Testing the supersymmetric transformed potential with nuclear multipole responses

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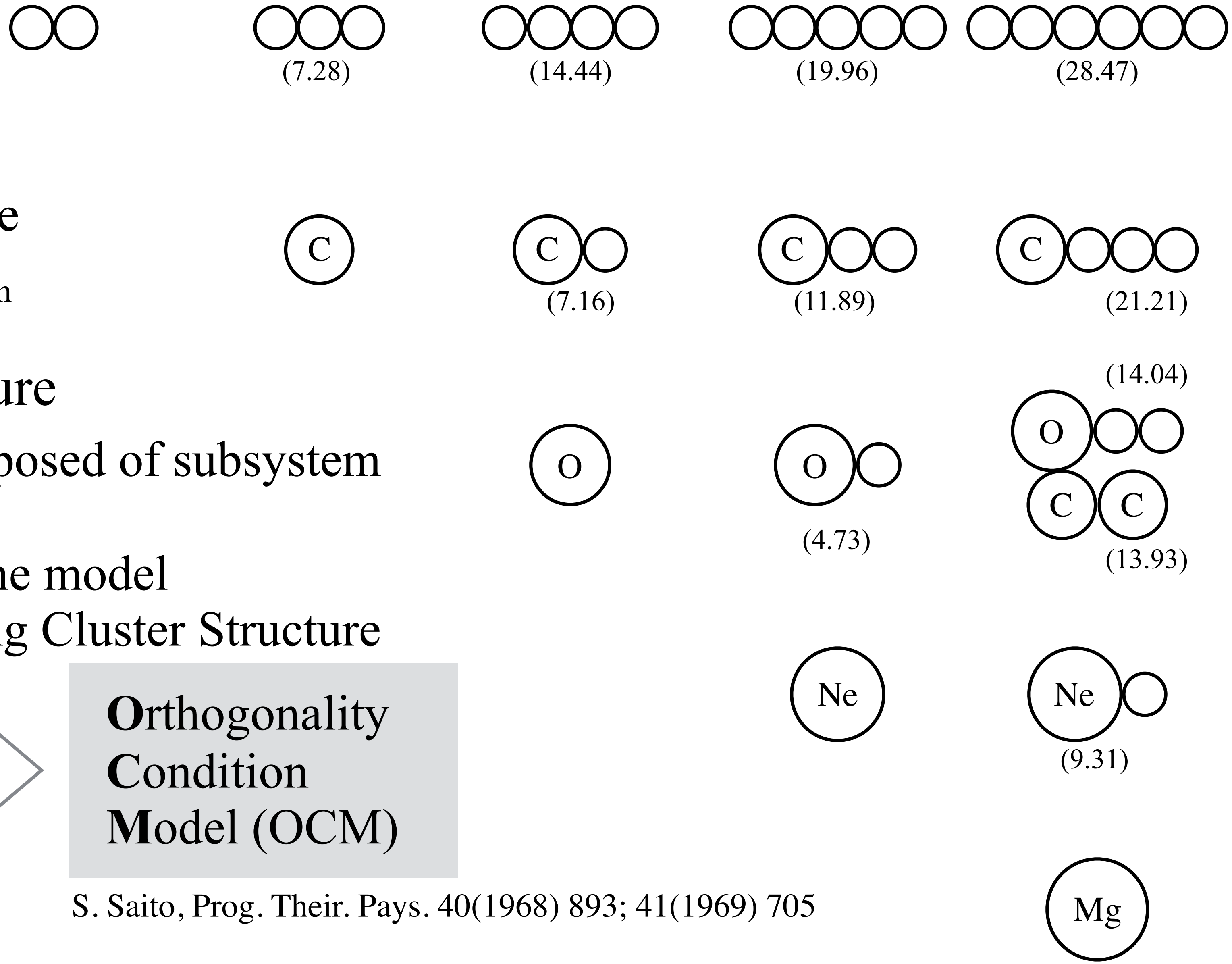
Background

Ikeda diagram

K. Ikeda et al., PTP suppl. Extra num., 464 (1968).

that conjectures various cluster states.



Nuclear Structure

one in them

Cluster Structure

Nuclear is composed of subsystem

One of the model
describing Cluster Structure

Orthogonality
Condition
Model (OCM)

S. Saito, Prog. Their. Pays. 40(1968) 893; 41(1969) 705

Background

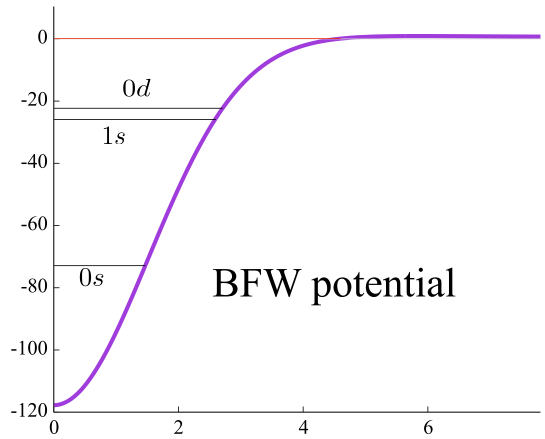
OCM (deep potential)

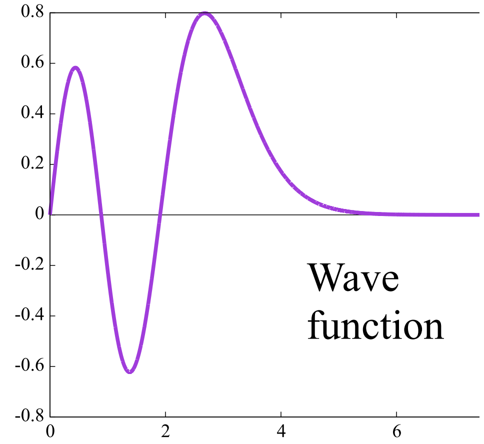
e.g.) BFW potential*

*B.Buck, H. Friedrich and C.Wheatley, Nucl. Phys. A 275 (1977) 246

- A number of Pauli forbidden state is presented.
- The relative wave function is complicated because of orthogonality condition with redundant state.

