Efimov Physics in Ultracold Atomic Gases

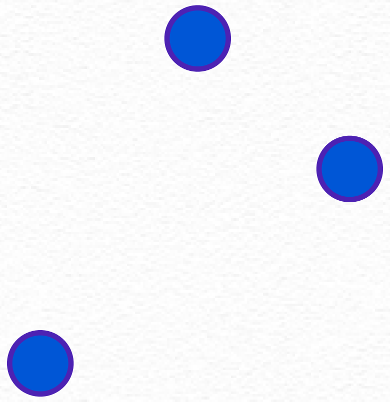
Hui Zhai

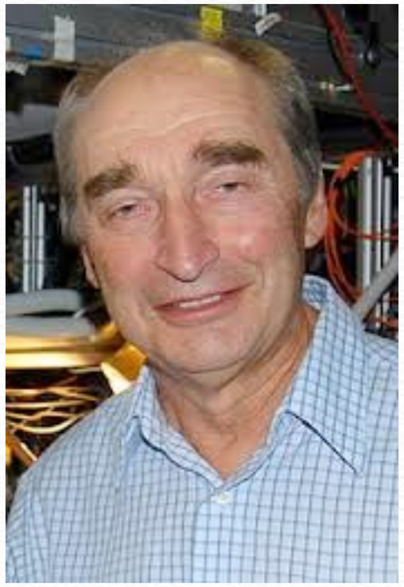
Institute for Advanced Study
Tsinghua University
Beijing, China

The 7th Asia-Pacific Conference on Few-Body Problems in Physics (APFB 2017)
Gulin
August 2017

Efimov Physics with Three Bosons

Problem: Three bosons interacting through a short-range interaction





1970

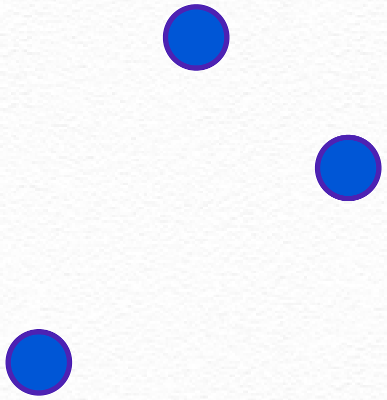
$$\left(-\frac{\nabla_1^2}{2m}-\frac{\nabla_2^2}{2m}-\frac{\nabla_3^2}{2m}\right)\psi = E\psi$$

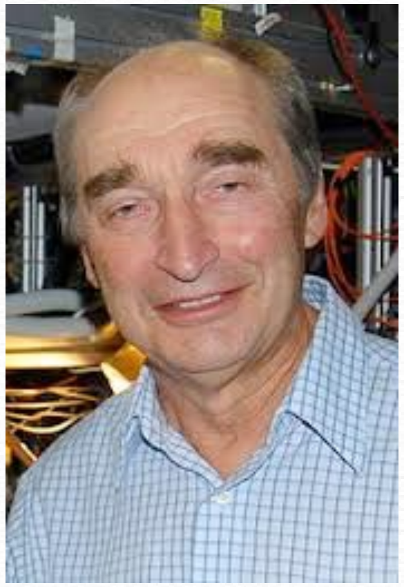
Bethe-Peierls Boundary Condition

$$\psi(r_i - r_j \to 0) = \frac{1}{|r_i - r_j|} - \frac{1}{a_s}$$

Efimov Physics with Three Bosons

Problem: Three bosons interacting through a short-range interaction





1970

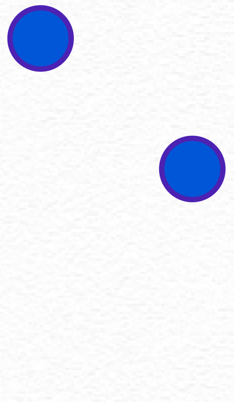
$$\left[-\frac{\hbar^2 \mathrm{d}^2}{2m\mathrm{d}\rho^2}-\frac{s_0^2+1/4}{m\rho^2}\right]\psi=E\psi$$

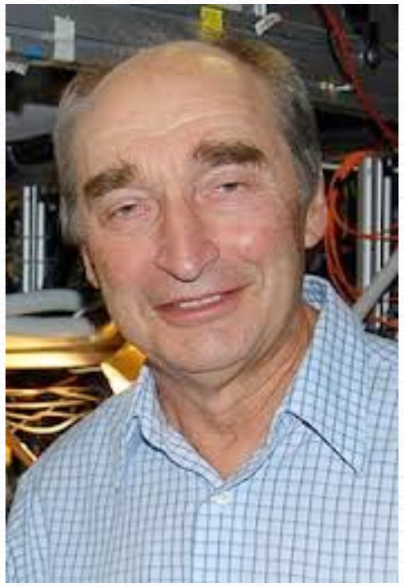
- Scale invariance

- Thomas collaps

Efimov Physics with Three Bosons

Problem: Three bosons interacting through a short-range interaction





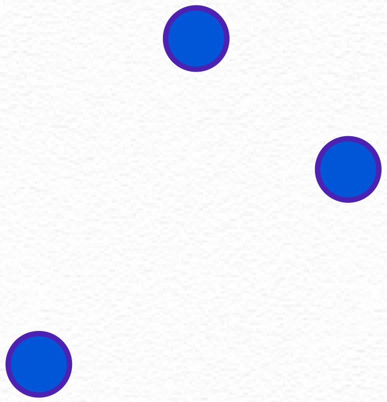
1970

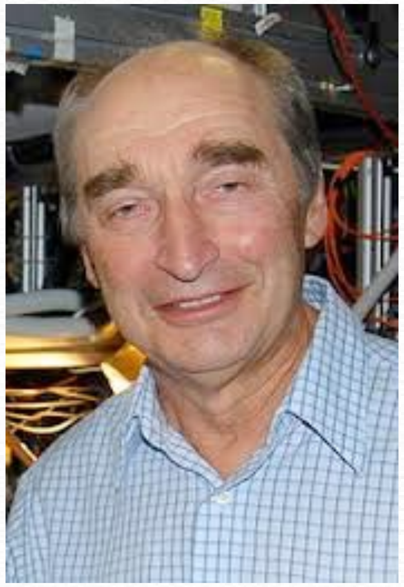
$$\left[-\frac{\hbar^2 \mathrm{d}^2}{2m\mathrm{d}\rho^2}-\frac{s_0^2+1/4}{m\rho^2}\right]\psi=E\psi$$

- **Scale invariance**
- **Thomas collaps** → **Short-range cutoff**

Efimov Physics with Three Bosons

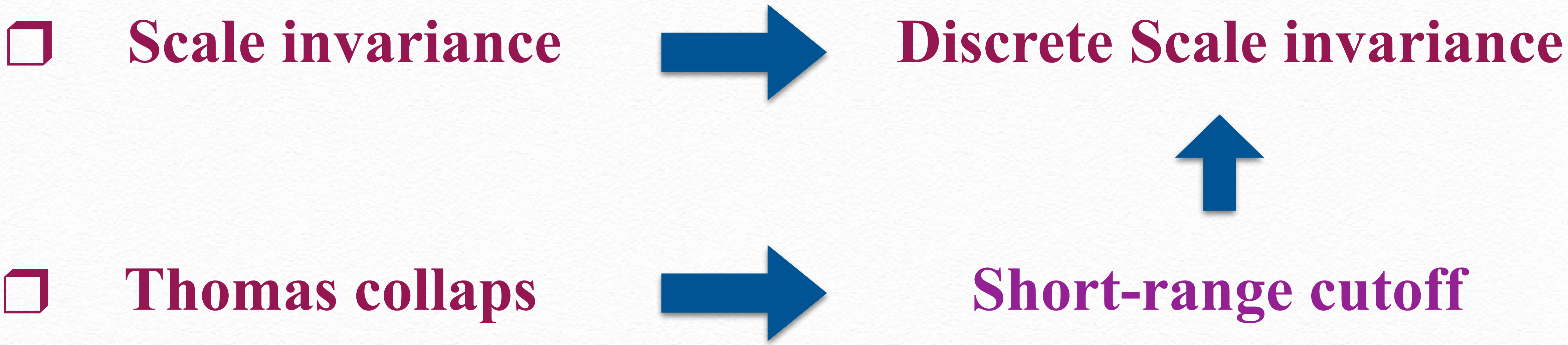
Problem: Three bosons interacting through a short-range interaction





1970

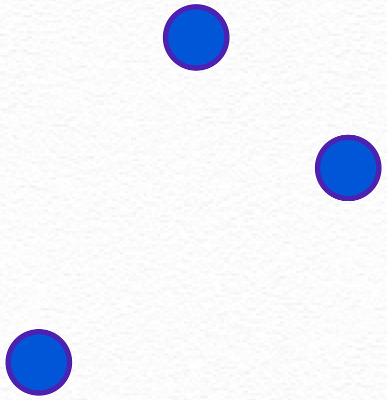
$$\left[-\frac{\hbar^2\mathrm{d}^2}{2m\mathrm{d}\rho^2}-\frac{s_0^2+1/4}{m\rho^2}\right]\psi=E\psi$$

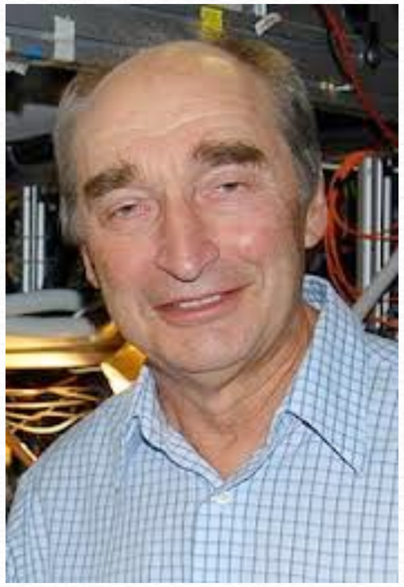


- ☐ **Scale invariance** → **Discrete Scale invariance**
- ☐ **Thomas collaps** → **Short-range cutoff**

Efimov Physics with Three Bosons

Problem: Three bosons interacting through a short-range interaction





1970

$$\left[-\frac{\hbar^2 \mathrm{d}^2}{2m\mathrm{d}\rho^2}-\frac{s_0^2+1/4}{m\rho^2}\right]\psi = E\psi$$

$$\boxed{\psi = \sqrt{\rho}\cos[s_0\log(\rho/\rho_0)]}$$

$$\rho \longrightarrow e^{2\pi/s_0}\rho$$

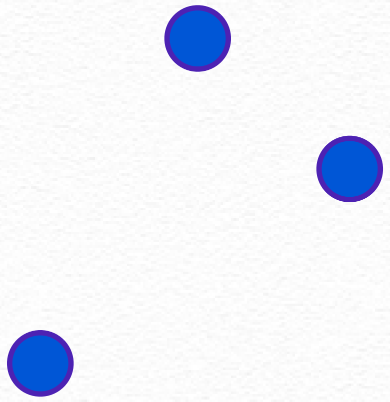
$$E_{\mathrm{T}}^{(n+1)}/E_{\mathrm{T}}^{(n)} \simeq e^{-2\pi/s_0}$$

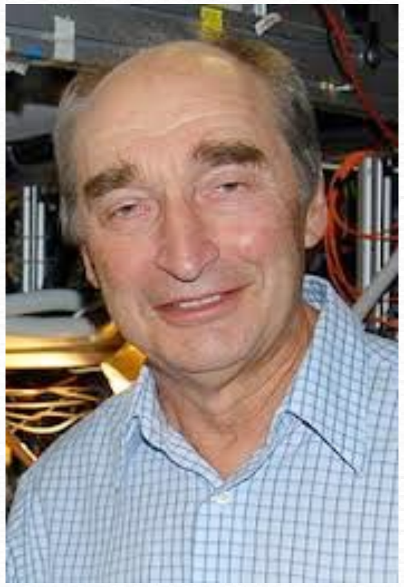
Discrete Scaling Symmetry

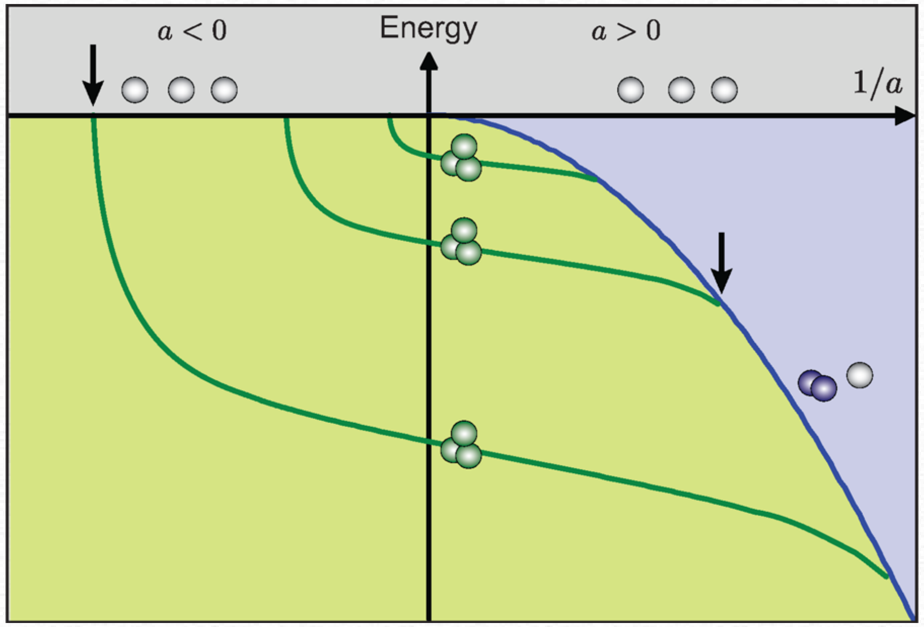
Efimov Physics with Three Bosons

Problem: Three bosons interacting through a short-range interaction

1970



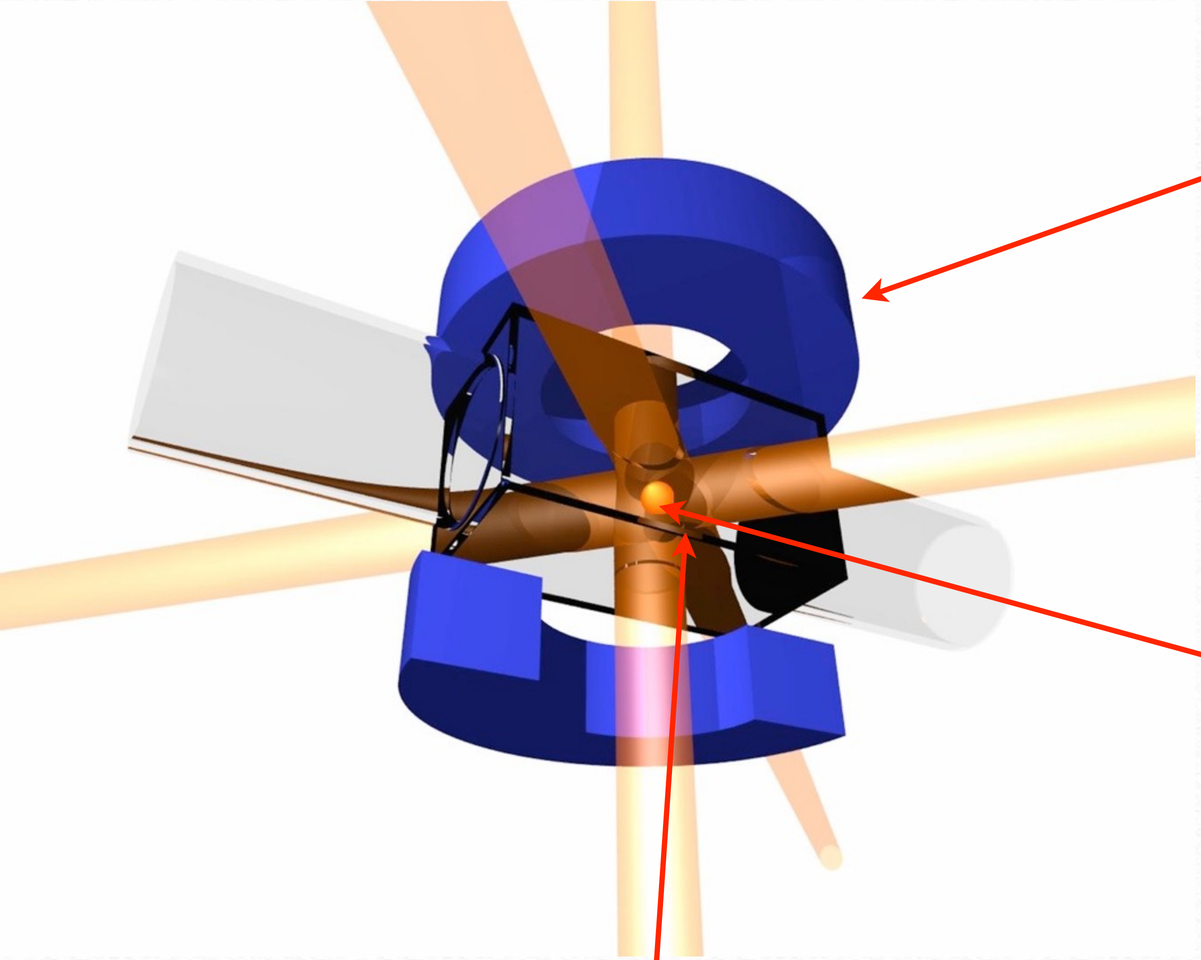




Universal Discrete Scaling Symmetry

Renormalization Group View: Limit Circle Solution

Ultracold Atomic Gases



Magnetic field

Laser

The Gas: Rb; K, Li,...

