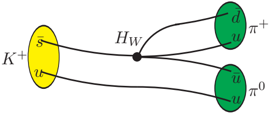


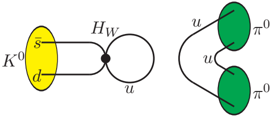

$K^0$-$\overline{K^0}$ mixing $\to B_K$

- single local operator insertion
- only single stable hadron or vacuum in the initial/final state
- spatial momenta carried by particles need to be small compared to $1/a$ (not a problem for Kaon physics, but essential for $B$ decays)

**Go beyond "standard"**, e.g.

- $K \to \pi\pi$ decay: $\langle \pi\pi|H_W|K\rangle$

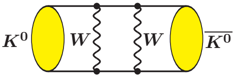


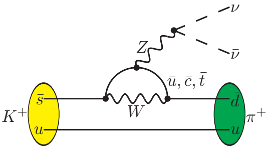




- LD processes and non-local hadronic matrix elements

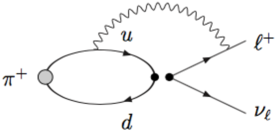
# LD processes and non-local matrix elements $\langle f|O_1\,O_2|i\rangle$

$2^{nd}$ order electroweak interaction

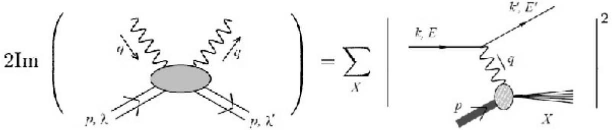


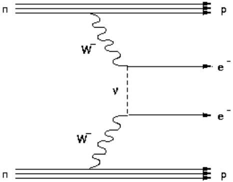

Electromagnetic correction



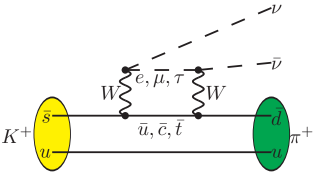

Inclusive decay and deep inelastic scattering [R. Young's plenary talk]

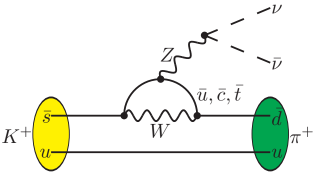


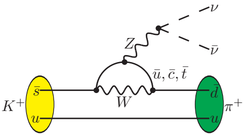

Neutrinoless double beta decay



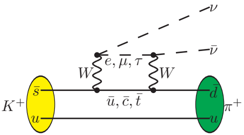
# Start with flavor physics
# – use $K^+ \to \pi^+ \nu\bar{\nu}$ as an example





$Z$-exchange



$W$-$W$

$K^+ \to \pi^+ \nu \bar{\nu}$: largest contribution from top quark loop, thus theoretically clean

$$\mathcal{H}_{eff} \sim \frac{G_F}{\sqrt{2}} \cdot \underbrace{\frac{\alpha_{\rm EM}}{2\pi \sin^2\theta_W} \lambda_t X_t(x_t)}_{\mathcal{N} \sim 2\times 10^{-5}} \cdot (\bar{s}d)_{V-A} (\bar{\nu}\nu)_{V-A}$$

Probe the new physics at scales of $\mathcal{N}^{-\frac{1}{2}} M_W = O(10\ \text{TeV})$

Past experimental measurement is 2 times larger than SM prediction

$$\text{Br}(K^+ \to \pi^+ \nu \bar{\nu})_{\text{exp}} = 1.73^{+1.15}_{-1.05} \times 10^{-10} \qquad \text{arXiv:0808.2459}$$

$$\text{Br}(K^+ \to \pi^+ \nu \bar{\nu})_{\text{SM}} = 9.11 \pm 0.72 \times 10^{-11} \qquad \text{arXiv:1503.02693}$$

but still consistent with $> 60\%$ exp. error

# New experiments

**New generation of experiment:** NA62 at CERN

- aims at observation of $O(100)$ events [2014-2018]
- 10%-precision measurement of $\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})$

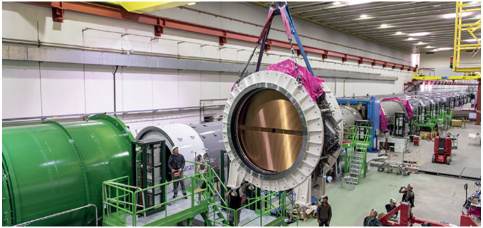
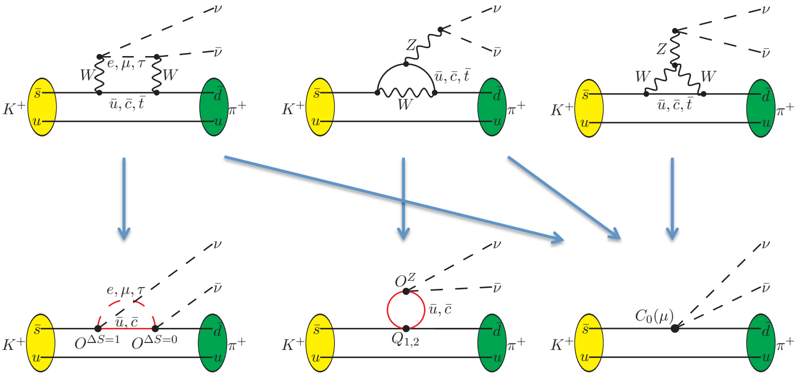