→ It is hard to apply to many body system.





Having a number of nodes

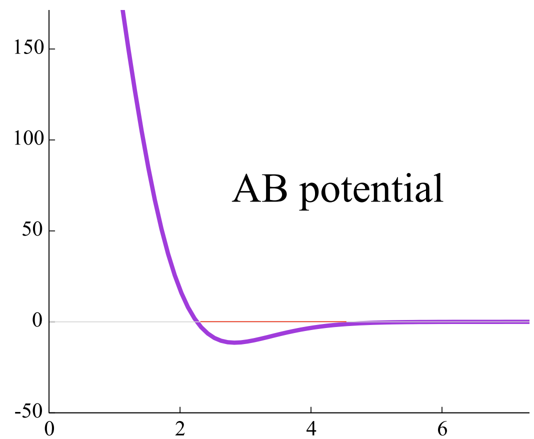
Shallow potential

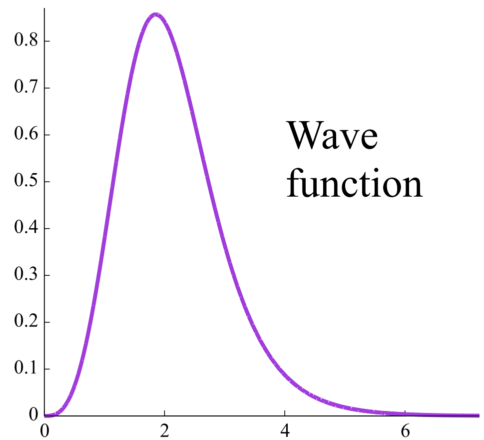
e.g.) Ali-Bodmer potential**

** S.Ali, A. R. Bodmer, Nucl. Phys. 80 (1966) 99

- The potential doesn't present Pauli forbidden state.
- the wave function is simple.

→ Application to many-body system is easier.





No node

Background

OCM (deep potential)

e.g.） BFW potential*

*B.Buck, H. Friedrich and C.Wheatley, Nucl. Phys. A 275 (1977) 246

Shallow potential

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Supersymmetric (SUSY) transformation

The method to generate a phase equivalent potential
that is eliminated the Pauli forbidden states
from a potential which have a number of the redundant state.

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Shallow potential

e.g.) Ali-Bodmer potential**

** S.Ali, A. R. Bodmer, Nucl. Phys. 80 (1966) 99

Supersymmetric (SUSY) transformation

The method to generate a phase equivalent potential
that is eliminated the Pauli forbidden states
from a potential which have a number of the redundant state.

However, even though the potential is phase equivalent,
there is no guarantee that other observables don't change.

e.g.) electric-dipole response*

*E.C. Pinilla, D. Baye, P. Descouvemont, W. Horiuchi, Y. Suzuki, Nucl. Phys. A 865 (2011) 43

Purpose

Testing the relative wave functions between clusters
generated from the original deep and transformed SUSY potentials.

Preparing phenomenological deep potentials of
$^{16}O + {}^{4}He$ and $^{40}Ca + {}^{4}He$ systems

Generating their SUSY potentials

Comparing the relative wave functions generated
from the deep and SUSY potential
and quantify how those differences appear in observables

- Root-mean-square radius
- Electric quadrupole transitions

SUSY transformation…Elimination of the lowest bound state

$* \ \frac{\hbar^2}{2m} = 1$

Factorizing H_0 using intertwining operator $L_0, L_0{}^\dagger$

$$H_0 = -\frac{d^2}{dr^2} + V_0 \quad \Rightarrow \quad H_0 = L_0^\dagger L_0 + \mathcal{E}_0$$

$\mathcal{E}_0$: Factorization energy

Intertwining operator

$$L_0 = -\frac{d}{dr} + \frac{d}{dr}\ln\psi_0(\mathcal{E}_0)$$

$$L_0^\dagger = \frac{d}{dr} + \frac{d}{dr}\ln\psi_0(\mathcal{E}_0)$$

Using intertwining relation, a SUSY parter H_1 is defined.

$$H_1 = L_0 L_0^\dagger + \mathcal{E}_0 = -\frac{d^2}{dr^2} + V_1$$

$$H_0\psi_0(E) = E\psi_0(E)$$

$$\downarrow$$

$$L_0 H_0\psi_0(E) = L_0 E\psi_0(E)$$

$$\downarrow$$

$$H_1\psi_1(E) = E\psi_1(E)$$

$$(\psi_1(E) = L_0\psi_0(E))$$

Intertwining relation

$$L_0 H_0 = H_1 L_0$$

Transformed potential

$$V_1 = V_0 - 2\frac{d^2}{dr^2}\ln\psi_0(\mathcal{E}_0)$$

Elimination of the lowest bound state

$$L_0^\dagger L_0 = H_0 - \mathcal{E}_0$$

$$\downarrow$$

$$\int dr\psi_0^* L_0^\dagger L_0\psi_0 = \int dr\psi_0^*(H_0 - \mathcal{E}_0)\psi_0$$

$$\downarrow$$

$$\int dr|L_0\psi_0|^2 = (E_0 - \mathcal{E}_0)\int dr|\psi_0|^2$$

$$\therefore E_0 \geq \mathcal{E}_0$$

Taking $\mathcal{E}_0 = E_0 = -\gamma_0^2$ (lowest bound state energy)

$$\psi_1(E_0) = L_0\psi_0(E_0) = 0$$

The bound state eliminated!

D. Baye, J-M. Sparenberg, A M. Pupasov, B F. Samsonov, J. Phys. A: Math. Theor. 47, 243001 (2014)

SUSY transformation…Adjustment of the phase shift

$* \ \frac{\hbar^2}{2m} = 1$

However, the phase shift is not identical.