Vacuum Chamber

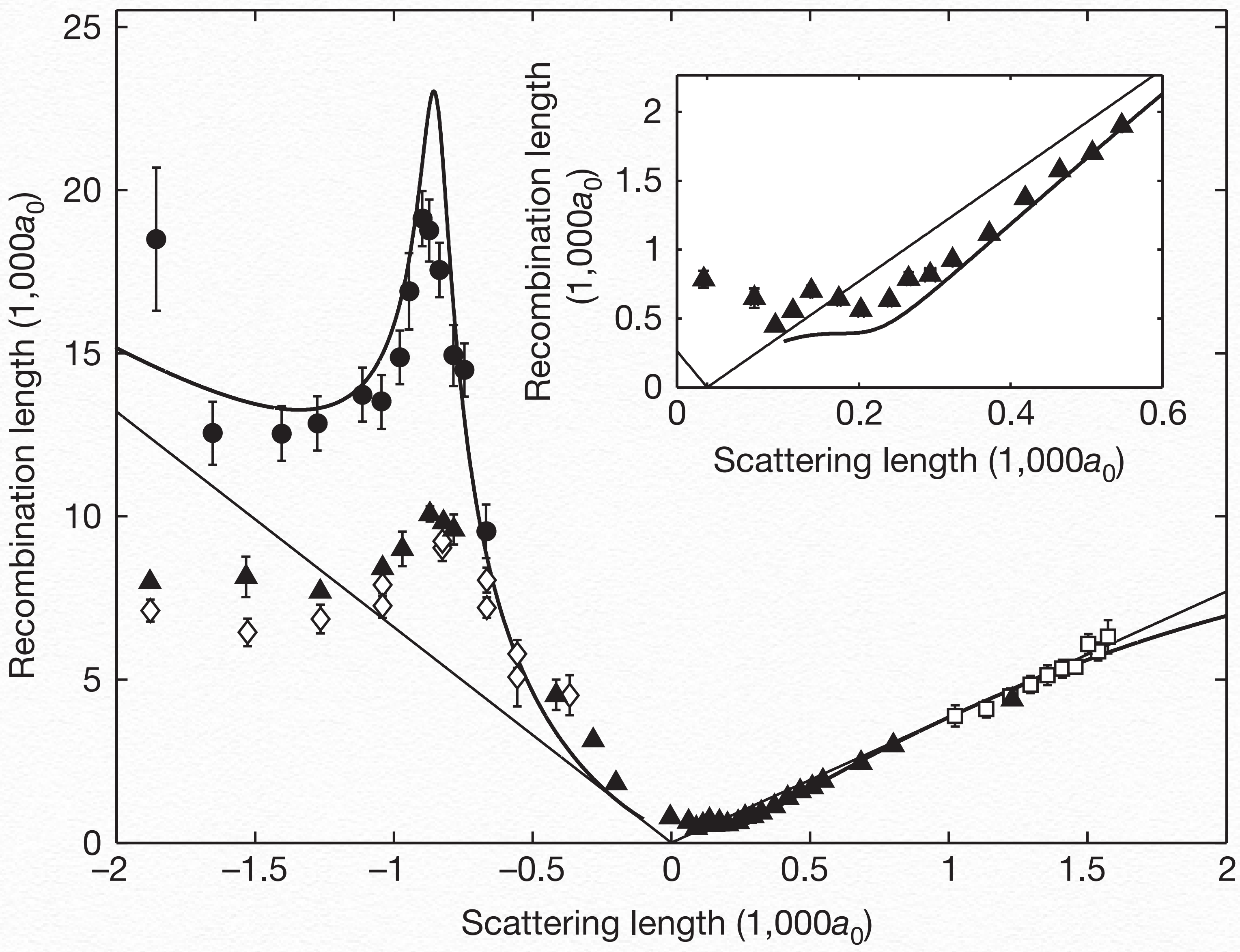
Low-density: $10^{14} cm^{-3}$

Ultralow-temperature: $10^{-9} - 10^{-7} K$

Ultraclean

Short-range (strong) interaction

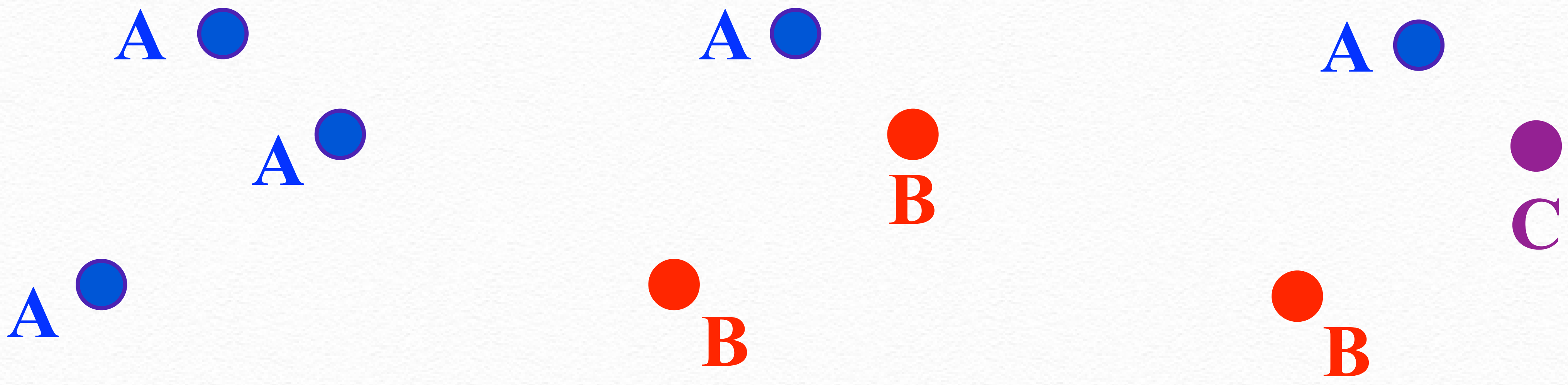
First Experimental Evidence of Efimov Effect



Three bosons of Cs atoms

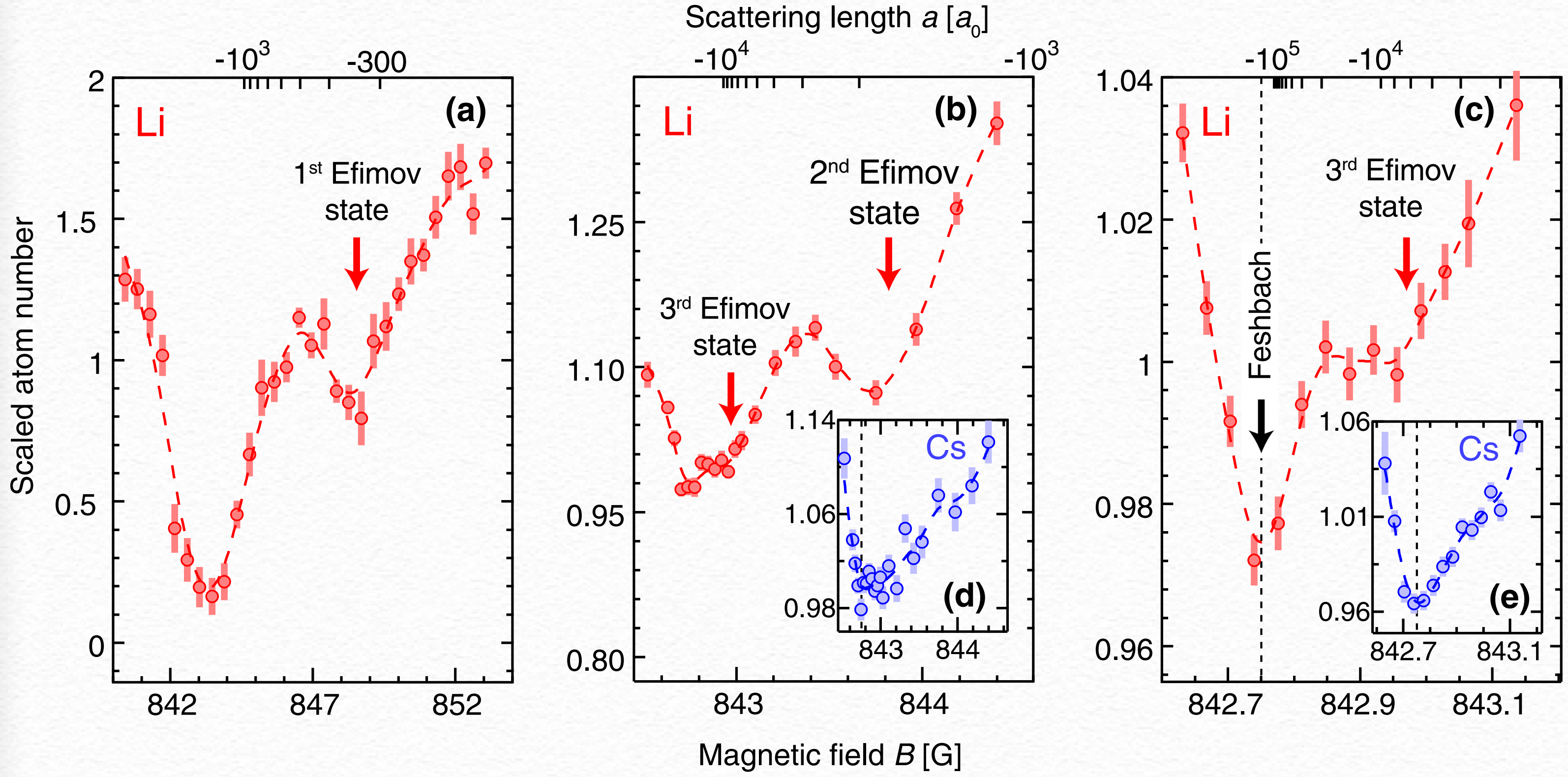
Grimm's group @ Innsbruck, Nature 2006

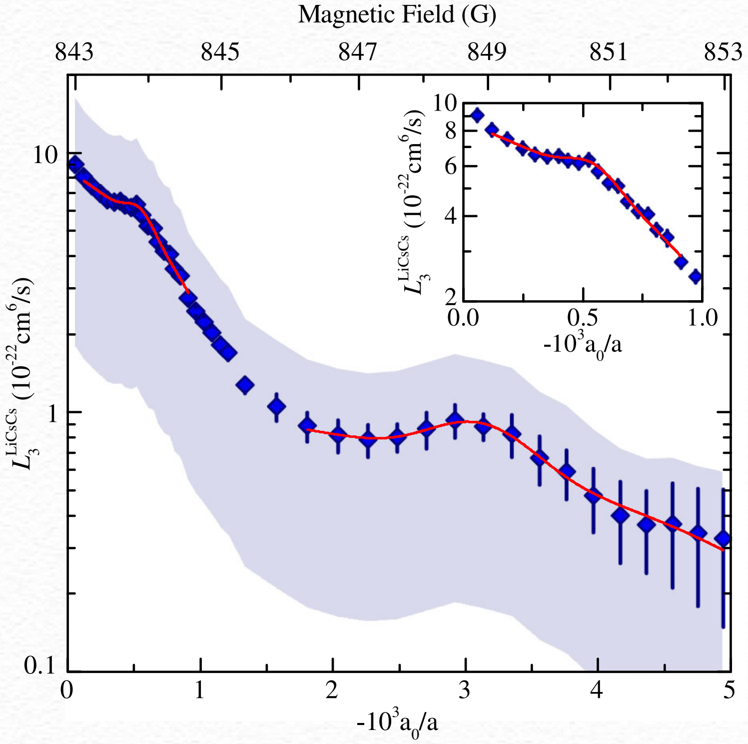
Experimental Evidences of Efimov Effect



AAA	ABB	ABC
^{133}Cs(Innsbruck, 2006)	^{41}K-^{87}Rb (Florence 2009? Osaka City 2017)	^{6}Li(Heidelberg 2008, Penn State 2009, Tokyo 2010)
^{39}K(Florence, 2009)	^{40}K-^{87}Rb (JILA, 2013)	
^{7}Li(Rice, Bar Ilan, 2009)	^{6}Li-^{133}Cs (Heidelberg and Chicago, 2014)	
^{85}Rb(JILA, 2012)	^{7}Li-^{87}Rb (Tubingen, 2015)	

Experimental Evidences of Efimov Effect





Experimental Confirmation of the Universal Scaling

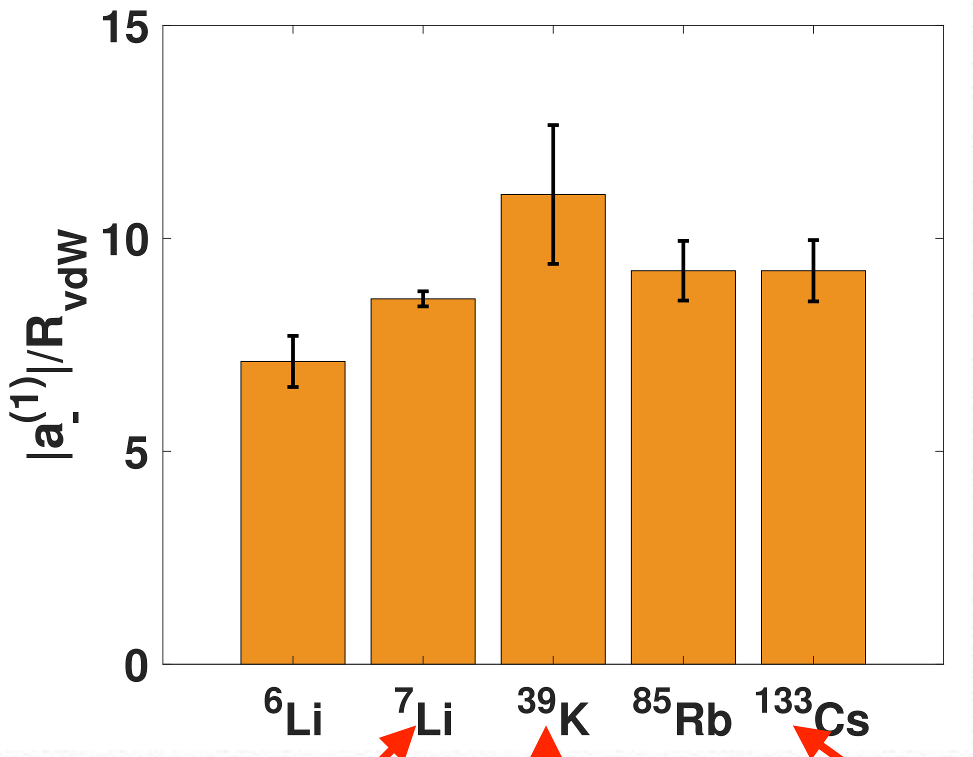
Chin’s group @ Chicago

Weidemuller’s group @ Heidelberg

Grimm’s group @ Innsbruck

PRL 2014

van der Walls Universality



$\lvert a_-^{(1)}\rvert/\ell_{\rm vdW}$
8.52 ± 0.35
8.65 ± 0.39

$\lvert a_-^{(1)}\rvert/\ell_{\rm vdW}$
10.0± 1.6
14.7± 3.9
14.7± 2.3
10.7± 0.6
12.9±2.2
10.0±1.4
11.3±1.9

$\lvert a_-^{(1)}\rvert/\ell_{\rm vdW}$
8.63 ± 0.22
10.19 ± 0.57
9.48 ± 0.79
9.45 ± 0.28

See review, Greene, Gianakeas and Perez-Rios, arXiv: 1704.02029

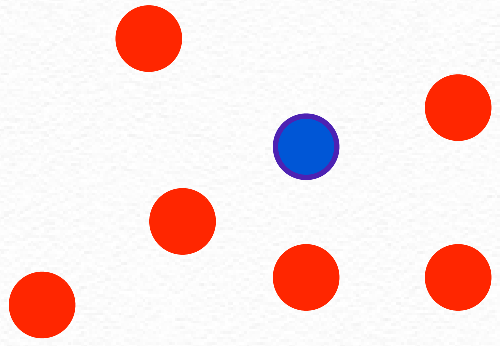
Grimm's group @ Innsbruck **PRL 2011**

Super-Efimov Effect

	Efimov	SuperEfimov
Statistics	Bosons/mixture	Fermions
Dimension	3D	2D
Interaction	s-wave resonance	p-wave resonance
Scaling	$e^{-2\pi n/\lvert s_0\rvert}$	$\exp(-2e^{3\pi n/4+\theta})$
Effective Potential	$-\dfrac{\lvert s_0\rvert^2+1/4}{R^2}$	$-\dfrac{1}{4\rho^2}-\dfrac{s_0^2+1/4}{\rho^2\ln^2(\rho/r_0)}$

Y. Nishida, S. Moroz and D. T. Son, PRL 2013
C. Gao, J. Wang and Z. Yu, PRA 2015
A. G. Voloseiev, D. V. Fedorov, A. S. Jensen and N. T. Zinner, JPB 2014