Fig: 09/2014, the final straw-tracker module is lowered into position in NA62

**$K_L \to \pi^0 \nu\bar{\nu}$**

- even more challenging since all the particles involved are neutral
- only upper bound was set by KEK E391a in 2010
- new KOTO experiment at J-PARC designed to observe $K_L$ decays
  - one candidate event is found very recently [arXiv:1609.03637]

# OPE: integrate out heavy fields $Z$, $W$, $t$, $\cdots$




Local SD contribution

Bilocal LD contribution

Hadronic part known: $\langle \pi^+ | V_\mu | K^+ \rangle$

$\langle \pi^+ \nu \bar{\nu} | Q_A(x) Q_B(0) | K^+ \rangle$: need lattice QCD

# $2^{\mathrm{nd}}$-order weak interaction and bilocal matrix element

Hadronic matrix element for the $2^{\mathrm{nd}}$-order weak interaction

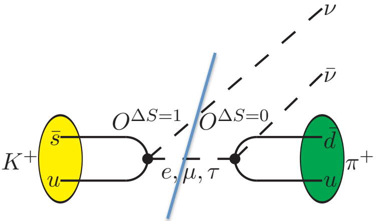
$$\int_{-T}^{T} dt\, \langle \pi^+ \nu \bar{\nu} | T\left[Q_A(t) Q_B(0)\right] | K^+ \rangle$$
$$= \sum_n \left\{ \frac{\langle \pi^+ \nu\bar{\nu} | Q_A | n \rangle \langle n | Q_B | K^+ \rangle}{M_K - E_n} + \frac{\langle \pi^+ \nu\bar{\nu} | Q_B | n \rangle \langle n | Q_A | K^+ \rangle}{M_K - E_n} \right\} \left(1 - e^{(M_K - E_n)T}\right)$$

- For $E_n > M_K$, the exponential terms exponentially vanish at large $T$
- For $E_n < M_K$, the exponentially growing terms must be removed
- $\sum_n$: principal part of the integral replaced by finite-volume summation
  - possible large finite volume correction when $E_n \to M_K$

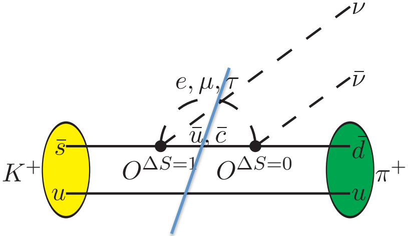
[Christ, XF, Martinelli, Sachrajda, PRD 91 (2015) 114510]

# Low lying intermediate states

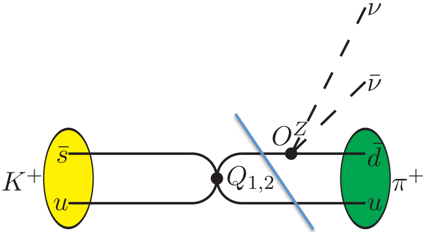
[Christ, XF, Portelli, Sachrajda, PRD 93 (2016) 114517]



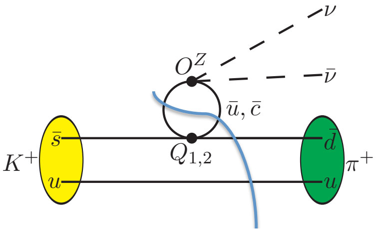

$K^+ \to \ell^+\nu \quad \& \quad \ell^+ \to \pi^+\bar{\nu}$




$K^+ \to \pi^0\ell^+\nu \quad \& \quad \pi^0\ell^+ \to \pi^+\bar{\nu}$




$K^+ \xrightarrow{H_W} \pi^+ \quad \& \quad \pi^+ \xrightarrow{V_\mu} \pi^+$



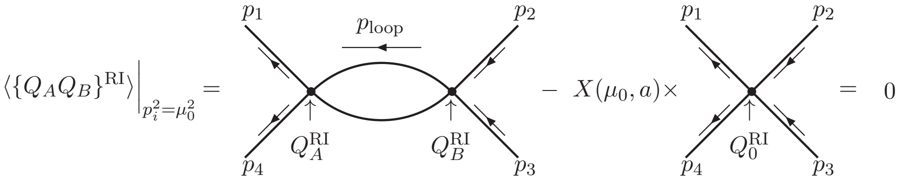

$K^+ \xrightarrow{H_W} \pi^+\pi^0 \quad \& \quad \pi^+\pi^0 \xrightarrow{A_\mu} \pi^+$