Iteration SUSY transformation due to adjustment of phase shift.

Asymptotic form of a scattering wave function

$$\psi_0(E) \xrightarrow{r\to\infty} \sin\left(kr - \frac{l\pi}{2} - \eta\ln 2kr + \delta\right)$$

$$\psi_1 \propto L_0\psi_0(E) \xrightarrow{r\to\infty} \sin\left(kr - \frac{l\pi}{2} - \eta\ln 2kr + \delta + \tan^{-1}(k/\gamma_0)\right)$$

k : wavenumber, η : Sommerfeld parameter, δ: phase shift

Factorizing H_1 using intertwining operator $L_1, L_1{}^\dagger$

$$H_1 = -\frac{d^2}{dr^2} + V_1 \quad \Rightarrow \quad H_1 = L_1^\dagger L_1 + E_0$$

Taking the factorization energy = E_0

$$\psi_1(E_0) = [\psi_0(E_0)]^{-1}\int_0^r [\psi_0(E_0)]^2 dt$$

Intertwining operator

$$L_1 = -\frac{d}{dr} + \frac{d}{dr}\ln\psi_1(E_0)$$

$$L_1^\dagger = \frac{d}{dr} + \frac{d}{dr}\ln\psi_1(E_0)$$

Using intertwining relation, a SUSY parter H_2 is defined.

$$H_2 = L_1 L_1^\dagger + E_0 = -\frac{d^2}{dr^2} + V_2$$

Intertwining relation

$$L_1 H_1 = H_2 L_1$$

Transformed potential

$$V_2(r) = V_0(r) - 2\frac{d^2}{dr^2}\ln\int_0^r [\psi_0(E_0)]^2 dt$$

Asymptotic form of a scattering wave function

$$\psi_2 \propto L_1 L_0 \psi_0$$

$$\xrightarrow{r\to\infty} \left(\frac{d^2}{dr^2} + E_0\right)\sin\left(kr - \frac{l\pi}{2} + 2\eta\ln kr + \delta\right)$$

phase equivalent !

D. Baye, J-M. Sparenberg, A M. Pupasov, B F. Samsonov, J. Phys. A: Math. Theor. 47, 243001 (2014)

A phase equivalent potential with SUSY transformation

$* \ \frac{\hbar^2}{2m} = 1$

Elimination of the lowest bound state

$$H_0 = -\frac{d^2}{dr^2} + V_0 = L_0^\dagger L_0 + E_0$$

Intertwining operator

$$L_0 = -\frac{d}{dr} + \frac{d}{dr}\ln\psi_0(E_0)$$

$$L_0^\dagger = \frac{d}{dr} + \frac{d}{dr}\ln\psi_0(E_0)$$

Intertwining relation $L_0 H_0 = H_1 L_0$

SUSY partner

$$H_1 = -\frac{d^2}{dr^2} + V_1 = L_0^\dagger L_0 + E_0$$

$\psi_0(E_0)$: Wave function of the lowest bound state

The phase shift is increased by $\tan^{-1}(k/\gamma_0)$

Adjustment of the phase shift

$$H_1 = -\frac{d^2}{dr^2} + V_1 = L_1^\dagger L_1 + E_0$$

Intertwining operator

$$L_1 = -\frac{d}{dr} + \frac{d}{dr}\ln\psi_1(E_0)$$

$$L_1^\dagger = \frac{d}{dr} + \frac{d}{dr}\ln\psi_1(E_0)$$

Intertwining relation $L_1 H_1 = H_2 L_1$

SUSY partner

$$H_2 = -\frac{d^2}{dr^2} + V_2 = L_1 L_1^\dagger + E_0$$

$$\psi_1(E_0) = [\psi_0(E_0)]^{-1}\int_0^r [\psi_0(E_0)]^2 dt$$

The phase shift is decreased by $\tan^{-1}(k/\gamma_0)$

Phase equivalent potential (V_0 : the number of the forbidden state is n)

$$V_{\mathrm{SUSY}}(r) = V_0(r) - 2\sum_n \frac{d^2}{dr^2}\ln\int_0^r [\psi_n(E_n)]^2 dt \qquad V_{\mathrm{SUSY}}(r) \xrightarrow{r\to 0} V_0(r) + \frac{(2n+l)(2n+l+1)}{r^2}$$

D. Baye, J-M. Sparenberg, A M. Pupasov, B F. Samsonov, J. Phys. A: Math. Theor. 47, 243001 (2014)

^{16}O + ^{4}He potential

$$V_{^{16}\mathrm{O}+^{4}\mathrm{He}} = V_N + V_C$$

$$V_N = (A + B\hat{P}_r)\exp(-\mu r^2)$$

$$V_C = 8 \times 2 \cdot \frac{e^2}{r}\mathrm{erf}(\beta r) \quad \beta = 0.25937[\mathrm{fm}^{-1}]$$