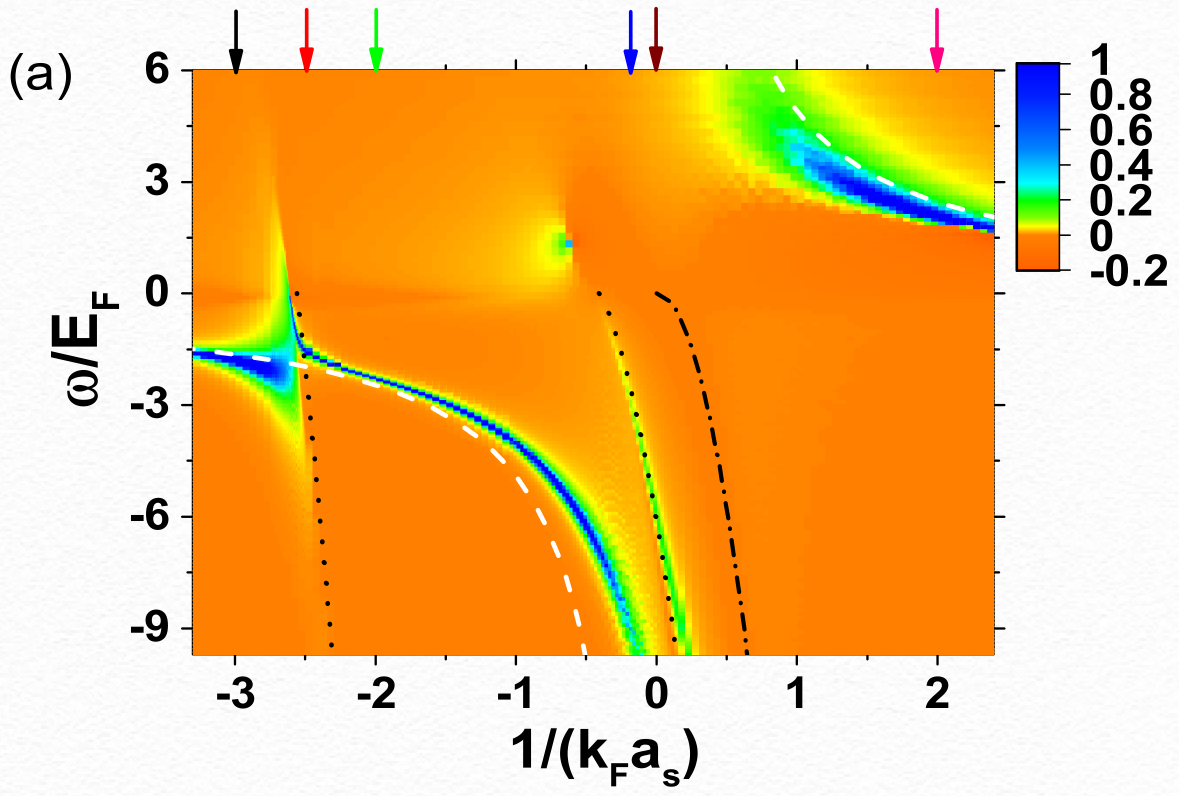
Manifestation in Many-Body System



Bose Polaron Problem

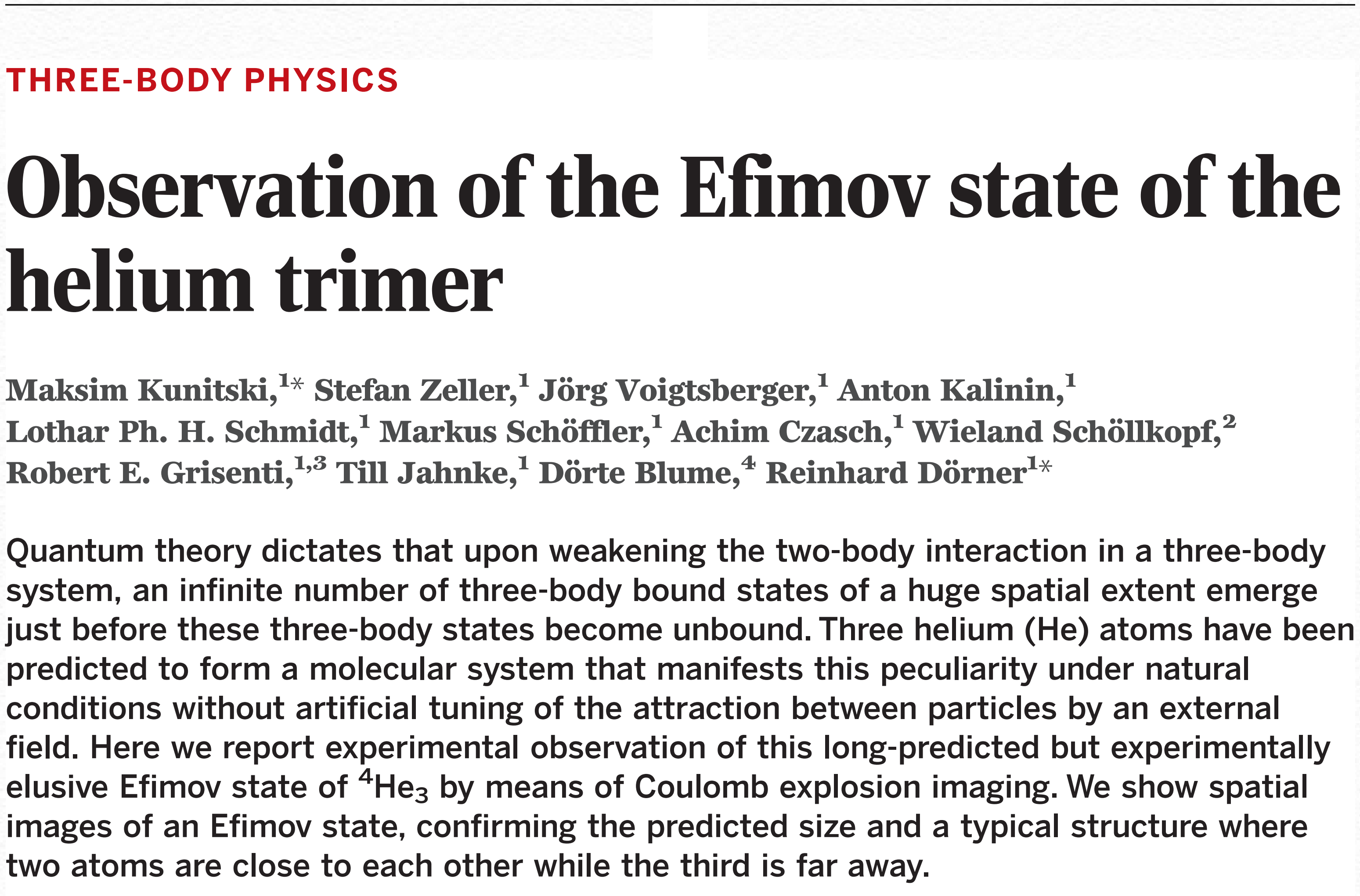
$$\epsilon^{(n)} \equiv E_T^{(n)}/E_F$$

Impurity-boson	$\epsilon^{(1)}$	$\epsilon^{(2)}$	$\epsilon^{(3)}$
^{39}K-^{39}K [58]	1.51×10^{-4}	3.84×10^{-11}	9.73×10^{-18}
^{40}K-^{87}Rb [59]	1.40×10^{-2}	9.32×10^{-7}	6.19×10^{-11}
^{6}Li-^{133}Cs [18,19]	185.9	6.09	0.25



M. Sun, H. Zhai and X. L. Cui, PRL 2017

The Efimov Effect in Helium Trimer

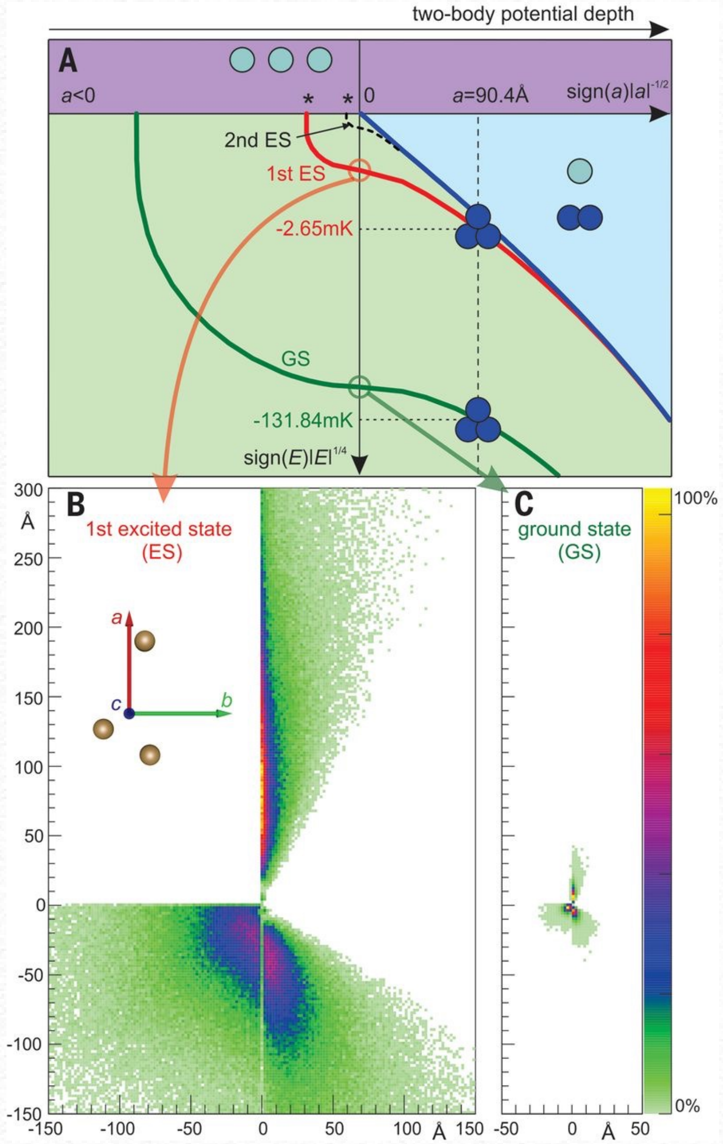


THREE-BODY PHYSICS

Observation of the Efimov state of the helium trimer

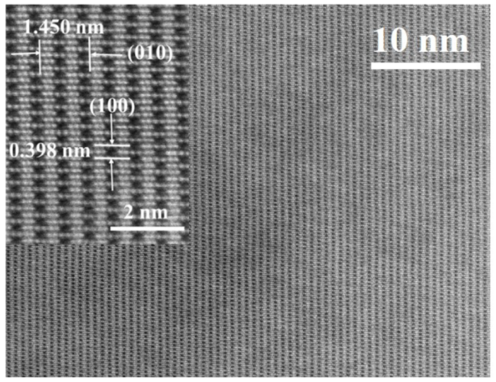
**Maksim Kunitski,[1]* Stefan Zeller,[1] Jörg Voigtsberger,[1] Anton Kalinin,[1]
Lothar Ph. H. Schmidt,[1] Markus Schöffler,[1] Achim Czasch,[1] Wieland Schöllkopf,[2]
Robert E. Grisenti,[1,3] Till Jahnke,[1] Dörte Blume,[4] Reinhard Dörner[1]***

Quantum theory dictates that upon weakening the two-body interaction in a three-body system, an infinite number of three-body bound states of a huge spatial extent emerge just before these three-body states become unbound. Three helium (He) atoms have been predicted to form a molecular system that manifests this peculiarity under natural conditions without artificial tuning of the attraction between particles by an external field. Here we report experimental observation of this long-predicted but experimentally elusive Efimov state of $^4He_3$ by means of Coulomb explosion imaging. We show spatial images of an Efimov state, confirming the predicted size and a typical structure where two atoms are close to each other while the third is far away.