# Short-distance divergence

[Christ, XF, Portelli, Sachrajda, PRD 93 (2016) 114517]

## SD divergence appears in $Q_A(x)Q_B(0)$ when $x \to 0$

- Introduce a counter term $X \cdot Q_0$ to remove the SD divergence




The coefficient $X$ is determined in the RI/(S)MOM scheme

- The bilocal operator in the $\overline{\mathrm{MS}}$ scheme can be written as

$$\left\{\int d^4x\, T[Q_A^{\overline{\mathrm{MS}}}(x)\,Q_B^{\overline{\mathrm{MS}}}(0)]\right\}^{\overline{\mathrm{MS}}}$$

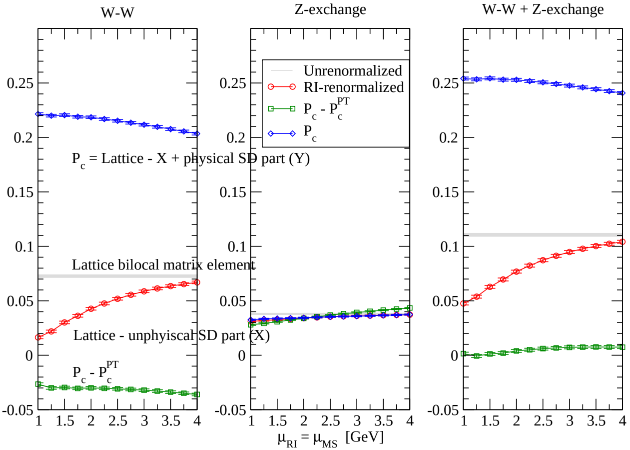
$$= Z_A Z_B \left\{\int d^4x\, T[Q_A^{\mathrm{lat}}\,Q_B^{\mathrm{lat}}]\right\}^{\mathrm{lat}} + \left(-X^{\mathrm{lat}\to\mathrm{RI}} + Y^{\mathrm{RI}\to\overline{\mathrm{MS}}}\right) Q_0(0)$$

  - $X^{\mathrm{lat}\to\mathrm{RI}}$ is calculated using NPR and $Y^{\mathrm{RI}\to\overline{\mathrm{MS}}}$ calculated using PT

First results @ $m_\pi = 420$ MeV, $m_c = 860$ MeV

[Bai, Christ, XF, Lawson, Portelli, Schrajda, PRL 118 (2017) 252001 ]

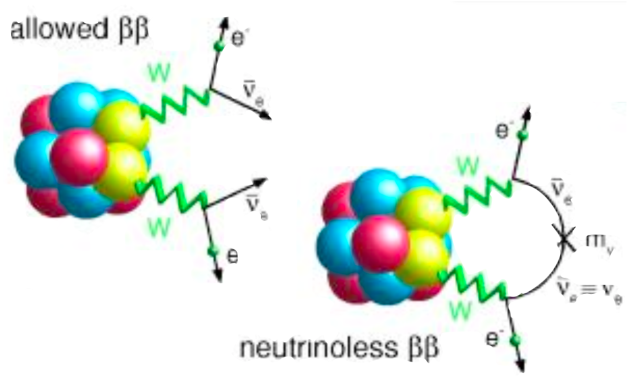
$$P_c = 0.2529(\pm 13)_{\rm stat}(\pm 32)_{\rm scale}(-45)_{\rm FV}$$




Lattice QCD is now capable of first-principles calculation of rare kaon decay

- The remaining task is to control various systematic effects

# From flavor physics to nuclear physics —— use double $\beta$ decay as an example



# Double $\beta$ decay

**$2\nu\beta\beta$ decay is the rarest SM process that has been measured**

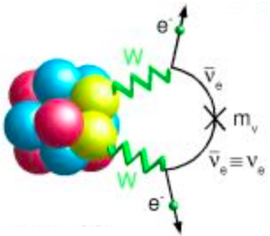
- Small mass difference $\Delta M$ between initial and final state
  $\Rightarrow$ suppressed by phase space
- $2^{\text{nd}}$-order EW interaction $\Rightarrow$ suppressed by $\frac{\Delta M^2}{M_W^2}$

**$2\nu\beta\beta$ has been detected in total of 10 nuclei: $^{48}\mathrm{Ca}$, $^{76}\mathrm{Ge}$, $\cdots$ $^{238}\mathrm{U}$**