$\hat{P}_r$:Parity operator

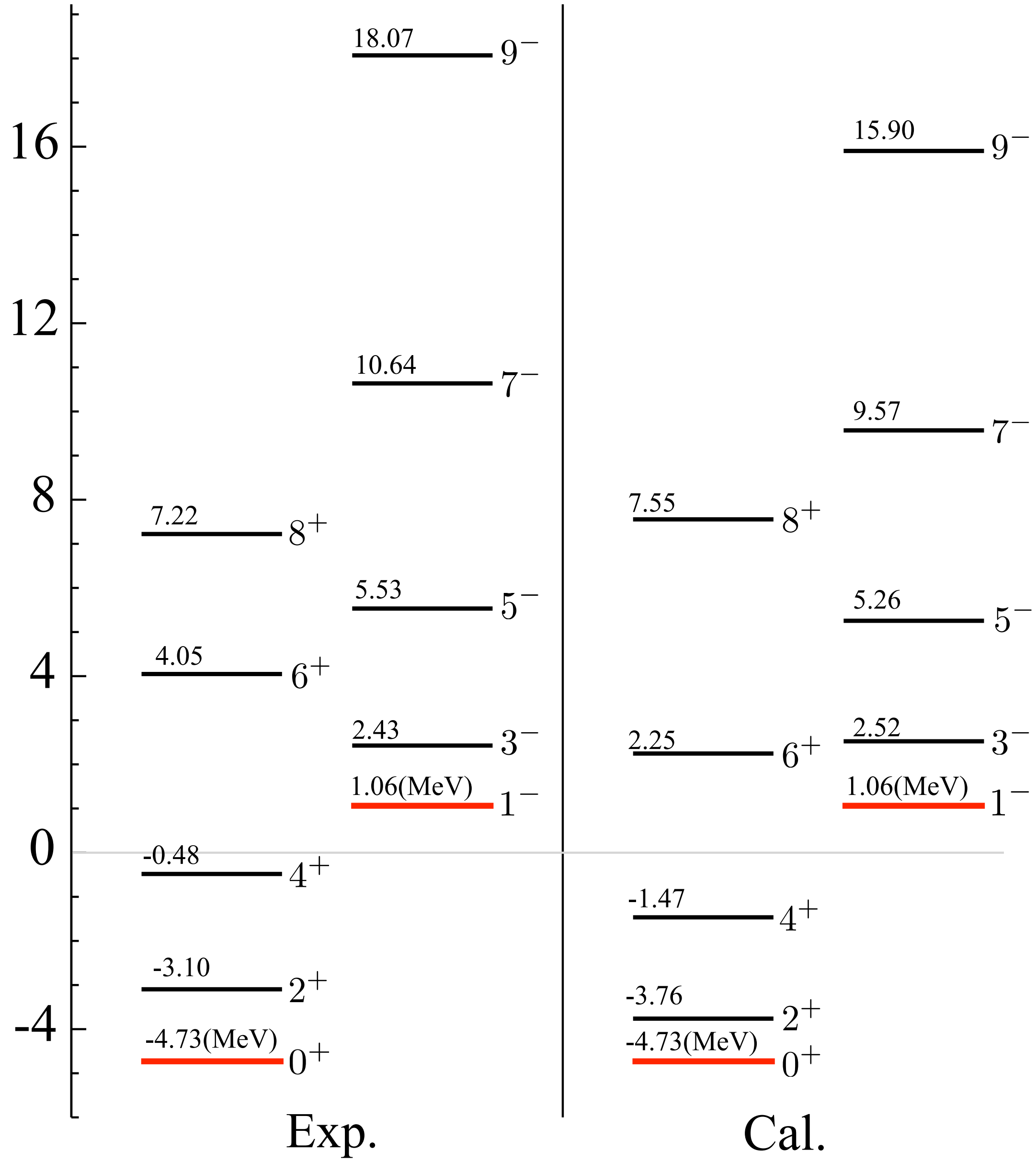
$\mathrm{erf}(x)$:Error function

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x \exp(-t^2)dt$$

$$A = -134.220\,[\mathrm{MeV}] \quad \mu = 0.09475\,[\mathrm{fm}^{-2}]$$

$$B = 0.415\,[\mathrm{MeV}]$$

- Referring to BFW potential
- Pauli forbidden state which follow $2N + l < 8$
- Parameters are determined to fit
  1. Energy of ground state (0^+)
  2. Root mean square radius
  3. Excitation energy (1-)



Rotational band $K^\pi = 0^+, 0^-$ in ^{20}Ne

^{40}Ca + ^{4}He potential

$$V_{^{40}\mathrm{Ca}+^{4}\mathrm{He}} = V_N + V_C$$

$$V_N = (A + B\hat{P}_r)\exp(-\mu r^2)$$

$$V_C = 20 \times 2 \cdot \frac{e^2}{r}\mathrm{erf}(\beta r) \quad \beta = 0.11188[\mathrm{fm}^{-1}]$$