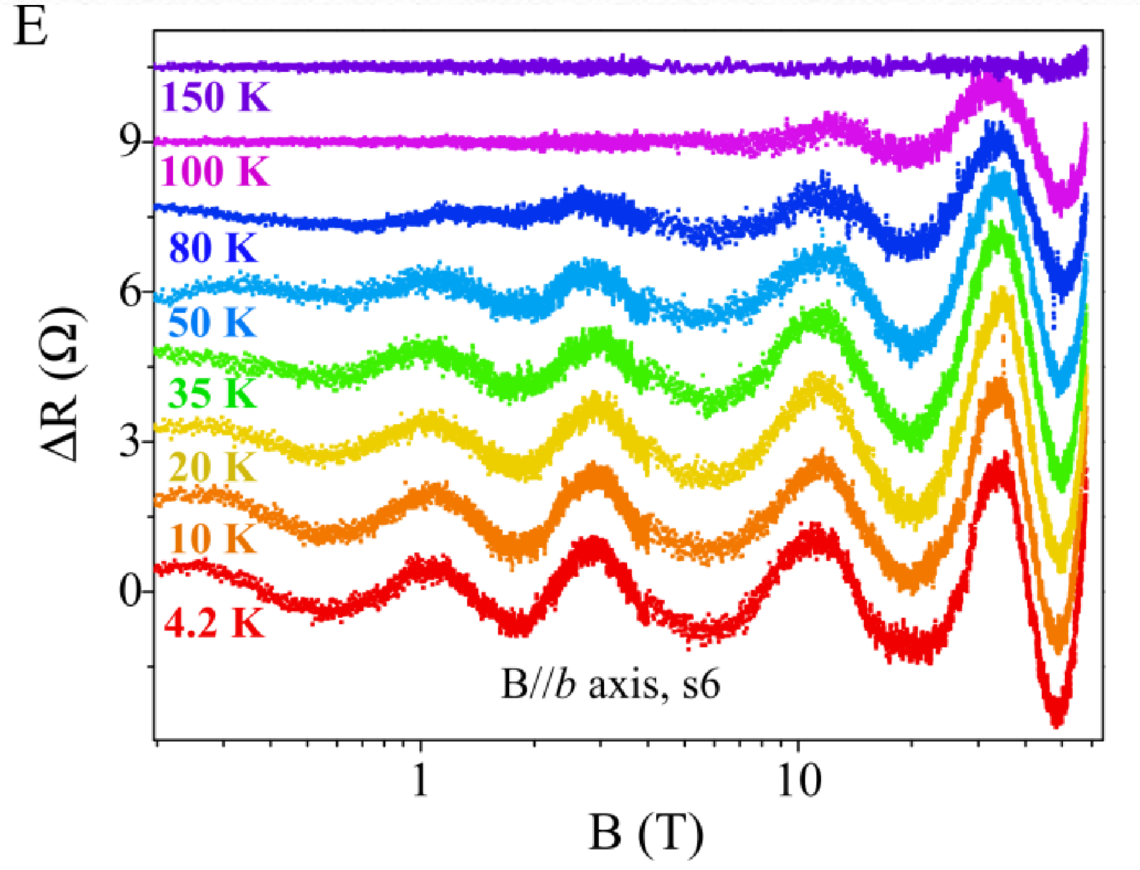


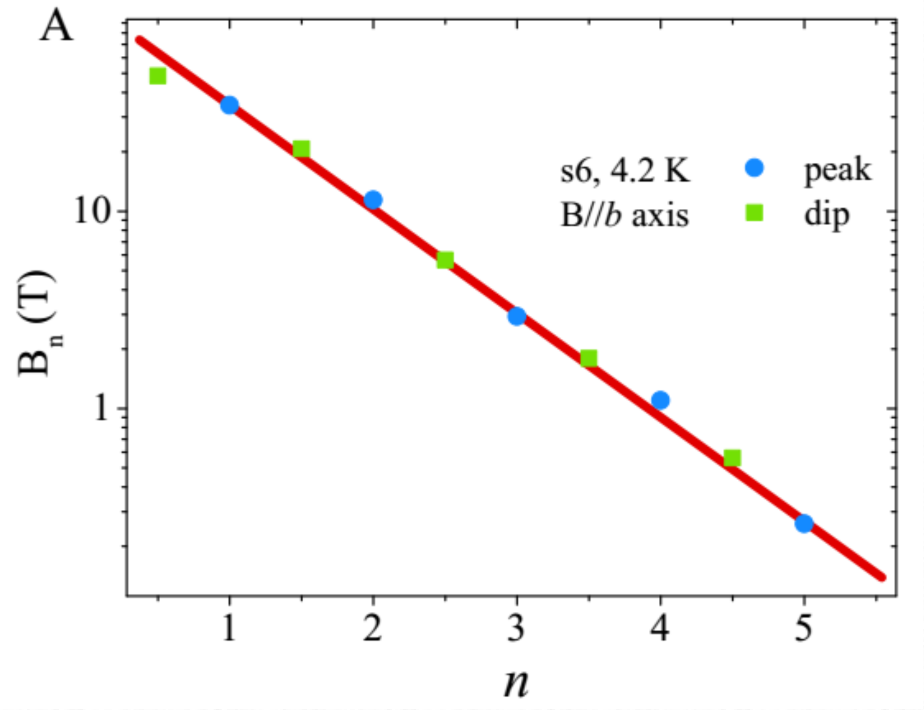
Science 2015

The Efimov Effect of Electron-Hole Pairs



Magnetoresistance measurement





Jian Wang'g group in PKU

arXiv: 1704.00995

Generalized Definition of Efimov Effect

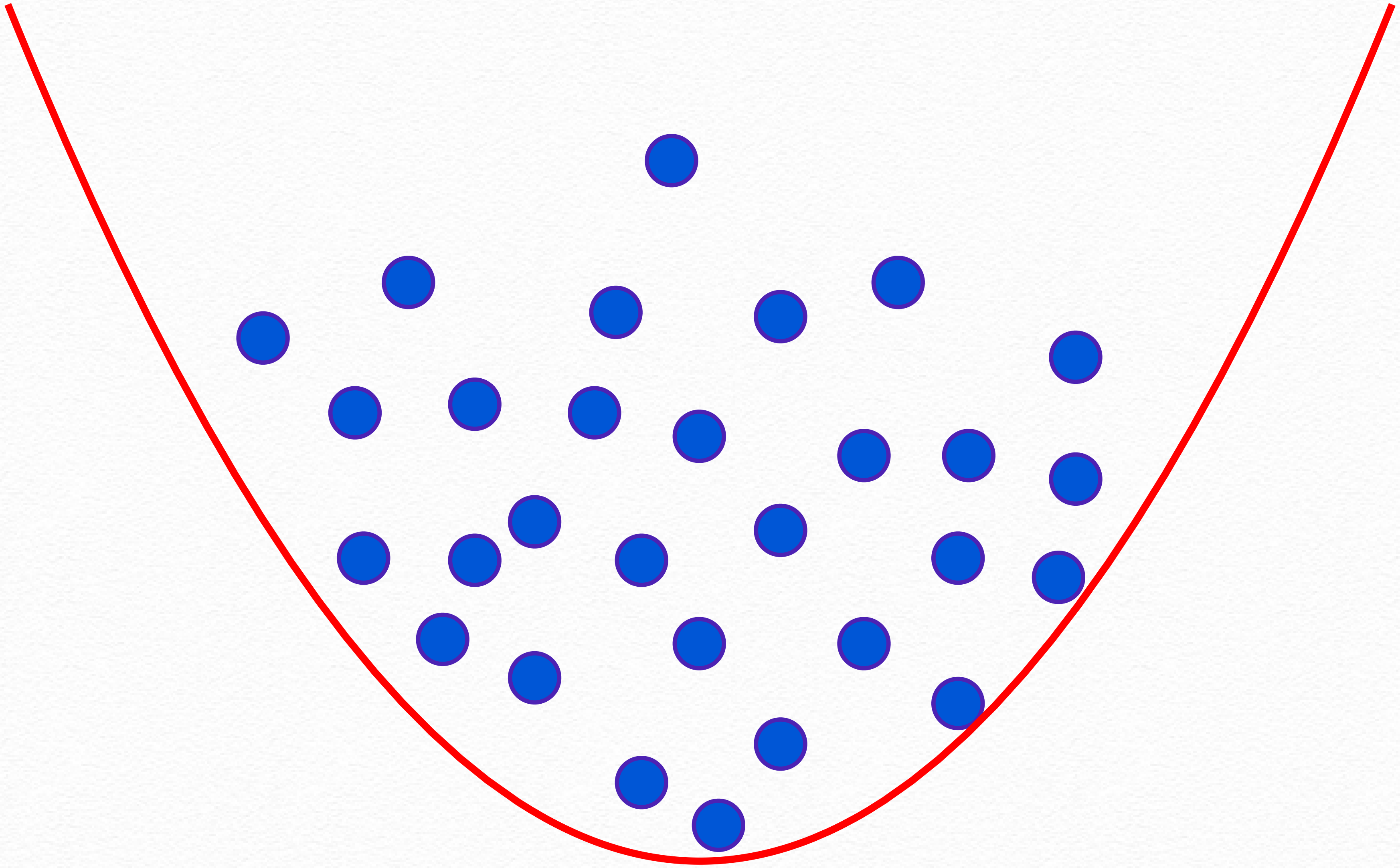
Phenomenon that has discrete scaling symmetry with universal scaling factor

Generalized Definition of Efimov Effect

Phenomenon that has discrete scaling symmetry with universal scaling factor

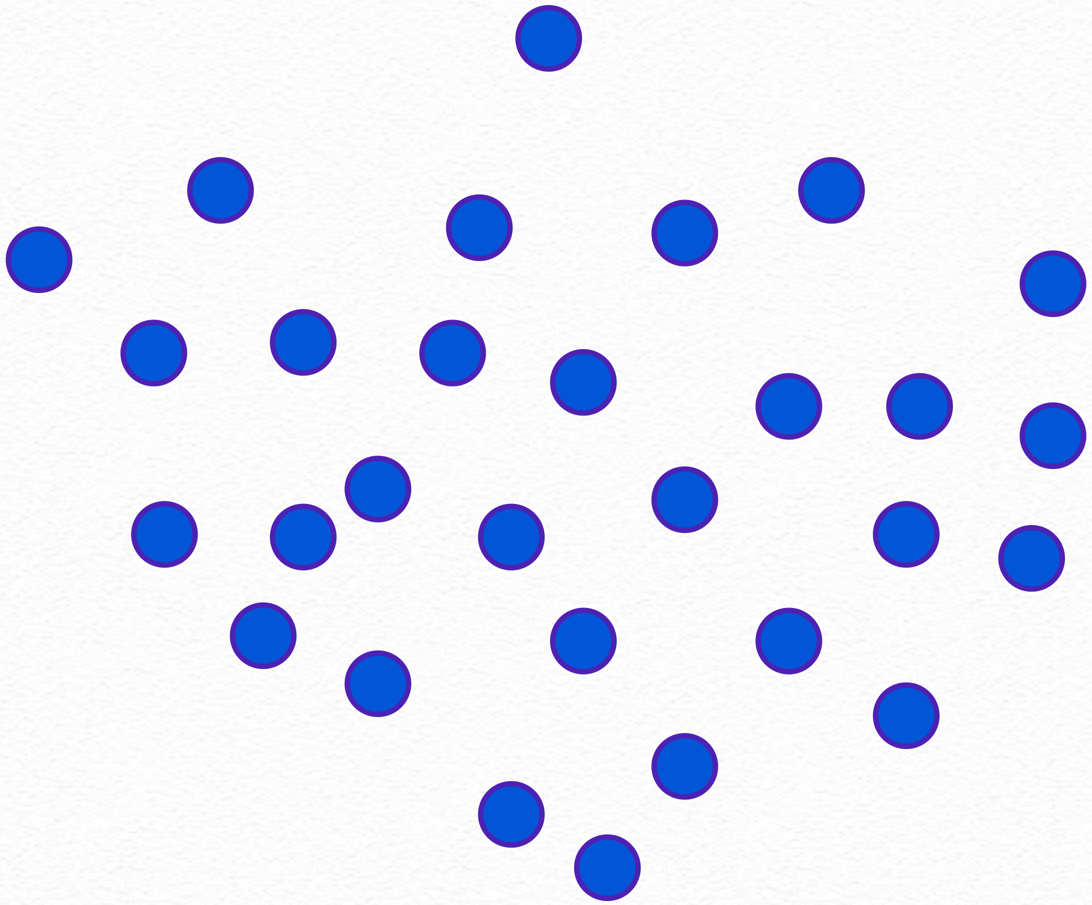
Whether such effect exists beyond few-body context ?

Expansion Experiment with Cold Atoms



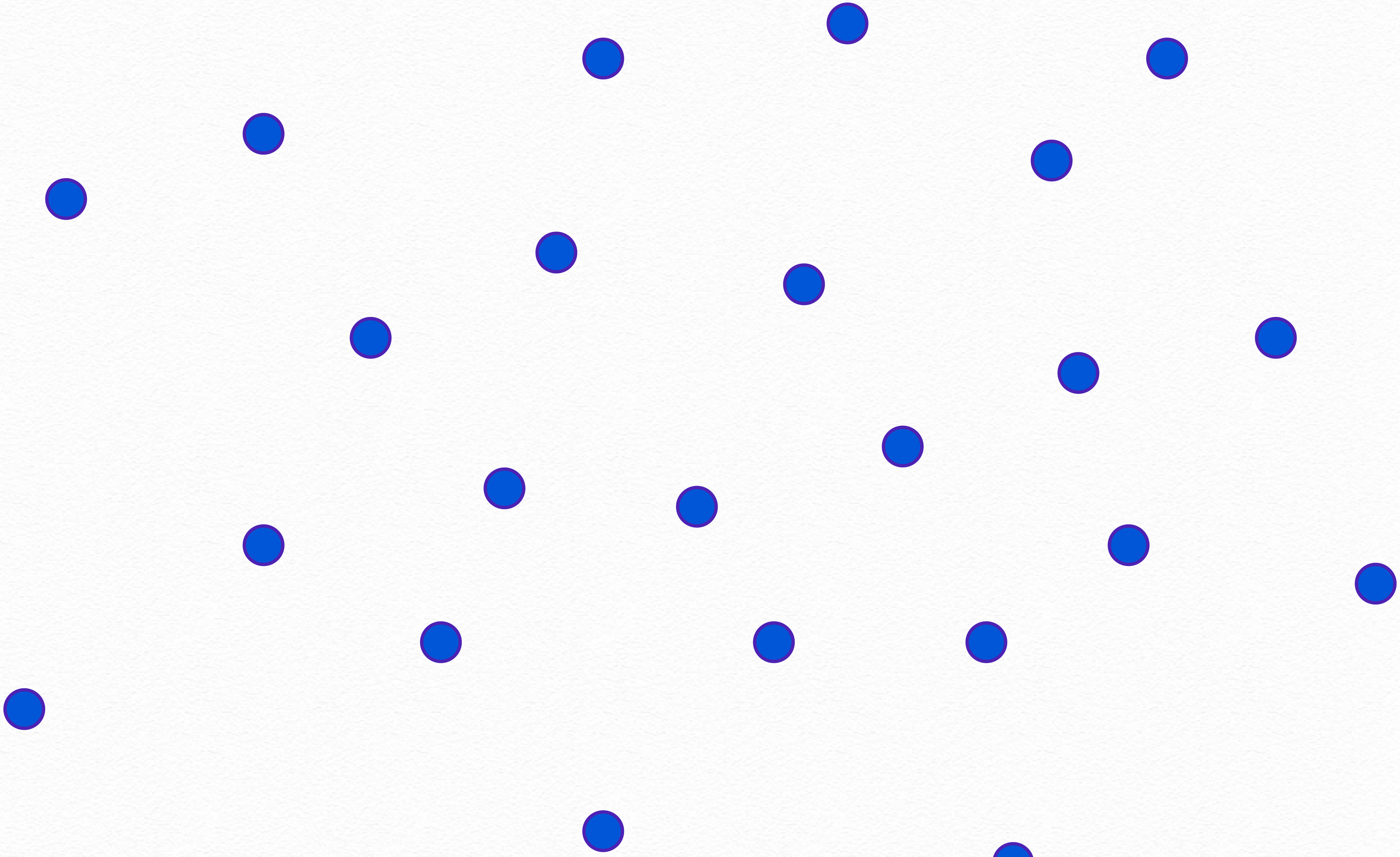
Gas hold in a trap

Expansion Experiment with Cold Atoms



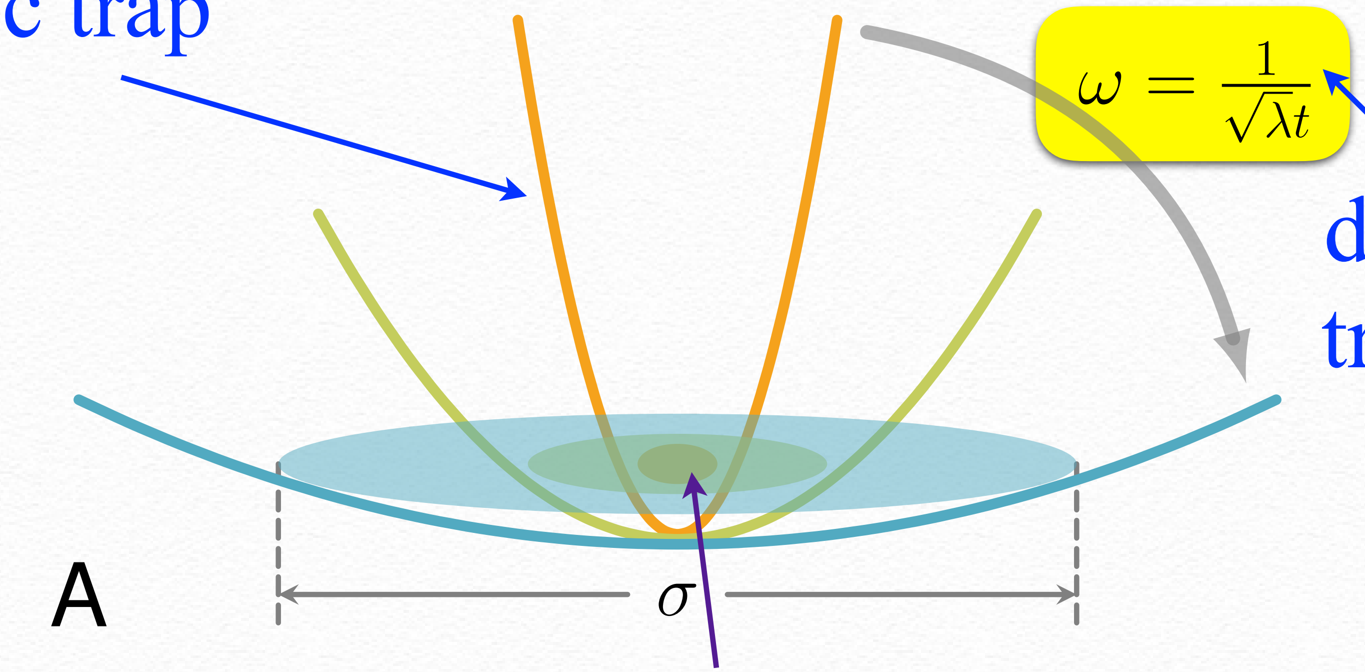
Turn off the trap

Expansion Experiment with Cold Atoms



Expansion

Our Proposal



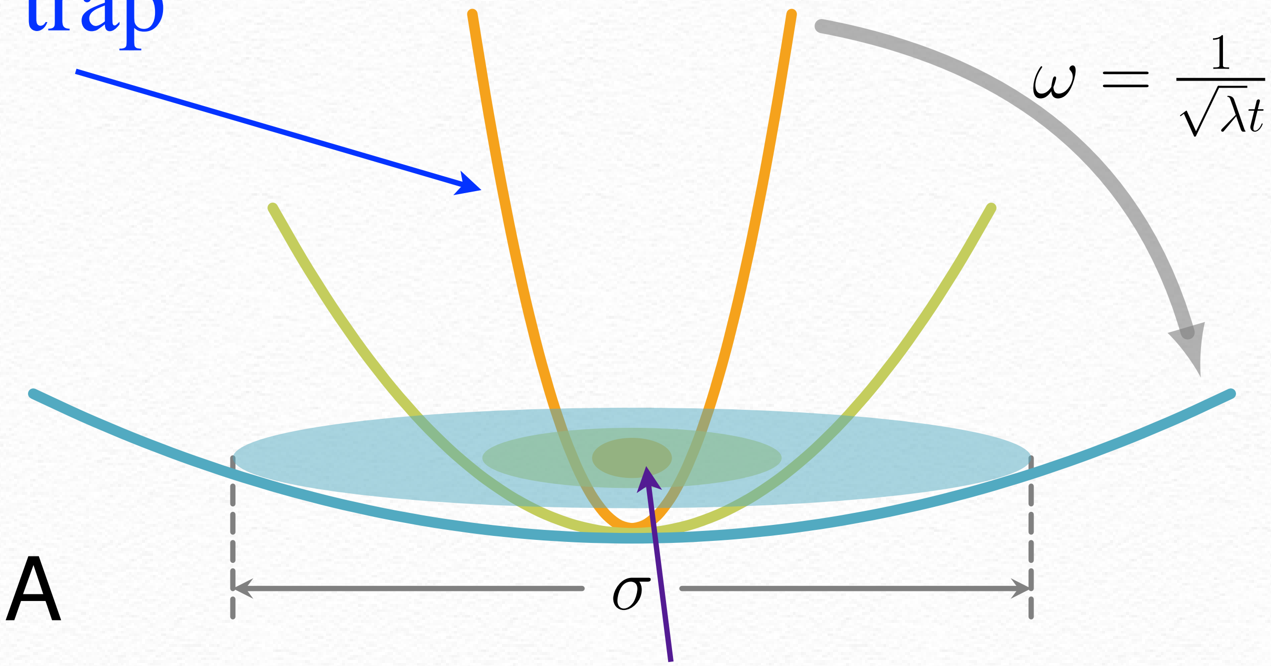
Harmonic trap

$\omega = \frac{1}{\sqrt{\lambda t}}$

decreasing the trap frequency

Scale Invariant Quantum Gas

Our Proposal



Harmonic trap

$\omega = \frac{1}{\sqrt{\lambda t}}$

decreasing the trap frequency

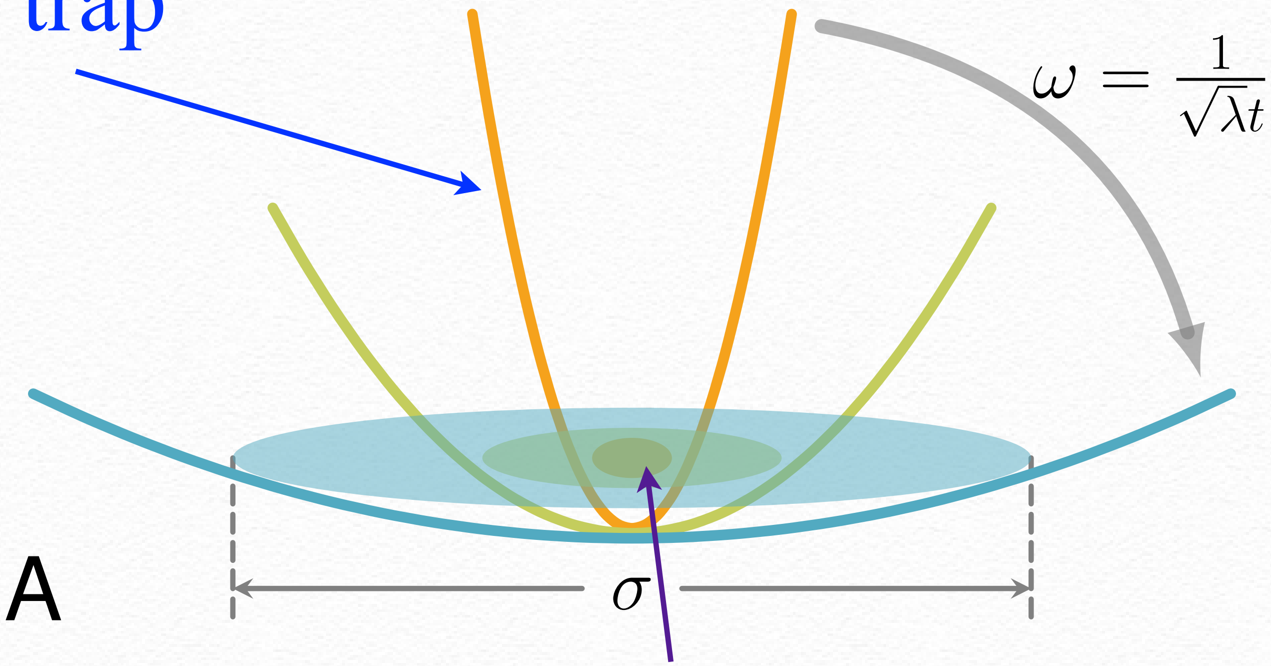
$$R = \sqrt{\langle \sum r_i^2 \rangle}$$