- For all decays, half-life time: $10^{18} - 10^{21}$ yr (Age of universe: $1.38 \times 10^{10}$ yr)

**$0\nu\beta\beta$ is prohibited by SM $\Leftarrow$ violation of lepton number conservation**

- Possible scenario: a virtual Majorana neutrino mediates double $\beta$ decay
- Probe the absolute mass scale of neutrinos



# Double $\beta$ decay : $nn \to ppee\bar{\nu}\bar{\nu}$

**At present, lattice QCD mainly targets on light nuclei**
**– because of two exponential difficulties**

- For nucleus A: $\frac{\text{signal}}{\text{noise}} \sim \exp\left[-A(M_N - 3/2 m_\pi)t\right] \quad \Rightarrow \quad$ a sign problem!
- Complexity: number of Wick contractions = $N_u!N_d!N_s!$
  - e.g. $^4\mathrm{He} \Rightarrow$ naively $5 \times 10^5$ contractions!

**First, study the simple $nn \to ppee\bar{\nu}\bar{\nu}$ decay**

[NPLQCD, arXiv:1702.02929, 1701.03456, 1610.04545]

- Such decay is not observed in nature because $nn$ is not bound
- However, hadronic matrix element is well defined for $nn \to ppee\bar{\nu}\bar{\nu}$
- At $m_\pi = 800$ MeV, $nn$ can form a bound state

[NPLQCD, PRD 87 (2013) 034506]

# Non-local hadronic matrix element

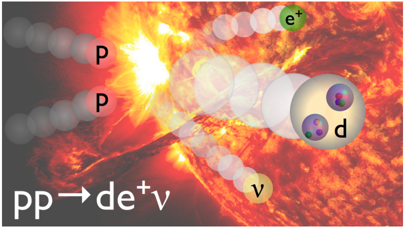
- Hadronic matrix element

$$M^{2\nu}_{GT} = \int d^4x \, \langle pp | J_3^+(x) J_3^+(0) | nn \rangle$$
$$= \sum_\alpha \frac{\langle pp | J_3^+ | \alpha \rangle \langle \alpha | J_3^+ | nn \rangle}{E_\alpha - (E_{nn} + E_{pp})/2}$$

- Here $J_3^+ = \bar{q}\gamma_3 \frac{1-\gamma_5}{2} \tau^+ q$ , $\tau^+$ is isospin Pauli matrix, allowing $n \to p$
- Initial $nn$ and final $pp$ are all ${}^1S_0$ states
- Lightest intermediate state is ${}^3S_1$ $pn$ bound state ⇒ a deuteron ($\alpha = d$)
  - The hadronic matrix elements are related to $pp$ fusion (an energy production mechanism for the Sun) and neutrino-deuteron collision

$$\langle pp | J_3^+ | d \rangle \Rightarrow pp \to de^+\nu$$

$$\langle d | J_3^+ | nn \rangle \Rightarrow \bar{\nu} d \to nne^+$$



# Lattice results

- Approximation: neglect the interaction in the *pp* and *nn*-system

$$\langle pp|J_3^+|d\rangle \quad \to \quad \langle p|J_3^0|p\rangle \quad \to \quad g_A$$

  then the contribution from ground-state deuteron is $g_A^2/\Delta$, where $\Delta = E_d - (E_{nn} + E_{pp})/2$

- Lattice results@$m_\pi = 800$ MeV [NPLQCD, arXiv:1702.02929]

$$\begin{aligned}
\frac{\Delta}{g_A^2}\frac{|\langle pp|J_3^+|d\rangle|^2}{\Delta} &= 1.00(3)(1) \\
\frac{\Delta}{g_A^2}\sum_{\alpha\neq d}\frac{\langle pp|J_3^+|\alpha\rangle\langle\alpha|J_3^+|nn\rangle}{E_\alpha - (E_{nn}+E_{pp})/2} &= 0.04(4)(2) \\
\frac{\Delta}{g_A^2}M_{GT}^{2\nu} &= 1.04(4)(4)
\end{aligned}$$

- Total contribution from all excited state $\sim 4\%$ $\Rightarrow$ a highly non-trivial result

Although very complicated $nn \to ppee\bar{\nu}\bar{\nu}$ is, lattice QCD start to deal with it

# Outlook

**Today, LQCD is entering Exaflop generation**

- Standard quantity: expect the precision significantly enhanced
- Non-standard quantity, such as LD processes: worthwhile for study

**For flavor physics:**

- lattice QCD provides useful low-energy QCD information
- plays important role in high-precision frontier

**The techniques developed in flavor physics can be used in nuclear physics**

- help to study the rare processes related to nuclear matter
- Can one day, nuclear physics become a new flavor physics?