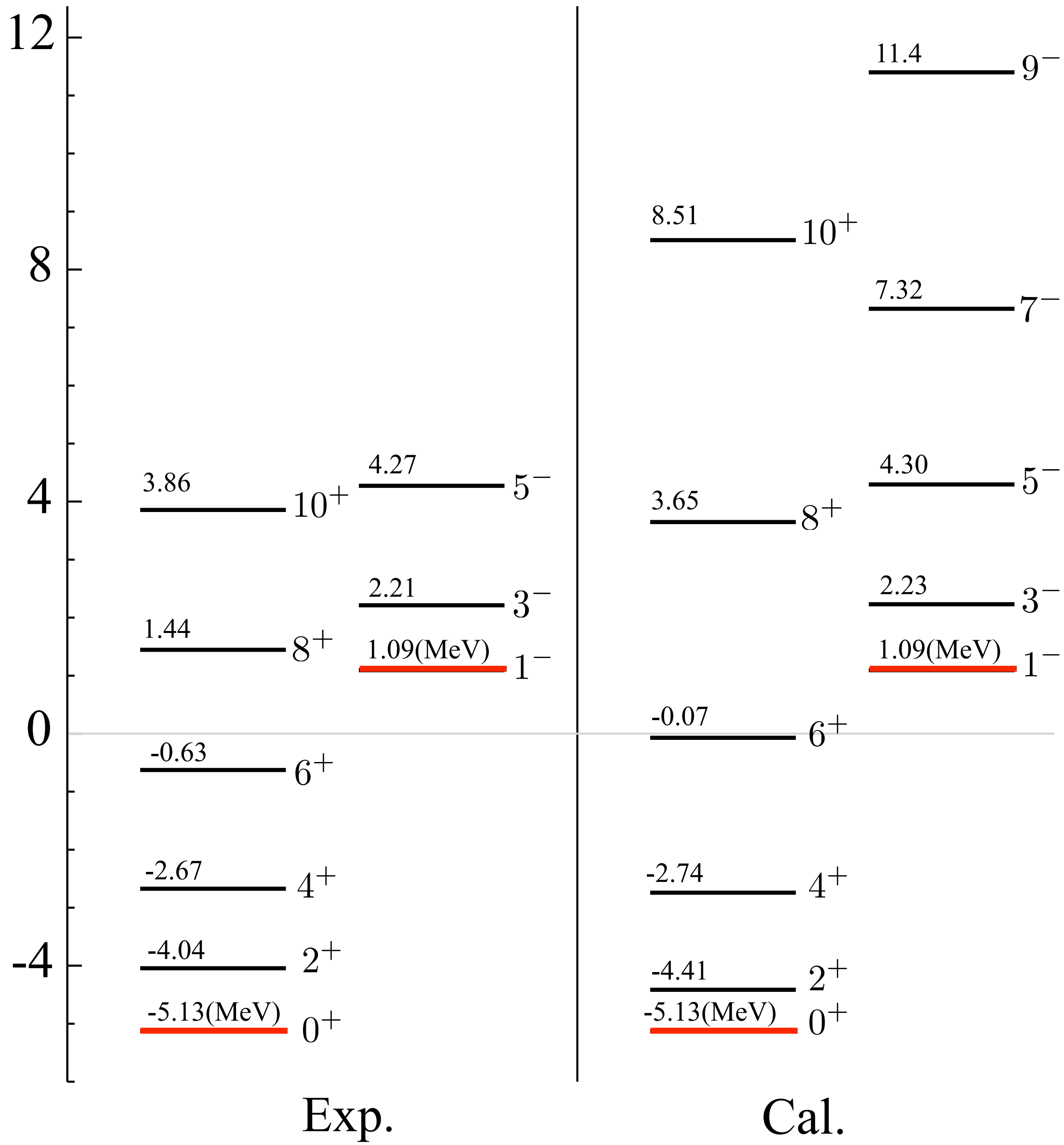
$\hat{P}_r$:Parity operator

$\mathrm{erf}(x)$:Error function

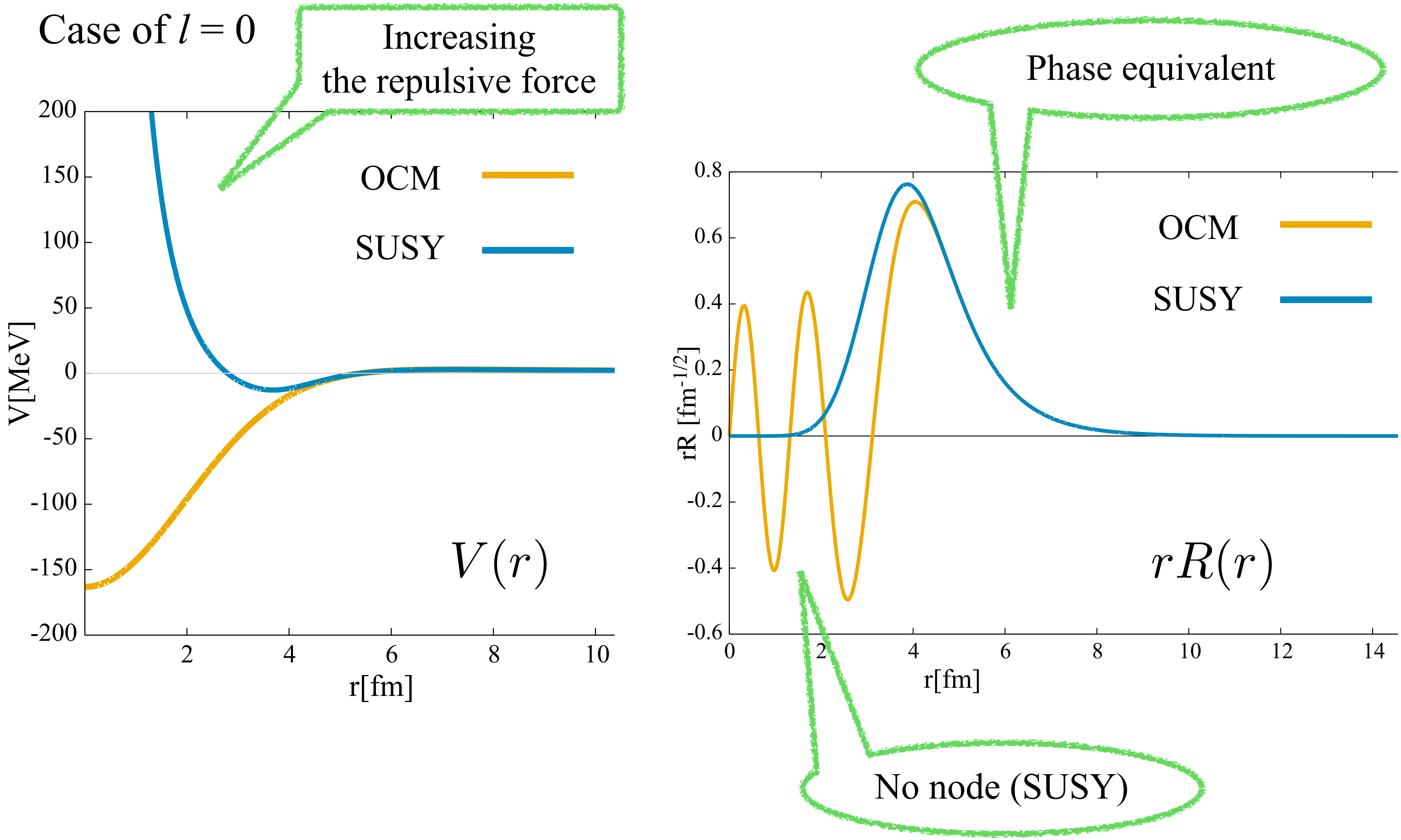
$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x \exp(-t^2)dt$$

$$A = -192.878\,[\mathrm{MeV}] \quad \mu = 0.08012\,[\mathrm{fm}^{-2}]$$
$$B = 0.539\,[\mathrm{MeV}]$$