A

Scale Invariant Quantum Gas

Our Proposal

Harmonic trap



$\omega = \frac{1}{\sqrt{\lambda t}}$

decreasing the trap frequency

$R = \sqrt{\langle \sum r_i^2 \rangle}$

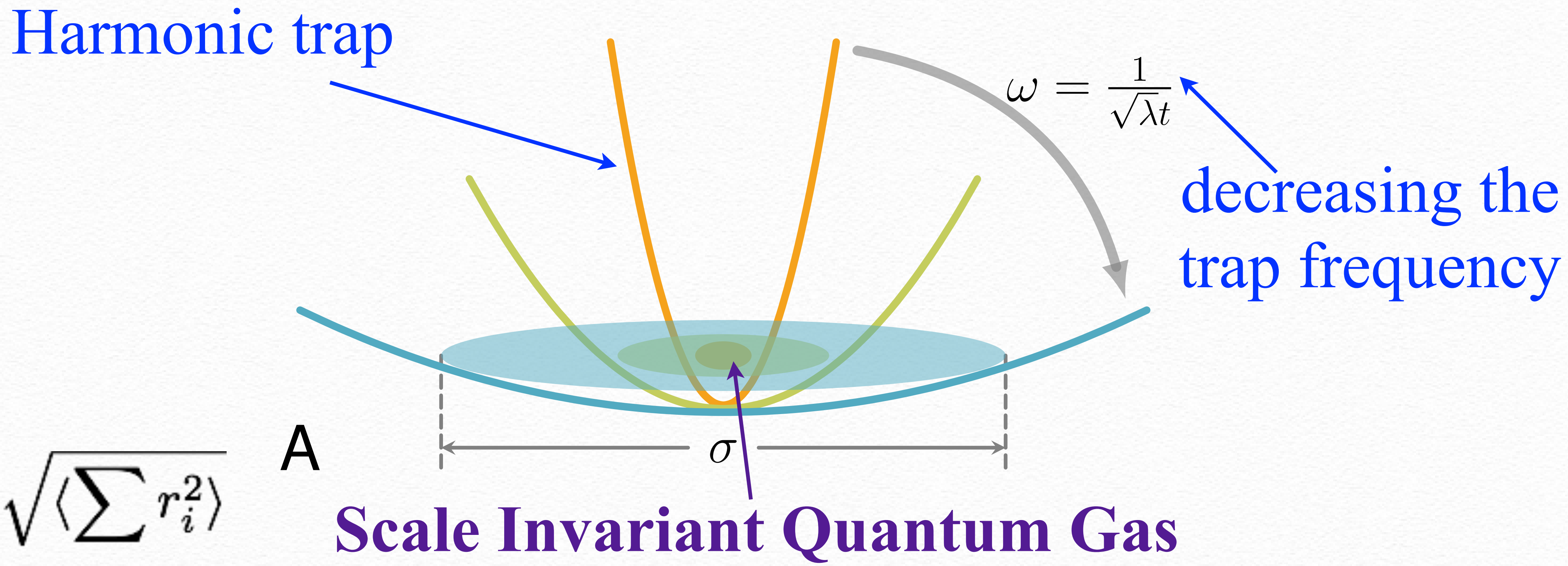
A

Scale Invariant Quantum Gas

Harmonic length: $$a = \sqrt{\frac{\hbar}{m\omega}}$$

By dimension analysis: $$\mathcal{R} \sim \sqrt{t}$$

Our Proposal



Harmonic trap

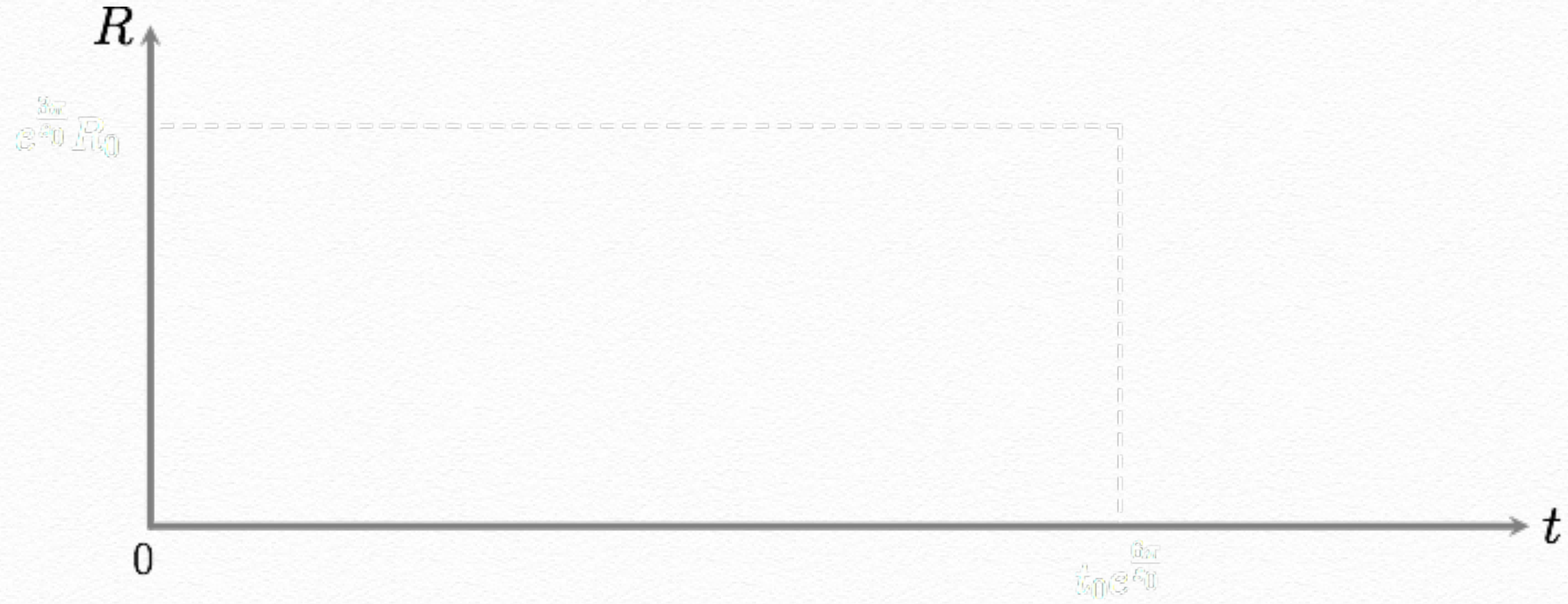
$\omega = \frac{1}{\sqrt{\lambda t}}$

decreasing the trap frequency

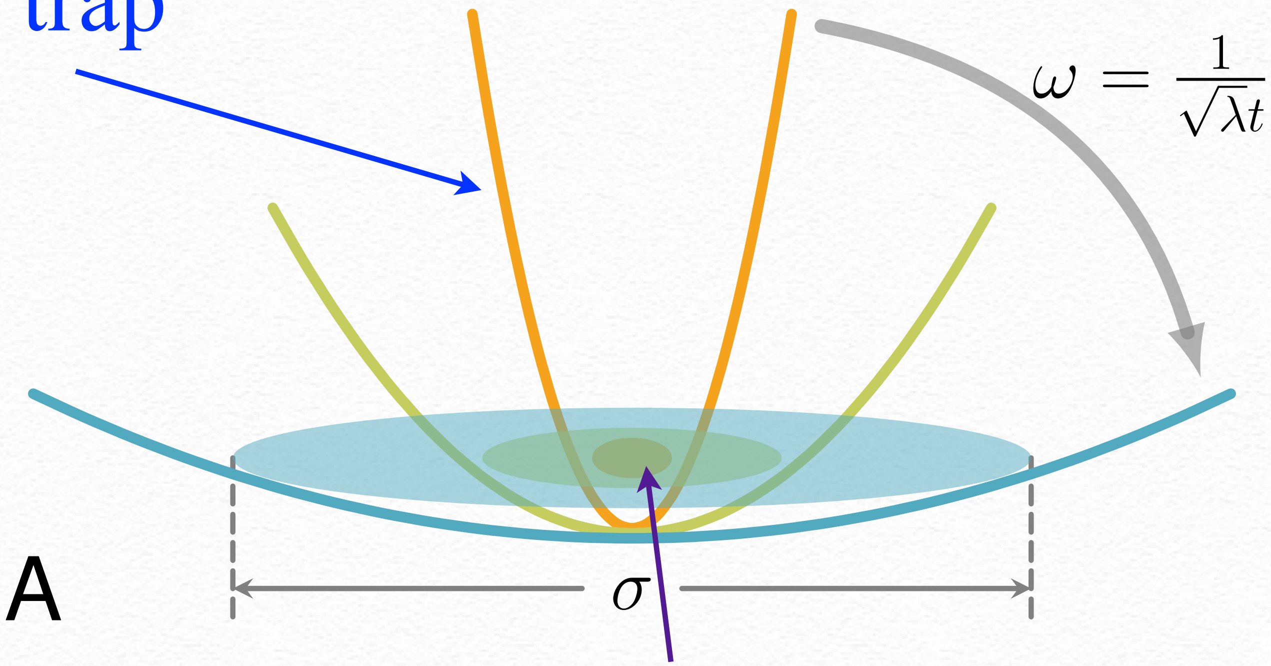
Scale Invariant Quantum Gas

$$R = \sqrt{\langle \sum r_i^2 \rangle}$$

A



Our Proposal



Harmonic trap

$\omega = \frac{1}{\sqrt{\lambda}t}$

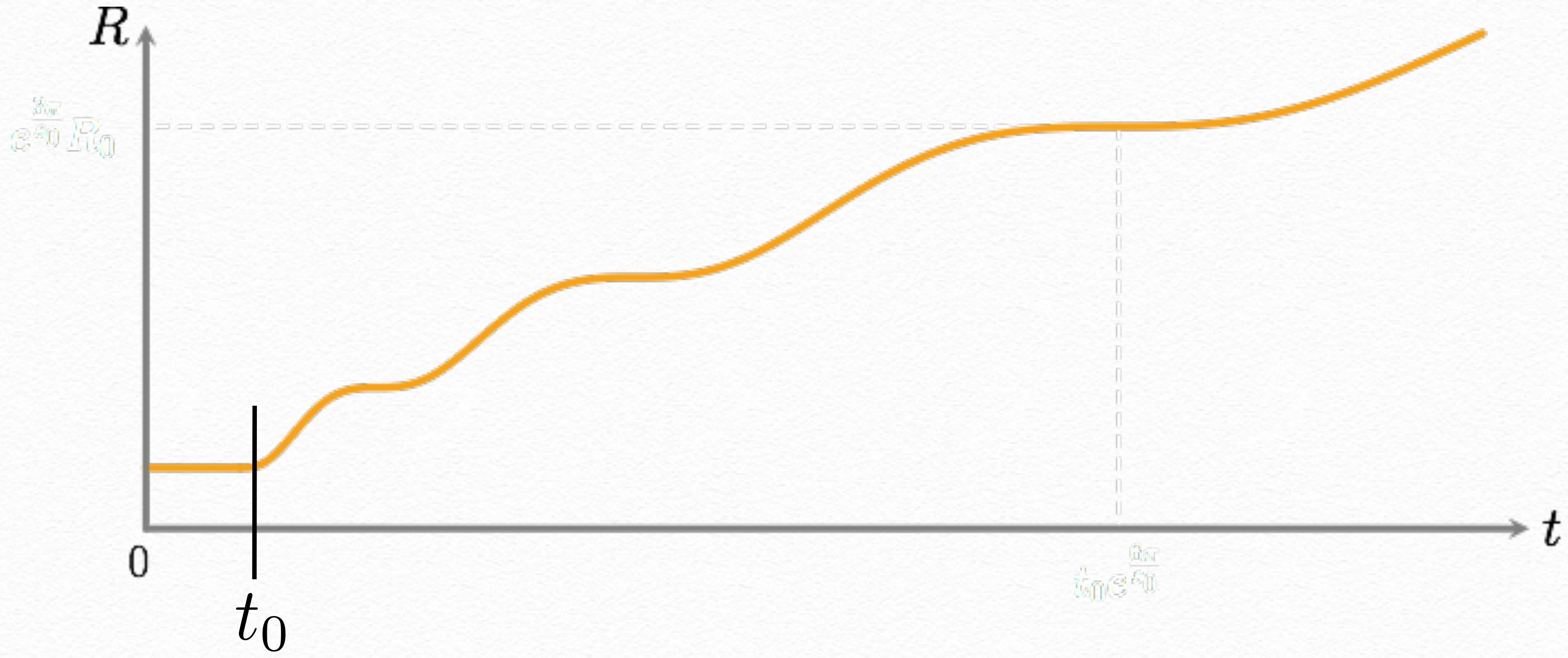
decreasing the trap frequency

σ

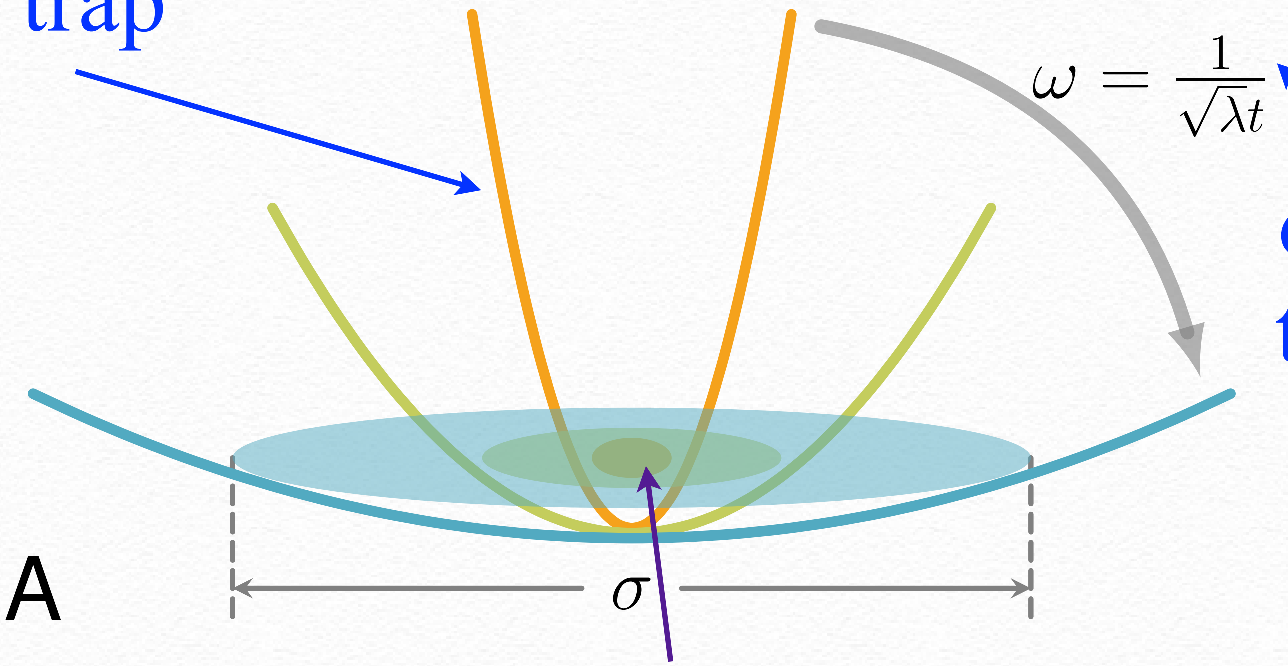
Scale Invariant Quantum Gas

$$R = \sqrt{\langle \sum r_i^2 \rangle}$$

A



Our Proposal



Harmonic trap

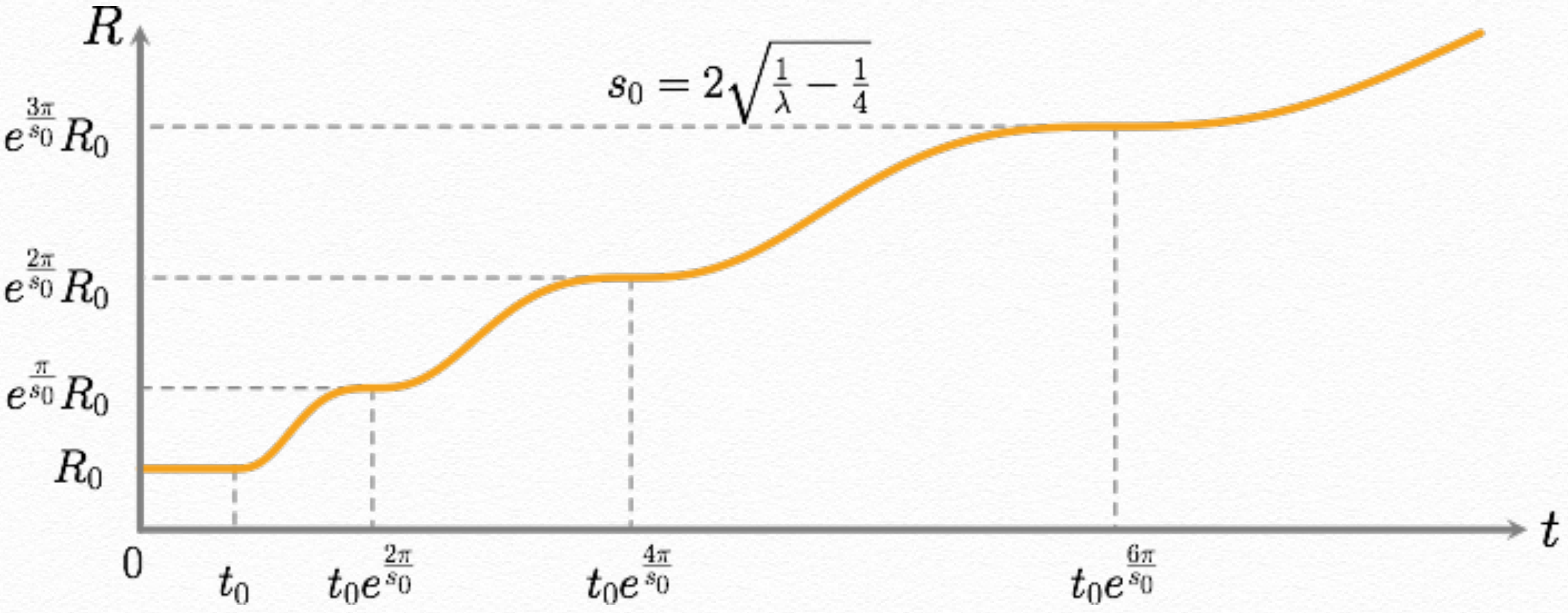
$$\omega = \frac{1}{\sqrt{\lambda} t}$$

decreasing the trap frequency

Scale Invariant Quantum Gas

$$R = \sqrt{\langle \sum r_i^2 \rangle}$$

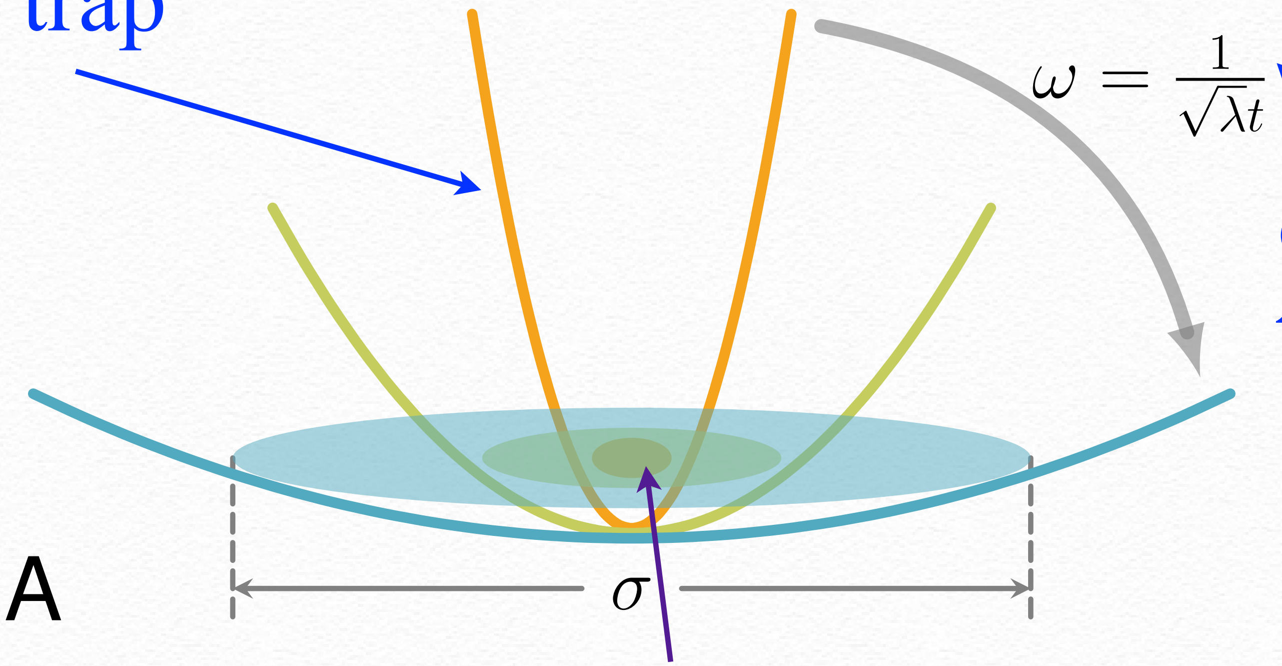
A



$$s_0 = 2\sqrt{\frac{1}{\lambda} - \frac{1}{4}}$$

Our Proposal

Harmonic trap

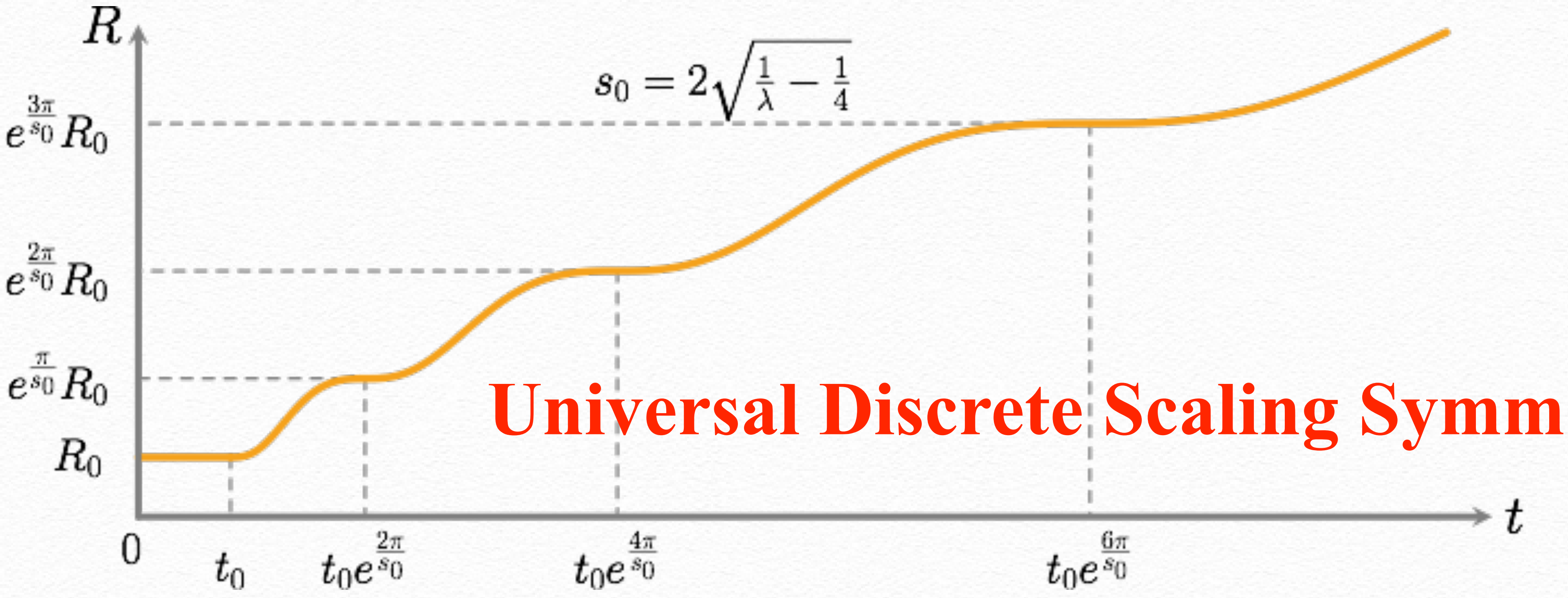


decreasing the trap frequency

$$R = \sqrt{\langle \sum r_i^2 \rangle}$$

A

Scale Invariant Quantum Gas



Universal Discrete Scaling Symmetry

Scale Invariant Quantum Systems

$$i\hbar\underset{\begin{array}{c}\Big\downarrow\\ \frac{1}{\Lambda^2}\end{array}}{\frac{\partial}{\partial t}}\Psi = -\sum_i \frac{\hbar^2}{2m}\underset{\begin{array}{c}\Big\downarrow\\ \frac{1}{\Lambda^2}\end{array}}{\nabla_i^2}\Psi$$

Scale Transformation

$$\boxed{\begin{array}{r} \mathbf{r}_i \longrightarrow \Lambda\mathbf{r}_i \\ t \longrightarrow \Lambda^2 t \end{array}}$$

No other energy scale except for the kinetic energy

Zoo of Scale Invariant Quantum Gases

Non-interacting bosons/ fermions at any dimension	No other length scale except for density
Unitary Fermi gas at three dimension	Density and a_s $a_s = \infty$
Tonks gas of bosons/ fermions at one dimension	Density and g_{1D} $g_{1D} = \infty$

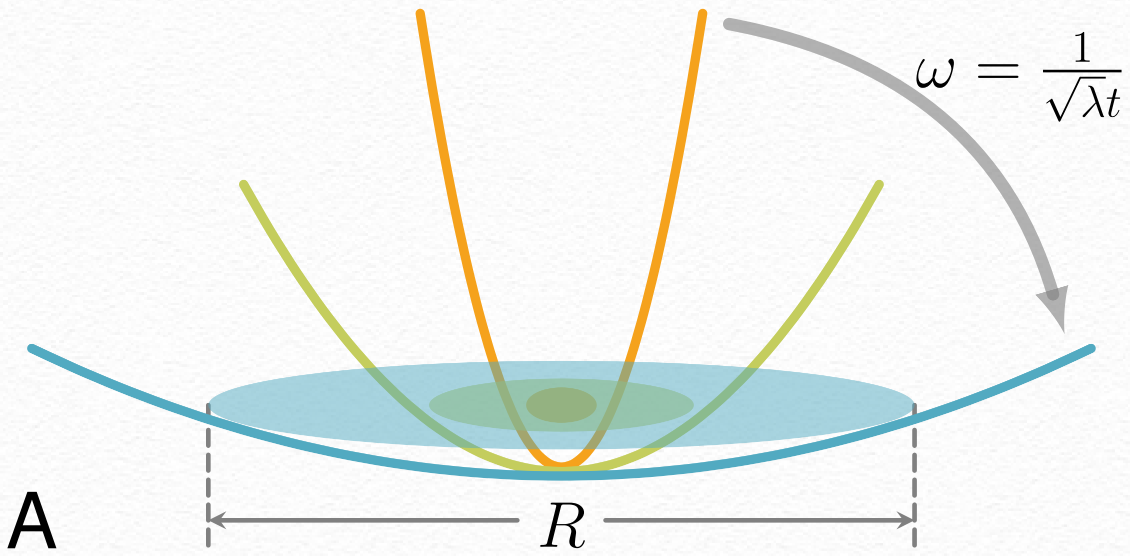
Universal behavior:

$$\boxed{\langle V \rangle = \alpha \langle T \rangle}$$

Significance

- Universal
  - Independent of Temperature
  - Independent of State of Matter
  - Independent Dimension

Scaling Symmetry in a Harmonic Trap



$$i\hbar\frac{\partial}{\partial t}\Psi = \left[H + \sum_i \frac{1}{2}m\omega^2 r_i^2\right]\Psi$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$\frac{1}{\Lambda^2} \qquad \frac{1}{\Lambda^2} \qquad \frac{1}{\Lambda^2}$$

Scale Transformation

$$\mathbf{r}_i \to \Lambda \mathbf{r}_i$$

$$t \to \Lambda^2 t$$

This scaling symmetry exists only if

$$\omega = \frac{1}{\sqrt{\lambda}t}$$

Expansion Dynamics

$$i\frac{d}{dt}R^2 = \sum_i \langle [r_i^2, H] \rangle = 2i\langle \hat{D} \rangle$$

$$i\frac{d}{dt}\langle \hat{D} \rangle = \langle [\hat{D}, H] \rangle = 2i\left(\langle H \rangle - \omega^2 R^2 \right)$$

$$\frac{d}{dt}\langle H \rangle = \langle \frac{\partial}{\partial t} H \rangle = \omega\dot{\omega} R^2$$

Expansion Dynamics

$$i\frac{d}{dt}R^2 = \sum_i \langle [r_i^2, H] \rangle = 2i\langle \hat{D} \rangle$$

$$i\frac{d}{dt}\langle \hat{D} \rangle = \langle [\hat{D}, H] \rangle = 2i\Big(\langle H \rangle - \omega^2 R^2 \Big)$$

$$\frac{d}{dt}\langle H \rangle = \langle \frac{\partial}{\partial t} H \rangle = \omega\dot{\omega}R^2$$

Scale Invariant Equation

$$\boxed{\frac{d^3}{dt^3}R^2 + 4\omega^2\frac{d}{dt}R^2 + 4\omega\dot{\omega}R^2 = 0}$$

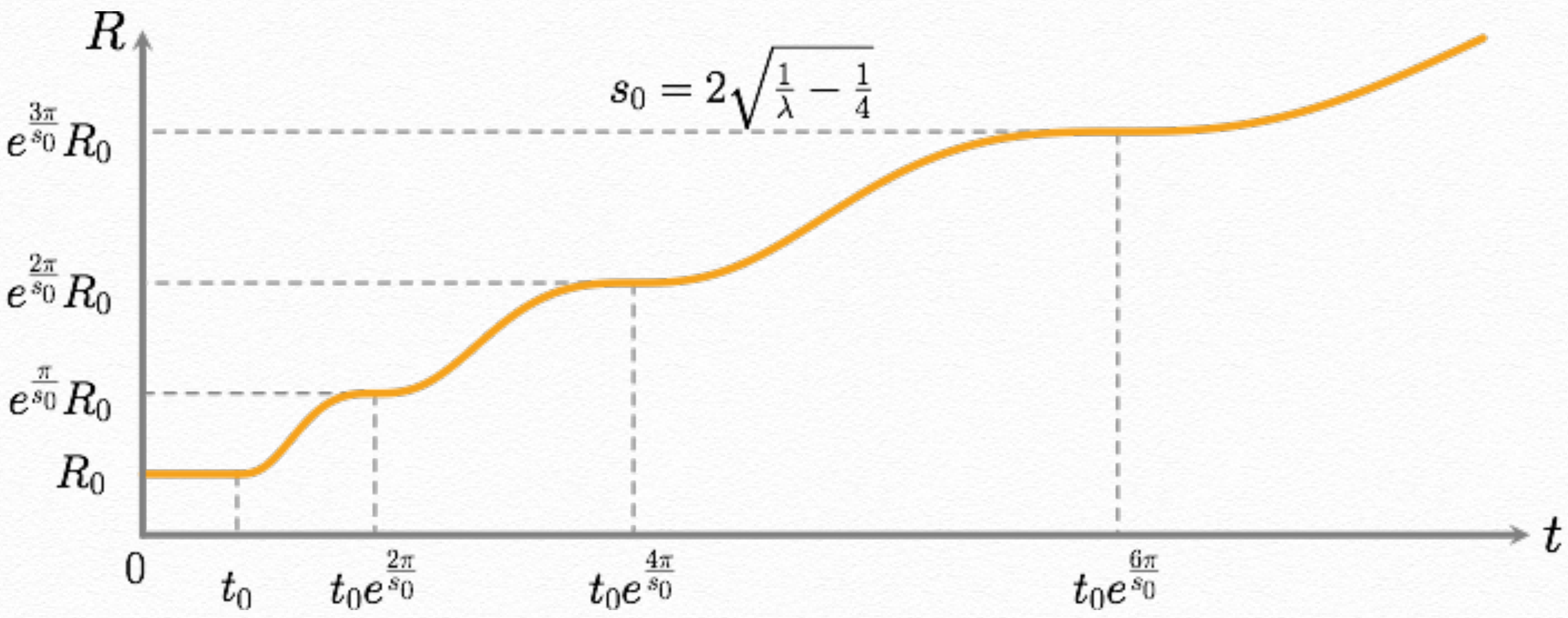
Initial Condition

$$\frac{d^n}{dt^n}\langle \hat{R}^2 \rangle|_{t=t_0} \;=\; 0$$

Expansion Dynamics

$$\lambda < 4$$

$$\frac{\langle \hat{R}^2 \rangle(t)}{R_0^2} = \frac{t}{t_0}\frac{1}{\sin^2\varphi}\left[1 - \cos\varphi \cdot \cos\left(s_0 \ln\frac{t}{t_0} + \varphi\right)\right]$$



Why plateaus ?