- Referring to BFW potential
- Pauli forbidden state which follow $2N + l < 12$
- Parameters are determined to fit
  1. Energy of ground state (0^+)
  2. Root mean square radius
  3. Excitation energy (1-)



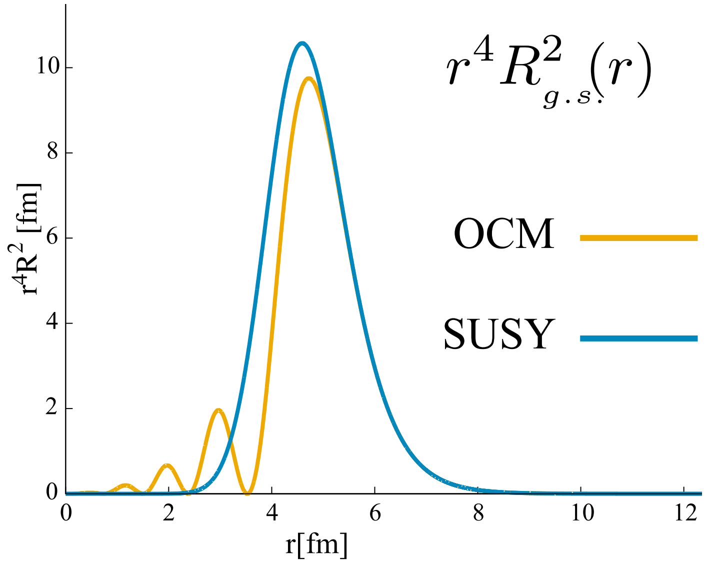
SUSY potential and wave function… $^{16}O + {}^{4}He$ potential



Root-mean-square radii

$$\frac{1}{A}\sum_{i=1}^{A}(\mathbf{r}_i - \mathbf{R}_{cm})^2 = \frac{A_1 r_{C_1}^2 + A_2 r_{C_2}^2}{A} + \frac{A_1 A_2}{A^2}\mathbf{r}^2$$

$^{16}O + {}^{4}He$

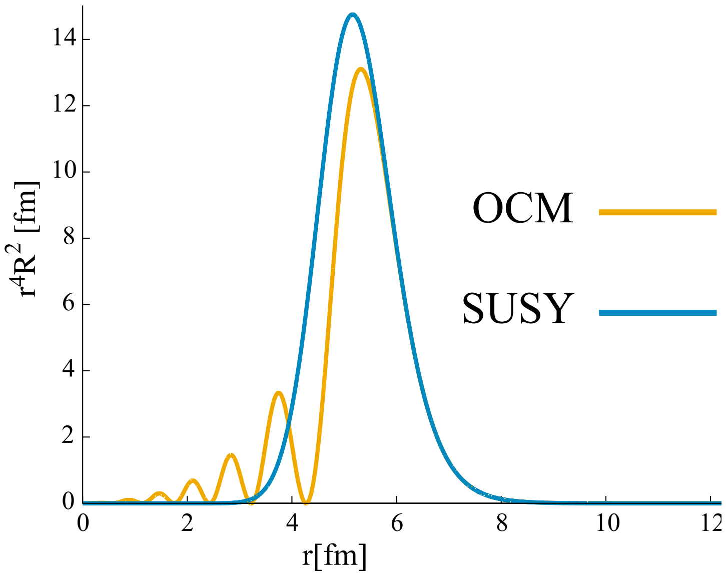


^{16}O ^{4}He

$r_{C_1} = 2.57\,\text{fm},\ r_{C_2} = 1.46\,\text{fm}$

	$\sqrt{\langle r^2 \rangle}$ [fm]	
OCM	**4.23**	
SUSY	**4.58**	**+8.2%**

$^{40}Ca + {}^{4}He$



^{40}Ca ^{4}He

$r_{C_1} = 3.38\,\text{fm},\ r_{C_2} = 1.46\,\text{fm}$

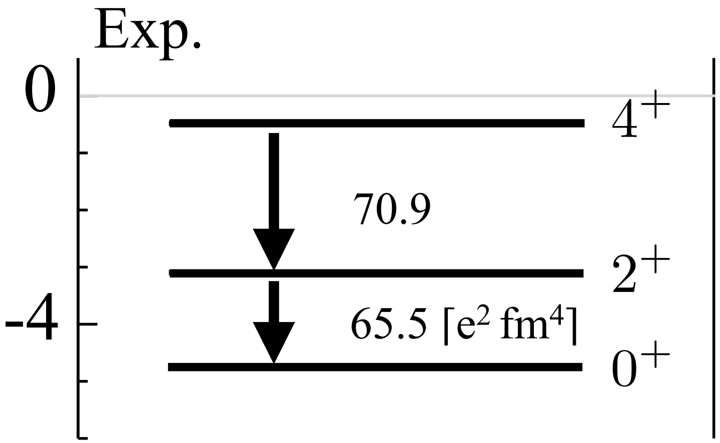
	$\sqrt{\langle r^2 \rangle}$ [fm]	
OCM	**4.66**	
SUSY	**5.14**	**+10.3%**

The rms radius calculated by the SUSY potential is larger than the OCM one.