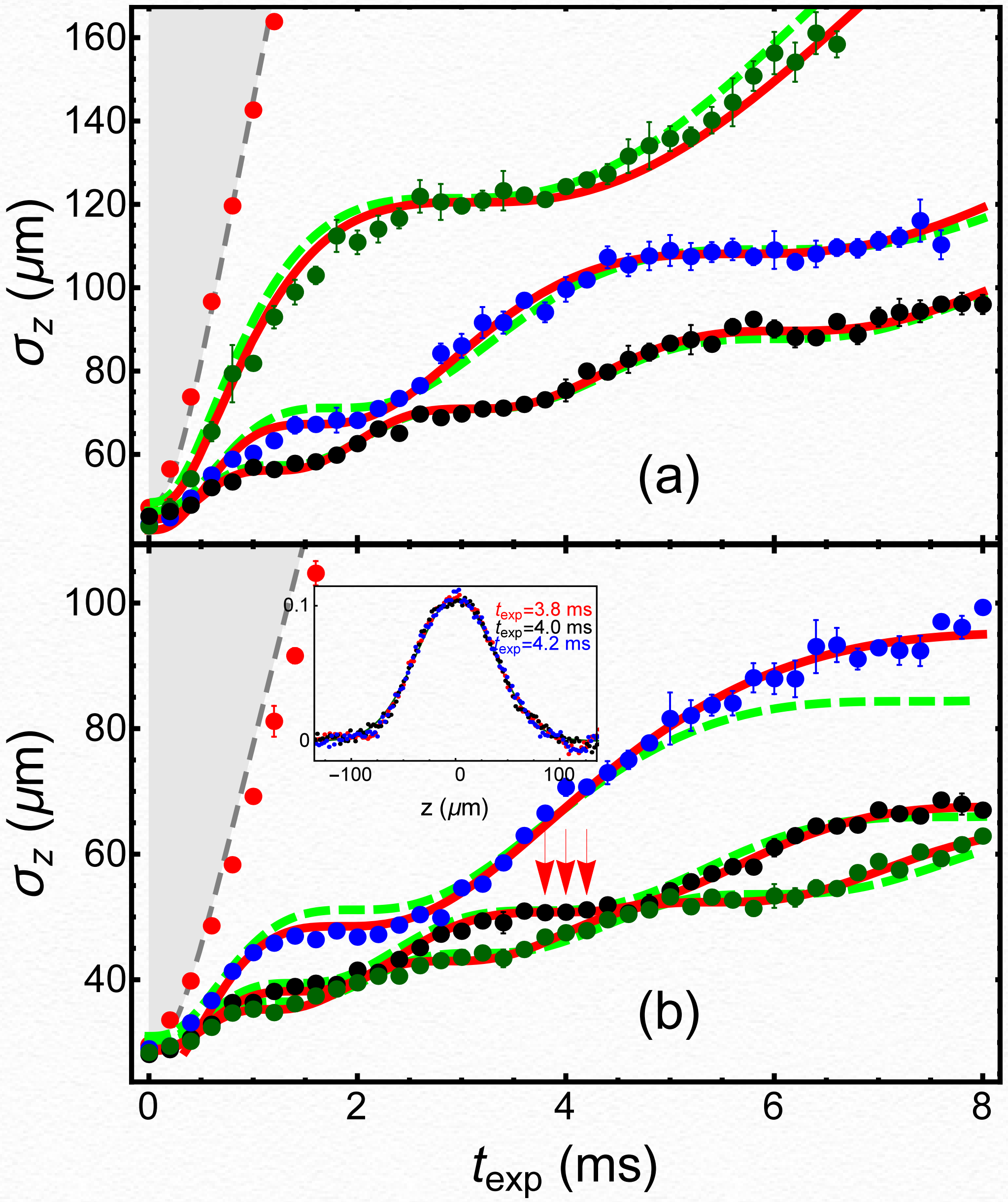
$$\frac{d^n}{dt^n}\langle \hat{R}^2 \rangle|_{t=t_0} = 0$$

Connection to the Efimov Effect

The Efimov Effect	The “Efimovian” Expansion
$-\frac{\hbar^2 d^2}{2md^2\rho}\psi - \frac{\lambda}{\rho^2}\psi = E\psi$	$\frac{d^3}{dt^3}\langle\hat{R}^2\rangle + \frac{4}{\lambda t^2}\frac{d}{dt}\langle\hat{R}^2\rangle - \frac{4}{\lambda t^3}\langle\hat{R}^2\rangle = 0.$
Spatial continuous scaling symmetry	Temporal continuous scaling symmetry
Short-range boundary condition	Initial time
$\psi = \sqrt{\rho}\cos[s_0\log(\rho/\rho_0)]$	$\frac{\langle\hat{R}^2\rangle(t)}{R_0^2} = \frac{t}{t_0}\frac{1}{\sin^2\varphi}\left[1 - \cos\varphi\cdot\cos\left(s_0\ln\frac{t}{t_0} + \varphi\right)\right]$
Spatial discrete scaling symmetry $\rho \longrightarrow e^{2\pi/s_0}\rho$	Temporal discrete scaling symmetry $t \longrightarrow e^{2\pi/s_0}t$

Experimental Observation

by Haibin Wu in East China Normal University



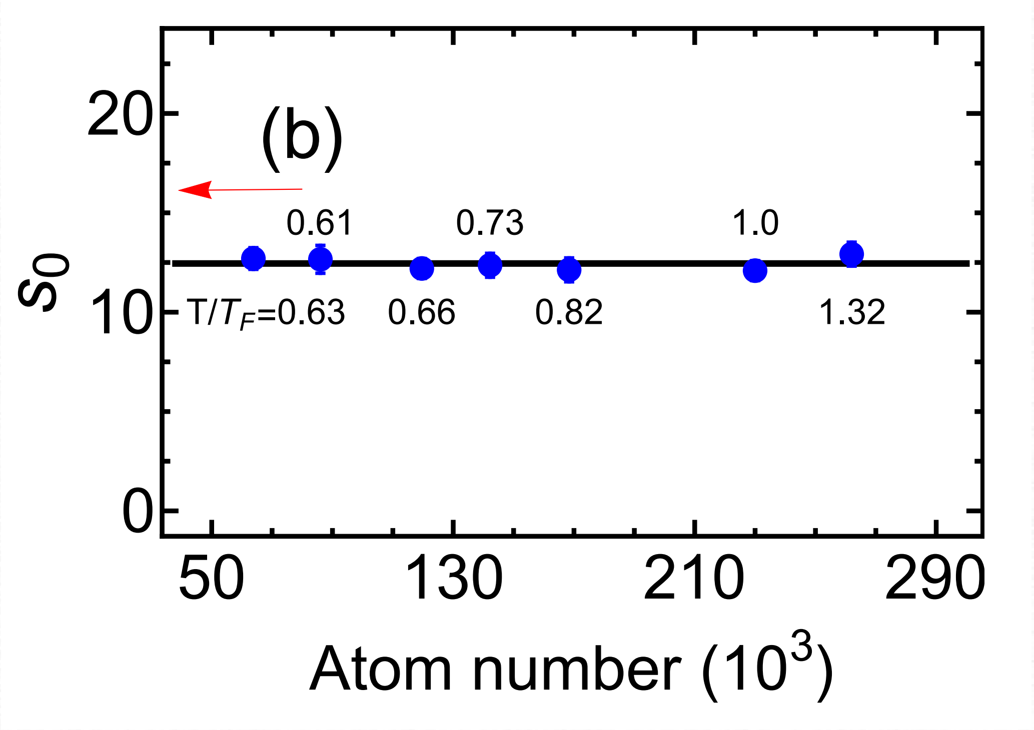
Non-interacting

Unitary Fermions

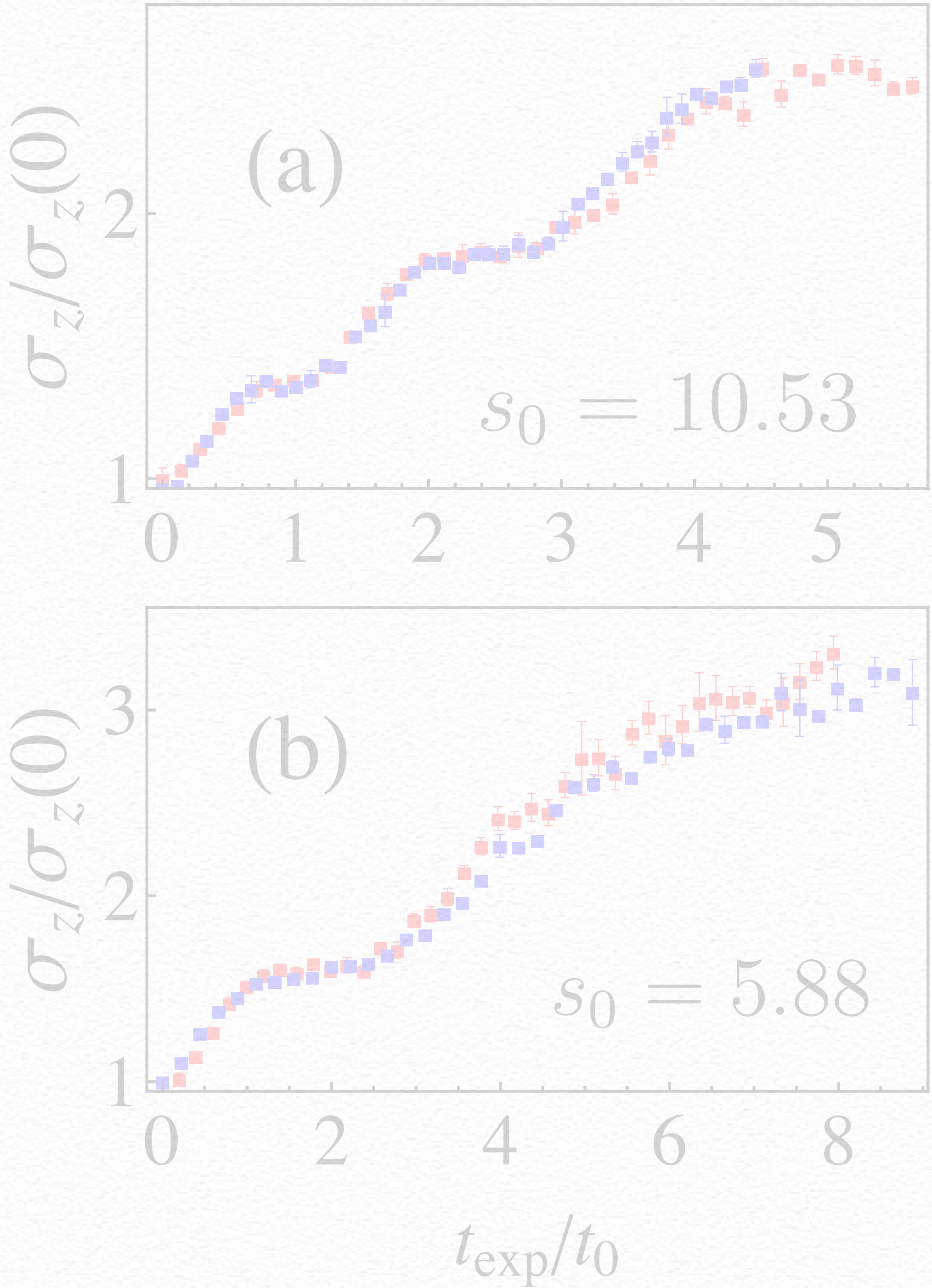
$$\frac{\langle \hat{R}^2\rangle(t)}{R_0^2} = \frac{t}{t_0}\frac{1}{\sin^2\varphi}\left[1-\cos\varphi\cdot\cos\left(s_0\ln\frac{t}{t_0}+\varphi\right)\right]$$