$B(E2)$

$$B(E2; I_i \to I_f) = \frac{1}{2I_i+1} \sum_{\mu M_i M_f} |\langle I_f M_f | \mathcal{M}(E2\mu) | I_i M_i \rangle|^2$$

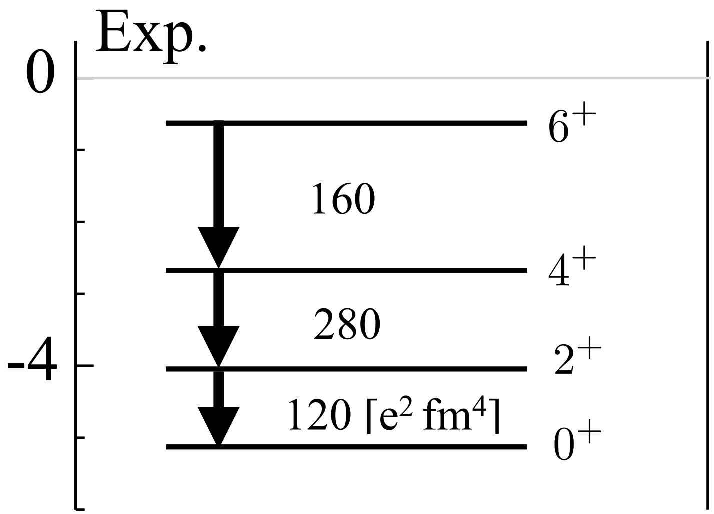
$^{16}O + {}^4He$



Cal.

B(E2)	$2^+ \to 0^+$	$4^+ \to 2^+$
OCM	63.8	75.2
SUSY	87.6	104.7
Exp.	65.5	70.9

$^{40}Ca + {}^4He$



Cal.

B(E2)	$2^+ \to 0^+$	$4^+ \to 2^+$	$6^+ \to 4^+$
OCM	123	168	171
SUSY	181	242	238
Exp.	120	280	160

The $B(E2)$ values is also larger because the rms radius is larger.

Another phase equivalent potential

$* \ \frac{\hbar^2}{2m} = 1$

Elimination of the ground state

$$H_0 = -\frac{d^2}{dr^2} + V_0 = L_0^\dagger L_0 + E_0$$

Intertwining operator

$$L_0 = -\frac{d}{dr} + \frac{d}{dr}\ln\psi_0(E_0)$$

$$L_0^\dagger = \frac{d}{dr} + \frac{d}{dr}\ln\psi_0(E_0)$$

Intertwining relation $L_0 H_0 = H_1 L_0$

SUSY partner

$$H_1 = -\frac{d^2}{dr^2} + V_1 = L_0^\dagger L_0 + E_0$$

$\psi_0(E_0)$: Wave function of the ground state

The phase shift is increased by $\tan^{-1}(k/\gamma_0)$

Addition of the ground state

$$H_1 = -\frac{d^2}{dr^2} + V_1 = L^\dagger L + E_0$$

Intertwining operator

$$L = -\frac{d}{dr} + \frac{d}{dr}\ln\varphi(E_0)$$

$$L^\dagger = \frac{d}{dr} + \frac{d}{dr}\ln\varphi(E_0)$$

Intertwining relation $LH_1 = H_2 L$

SUSY partner

$$H_2 = -\frac{d^2}{dr^2} + V_2' = LL^\dagger + E_0$$

$$\varphi(E_0) = [\psi_0(E_0)]^{-1}\ln\left(\alpha + \int_r^\infty [\psi_0(E_0)]^2 dt\right)$$

The phase shift is decreased by $\tan^{-1}(k/\gamma_0)$

Another phase equivalent potential

$$V_2'(r) = V_0(r) - 2\frac{d^2}{dr^2}\ln\left|\alpha + \int_r^\infty [\psi_0(E_0)]^2 dt\right|$$

If $\alpha = -1$,

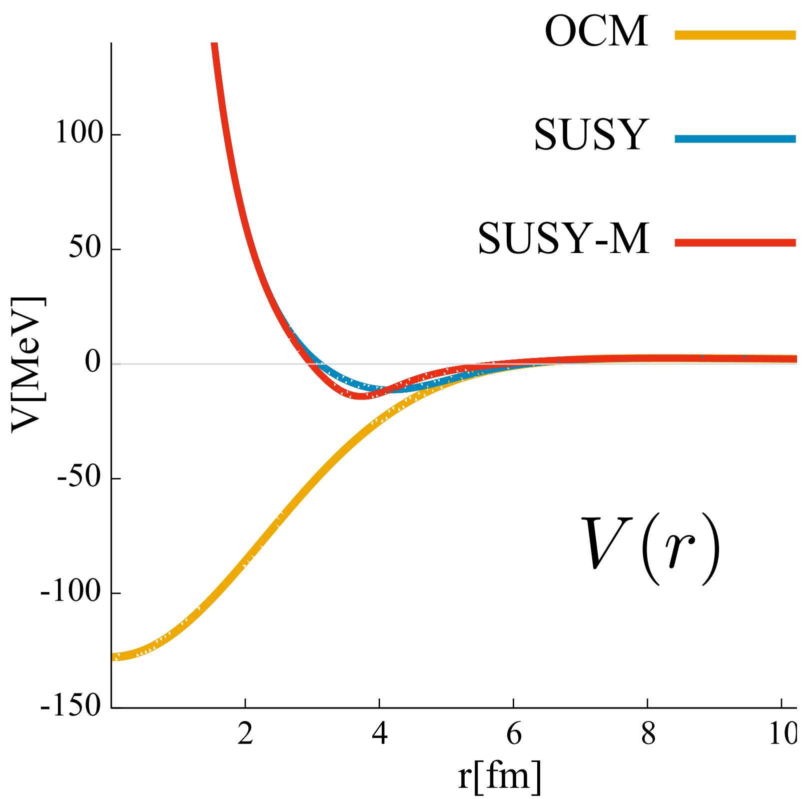
$$V_2(r) = V_0(r) - 2\frac{d^2}{dr^2}\ln\int_0^r [\psi_0(E_0)]^2 dt$$

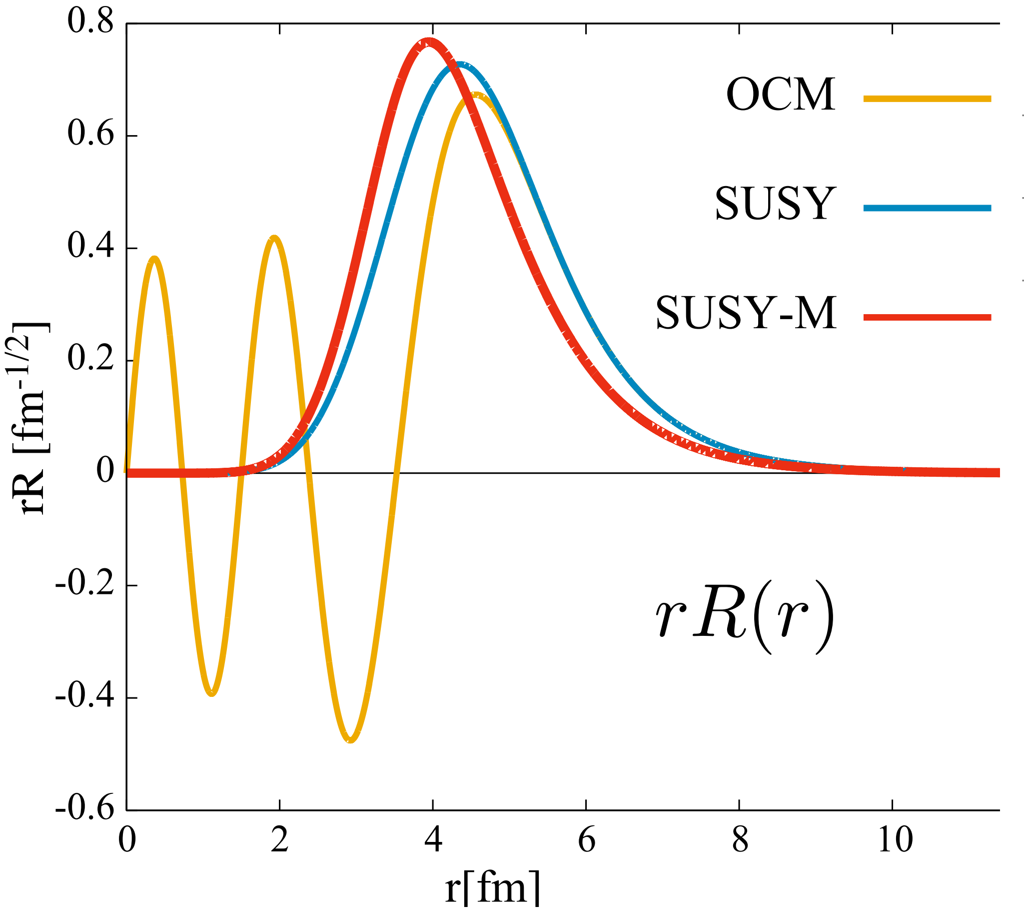
Another phase equivalent potential

$$V'(r) = V(r) - 2\frac{d^2}{dr^2}\ln\left|\alpha + \int_r^\infty [\psi_0(E_0)]^2 dt\right|$$

Fit the parameter α
to reproduce the rms radius (SUSY-M)

The case of $^{16}O + {}^{4}He$





To make rms radius smaller, the potential and the wave function are attracted inside.

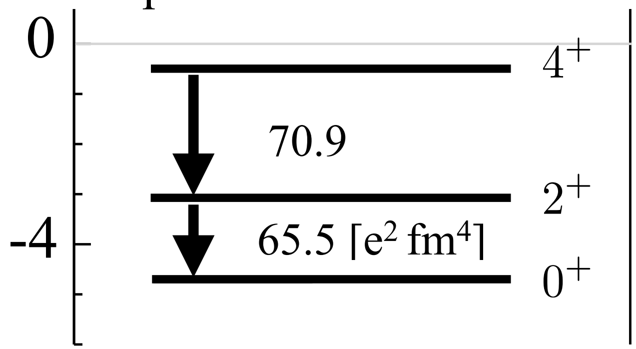
Another phase equivalent potential

$$V'(r) = V(r) - 2\frac{d^2}{dr^2}\ln\left|\alpha + \int_r^\infty [\psi_0(E_0)]^2 dt\right|$$

Fit the parameter α
to reproduce the rms radius （SUSY-M）

^{16}O + ^{4}He

Exp.



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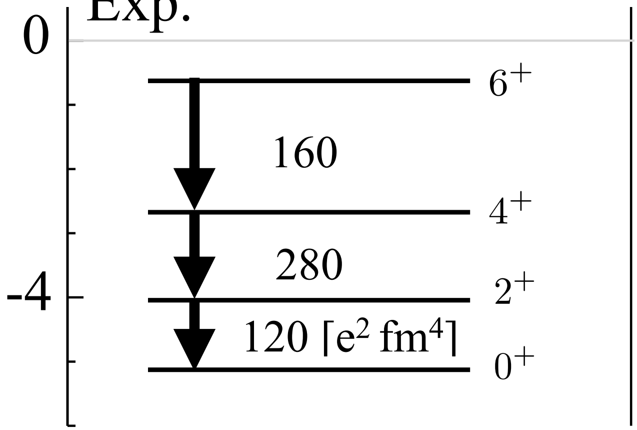
^{40}Ca + ^{4}He

Exp.



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B(E2)	$2^+ \to 0^+$	$4^+ \to 2^+$	$6^+ \to 4^+$
OCM	123	168	171
SUSY	181	242	238
SUSY-M	123	169	172
Exp.	120	280	160

The $B(E2)$ values are also reproduce OCM values by reproducing the rms radii.

Summary

- We generate SUSY potential from phenomenological deep potentials of $^{16}O+^{4}He$ and $^{40}Ca+^{4}He$ systems.
- The root-mean-square radius and B(E2) values calculated by the SUSY potential is larger than the OCM potential.
- Using another SUSY transformation, we fit other observable.
  Fitting the parameter to restore rms radius value
  → B(E2) is also reproduce previous values.

Future work

- Applying many body system e.g.) $^{24}Mg \rightarrow {}^{16}O + {}^{4}He + {}^{4}He$
  $^{48}Cr \rightarrow {}^{40}Ca + {}^{4}He + {}^{4}He$