Experimental Observation

by Haibin Wu in East China Normal University



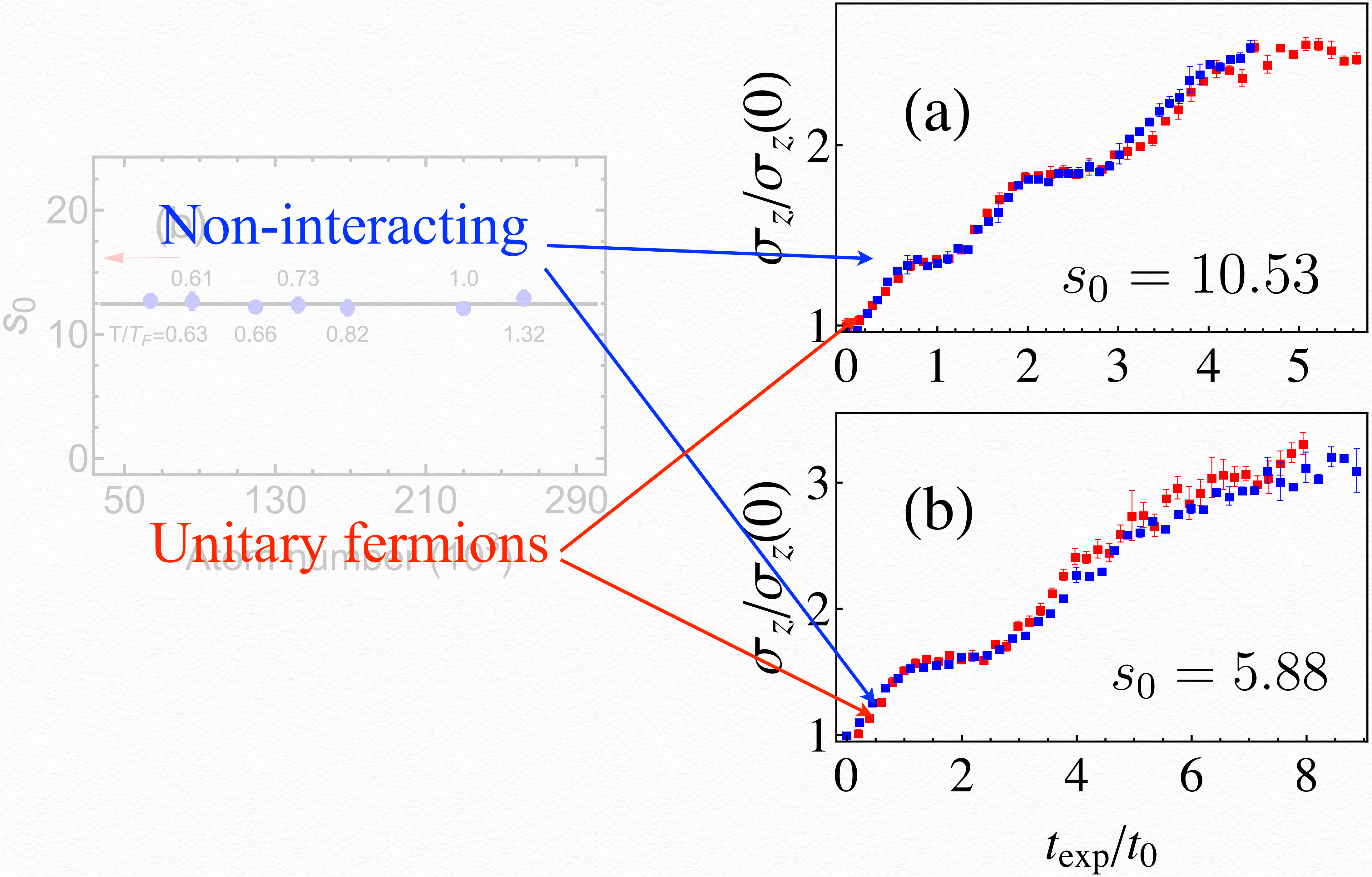
Independent of Temperature



Independent of State of Matter

Experimental Observation

by Haibin Wu in East China Normal University



Non-interacting

Unitary fermions

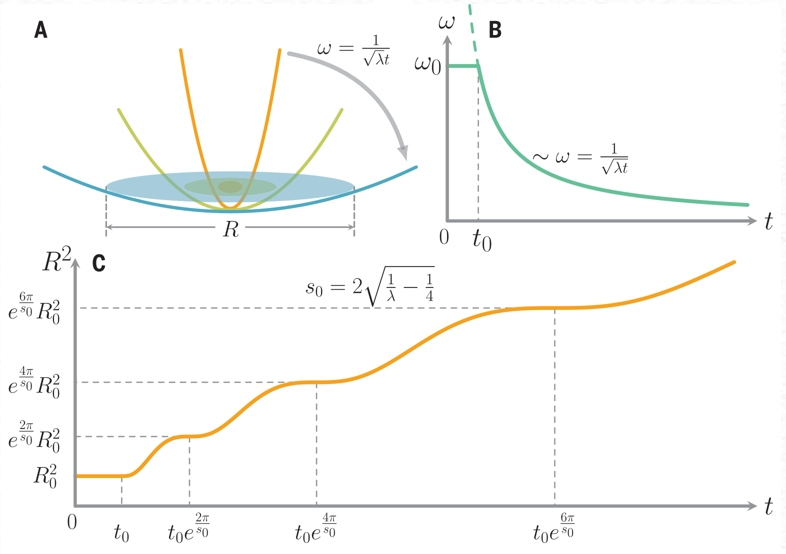
Independent of Temperature

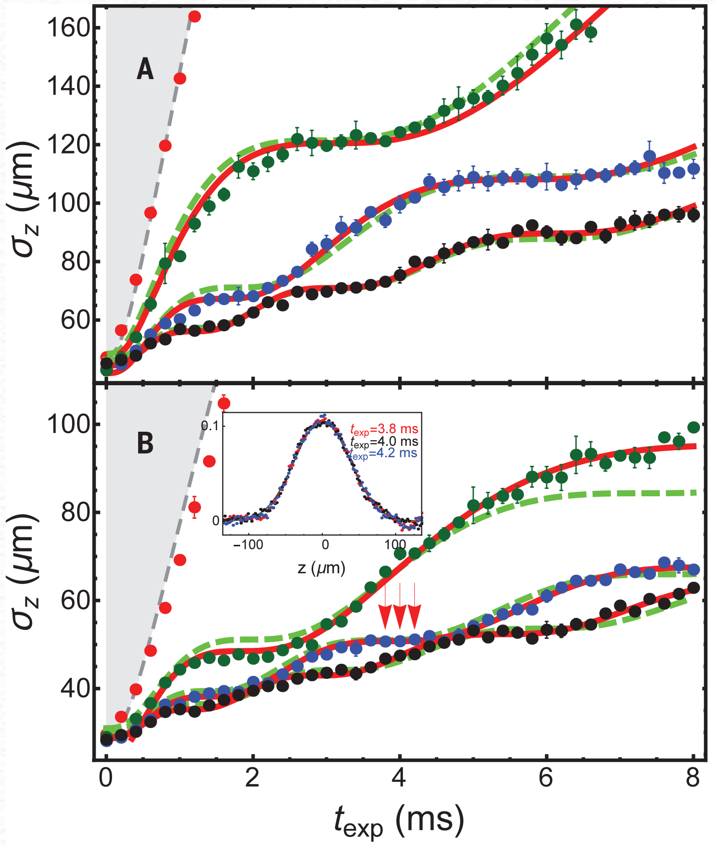
Independent of State of Matter

QUANTUM GASES

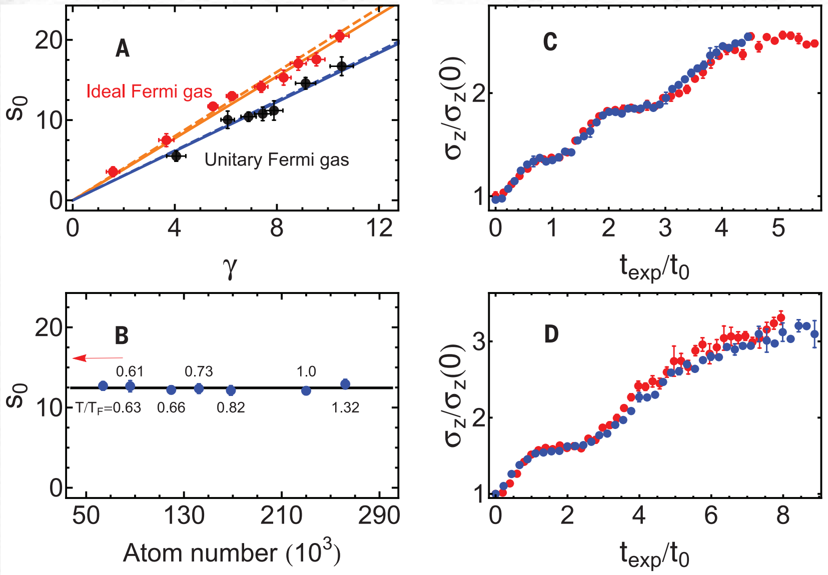
Observation of the Efimovian expansion in scale-invariant Fermi gases

Shujin Deng,[1]* Zhe-Yu Shi,[2]* Pengpeng Diao,[1] Qianli Yu,[1] Hui Zhai,[2] Ran Qi,[3]† Haibin Wu[1,4]†





Dr. Zheyu Shi



**Prof. Ran Qi
at Renmin University**

Science, 371, 353 (2016)

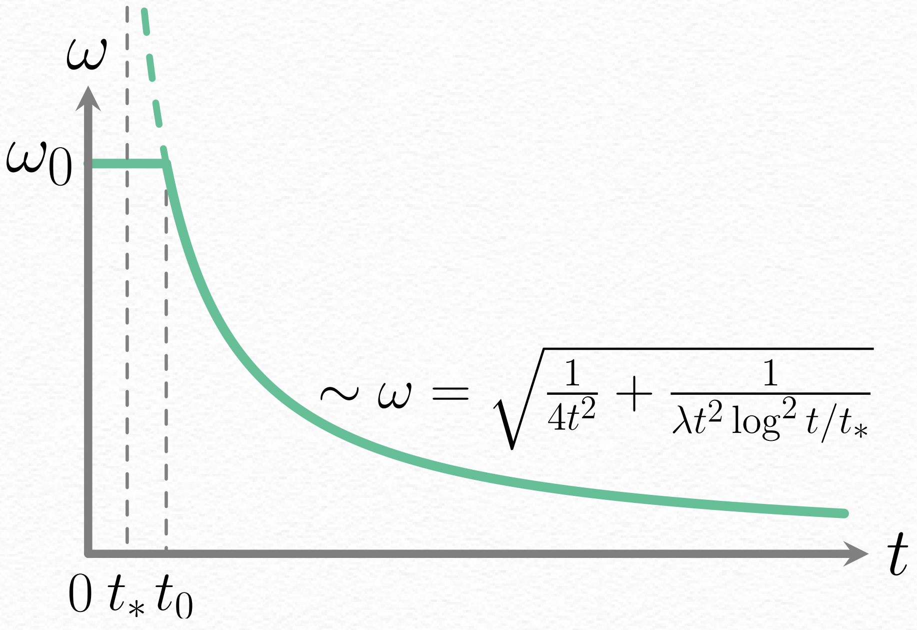
Haibin Wu'g group in ECNU

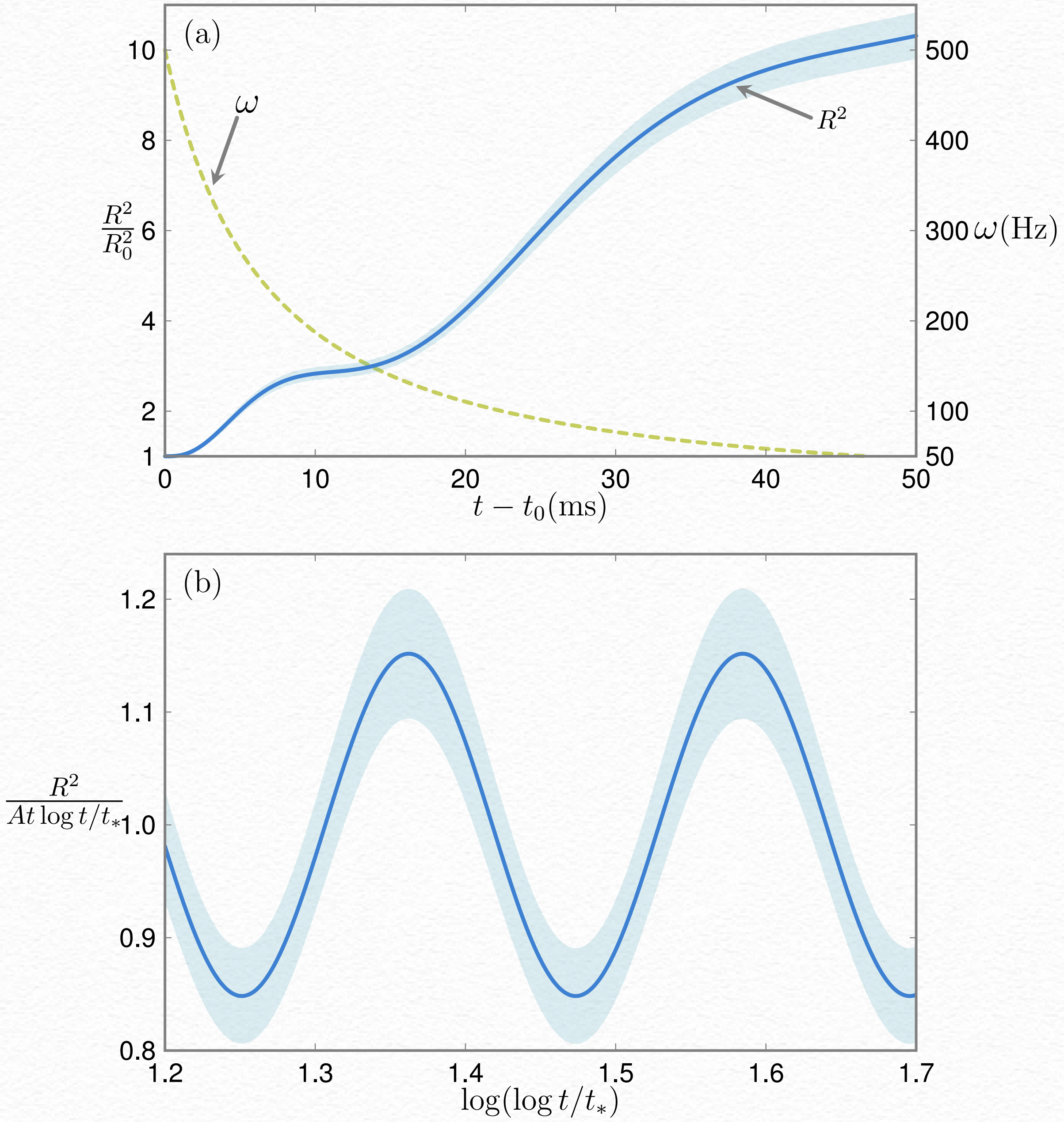
Super-Efimov Effect

	Efimov	SuperEfimov
Statistics	**Bosons/mixture**	**Fermions**
Dimension	**3D**	**2D**
Interaction	**s-wave resonance**	**p-wave resonance**
Scaling	$\mathrm{e}^{-2\pi n/\|s_0\|}$	$\exp(-2e^{3\pi n/4+\theta})$
Effective Potential	$-\frac{\|s_0\|^2+1/4}{R^2}$	$-\frac{1}{4\rho^2}-\frac{s_0^2+1/4}{\rho^2\ln^2(\rho/r_0)}$

Not yet been detected in few-body system !

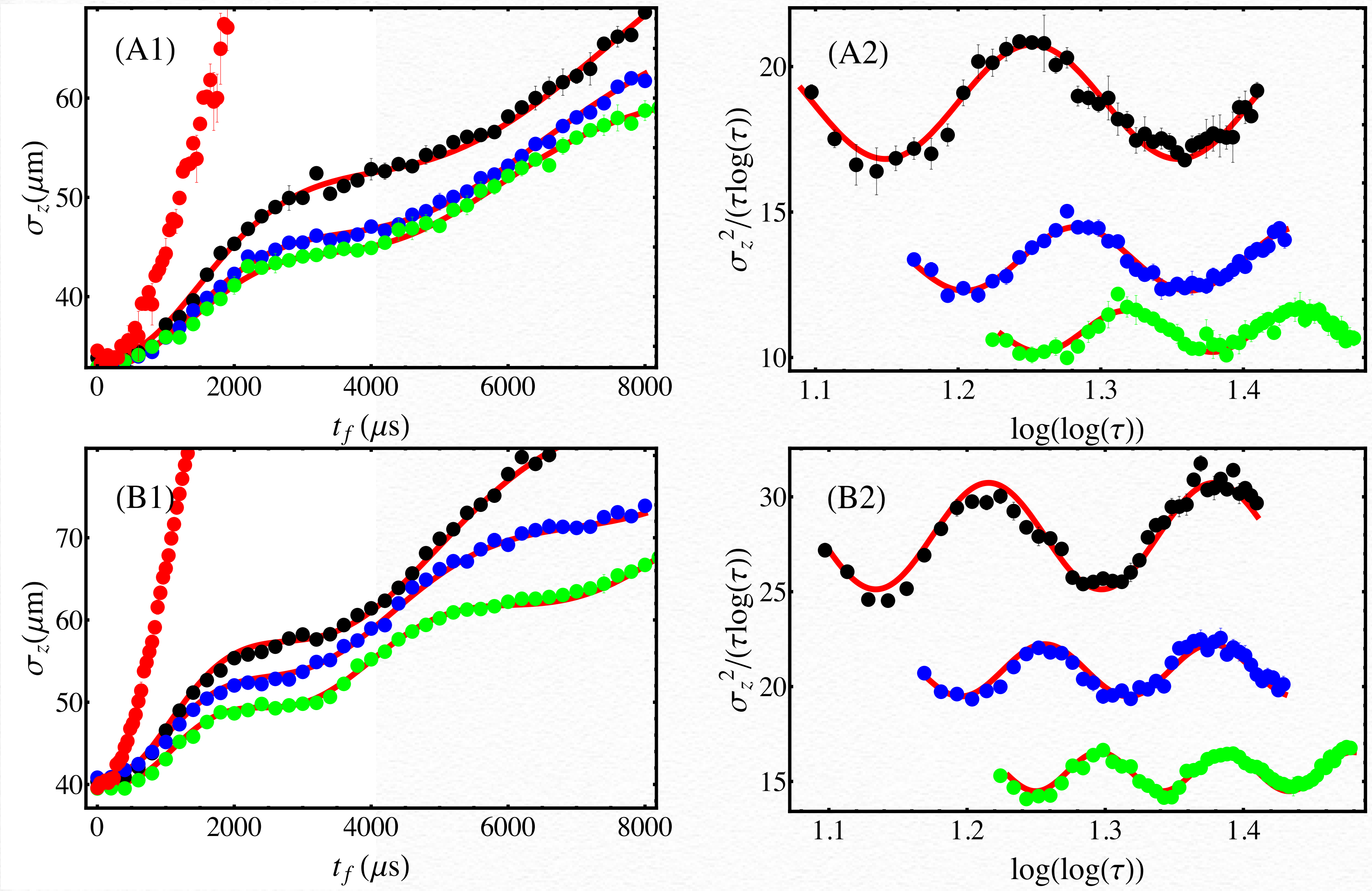
Dynamical Super-Efimov Effect

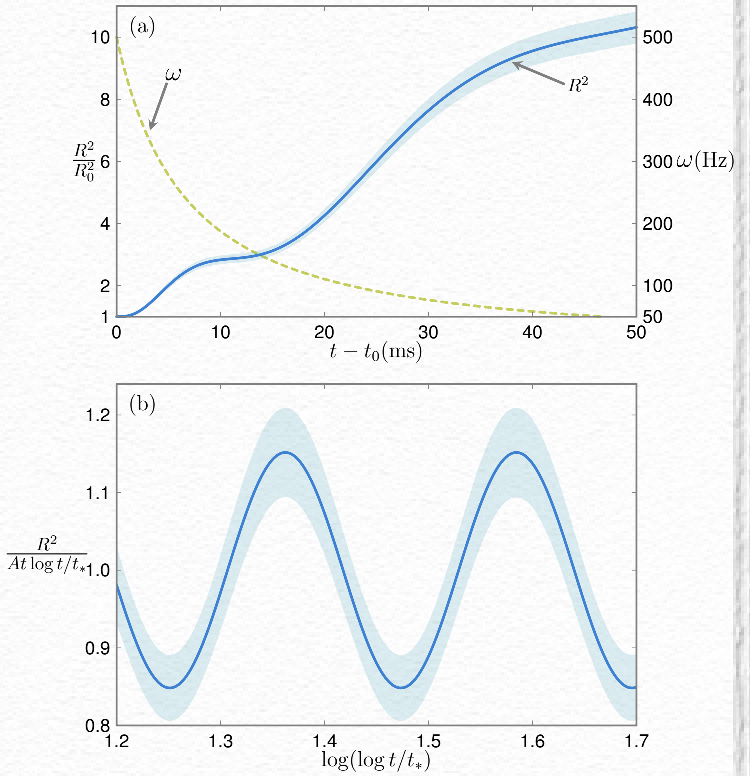




Z. Shi, R. Qi, H. Zhai, and Z. H. Yu, arXiv: 1608.05799

Dynamical Super-Efimov Effect





Haibin Wu's group in ECNU, arXiv: 1707.06732

Z. Shi, R. Qi, H. Zhai, and Z. H. Yu, arXiv: 1608.05799

Take Home Message

Efimov Effect not only has rich context as a few-body effect, but also will show broad impact beyond few-body physics.

Thank You Very Much for Attention !